Manufacturing [31] is the process of converting raw materials (such as iron, glass or polymer) into useful products, ranging from goods such as kettles and telephones to machinery such as railway locomotives and aircrafts. Computer-aided design (CAD) and computer-aided manufacturing (CAM) have automated these manufacturing processes, both in the design phase and the construction phase. Due to the geometric nature of manufacturing processes, many geometric problems arise in the automation of manufacturing. Computational geometry arises at all levels of manufacturing, from design, modeling and simulation to process planning, on-line verification and testing. The survey by Bose and Toussaint [11, 14] gives an overview of geometric problems and algorithms relevant to manufacturing.

In this thesis we study some geometric aspects of the casting process, a commonly used manufacturing process for plastic and metal objects, and give algorithms to solve several geometric problems arising in casting. This introduction provides the necessary background, overview, and definitions to appreciate the following chapters of this thesis.

Section 1.1 gives a brief introduction to manufacturing, and introduces several processes in the manufacturing industry. We briefly introduce casting, stereolithography and extrusion.

Section 1.2 introduces the casting process. We introduce sand casting, injection moulding and die casting. The adequate process is chosen depending on factors such as the material, the feeding system for the material, required quality standards, whether the object will be
mass-produced, and so on.

Section 1.3 shows how computers have become an essential element in the manufacturing process, from primitive systems for 2-dimensional drawing and drafting (the Sketchpad system of the early 1960s) to the current sophisticated systems for 3-dimensional modeling and simulation.

Section 1.4 introduces geometric aspects of the casting process. The fundamental question arising during the design of an object is whether the object can actually be manufactured using a casting process. We focus on a geometric decision at the basis of the problem and define the problem we address in this thesis. We also briefly introduce features of an object that facilitate the geometric decision process.

Sections 1.5–1.9 provide an overview on the five geometry problems that are studied in Chapters 3–7. We give definitions of problems and summarize our results.

1.1 Manufacturing processes

Manufacturing industries have been considered one of the competitive technologies in today’s economy. Even though the word manufacture itself comes from the Latin words manus and facere meaning “to make by hand,” most products today are mass-produced with the help of machines.

There are many different production processes for constructing objects in the manufacturing industry, including gravity casting, injection moulding [15, 21, 45], layered manufacturing (as, for instance, stereolithography [9]), material removal via conventional (or chemical or electrical) machining [25], deformation (forging, rolling, extrusion, bending), composition (as in composite materials, sintered ceramics, and the like), and spray deposition.

The casting process is used extensively to mass-produce a wide variety of products. In the process, liquid is fed into a cast (mould) that has a cavity with the shape of the object to be manufactured. After the liquid has hardened, the cast parts are removed from the object. Depending on the material and the feeding system for the molten material (either using gravity or by force), different casting methods are used. More details are discussed in Section 1.2.

Stereolithography [9] is a layer-deposition manufacturing process using a vessel of photosensitive liquid plastic, a table controlled by a computer, and a laser (see Figure 1.1). The laser lies above the vessel and shines on the surface of the liquid plastic. At the first step the table in the vessel is just below the surface of plastic. The laser moves horizontally, solidifying this layer of plastic. This is the bottom-most layer of the object. At the next step the table is lowered a little bit so that liquid covers the hardened layer, and the laser then draws the next layer. This process is repeated for subsequent layers until the entire
object is formed.

![Figure 1.1: Stereolithography](image1)

![Figure 1.2: Extrusion](image2)

The extrusion process is a widely spread manufacturing technology to produce parts used mainly in the construction industry (such as PVC window profiles, pipes and tubes), automotive applications (such as rubber seals and gas conducts), biomedical applications (such as medical tubings), etc. The material is softened by heat prior to extrusion. The heated material is placed into an extrusion press, where a powerful hydraulic ram or a rotating screw forces the softened material through a precision opening, known as a die, to produce the desired shape (See Figure 1.2). Bakers, for example, use a collection of
shaped nozzles to decorate cakes with fancy bands of icing. They are producing extruded shapes. As suggested by these nozzles, the shape of the extrusion is determined by the shape of the opening (die).

### 1.2 The casting process

The casting process has been widely used for a long time to make household utensils, kitchenware, works of art, etc. For example, the bronze statue of Zeus shown in Figure 1.3 was made using the casting process in 470 BC. Figure 1.4 shows the process in which

![Figure 1.3: Zeus throwing lightnings, Bronze, ca. 470 BC](image)

this statue was made by hand: A sculptor carved a prototype of the statue in wax. The prototype was covered by clay, leaving a pin gate. To make a cavity inside the clay mould, the wax prototype was molten and passed away through the pin gate. Molten bronze was poured into the cavity with the shape of Zeus using gravity as in Figure 1.4(c). After the bronze had hardened the clay mould was broken. As shown in Figure 1.4, the cast itself is broken at the end of the process, and to make another duplicate one had to restart from the beginning, making a new cast.

Today, the prevailing mode of production is called “mass production”: one wants to reuse
Figure 1.4: The casting process of old days: making a bronze statue of the Zeus: (a) prototype in wax, (b) the prototype is completely covered by clay, after which wax melts and comes out of the clay cover leaving cavity, (c) molten bronze is poured into the cavity, (d) the outer clay cover is broken to get the bronze statue.

The industrial casting process consists of two stages. First, liquid is filled into a cavity formed by two cast parts. After the liquid has hardened, one cast part retracts, carrying the object with it. Afterwards, the object is ejected from the retracted cast part (see Figure 1.5). In both retraction and ejection steps, the cast parts and the object should not be damaged, so that the quality of final object is guaranteed and the cast parts can be reused to produce another object.

Depending on the materials (iron, aluminum, polymer, zinc, etc.) being used, the mass
Figure 1.5: The casting process in the real world

producibility of moulds, and the feeding systems (gravity or pressure), there are many different methods for the process.

Most castings of metals, especially large ones, are made in sand moulds. In sand casting a prototype of the object is first obtained, after which the prototype is divided into two parts along a plane (called a parting plane). Sand, mixed with a binder to hold it together, is pressed around the prototype. The sand mould is divided into two along the parting plane, and the prototype is removed from the mould leaving a cavity in the sand mould. Then two sand mould parts are placed together along the parting plane (See Figure 1.6 (c)). To build an object, liquid metal is poured into the cavity through the pin gate using gravity. After the metal has solidified, the sand mould is usually broken and leaves the object.

Injection moulding is a method of casting where plastic is forced into a mould cavity under pressure. The cavity is filled with plastic, and the plastic changes phase to a solid, resulting in an object. Because of the high pressures involved, the mould must be clamped shut during injection and cooling.

Large numbers of small, precise metal parts that have a low melting point, such as zinc, are made by die casting using permanent steel moulds. Die casting is accomplished by
forcing molten metal alloy into a steel mould under high pressure. The heat from the molten metal flows by conduction into the steel mould, which causes the molten metal to solidify. The process is often described as “the shortest distance between raw material and finished product.” Unlike sand casting, die casting is used for mass-producing high quality objects, such as handles, brackets, camera bodies and telephone parts, with high speed.

### 1.3 Automated computer-aided design systems

Computer-aided manufacturing automates manufacturing processes by letting computers communicate instructions directly to the manufacturing machinery. A single computer can control banks of robotic milling machines, lathes, welding machines, and other tools, moving the product from machine to machine as each step in the manufacturing process is completed.
The first phase of manufacturing processes is the design of a product. Computer-aided design (CAD) is a form of automation that helps designers prepare drawings, specifications, parts lists, and other design-related elements using special graphics and calculations intensive computer programs. Computer-aided design systems have considerably simplified industrial design, the first phase in the life of a new product.

**Modeling.** The first CAD system was the *Sketchpad* system [51], developed by Ivan Sutherland in the early 1960s. Although CAD systems originally automated 2-dimensional drawing and drafting, they now usually include 3-dimensional modeling and computer-simulated operation of the object. The history of modelings in CAD systems can be summarized as follows:

- **2-dimensional projections:** Entities (line, circle, arc and text) are projected on 2-dimensional planes. Several 2-dimensional views represent a 3-dimensional object. When you modify one of these drawings, you also need to change the others manually.

- **wire-frame:** The first modeling in 3-dimensional space. Edges are defined by lines or arcs connecting points in 3-dimensional space. This is the easiest and simplest way of representing a 3-dimensional model, but models need to be simple and clear. A 2-dimensional view of an object can easily be displayed.

- **surface modeling:** Surfaces interpolate edges of wire-frames. Curved surfaces can be represented with shading. Surface modeling is widely used where the quality of surfaces is important, for example, for the surfaces of the external bodies of cars and planes. It is impossible to extract physical properties of models. Figure 1.8 shows an example of surface modeling: an oilpump.

- **solid modeling:** A solid is a volume enclosed by surfaces that are represented as a quilt of vertices, edges, and faces. The major representations of solids include
constructive solid geometry, boundary representations, and spatial subdivision representations, all of which support the unambiguous, algorithmic determination of point membership: given any point \( p = (x, y, z) \), there must be an algorithm that determines whether the point is inside, outside, or on the surface of the solid. Solid modeling maintains additional information on the interior and the exterior of the volume.

Solid modeling is in transition. As Hoffmann [27] writes, classical design paradigms that concentrated on obtaining one specific final shape are being supplanted by feature-based, constraint-based design paradigms that are oriented more toward the design process and define classes of shape instances. One of these new paradigms is parametric solid modeling which is a key technology to define and manipulate solid models through high-level, parameterized steps. A parametric solid can be defined as a solid whose actual shape is a function of a given set of parameters and constraints. The shape designer can define entire families of shapes, not just specific instances. Hoffmann’s survey [28, 27] provides an excellent overview of solid modeling and parametric modeling.
There are plenty of commercial CAD packages, such as AutoCAD\textsuperscript{1}, UniGraphics\textsuperscript{2}, SolidWorks\textsuperscript{3}, Helix Design System\textsuperscript{4}, SOLIDCAM\textsuperscript{5} and I-DEAS\textsuperscript{6}, and most of them provide integrated features for surface modeling and solid modeling. Nowadays these packages have features to publish CAD drawings to the Web.

**Verification and Simulation.** Once an object has been designed, it has to be manufactured using the intended technique. As Bose and Toussaint [11, 14] write, it is desirable to design the object in such a way that manufacturing can be performed easily and cheaply. A fundamental question arises concerning every type of manufacturing process: Given an object, can it be constructed using a particular process?

The geometry of the object, coupled with the restrictions imposed by the particular manufacturing process under consideration, plays a vital role in determining the answer to the question. To answer the question, computer-aided design systems must be augmented with a component that verifies on-line whether an object being designed can actually be manufactured using the intended techniques long before the fabrication of costly physical models. Algorithms in such verification systems need to deduce the feasibility of manufacturing techniques purely on the basis of a CAD model of the object. Not only should they answer whether production is feasible, they can provide more information such as a list of possible orientations of the object that can build the object in the technique, a list of possible sequences of movements for manufacturing parts, and a simulation of the building process. In case the object is not feasible, they should point out what is wrong with the object.

Such algorithms have been proposed for a number of manufacturing processes, such as injection moulding [15, 21, 45], NC-machining [25], and stereolithography [10].

The importance of these verification components is quite evident. For example, when designing an object to be built using a certain technique, an engineer can check on-line whether the object can be built or not. By employing such components, computer-aided design systems help designers minimize scraps, reduce design time and eliminate wasted or redundant operations. These systems enable engineers to considerably reduce product-development costs and greatly shorten the design cycle.

1.4 **Geometric aspects of the casting process**

We now concentrate on the casting process. As discussed in the previous section, industrial computer-aided design systems could aid a part designer in verifying already during

\begin{footnotesize}
\begin{enumerate}
\item AutoCAD\textsuperscript{TM} is a trademark of Autodesk, http://www.autodesk.com
\item UniGraphics\textsuperscript{TM} is a trademark of UGS, http://www.ugs.com
\item SolidWorks\textsuperscript{TM} is a trademark of SolidWorks Corporation, http://www.solidworks.com
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the design of an object whether the object in question can actually be manufactured using a casting process. At the basis of this verification is a geometric decision: is it possible to enclose the object in a mould that can be split into two parts, such that these cast parts can be removed from the object without colliding with the object or each other? These geometric problems can generally be termed separability problems [53]. (We are not interested in casting processes where the mould has to be destroyed in order to remove the object, but only in situations where the given object can be mass-produced by re-using the same cast parts.) The casting process may fail in the removal of the cast parts: if the cast is not designed properly, then one or more of the cast parts may be stuck during the removal phase, as in Figure 1.9. The problems we address here concern this aspect: Given a 3-dimensional object, is there a cast for it whose parts can be removed after the liquid has solidified? An object for which this is the case is called castable. Note that this is a preliminary decision meant to aid in part design—to physically create the mould for a part one needs to take into account other factors such as heat flow and how air can evade from the cavity.

![Figure 1.9: The top part of the cast is stuck.](image)

In Chapter 3, 4, and 5, we consider the castability problem for three different casting models, give complete characterizations of castability in those models, and obtain algorithms to verify these conditions for polyhedral parts.

In manufacturing, features of an object imply manufacturing information that facilitates the process of analyzing manufacturability and the automated design of a cast for the object [44, 23]. Informally, features are a product’s generic shapes or characteristics that are associated with engineering knowledge about the product [46, 47]. A small hole or a depression on the boundary of an object, for example, restricts the set of removal directions for which this object is castable, since the portion of the cast in the hole or in the depression must be removed from the object without breaking the object. Identifying such features not only facilitates the decision process, but also reduces the search space for castable directions. Features can also make the automated design of cast much easier. In Chapter 6 and 7, we define a geometric feature, the cavity, which is related to the castability of objects, and provide algorithms to extract it from objects.
Our approach is to extract the geometric essence of the object we designed and answer the question based on a purely geometric perspective. Most geometric components in commercial CAD packages serve as front-end processors. Once CAD models are created, geometric components import geometric data from CAD models, filter redundant edges, repair geometric and topological irregularities, and generate meshes.

1.5 Casting with opposite cast removal

In Chapter 3, we consider a casting model where the cast (mould) consists of two parts and these parts must be removed in opposite directions without damaging the parts or the object. This chapter is based on a paper with Mark de Berg, Prosenjit Bose, Siu-Wing Cheng, Dan Halperin, Jiří Matoušek, and Otfried Schwarzkopf [7].

Contrary to the sand casting model studied by Bose et al. [12] where the partition of the cast into two parts must be done by a plane, the cast is partitioned by a polygonal parting surface. In this casting model all convex polyhedra are castable. (In the sand casting model, even convex polyhedra are not always castable.)

In Section 3.2 we consider the case where the orientation of the object in the cast and the removal direction \(d\) are specified in advance. The problem then is to decide whether the object is castable in that direction, that is, whether the cast can be partitioned into two parts that can be removed in direction \(d\) and \(-d\), respectively. We give a necessary and sufficient condition under which such a partition exists: the object is monotone in the removal direction \(d\). In other words, an object \(Q\) is castable in direction \(d\) if and only if every line with direction \(d\) intersects the interior of \(Q\) in at most one connected component. The class of objects we allow in this characterization is more general than in previous works: the object need not be polyhedral and may have arbitrary genus. We give a simple way to verify the condition for polyhedral objects of arbitrary genus: a necessary and sufficient condition under which a polyhedron \(P\) is monotone in direction \(d\), and therefore castable in \(d\) is that \(P\) has no reflex silhouette elements and its shadow edges form a set of non-crossing curves. Silhouette elements are silhouette edges in \(d\), edges parallel to \(d\), and parts of facets parallel to \(d\). Shadow edges are the projection of convex silhouette elements onto a plane whose normal direction is parallel to \(d\). This condition leads to an \(O(n \log n)\) time algorithm, where \(n\) is the combinatorial complexity of the polyhedron. We also give an algorithm that computes a partitioning of the cast into two removable parts (provided the polyhedron is castable, of course).

In Section 3.3 we consider the case where the removal direction is not specified in advance. Here the problem is to find all combinatorially distinct directions in which the object is castable (we postpone a formal definition of “distinct directions” to Section 3.3). One way of doing this is to generate a large set of sample directions, and to test each direction with the \(O(n \log n)\) algorithm. This is the approach we take in the experimental section (Section 3.4) and it turns out to work well in practice. Such a sampling approach
is not complete, however: it might erroneously report that there are no good directions. Hence, in Section 3.3 we give an exact algorithm that computes all combinatorially distinct casting directions in $O(n^4)$ time: If we imagine the direction $\vec{d}$ changing continuously, there are events that may influence the castability of the object. These critical events can be represented by great circles and arcs of great circles on the unit sphere. These curves form an arrangement of complexity $\Theta(n^4)$ in the worst case. The algorithm makes use of the fact that the difference between two adjacent faces in the arrangement is quite small. It traverses the arrangement, and updates the castability information in constant time per edge involved, except that the computation at the starting point takes $O(n^2 \log n)$ time. We also show that there exist polyhedra for which there are $\Omega(n^4)$ combinatorially distinct casting directions. This implies that our algorithm is optimal in the worst case if we want to report all such directions.

1.6 Casting with directional uncertainty

The casting algorithms mentioned in the previous section assume perfect control of the casting machinery. When a cast part is removed, it is required that the part moves exactly in the specified direction. In practice, however, this will rarely be the case. As in all applications of robotics, we have to deal with imperfect control of the machinery, and a certain level of uncertainty in its movement. When a facet of the object or of a cast part is almost parallel to the direction in which the cast parts are being moved, then the two touching surfaces may damage each other when the mould is being opened. This can make the resulting object worthless, or it may wear away the surface of the mould so that it cannot be reused as often.

In Chapter 4, we consider a casting model that is identical to the model with opposite cast removal in Chapter 3, except that the cast machinery has a certain level of uncertainty in its directional movement: Given a removal direction $\vec{d}$ for a cast part, the part may move in a direction $\vec{d}'$ such that the angle between two directions is within a certain level of uncertainty. This chapter is based on a paper with Otfried Cheong and René van Oostrum [6].

In Section 4.3 we consider the case where the orientation of the object, the removal direction $\vec{d}$, and the level of directional uncertainty angle $\alpha$ are specified in advance. The problem then is to decide whether the object is castable in that direction with uncertainty $\alpha$, that is, whether the cast can be partitioned into two parts that can be removed in any direction $\vec{d}'$ and $\vec{d}''$, respectively, such that the angles between $\vec{d}$ and $\vec{d}'$, and between $-\vec{d}$ and $\vec{d}''$ are smaller than or equal to $\alpha$. We give a necessary and sufficient condition under which such a partition exists: the polyhedron is $\alpha$-monotone and $\alpha$-safe. We say that a polyhedron $\mathcal{P}$ is $\alpha$-monotone in direction $\vec{d}$ for an angle $\alpha$ if $\mathcal{P}$ is monotone in direction $\vec{d}'$ for all directions $\vec{d}'$ with $\angle(\vec{d}, \vec{d}') \leq \alpha$. A polyhedron $\mathcal{P}$ is called $\alpha$-safe in direction $\vec{d}$ if none of the normals of its facets make angles $\beta$ with $\vec{d}$ in the range $\pi/2 - \alpha \leq \beta \leq \pi/2 + \alpha$. 15
The casting model we consider is more practical than the models in previous works, since most of the existing machinery bears a certain level of uncertainty. We give a simple way to verify the condition for polyhedral objects of arbitrary genus, leading to an $O(n \log n)$ time algorithm, where $n$ is the combinatorial complexity of the polyhedron. We also give an algorithm that computes a partitioning of the cast into two removable parts.

In Section 4.4 we consider the case where the removal direction is not specified in advance. Depending on whether the uncertainty is specified in advance or not, we consider two problems. One of them is, for given uncertainty $\alpha$, to find all combinatorially distinct directions in which the object is castable with directional uncertainty $\alpha$. In Section 4.4 we give an exact algorithm that computes all combinatorially distinct casting directions in time $O(n^2 \log n/\alpha^2)$. We also consider an approximative solution, and give a heuristic that runs in time $O(n \log n)$ for constant $\alpha$.

The other problem we consider is to find the best removal direction in which the object is castable. In this problem the best is qualified in the way that the directional uncertainty is as large as possible with which the object is castable. In Section 4.4 we give an exact algorithm that computes the best casting directions in $O(n^4)$ time. If it is known that $P$ is $\alpha$-castable for a certain angle $\alpha$, we can compute the largest feasible uncertainty in time $O(n^2 \log n/\alpha^2)$. We also give a heuristic approach to approximate the largest feasible uncertainty.

1.7 Casting with skewed ejection direction

In most existing machinery, the retraction and ejection directions are identical as in Figure 1.5. Previous work on this problem has also assumed this restriction on casting. Existing technology for injection moulding, however, already has the flexibility to accommodate an ejection direction that is different from the retraction direction of the moving cast part. Exploiting this possibility allows to cast more parts, or to cast parts with simpler moulds. This is a generalization of the opposite casting model in the sense that the restriction on the removal directions of cast parts are removed.

In Chapter 5, we consider a casting model where the two cast parts are to be removed in two given directions and these directions need not be opposite. In contrast with previous works, the ordering of removal is important in this casting model. This chapter is based on a paper with Siu-Wing Cheng and Otfried Cheong [2].

We give a complete characterization of castability in this casting model, under the assumption that the cast has to consist of two parts that are to be removed in two not necessarily opposite directions. We also give an algorithm to verify this condition for polyhedral objects. We do not assume any special separability of the two cast parts, and allow parts of arbitrary genus. The running time of our algorithm for determining the castability of an object with a given pair of directions is $O(n^2 \log n)$.
All the results for opposite cast parts removal in [7, 30, 34] rely on the property that an object is castable if its boundary surface is completely visible from the two opposite removal directions. This is not true when the removal directions are non-opposite: there are polyhedra whose whole boundary is visible from the removal directions but which are not castable with respect to those directions [7].

For completeness, we also give an $O(n^{14} \log n)$-time algorithm for finding all combinatorially distinct feasible pairs of removal directions: we consider a 4-dimensional parameter space formed by the set of all pairs of directions, and construct a set of algebraic surfaces which correspond to a number of critical events that may influence the castability of the object. There are $O(n^3)$ surfaces and their arrangement has complexity $O(n^{12})$. We test at most $O(n^{12})$ pairs of directions using the algorithm of time complexity $O(n^2 \log n)$ for determining the castability. Though the running time is polynomial, the algorithm is clearly of theoretical interest only.

1.8 The reflex-free hull

Computational geometers have defined many classes of 2-dimensional polygons, but few classes of 3-dimensional polyhedra. Perhaps the fact that 3-dimensional polyhedra support a rich class of topological structure in the form of knots and links has overshadowed the identification of geometric structure.

A small hole or depression on the boundary of an object, for example, restricts the set of directions for which this object is castable, since the portion of the cast in the hole or the depression must be removed without breaking the object. Most parts used in industry, such as engine rooms, telephone bodies, and small parts for cars and aircrafts, have such features.

This suggests a new approach to castability analysis: For a given pair of removal directions, we first identify such features (holes and depressions) of an object and if any such features cannot be accommodated with the given removal directions, then we can conclude that the object is not castable with the given removal directions. This idea can drastically reduce the size of the search space for feasible casting directions. For example, a hole with the shape of a cylinder in an object reduces the search space into a pair of two opposite directions parallel to the generators of the cylinder. Features, furthermore, can also be used for computing the minimum number of casting parts. In other words, the minimum number of additional casting parts (called side cores), together with two main parts, can be obtained from features.

In Chapter 6, we study features of a polyhedron related to casting, and define three geometric structures: plane-cavities, cavities, and the reflex-free hull. These definitions can also be applied to a 3-dimensional general shape. This chapter is based on work with Siu-Wing Cheng, Otfried Cheong, and Jack Snoeyink [4, 5].
In Section 6.3, we show several properties of the reflex-free hull of a polyhedron. One of the interesting properties of the reflex-free hull is that its complexity is linear in the size of the input polyhedron.

We currently have no algorithms for constructing the reflex-free hulls and cavities. In Chapter 7, we show how these geometric structures can be made use of with application to casting.

1.9 Coloring algorithm for finding cavities

Based on the definition of the reflex-free hull and cavities in Chapter 7, we consider applications of these geometric structures to casting. Cavities and the reflex-free hull are important features in applications such as manufacturing and molecular analysis. Unfortunately, we are currently unable to construct the reflex-free hulls and cavities. Nevertheless, we are able to prove that given a castable polyhedron, the bounding faces of a cavity necessarily belong to the same mould part. So we can make use of cavities in automatic mould part construction.

In Chapter 7, we present an algorithm to partition the faces of a polyhedron into disjoint subsets such that each subset must belong to the same mould part. Furthermore, we prove that the bounding faces of each cavity belong to the same subset. Thus, our algorithm is an effective method to restrict the search space for feasible casting directions. In fact, we conjecture that this algorithm can be extended so that, in the end, for any two distinct subsets, there is a feasible casting direction in which the mould is removed from the corresponding faces in opposite directions. This chapter is based on a paper with Siu-Wing Cheng, Otfried Cheong, and Jack Snoeyink [3].