CHAPTER 3

Active and passive ocean regimes in a low-order climate model


Abstract. A low order climate model is studied which combines the Lorenz-84 model for the atmosphere on a fast time scale and a box model for the ocean on a slow time scale. In this climate model, the ocean is forced strongly by the atmosphere. The feedback to the atmosphere is weak. The behaviour of the model is studied as a function of the feedback parameters. We find regions in parameter space with dominant atmospheric dynamics, i.e. a passive ocean, as well as regions with an active ocean, where the oceanic feedback is essential for the qualitative dynamics. The ocean is passive if the coupled system is fully chaotic. This is illustrated by comparing the Kaplan-Yorke dimension and the correlation dimension of the chaotic attractor to the values found in the uncoupled Lorenz-84 model. The active ocean behaviour occurs at parameter values between fully chaotic and stable periodic motion. Here, intermittency is observed. By means of bifurcation analysis of periodic orbits, the intermittent behaviour, and the role played by the ocean model, is clarified. A comparison of power spectra in the active ocean regime and the passive ocean regime clearly shows an increase of energy in the low frequency modes of the atmospheric variables. The results are discussed in terms of itinerancy and quasi-stationary states observed in realistic atmosphere and climate models.

1. Introduction

On a time scale of days or weeks, the atmospheric component of the earth’s climate system is dominant. Therefore, for short range weather forecasts, oceanic variables, such as the sea surface temperature, can often be considered fixed. On a much longer time scale the ocean’s dynamics can play an important role. It has to be taken into account when studying for instance decadal climate variability or anthropogenic influences like the enhanced greenhouse effect. For such purposes state-of-the-art climate models are often used, which possess millions of degrees of freedom. The results of experiments with such models are analysed statistically, as they are out of reach of the ordinary analysis of dynamical systems theory. As much understanding of atmosphere models has been gained by looking at extremely low dimensional truncations, our aim is to do the same for coupled models.

The issue we will focus on is the interplay of the short time scale variability of the atmospheric, intrinsically chaotic, component, and the long time scale of the oceanic component. In climatological terms, the question is whether the ocean is passive or active. If it is passive, it simply integrates the atmospheric signal, as if there is a one
way coupling. If it is active, there are notable feedback effects in the atmospheric dynamics. Even though the forcing of the atmosphere by the ocean is intrinsically weak, it may produce, for instance, decadal variability in atmospheric observables. Whether the ocean really is passive or active is still a matter of debate. Latif and Barnett [1994] found decadal variability induced by an active ocean. Other studies, such as Frankignoul et al. [2000], using complex, state-of-the-art climate models, or Saravanan and Mc Williams [1997] and Selten et al. [1999], who used models of intermediate complexity, give evidence for a passive ocean. In this paper we investigate the atmosphere-ocean interaction in a low-dimensional coupled model applicable to midlatitudes. We will show, that the ocean can be passive or active, depending on small changes in the coupling parameters. This may explain the different conclusions reached in studies with more realistic, complex models.

The low-order model studied in this paper is based on a proposal by Roebber (1995). He coupled the Lorenz-84 model, which is a metaphor for the general circulation of the atmosphere [Lorenz, 1984], to Stommel’s box model for a single ocean basin [Stommel, 1961]. Roebber uses numerical integrations and power spectra to characterise the coupled dynamics, without the exploration of bifurcation analysis and other tools of dynamical systems theory. In the fully coupled system, subject to periodic forcing, he finds increased energy in low frequency atmospheric modes when compared to one way coupling, i.e. without oceanic feedback to the atmosphere. In this paper, we investigate the coupled dynamics in detail.

In sections (2) and (3) the Lorenz-84 model and Stommel’s box model are briefly introduced. When coupling these models, as described in section (4), we take into account that experiments with realistic climate models, such as Grötzner et al. [1998], indicate that the circulation of the ocean is largely driven by atmospheric dynamics and solar forcing. In contrast, the oceanic feedback to the atmosphere is rather weak, and only notable on intrinsic time scales of the ocean. Therefore, we assume that the coupling terms in the ocean model are of the same order of magnitude as its internal dynamics, while the coupling terms in the atmosphere model are taken to be small perturbations of the atmosphere’s internal dynamics. Thus, a small parameter is introduced into the equations. The ratio of time scales of the atmosphere and the ocean model is a second small parameter. The consequences of the presence of small parameters, in the light of perturbation theory, are discussed in section (5). The behaviour of the coupled system is then investigated as a function of the coupling parameters in the atmosphere model.

In section (6) a bifurcation analysis of the equilibria of the coupled model is presented. The coexistence of attracting equilibria is inherited from the uncoupled box model. These equilibria describe two different orientations of the thermohaline circulation (THC).

For a range of parameter values chaotic attractors exist. In section (7), numerical estimates of the Kaplan-Yorke dimension and the correlation dimension of these attractors are given. They are compared to the corresponding quantities for the uncoupled Lorenz-84 model. It is shown that, in the chaotic regime, the ocean can be considered passive. It is slaved by the atmospheric forcing.
2. The Lorenz-84 general circulation model

Like the Lorenz-63 model, the Lorenz-84 model is related to a Galerkin truncation of the Navier-Stokes equations. Where the ‘63 model describes convection, the ‘84 model gives the simplest approximation to the general atmospheric circulation at midlatitude. The approximation is applicable on an $f$-plane, placed over the North Atlantic ocean.

We can give a physical interpretation of the variables of the Lorenz-84 model: $x$ is the intensity of the westerly circulation, $y$ and $z$ are the sine and cosine components of a large traveling wave. The time derivatives are given by

$$\dot{x} = -y^2 - z^2 - ax + aF$$
$$\dot{y} = xy - bxz - y + G$$
$$\dot{z} = bxy + xz - z$$

where $F$ and $G$ are forcing terms due to the average north-south temperature contrast and the earth-sea temperature contrast, respectively. Conventionally we take $a = 1/4$ and $b = 4$.

The behaviour of this model has been studied extensively since its introduction by Lorenz [1984]. Numerical and analytical explorations can be found in Masoller et al. [1995] and Sicardi and Masoller [1996], a bifurcation analysis is presented in Shilnikov et al. [1995]. The bifurcation diagram of this model is quite rich. It brings forth equilibrium points, periodic and quasi periodic orbits as well as chaotic motion. Qualitatively the behaviour can be sketched by looking at the energy transfer between the westerly circulation and the traveling wave. The energy content of the westerly circulation tends to grow, forced by solar heating. Above a certain value however this circulation becomes unstable and energy is transferred to traveling waves, and then dissipated. The energy content of the westerly circulation decreases rapidly and the cycle repeats itself in a periodic or irregular fashion. In figure (1) one can see that the orbit tends to spiral around the $x$-axis towards a critical value of $x$, then drops towards the $y, z$-plane.

At parameter values $(F, G) = (6, 1)$ two stable periodic solutions coexist. These parameter values are called summer conditions. For $(F, G) = (8, 1)$ the behaviour is chaotic (see figure (1)). These parameter values are called winter conditions. As argued in Lorenz [1990], the north-south temperature contrast, $F$, is larger during
winters. This results in strong baroclinic wave activity, reflected by chaotic motion in the Lorenz-84 model. The periodic motion under summer conditions reflects less turbulent large scale dynamics due to a smaller north-south temperature contrast.

If we fix these forcing parameters to summer conditions in the coupled model, described below, no complex dynamics arise. When varying the coupling parameters we see only equilibrium points and periodic solutions. In our investigations we will take \((F, G) = (8, 1)\), i.e. we will consider perpetual winter conditions.

3. The box model for a single ocean basin

The ocean-box model was introduced by Stommel [1961]. It is a simple model of a single ocean basin, the North Atlantic. This basin is divided in two boxes, one at the equator and one at the north pole. Within the boxes the water is supposed to be perfectly mixed, so that the temperature and salinity are constant within each box but may differ between them. This drives a circulation between the boxes which represents the THC. Water evaporates from the equatorial box and precipitates into the polar box. Thus the salinity difference between the boxes is enhanced. The temperature difference is maintained by the difference in heat flux from the sun. Thus, the salinity and the temperature difference drive a circulation in opposite directions. For a suitable choice of parameters, both the circulation driven by salinity and the circulation driven by temperature occur as stable solutions in this model [Stommel, 1961]. In contrast to the Lorenz model, no complex dynamics arise.

Figure (2) shows the setting of the model. The volume of water is kept equal, but its density may differ between the boxes. Using a linearised equation of state and some assumptions on the damping, dynamical equations for the temperature difference \(T = T_e - T_p\) and the salinity difference \(S = S_e - S_p\) can be derived. They
**Figure 2.** The two box model. Water evaporates from the warmer equatorial box on the left and is transported through the atmosphere to the polar box on the right. The flow $f$ is positive when directed northward.

The coupled equations

\[
\dot{T} = k_a(T_a - T) - |f(T, S)|T - k_wT \\
\dot{S} = \delta - |f(T, S)|S - k_wS \\
f = \omega T - \xi S
\]

(2.1) (2.2) (2.3)

where $k_a$ is the coefficient of heat exchange between ocean and atmosphere, $k_w$ is the coefficient of internal diffusion and $\omega$ and $\xi$ derive from the linearised equation of state. The flow, $f$, represents the THC. It is positive when temperature driven and negative when salinity driven. The inhomogeneous forcing by solar heating and atmospheric water transport are given by $T_a$ and $\delta$, respectively. When coupling the box model to the Lorenz-84 model, we will use the estimates in Roebber [1995] for the parameters in equations (2). The volume of the deep ocean box, not present in our model, is simply divided between the polar and the equatorial box.

The absolute value in (2.1) and (2.2) was put there by Stommel, arguing that the mixing of the water should be independent of the direction of the flow. A more straightforward derivation of the equations of motion of a simple ocean model related to the box model indicates that this is indeed the case, although the term comes out quadratic instead of piecewise linear [Maas, 1994]. If we take this term to be quadratic in the coupled model, described below, the average values of $T$ and $S$ change significantly, but we find qualitatively the same behaviour.

**4. The coupled equations**

Having described these simple models for atmospheric and oceanic circulation, and the physical interpretation of their variables, we can now identify three mechanisms by which they interact:
(1) The atmospheric pole-equator temperature contrast is supposed to be in permanent equilibrium with the zonal wind strength $x$, i.e. we put $T_a \propto x$. Also, the forcing of the atmosphere by the north-south temperature contrast in (1.1) is modified by the ocean temperature contrast, so we put $F \rightarrow F_0 + F_1 T$. This expresses the simplest geostrophic equilibrium: a north-south temperature gradient which drives a east-west atmospheric circulation.

(2) The inhomogeneous forcing by land-sea temperature contrast in (1.2) should decrease with increasing temperature difference $T$. It is assumed that in the polar region the sea water temperature is higher than the temperature over land, while in the equatorial region it is lower. A higher temperature difference $T$ thus means a lower land-sea temperature contrast. This influence is described as a fluctuation upon a fixed forcing: $G \rightarrow G_0 + G_1 (T_{av} - T)$.

(3) The water transport through the atmosphere is taken to be linear in the energy content of the traveling wave: $\delta \rightarrow \delta_0 + \delta_1 (y^2 + z^2)$.

Combining (1) and (2) with the proposed coupling terms we obtain

\begin{align*}
\dot{x} &= -y^2 - z^2 - ax + a(F_0 + F_1 T) \\
\dot{y} &= xy - bxz - y + G_0 + G_1 (T_{av} - T) \\
\dot{z} &= bxy + xz - z \\
\dot{T} &= k_a (\gamma x - T) - |f(T, S)|T - k_w T \\
\dot{S} &= \delta_0 + \delta_1 (y^2 + z^2) - |f(T, S)|S - k_w S
\end{align*}

with $f$ as in (2.3). With the coupling some new constants have been introduced. They are $T_{av}$, the standard temperature contrast between the polar and the equatorial box, $\gamma$, the proportionality constant of the westerly wind strength and the north-south temperature contrast and $\delta_1$, a measure for the rate of water transport through the atmosphere. When exploring the dynamical behaviour of the model we take $F_1$ and $G_1$ as free parameters. As motivated in the introduction, we consider small coupling to the atmosphere model. This is the case if we take $(F_1, G_1) \in [0, 0.1] \times [0, 0.1]$. In table (1) the parameters are listed. In this scaling, one unit of time in the model corresponds to the typical damping time scale of the planetary waves, estimated to be five to ten days.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1/4</td>
</tr>
<tr>
<td>$b$</td>
<td>4</td>
</tr>
<tr>
<td>$F_0$</td>
<td>8</td>
</tr>
<tr>
<td>$G_0$</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma$</td>
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</tr>
<tr>
<td>$\delta_1$</td>
<td>$9.6 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>$T_{av}$</td>
<td>30</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>$7.8 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>$k_w$</td>
<td>$1.8 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>$k_a$</td>
<td>$1.8 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$1.1 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$1.3 \cdot 10^{-4}$</td>
</tr>
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Table 1. The constants of the coupled model. With these constants the ocean and the atmosphere model have time scales that differ by a factor of about one thousand. See Roebber [1995].
5. Perturbation theory

If we denote the atmospheric variables by $x \in \mathbb{R}^3$ and the oceanic variables by $y \in \mathbb{R}^2$, we can write the system (3) as

$$
\dot{x} = f_0(x) + \epsilon_1 f_1(y) \\
\dot{y} = \epsilon_2 g_0(x) + \epsilon_2 g_1(y)
$$

(4)

where $\epsilon_1$ and $\epsilon_2$ are small parameters. Putting $\epsilon_1 = \epsilon_2 = 0$ yields the Lorenz-84 model. Hyperbolic equilibria in the Lorenz-84 equations correspond to a first approximation of a slow manifold in the full system (4) as a consequence of Fenichel’s theorem [Wiggins, 1994]. In our case the slow manifold is unstable and not physically interesting.

Instead we focus on periodic and chaotic solutions of the Lorenz-84 equations and the phenomenon of intermittency in the coupled system. Note, that the existence of the periodic solutions and their corresponding Floquet spectrum, which determines the stability properties, is described by the Poincaré expansion theorem [see Verhulst, 1996]. The Floquet multipliers of periodic orbits in the coupled system are expected to be $O(\epsilon_1, \epsilon_2)$ perturbations of the corresponding multipliers in the uncoupled system. This information will be used when describing the intermittent behaviour.

6. Bifurcations of equilibrium points

The equilibrium points of system (3) can be found after some algebraic manipulations, as roots of a high order polynomial equation. For each set of parameter values, the equilibria can be calculated along with their spectra. In addition, the bifurcations of these equilibria can be found using the continuation package AUTO [Doedel et al., 1986]. On a plane in phase space, defined by $f = 0$, the vector field is not differentiable. There is an equilibrium point on this plane if

$$
G_1 = \frac{-G_0 \pm \sqrt{a(F_0 + F_1 T_0 - x_0)(1 - 2x_0 + (1 + b^2)x_0^2)}}{T_{av} - T_0}.
$$

(5)

with equilibrium values $x_0 = (\delta_0 + a\delta_1 F_0)(k_a + k_w)\xi/(\omega k_w k_a \gamma + a\delta_1 \xi[k_a + k_w - F_1 \gamma k_a])$ and $T_0 = k_a \gamma x_0/(k_a + k_w)$. On the curve in parameter space, defined by (5), a bifurcation occurs. When crossing it, increasing $G_1$, two equilibrium points appear, one with a positive value of $f$, and one with a negative value. The latter is stable. In fact, for any $G_1$ greater than the right hand side in (5) there is an attracting equilibrium or periodic solution on which $f$ is negative, i.e. the THC is salinity driven. At such an equilibrium, most of the energy in the atmosphere model is stored in the wave activity, whereas the jetstream intensity is low (i.e. $y^2 + z^2 \gg x^2$). This results in a large freshwater flux through the atmosphere. The ocean’s response is a weak, inverted, THC, a small negative temperature difference $T$ and a relatively large salinity difference $S$.

The results of the bifurcation and stability analysis are shown in diagram (3). The stability of the equilibrium points is indicated in the diagram. As can be seen,
only in a small window in parameter space there exists a stable equilibrium with positive flow. Other stable solutions with positive flow are periodic or chaotic.

7. Chaotic attractors

As mentioned in section (5), the coupled model can be regarded as a perturbation of the Lorenz-84 model. The parameters $F_0$ and $G_0$ of the uncoupled Lorenz-84 model have been chosen in a fully chaotic regime. Therefore, we cannot expect a complete insight in the bifurcations and the behaviour of the coupled model as a function of the coupling parameters. Still, through a combination of time integrations and bifurcation analysis, we can clarify our model’s dynamics to a large extent.

It is found that for many parameter values the behaviour is chaotic. Using the algorithm described by Wolf et al. [1985] we can approximate the Kaplan-Yorke dimension of the chaotic attractors. We can also numerically estimate their correlation dimension. A description of the algorithms and a discussion of the physical interpretation of these dimensions can be found in Nayfeh and Balachandran [1995].

For several parameter values the Kaplan-Yorke dimension is found to be about 4.3, compared to the typical value of about 2.4 for the uncoupled Lorenz-84 model. The Kaplan-Yorke dimension, however, only characterises the geometry of the attractor. Even though the Kaplan-Yorke dimension is increased by 1.9, there might be little variability in the oceanic variables. In order to see if the ocean model plays
an important role, we can calculate the correlation dimension. This quantity is calculated from an orbit on the chaotic attractor, sampled at equal time intervals. If there is a lower dimensional subset of the attractor which is visited relatively often, the correlation dimension will be smaller than the Kaplan-Yorke dimension. This is an indication that the Kaplan-Yorke dimension overestimates the variability of the system.

Indeed, for the uncoupled Lorenz model the correlation dimension is typically about 2.3 [Anastassiades, 1995], compared to 3.4 ± 0.2 for the coupled model. The difference in correlation dimension is significantly smaller than the difference in Kaplan-Yorke dimension. We conclude that the attractor of the coupled system is much more inhomogeneous than that of the Lorenz-84 system.

The ocean is passive in the chaotic regime. The qualitative behaviour of the atmospheric variables is similar to the uncoupled behaviour and the inhomogeneity of the attractor shows that there is little variability in the oceanic variables. In the following we describe a regime in parameter space in which the ocean is manifestly active, as it dictates the qualitative behaviour of the system.

8. Intermittency

Inbetween fully chaotic and stable periodic regions in parameter space, intermittency can be observed. In figure (4) an example of an intermittent time series of system (3) is shown. On some time intervals the curve looks chaotic, on the other time intervals it looks periodic. This behaviour occurs near a bifurcation point at which a periodic orbit loses its stability. The systematic study of this phenomenon
in dynamical systems was initiated by Pomeau and Manneville [1980]. Here, we describe the underlying bifurcation structure.

Note, that the phenomenon of intermittency is not restricted to alternating periodic and chaotic motion in low-order models. It is also found in models with multiplicative noise [Shapiro, 1993] or additive noise [Eckmann et al., 1981]. Furthermore, in so called cycling chaos [Dellnitz et al., 1995] two, or more, chaotic sets are involved, instead of a chaotic set and a periodic orbit. Our case, described below, is called type II intermittency [Pomeau and Manneville, 1980].

8.1. The theory of intermittency. In order to explain the intermittent behaviour and the role of the slow, oceanic, variables in detail we did a numerical bifurcation analysis of the relevant periodic orbit. This was done following the algorithm described in Simó [1989]. An approximation of the periodic orbit can be obtained from a time series such as the one shown in figure (4). This orbit is then followed in one parameter by a prediction-correction method. Generically the periodic orbit will undergo saddle node, period doubling and Neimark-Sacker bifurcations, at which its stability properties change [see, e.g. Wiggins, 1990, chapter 3].

The result of this analysis is shown in figure (5). At the saddle node bifurcation (SN) two periodic orbits come into existence. Initially both are unstable. Increasing parameter $F_1$ slightly, the upper branch passes through a Neimark-Sacker bifurcation (NS), after which all Floquet multipliers lie within the unit circle, indicating that the periodic orbit is stable. Beyond this point, for $F_1 > F_{NS}$, periodic motion sets

![Figure 5. Continuation of the periodic solution approached during the periodic interval in (4). The Floquet multipliers are drawn in the complex plane. The bifurcation points are indicated by SN for saddle node and NS for Neimark-Sacker. This picture was obtained applying the algorithm described in Simó [1989]. The stable branch has been marked with dots.](image-url)
The intermittent behaviour persists to the left of the saddle node point, i.e. for \( F_{SN} < F_1 < F_{NS} \) there is no stable periodic orbit for the system to settle on. Instead the periodic orbit has a stable and an unstable manifold attached to it [see Wiggins, 1990, chapter 1]. The solution of system (3.1)-(3.5) approaches the periodic orbit closely following the stable manifold, and moves away from it closely following the unstable manifold. This is what happens during a periodic interval. Sufficiently far away from the unstable periodic orbit in phase space, the solution wanders about chaotically untill it comes close to the stable manifold again and the cycle repeats.

The farther left of the saddle node point parameter \( F_1 \) is chosen, the less the influence of the ghost structure. This can be quantified by measuring the length of the periodic intervals, or rather its distribution, for a number of parameter values. To obtain these data, integrations of \( 5 \times 10^6 \) in units of \( t \) (about \( 9.6 \times 10^4 \) years) were done, during which more than 600 periodic intervals were registered for each parameter value. The length of the periodic intervals in each integration run is approximately normally distributed. In figure (6), the expectation value \( l \) has been plotted against parameter \( F_1 \). Beyond the Neimark-Sacker bifurcation, for \( F_1 > F_{NS} \), the periodic behaviour is stable and therefore \( l \) diverges. The rate
of divergence was numerically estimated by Pomeau and Manneville [1980]. They found the power law $l \propto \epsilon^{-\alpha}$, where $\epsilon$ is the distance to the bifurcation point, in our case $F_{NS} - F_1$, and $\alpha \approx 0.04$. In our experiments we find $\alpha \approx 0.06$, in reasonable agreement.

The Neimark-Sacker bifurcation, at which the behaviour becomes periodic, can be continued in two parameters in order to find a window in parameter space in which intermittency takes place. Such a window is shown in figure (7). The saddle-node and Neimark-Sacker bifurcations of figure (5) have been labeled SN1 and NS1, respectively. At SN2 another periodic orbit becomes stable, following the same scenario as described above. Directly to the right of NS1 and below NS2 the behaviour is periodic. In the shaded region the behaviour is intermittent.

**8.2. The driven Lorenz-84 model.** In order to study the behaviour of the slow variables, $T$ and $S$, in the intermittent regime, we made a Poincaré plot of the coupled system. The plane of intersection in phase space is defined as $S_x = \{(x, y, z, T, S) \in \mathbb{R}^5 | x = 1\}$. In figure (8) the intersection points are shown, projected onto the $T, S$-plane. The intersections of the unstable periodic orbit with $S$ have been marked with crosses. The arrows indicate the direction of the flow in $S_x$ near the intersections of the unstable periodic orbit. The qualitative behaviour, as described above, is neatly illustrated. On the left hand side of the picture the solution of equations (3) approaches the periodic solution, closely following its stable manifold. This happens during a periodic interval. At the top of the picture the solution moves away from the periodic solution closely following the unstable
manifold. The average values of $T$ and $S$ clearly differ between the periodic and the chaotic intervals.

It seems natural to study the uncoupled Lorenz-84 model (1.1)-(1.3) with the effective parameters $F = F_0 + F_{int}T_{eff}$ and $G = G_0 + G_{int}(T_{av} - T_{eff})$, where $F_{int}$ and $G_{int}$ are coupling parameters for which we observe intermittency. The effective temperature contrast $T_{eff}$ can be taken fixed, which would correspond to the limit of $\epsilon_2 \downarrow 0$ in section (5), or slowly varying. A bifurcation analysis of the Lorenz-84 model with these effective parameters, and $T_{eff}$ as the bifurcation parameter, yields an alternative explanation of the intermittency.

If we set $T_{eff}$ equal to the average value of $T$ during a periodic interval, we find a stable periodic orbit in the uncoupled Lorenz-84 model. This orbit is strongly attracting and the transient time from an arbitrary initial condition is short. If we let $T_{eff}$ vary on the time scale of the ocean model, as a second approximation to the coupled system, this orbit persists.

If we continue the attracting orbit to higher values of $T_{eff}$ it undergoes a period doubling cascade, resulting in chaotic behaviour. The first four period doublings have been marked $T_{1,2,3,4}$ in figure (8). At these period doublings, weakly attracting orbits of high period are created. The transient time from an arbitrary initial condition
is rather long. Therefore, these orbits do not persist in the second approximation, with a slowly varying $T_{eff}$. In the second approximation the behaviour is chaotic beyond the first period doubling.

To the right of the culmination point of the period doubling cascade, close to $T_4$, the behaviour is chaotic with fixed or slowly varying $T_{eff}$. If $T_{eff}$ follows the behaviour of $T$ during the intermittency, it repeatedly drives the Lorenz-84 model through the period doubling bifurcations, into the chaotic region, and back to the periodic region. This gives an intuitive picture of the intermittent behaviour.

9. Power spectra in the passive and the active regime

An illustrative way to show the influence of the active ocean component on the coupled dynamics is to look at power spectra. In figure (9) the power spectrum of the atmospheric variable $x$, averaged over $t_{av} = 15 \approx 3$ months, is shown. The data have been taken from integrations spanning some $7.7 \cdot 10^3$ yrs. The top picture shows the spectrum in the passive, chaotic, regime at parameter values $(F_1, G_1) = (0.02, 0.01)$. The bottom picture, on the same scale, shows the spectrum in the active, intermittent, regime, at parameter values $(F_1, G_1) = (0.021685, 0.01)$. The three sharp peaks on the left are caused by the periodic intervals. Compared to the spectrum in the passive regime, a lot of energy is present in low frequency modes, associated with the recurrence of periodic intervals. These are modes on the thermal damping time scale of the ocean model.

10. Conclusion and discussion

The behaviour of the coupled model has been studied as a function of the coupling parameters in the atmosphere model. A bifurcation analysis of the equilibrium states reveals that, for a range of parameter values, there exists a stable equilibrium which describes a salinity driven THC. This property is inherited from the uncoupled ocean box-model. Other attractors of the coupled model are chaotic or periodic. Depending on small changes in the coupling parameters, the ocean can be passive or active.

The passive behaviour occurs if the coupled model is fully chaotic. Here, the ocean basically integrates the atmospheric forcing. As illustrated by comparing attractor dimensions of the coupled model to those of the Lorenz-84 model, there is not much variability in the oceanic variables in this case.

Active behaviour is found near a bifurcation of the coupled system. On one side of this bifurcation the behaviour is periodic, on the other side it is intermittent. In the intermittent regime the slow time scale of the ocean model plays an important role in the dynamics, as illustrated by the power spectrum of the atmospheric variables. This can be explained from the theory of intermittency, and the motion along the stable and unstable manifolds of the periodic orbit. Alternatively, the intermittency can be described as the behaviour of the Lorenz-84 model with slowly varying, effective parameters. The effective parameters vary on the time scale of the ocean model and repeatedly push the Lorenz-84 model through a sequence of bifurcations at which the stability of periodic motion is lost.
Figure 9. Power spectra of $x(t)$ in the passive ocean (chaotic) regime (top) and the active ocean (intermittent) regime (bottom). Period in weeks (see section (4)).
In order to make a connection to more realistic climate models, two points
should be made clear. First of all, stable equilibria or periodic orbits are not com-
monly found in the state space of realistic, high resolution, models. The invariant
structures in high dimensional state spaces are usually much more complicated. A
useful analogy is provided by the method of Probability Density Functions [PDF’s,
see Molteni et al., 1990, and references therein]. Figure (8) is the analogue of a
bimodal PDF, with one regime near the saddle type periodic orbit (upper left cor-
ner) and one chaotic regime (bottom right corner). In fact, applying the method of
PDF’s for intermittent parameter values would yield such a bimodal distribution.
The expectation value of the length of the periodic intervals, plotted in figure (6),
then measures the residence time in the regimes. In this context, the intermittency
can be regarded as itinerancy, switching back and forth between climate regimes.
The time scale of motion within each of the regimes is set by the fast, atmospheric,
component of the model while the time scale of migration between the regimes is
set by the slow, oceanic component.

Itinerancy and the related notion of quasi-stationary states [see, e.g. Marshall
and Molteni, 1993], have been proposed as a possible mechanism generating low
frequency variability. Itoh and Kimoto [1996] found evidence for itinerancy in an
atmosphere model up to fairly high resolution (T21). In their case the transitions
between regimes are noise-induced, as described in De Swart and Grasman [1987],
and describe internal low-frequency variability of the atmosphere. Schopf and Suarez
[1988] found vacillation in a coupled model, generating variability on a time scale of
years, related to the transit time of traveling waves in the ocean models rectangular
domain. In our case, the long time scale is set by fluctuations in the THC, generating
decadal variability.

The second point is that, in our model, the coupling of the ocean to the atmo-
sphere is on a conceptual level and depends only on two parameters. In reality, the
ocean-atmosphere feedback is highly complex, which results in a large amount of pa-
rameterisations in realistic models. This makes it very hard to identify the relevant
parameters for a sensitivity experiment. The goal of such an experiment would be to
find transitions from unimodal to multimodal PDF’s, analogous to the bifurcation
scheme presented in section (8.1). Alternatively, an experiment in the fashion of
section (8.2) might be conducted, in which the dependence of the atmospheric PDF
on prescribed Sea Surface Temperature (SST) patterns is tested.

In classical studies such as Palmer and Sun [1985], the response of the atmo-
spheric circulation to SST anomalies is measured in terms of time mean quantities.
However, in order to detect a qualitative change in behaviour, namely a crossover
to another regime, it may be necessary to extract more information, in the form of
PDF’s. In the experiment we propose, the atmospheric PDF, projected onto a few
relevant indicators, should be measured as a function of the amplitude of a typi-
cal SST anomaly pattern. Relevant indicators can be, for instance, the amplitudes
of the leading empirical orthogonal functions of surface pressure. If a qualitative
change in the PDF is detected at a reasonable amplitude of the SST anomaly forc-
ing, this indicates that the atmosphere is sensitive to oceanic feedback. Whether or
not itinerancy, and the associated variability on oceanic time scales, will be found
in a subsequent coupled integration depends on the forcing of the ocean by the atmosphere in the preferred flow regimes. If, in an ocean only run, driven by one of the centroids of the atmospheric PDF, the amplitude of the SST anomaly is driven through its critical value, the feedback loop is complete and itinerancy can occur in a fully coupled integration.

Evidence for sensitivity of the atmospheric PDF to SST anomalies was found in Selten et al. [1999]. In an atmosphere only run they found that a North Atlantic Oscillation (NAO) related SST anomaly induced a preference for one sign of the model’s NAO pattern. Also, Kharin [1995] showed that SST can have a nonlinear effect on the atmospheric circulation.

The discussion about the active or passive role of the ocean in coupled climate models and in the real climate systems is still going on [for a review, see Latif, 1997]. We think, that the description of intermittent behaviour in a low-order climate model and the analogy to itinerancy in realistic models gives a possible explanation for the different results in recent literature. An experiment as proposed above might reveal that active and passive ocean behaviour can occur in one model at different parameter values. Recent sensitivity studies, such as performed by Rahmstorf and Ganopolsky [1999], indicate that at critical values of coupling parameters major climatic changes can occur. It is at such critical points that coupling of the atmosphere to other components of the climate system can be crucial.

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Bibliography


