

# Estimating the Gradient Direction of a Luminance Ramp

RODERIK G. F. ERENS,\* ASTRID M. L. KAPPERS,\* JAN J. KOENDERINK\*

Received 28 October 1992; in revised form 12 January 1993

---

**We report on the extent to which human observers are able to indicate the gradient direction of a luminance ramp. In our experiment modulation depth ranged from 1 to 64% and field sizes subtended 0.5 to 47.5° of visual angle. Observers are not able to indicate the gradient direction for modulation depths below 6%. For values above this threshold, the angular standard deviation in the responses decreases proportionally with the logarithm of the modulation depth and is about 11° for a modulation depth of 64%. The angular standard deviations for supra-threshold contrast are slightly increased for the field sizes of 0.5 and 47.5° but are constant over a range of field sizes of 1 to 25°. Thus, size invariance holds well for this range of field sizes.**

Shape from shading   Spatial contrast   Luminance gradients   Gradient direction   Stimulus size

---

## INTRODUCTION

In order to find out to what extent human observers are capable of estimating solid shape on the basis of shading, one has to know whether they can analyse such a luminance pattern quantitatively and qualitatively. The accuracy with which observers judge the direction as well as the variation in luminance gradients determines to what extent the surface shape can be reconstructed from shading (Koenderink & van Doorn, 1980; Pentland, 1982, 1984). For example the direction of the luminance gradient can be used to estimate the direction of the illumination (Pentland, 1982). Since only sparse data are available on how accurately human observers can estimate the direction as well as variations in luminance gradients we have begun a systematic study of the subject. In this paper we report on the degree to which human observers can estimate the gradient direction of a luminance ramp.

### *Visibility of luminance gradients*

The visibility of spatial contrast has been investigated extensively over the past three decades. This work has been well summarized by De Valois and De Valois (1988) and Graham (1989). Most of this research has concentrated on the detection and the discrimination of sinusoidally modulated luminance patterns. Only a few authors have done psychophysical work on the visibility of luminance gradients (McCann, Savoy, Hall & Scarpetti, 1974; van der Wildt, Keemink & van den Brink, 1976).

McCann *et al.* (1974) studied the visibility of a luminance ramp as a function of the spatial contrast and the steepness of the luminance gradient. They found that the detection threshold for the ramp was determined only by the global spatial contrast and not by the steepness of the luminance gradient. They varied the steepness of the luminance gradient by altering the distance between the viewer and the stimulus and thus altering its field size. In other words, the visibility of the ramp is invariant for size changes.

An experiment which reports on the ability of observers to indicate the direction of a luminance gradient, was performed by van der Wildt *et al.* (1976). The field sizes ranged from 0.1 to 20° of visual angle. They found a threshold modulation of 3% for field sizes of 1–5°, which increased up to 20% for smaller and larger field sizes. Thus in this paradigm the size invariance breaks down for the smallest and largest field sizes.

Electrophysiological studies about orientation selectivity indicate that the orientation bandwidth is very similar at all orientations. Estimates of the bandwidth depend on the type of stimulus and experiment. Values from 5 or 10° up to as much as 60–90° have been reported (see De Valois & De Valois, 1988; Graham, 1989).

In order to gain a better understanding of the extent to which human observers can estimate the gradient direction of a luminance ramp, we have investigated the influence of the spatial contrast and field size of the stimuli on orientation settings of subjects. This is done for the widest possible range of modulation depth and field sizes that we could obtain in our experimental setup.

---

\*Utrecht Biophysics Research Institute (UBI), University of Utrecht, Princetonplein 5, 3584 CC Utrecht, The Netherlands.

## METHOD

*Stimulus*

The stimulus is a linear, unidirectional luminance distribution. The luminance ( $L$ ) changes as a function of the position in Cartesian coordinates ( $x, y$ ) ( $\sqrt{x^2 + y^2} \leq 5.5$  cm):

$$L(x, y) = L_0(1 + M(x \cos \phi + y \sin \phi)) \quad (1)$$

where  $L_0$  is the mean luminance value,  $M$  is the modulation depth of the ramp and the angle  $\phi$  denotes the direction of the luminance gradient.

The stimulus is rendered on an Apollo DN590 computer and displayed in 8-bit grey-tone on a high resolution monitor screen (dimensions: 39.0 × 29.0 cm, with 1280 × 1024 pixels). The colour-lookup table is calibrated such that the screen luminances increase linearly as a function of the grey values. During the experiment we used a photometer (Tektronix J16 digital photometer) to check at regular intervals the linearity of the screen luminance.

The luminance ramp is oriented with unknown direction  $\phi$  behind a circular aperture with a diameter of 11.0 cm. The luminance ramp is defined by the modulation depth ( $M$ ) and the field size [visual angle ( $V$ )]. The modulation depth is defined as:

$$M = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} 100\%. \quad (2)$$

$I_{\max}$  and  $I_{\min}$  are the maximum and minimum intensity of the luminance ramp, respectively. The mean luminance value is 4.0 cd/m<sup>2</sup>. This is also the intensity of the background. The field size of the stimulus is varied by enlarging or reducing the distance between the viewer and the monitor. In this way we can be certain that the state of the monitor is identical in all viewing conditions. Distances ranged from 12.5 to 1200 cm. In the case of viewing distances of 800 and 1200 cm observers viewed the monitor screen via two mirrors in order to enlarge the visual pathway.

The screen is partially obscured by a circular black cardboard mask with a diameter of 22.5 cm. This mask enables us to minimize the possible influence of the shape of the target field surround on the responses of the subjects.

A sine-wave modulated annulus surrounds the luminance ramp (Fig. 1). This annulus is introduced to prevent subjects from using unwanted luminance information at the stimulus boundary. For instance, they might conceivably use the luminance extrema at the boundary to detect the gradient direction. The sine-wave modulated annulus induces light and dark regions at the boundary of the luminance ramp by which subjects are hampered to find the precise position of the luminance extrema. This discourages observers from using the information given by the extrema.

The annulus has a width of 2.5 cm, its mean luminance is 4.0 cd/m<sup>2</sup> and the modulation depth is 75%. This configuration ensured that subjects were practically un-

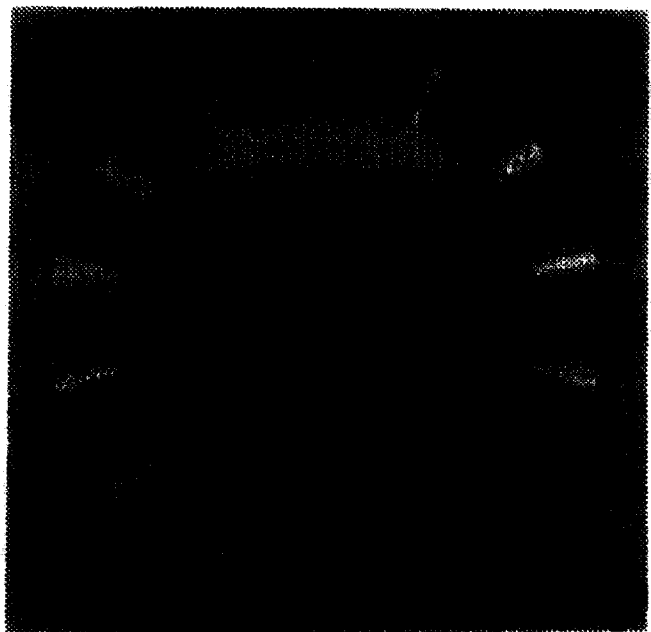


FIGURE 1. An example of the stimulus. The figure shows a luminance ramp with a modulation of 64%. The annulus around the stimulus is sine-wave modulated. One can see the induced bright and dark regions at the stimulus boundary, which discourages observers finding the precise position of the luminance extrema (see text). Hereby we prevent that subjects might use the extrema at the stimulus boundary to estimate the gradient direction. The actual stimulus is represented in 8-bit grey tone, so the luminance ramp is perceived as quite smooth. One can see in this figure that the gradient direction of the luminance ramp is almost aligned with the  $y$ -axis. Subjects were asked to set the "compass needle" with the gradient direction of the luminance ramp.

able to use the information given by the extrema at the boundary.

*Experiment*

Three male subjects performed the experiment; all have normal or corrected to normal vision. Natural pupil was used. They viewed the stimulus monocularly with the right eye. The subjects were asked to fixate loosely in the centre of the stimulus. No fixation mark was present in the stimulus. Head movements were restricted by a chin rest. The experiment was performed in a totally dark room.

Subjects had to align a compass needle, which was displayed on the monitor, with the gradient direction of the luminance ramp. The subject was asked to rotate the needle around the stimulus using the computer mouse. The needle did not cross the luminance ramp and sine-wave modulated annulus (Fig. 1). The subject confirmed his decision by pressing the mouse button.

We measured the deviation angle between the subject's response and the actual direction of the gradient. The deviation angle could vary maximally between  $-180$  and  $180^\circ$ . Modulations ranged from 1 to 64%. The field size of the stimulus was defined by its visual angle and extended from  $0.5$  to  $47.5^\circ$ . This corresponds to viewing distances ranging from 1200 cm for a field size of  $0.5^\circ$  to 12.5 cm for a field size of  $47.5^\circ$ . Three series of measurements were repeated on separate days. In each series the

modulation depth is drawn randomly from the range and each modulation was presented 20 times.

We performed two series of control measurements. Firstly we investigated whether the mean luminance value influenced the task of the subjects. Therefore a series of measurements was repeated at a lower mean luminance of  $0.4 \text{ cd/m}^2$ . The lower luminance was obtained by placing a 10% transmittance neutral filter between the observer and the monitor screen. Furthermore, we checked whether observers could indicate a well defined luminance step direction with high precision. A luminance step from minimum intensity ( $0 \text{ cd/m}^2$ ) to maximum intensity ( $8 \text{ cd/m}^2$ ) was displayed and subjects were asked to indicate the direction perpendicular to the step. Both series of control measurements were repeated for all viewing distances.

## RESULTS

The experimental data is expressed in values for the deviation angles as a function of the spatial contrast and the field size of the stimulus. From the data we calculate the *mean direction* ( $\phi$ ) and the *angular standard deviation* ( $\sigma$ ). The angular standard deviation is a measure for the accuracy with which subjects indicate the gradient direction.

For all experimental conditions the mean direction  $\phi$  did not deviate significantly from zero. Two-way ANOVA shows that there is no effect for any of the subjects [KG,  $F(7,199) = 1.62$   $P < 0.134$ ; MH,  $F(7,199) = 0.74$   $P < 0.635$ ; RE,  $F(6,130) = 0.78$   $P < 0.590$ ]. Thus, it is allowed to use  $\sigma$  to express the accuracy with which subjects indicate the gradient direction.

The deviation angles lie between  $-180$  and  $180^\circ$ . In the case of fully random responses every value within

this range is equally likely. This implies that the distribution of the responses is no longer Gaussian but will be uniform in a range of  $-180$  to  $180^\circ$ . It is now easy to predict the angular standard deviation for fully random responses:

$$\sigma_{\text{random}} = \int_{-180}^{180} \theta^2 \frac{1}{360} d\theta \approx 104^\circ. \quad (3)$$

Thus, when the angular standard deviation is about  $104^\circ$ , the orientation settings of the subjects are random and they are unable to indicate the gradient direction of the ramp. For series of 20 values of truly random responses the angular standard deviation ranged in most cases from  $100$  to  $120^\circ$ .

Subjects might conceivably indicate certain directions of the luminance gradient more accurately than others. In our experiment such effect of the orientation of the luminance ramp on the orientation settings of the subjects will average out because the direction of the luminance gradient is randomly selected from the range of  $0-360^\circ$ .

In Fig. 2 the angular standard deviations are plotted as a function of the modulation depth. The curves represent the results for various field sizes and a mean luminance value of  $4 \text{ cd/m}^2$ . The error bars indicate the standard deviation in the three repeated series of measurements. The three graphs show clearly that for the three subjects, below a modulation depth of about 6% the angular standard deviations lie between  $100$  and  $120^\circ$ . For these low modulations, subjects are not able to indicate the gradient direction and the results correspond to random responses. Above the threshold value of about 6% however, subjects respond to the gradient direction with a degree of accuracy, as is shown in Fig. 2. On a double logarithmic scale the angular standard deviation decreases proportionally with the modulation

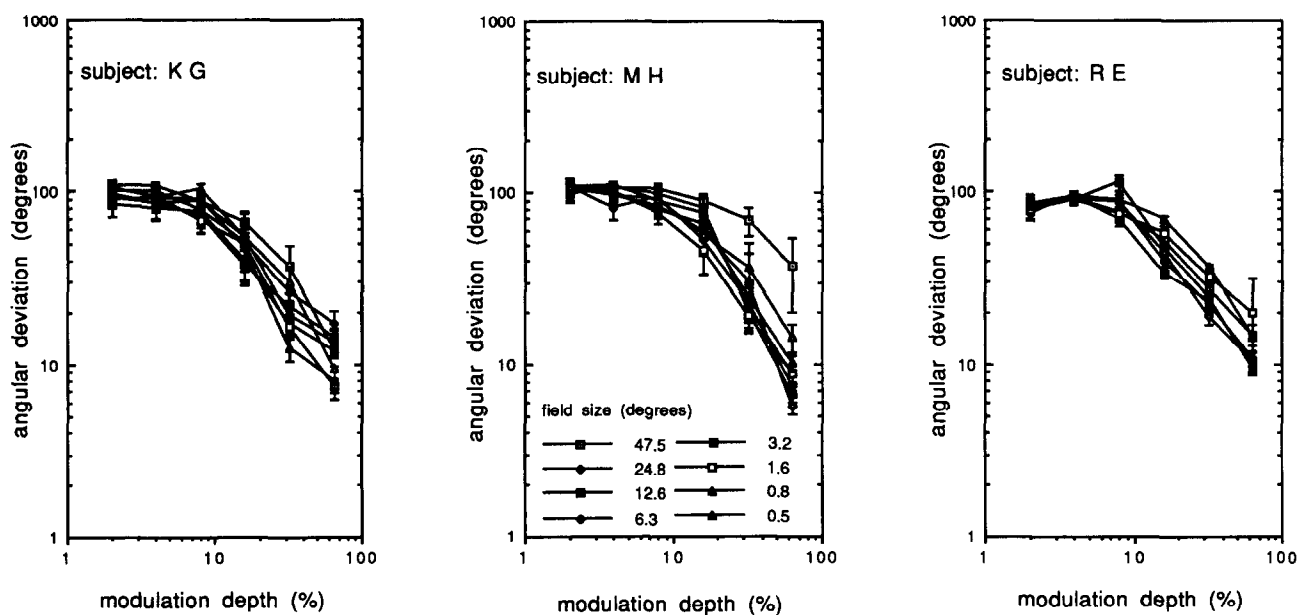


FIGURE 2. This figure shows the angular standard deviation as a function of the modulation depth for the mean luminance value of  $4 \text{ cd/m}^2$ . The three graphs show the results of each subject. An angular standard deviation larger than about  $100^\circ$  corresponds to random responses.

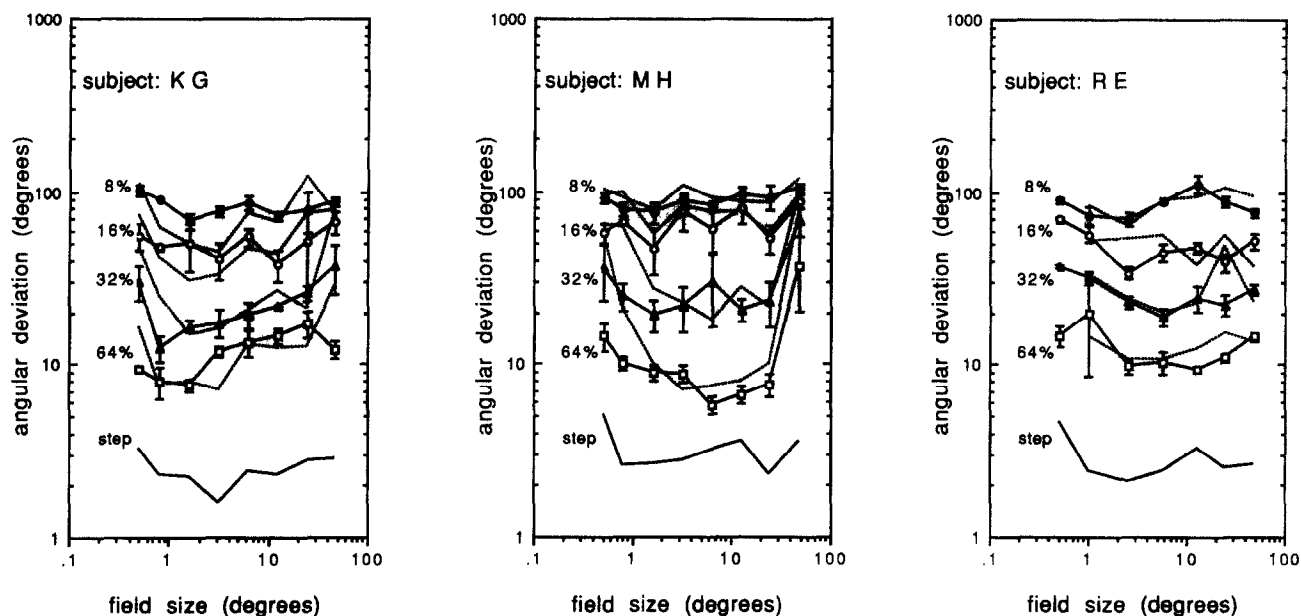


FIGURE 3. The results for the supra-threshold modulations ( $\geq 8\%$ ) for the three subjects are re-plotted, here the angular standard deviation is shown as a function of the field size of the stimulus. The solid curves show the angular standard deviation for the mean luminance level of  $4 \text{ cd/m}^2$ , whereas the dashed curves show the results for the lower mean luminance value of  $0.4 \text{ cd/m}^2$ . The solid curve without symbols shows the angular standard deviation for the experiment in which subjects indicated the direction perpendicular to a luminance step.

depth. In our experiment we found for a modulation depth of 64% an angular standard deviation of about  $11^\circ$ . For higher modulation values of the luminance ramp the accuracy of subjects in this task will probably increase even more.

Figure 2 illustrates also that especially the angular standard deviations for subject MH are increased significantly for both the smallest and largest field size of  $0.5^\circ$  and  $47.5^\circ$ , respectively. A two-way ANOVA shows that this increase is significant for all three subjects [KG,  $F(7,99) = 3.89$   $P < 0.000$ ; MH,  $F(7,99) = 8.69$   $P < 0.000$ ; RE,  $F(6,87) = 4.69$   $P < 0.000$ ]. The ANOVA tests are applied to the data of 8–64% modulation depth. The data for the smaller modulation depth are not taken into account because here the responses are already at random and thus no further increase of the angular standard deviation can be expected.

In Fig. 3 the results for the supra-threshold modulations ( $\geq 8\%$ ) are replotted as a function of the field size of the stimulus. The dashed curve in Fig. 3 presents the results obtained for the first control measurement at a lower luminance level of  $0.4 \text{ cd/m}^2$ . The solid curve without symbols shows the results for the second control experiment. Comparison shows that the subjects were able to indicate an edge direction with high precision, whereas the settings were much more variable for the smooth gradients.

Figure 3 illustrates clearly that the angular standard deviations are somewhat increased for the smallest and largest field sizes. However, within a range of about  $1\text{--}25^\circ$  the angular standard deviations for the supra-threshold modulations are about constant. For subjects KG and MH there is no effect of the field sizes in this range on the results [KG,  $F(5,75) = 1.80$   $P < 0.124$ ; MH,

$F(5,75) = 1.65$   $P < 0.160$ ]. However, there is a small effect for subject RE [ $F(4,63) = 3.35$   $P < 0.014$ ].

Furthermore, one can see in Fig. 3 that the results for the control experiment at a lower luminance level of  $0.4 \text{ cd/m}^2$  are nearly the same as for the luminance value of  $4 \text{ cd/m}^2$ . For subjects KG and RE there is no significant difference [KG,  $F(1,131) = 0.43$   $P < 0.513$ ; RE,  $F(1,99) = 0.10$   $P < 0.3119$ ], however for subject MH the angular standard deviations are slightly higher [ $F(1,131) = 7.18$   $P < 0.009$ ]. So, the screen luminances are in the photopic region.

The results indicate that in our paradigm the accuracy with which subjects estimate the gradient direction of a luminance ramp is determined only by the spatial contrast and hardly by the mean luminance level and field sizes in a range of  $1\text{--}25^\circ$  of the stimulus.

## DISCUSSION

The threshold modulation above which subjects are able to indicate the gradient direction is about 6% for all subjects. This corresponds to the average discriminability thresholds which van der Wildt *et al.* (1976) obtained for the gradient direction of a luminance ramp (between 3 and 20%). The higher threshold modulation of 20% occurred for the field sizes smaller than  $0.5^\circ$  and larger than  $20^\circ$ . In our experiment we also found that the angular standard deviations for the supra-threshold modulated stimuli were slightly higher for the field sizes of  $0.5^\circ$  and  $47.5^\circ$ . However, within a range of about  $1\text{--}25^\circ$  of visual angle the angular standard deviations were fairly constant. Thus "size invariance" holds well for this range of field sizes. This was also reported by McCann *et al.* (1974); however, in their experiment the range of

field sizes subtended only from 1° to about 5° visual angle. The "size invariance" has been found for the detection thresholds of a great variety of spatially modulated luminance distributions [e.g. sine-wave gratings (Campbell & Robson, 1968; Hoekstra, van de Goot, van den Brink & Bilsen, 1974; Savoy, 1975; Koenderink & van Doorn, 1982; Jamar & Koenderink, 1983) and complex bar patterns (Koenderink & van Doorn, 1978)]. Our results show that "size invariance" also holds with regard to the estimation of the direction of ramp patterns at supra threshold.

Orientation selectivity depends on the physical contrast of the stimulus. In the literature (De Valois & De Valois, 1988; Graham, 1989) there is some indication that the orientation selectivity improves for high spatial frequencies. Our stimulus was in the low frequency region of 0.01 c/deg (for the field size of 47.5) up to 1 c/deg (for the field size of 0.5). We found no indication that the angular standard deviations would further improve for higher spatial frequencies (=smaller field sizes), because subjects indicated the gradient direction already somewhat less accurately for the smallest field size.

Thus, only the modulation depth of the luminance ramp seems to influence the accuracy with which observers indicated the gradient direction. We found that this accuracy increased proportionally with the logarithm of the modulation depth. The orientation selectivity was about 11° for a modulation of 64%, however we expect that the orientation selectivity will improve still further for higher modulation values. From Fig. 2 we predict that subjects would indicate a gradient direction within about 6° precision for a modulation of 100%. This corresponds to the smallest bandwidth of orientation channels determined electrophysiologically (De Valois & De Valois, 1988; Graham, 1989).

What do our findings imply concerning the ability of human observers to judge solid "shape from shading"?

Pentland has proposed a shape-from-shading model in which the direction of the luminance gradient is used to find the direction of the illuminant. His psychophysical experiments indicated that human observers were able to estimate the luminance direction on a sphere (with high spatial contrast) within 5–10° accuracy. These results correspond very well with our findings. Recent work of Koenderink, van Doorn and Kappers (1992) shows that human observers can reproduce a tilt (tilt is direction of the surface normal in the image plane) setting of a surface normal on natural images within about 10° accuracy. Because of the varying contrast in the image they used it is hard to indicate whether the accuracy to reproduce a tilt setting was influenced by local spatial contrast or not.

The spatial contrast in a shaded image is determined by the surface geometry and the direction of the illuminant. Observers estimate the direction of a luminance gradient more accurately for high spatial contrast. This result implies that observers estimate the surface geometry better in regions with high spatial contrast. Thus in shaded images the surface geometry will be more apparent in the vicinity of the attached shadows.

Furthermore, we have found that the information about the direction of the luminance gradient will be invariant for size changes over quite a large range. Therefore we can expect that the estimation of solid shape will hardly change with size changes, because the same information is available.

## REFERENCES

- Campbell, F. W. & Robson, J. G. (1968). Application of Fourier analysis to the visibility of gratings. *Journal of Physiology, London*, *197*, 551–566.
- De Valois, R. L. & De Valois, K. K. (1988). *Spatial vision*, Oxford Psychology Series 14 (pp. 264–267). New York: Oxford University Press.
- Graham, N. V. S. (1989). *Visual pattern analyzers*, Oxford Psychology Series 16 (p. 452). New York: Oxford University Press.
- Hoekstra, J., van de Goot, D. P. J., van den Brink, G. & Bilsen, F. A. (1974). The influence of the number of cycles upon the visual contrast threshold for spatial sine wave patterns. *Vision Research*, *14*, 365–368.
- Jamar, J. H. T. & Koenderink, J. J. (1983). Sine-wave gratings: Scale invariance and spatial integration at suprathreshold contrast. *Vision Research*, *23*, 805–810.
- Koenderink, J. J. & van Doorn, A. J. (1978). Spatial summation for complex bar patterns. *Vision Research*, *20*, 169–176.
- Koenderink, J. J. & van Doorn, A. J. (1980). Photometric invariants related to solid shape. *Optica Acta*, *27*, 981–996.
- Koenderink, J. J. & van Doorn, A. J. (1982). Invariant features of contrast detection: An explanation in terms of self-similar detector arrays. *Journal of the Optical Society of America*, *72*, 83–87.
- Koenderink, J. J., van Doorn, A. J. and Kappers, A. M. L. (1992). Surface perception in pictures. *Perception & Psychophysics*, *52*, 487–496.
- McCann, J. J., Savoy, R. L., Hall, J. A. Jr & Scarpetti, J. J. (1974). Visibility of continuous luminance gradients. *Vision Research*, *14*, 917–927.
- Pentland, A. P. (1982). Finding the illuminant direction. *Journal of the Optical Society of America, A*, *72*, 448–455.
- Pentland, A. P. (1984). Local shading analysis. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, *6*, 170–187.
- Savoy, R. L. & McCann, J. J. (1975). Visibility of low-spatial-frequency sine-wave targets: Dependence on number of cycles. *Journal of the Optical Society of America*, *65*, 343–350.
- van der Wildt, G. J., Keemink, C. J. & van den Brink, G. (1976). Gradient detection and contrast transfer by the human eye. *Vision Research*, *16*, 1047–1053.

---

*Acknowledgement*—This research is supported by the SPIN project "3D-Computer Vision" of the Dutch Ministry of Economic Affairs.