Chapter 3
Teaching multiplication to low math performers: Guided versus structured instruction[1]

Abstract

The results of an intervention program for students with difficulties learning mathematics are reported. Two kinds of math intervention, guided versus structured instruction, were compared to regular math instruction. A total of 75 students from regular and special education, aged seven to thirteen, participated. Ability and automaticity multiplication tests were administered before and after the four-month training period. The results show that the students in both of the experimental conditions improved more than the students in the control condition. Some additional differences were found between the two experimental interventions. Guided instruction appeared to be more effective for low performing students than structured instruction and especially for those students in regular education. Special education students appear to benefit most from structured instruction for the automaticity of multiplication problems. A three-month follow-up test showed the acquired knowledge to be well established in both groups.

[1] Kroesbergen, E.H. & Van Luit, J.E.H. (in press). Teaching multiplication to low math performers: Guided versus structured instruction. *Instructional Science.*

3.1 Introduction

In the last decades, mathematics education has been the topic of considerable research and many changes resulting in new teaching approaches. The teaching and learning of mathematics has shifted worldwide towards more realistic education. In the Netherlands, almost every regular elementary school now uses a so-called "realistic mathematics method", which is a method based on the principles of Realistic Mathematics Education (RME). In the US, more than ten years ago, a comparable process was initiated on the basis of the curriculum and evaluation standards of the National Council of Teachers of Mathematics (NCTM, 1989). More than thirty years ago, the foundations for RME were laid by Freudenthal (1968) and his colleagues, who stressed the idea of mathematics as a human activity. Education should give students the "guided" opportunity to "re-invent" mathematics by doing it. The term "realistic" refers to the emphasis that RME places on providing students with problem situations that they can clearly imagine, make real in their minds (Freudenthal, 1991). This can be achieved by offering problems in real-world contexts although such contexts are not a necessary prerequisite. The use of context problems nevertheless plays a significant role in RME and is also promoted by the NCTM standards. They can function as a source for the learning process. By practicing with several different mathematical problems in several different contexts, students learn to use diverse strategies (Van Luit & Naglieri, 1999). During the learning process, not only the interactions between the teacher and the student are important but also those between the students and their peers. Students can learn from each other and each other's strategies. Interaction with peers also promotes conceptual learning (Cobb, 1994).

Another characteristic of RME, which is also promoted by the NCTM standards, is the use of students' own productions and constructions. Students must actively participate in the learning process to become active learners. Whereas the teacher formerly passed on mathematics knowledge in small and basically meaningless parts, students now play an important role in the construction of their own knowledge base. This vision of RME is in line with (socio-) constructivist methods of education (Cobb, 1994; Klein, Beishuizen, & Treffers, 1998). The challenge of teaching from such a constructivist perspective is to create experiences that engage students and encourage them to invent new strategies and learn to understand math. Socio-constructivists consider learning mathematics as a process of becoming familiar with the ideas and methods of the mathematical community (Grave-meijer, 1997). The classroom becomes a community that develops its own mathematics. Children in the classroom acquire new knowledge together. Students work together, learn together, and discuss possible solutions to a problem with each other. They develop problem-solving strategies, which they must then explain and justify to each other. Such learning can be promoted by guided instruction, which forms the focus of this study.

Instruction based on constructivist principles asks students to be proactive participants in the learning process. In practice, the task of the teacher is to structure interaction in such a manner that the students discover new knowledge. This is realized by offering meaningful problems and not teaching strategies. By finding ways to solve problems, the purpose of various strategies becomes apparent to students along with how to apply the necessary strategies. The teacher structures the lessons by asking questions and posing problems. The contribution of the students consists of thinking and discussing possible solutions. Although the teacher can lead the discussion by posing more questions and summarizing what the students say, the students are responsible for bringing forth the strategies to be discussed. By discussing different strategies and solution procedures, students can discover how to solve the relevant problem and thus acquire new knowledge as a result.

Research suggests that instruction based on constructivist principles leads to better results than more direct, traditional mathematics education (Cobb et al., 1991; Gravemeijer et al., 1993; Klein, 1998). And many researchers have observed that learning in such a manner is more motivating, exciting, and challenging (Ginsburg-Block & Fantuzzo, 1998). Students who learn to apply active learning strategies are also expected to acquire more useful and transferable knowledge because, for example, problem solving requires active participation on the part of the learner (Gabrys, Weiner, & Lesgold, 1993).

The question that remains is whether constructivist approaches and RME are also beneficial for *low* performing students (Klein et al., 1998). Van Zoelen, Houtveen, and Booij (1997) conclude that although the average and good students profit from RME, the weaker students appear to benefit much less from this method. Woodward and Baxter (1997) also state that special educators have raised objections to the instructional methods and materials put forth in the NCTM standards because they are too discovery-oriented, and not very sensitive to teaching students with math difficulties. In research, they also show this kind of instruction to benefit most students but those with learning disabilities and low-achievers to a much lesser extent.

Children with learning disabilities and low-performers need special attention to acquire basic mathematics skills (Geary, 1994). Researchers have documented specific mathematical deficiencies. In the domain of computation, for example, students may exhibit deficits and limited proficiency related to fact retrieval, problem conceptualization, speed of processing, and use of effective calculation strategies (Rivera, 1997). These same students also show deficits in the domain of metacognitive skills. Geary, Brown, and Samaranayake (1991), for example, have shown that children with math difficulties frequently use strategies normally used by younger children. More specifically, they usually have difficulties with the use of effective cognitive and metacognitive problem-solving strategies, various memory and retrieval processes, and generalization or transfer. These difficulties express themselves as an inability to acquire and apply computational skills, concepts, reasoning skills, and problem-solving skills. Due to the special needs of children with difficulties learning math, thus, this group of children needs specific instruction (Rivera, 1997).

Instruction should obviously take the particular difficulties of children with special needs into account and especially the lack of spontaneous use of adequate information processing strategies. Van Luit and Naglieri (1999) suggest that teaching step-by-step from concrete to abstract, working with materials to mental representations and providing task-relevant examples can certainly help. Many researchers (e.g., Jones, Wilson, & Bhojwani, 1997; Wood, Frank, & Wacker, 1998) also state that the instruction for children with special needs looks different from regular instruction. Students with math learning difficulties, whether severe or mild, clearly need structured and detailed instruction, explicit task analysis, and explicit instruction for generalization and automatization. This can be realized with direct instruction.

The main characteristic of direct instruction is, in fact, that it is very structured. In practice, direct instruction is teacher-led, because the teacher provides systematic explicit instruction (Jones et al., 1997). New steps in the learning process are taught one at a time, and the teacher decides (guided by the instructional program) when new steps are taught. The lessons are generally built up following the same pattern (e.g., Archer & Isaacson, 1989). In the opening phase, the students' attention is gained, previous lessons are reviewed, and the goals of the lesson are stated. In the main part of the lesson, the teacher demonstrates how a particular task can be solved and then allows the students to work together on the task. When the students appear to have sufficient understanding of the task, they are given new tasks to practice independently. The teacher monitors the students during such practice and provides feedback on completed tasks. Interventions in which students receive direct structured instruction have been frequently found to be very effective (e.g., Harris, Miller, & Mercer, 1995; Jitendra & Hoff, 1996; Van Luit, 1994; Wilson, Majsterek, & Simmons, 1996).

The recommendations mentioned in the literature for teaching students with learning disabilities or low-performing students appear to be in clear opposition to the constructivist principle of guided re-invention. The question is whether teachers can ask low-performing children to actively contribute to lessons by inventing new strategies. Asking for such a contribution actually appears to deny the special status of these children, who clearly have more difficulties with knowledge generalization, connecting new information to old, and the automatization of basic facts.

Recent studies nevertheless suggest that children with learning difficulties can benefit from interventions based on instructions that give students more freedom to develop and use their own strategies (Kroesbergen & Van Luit, in press; Woodward & Baxter, 1997). In other words, RME can perhaps be adjusted in such a manner that it is also very effective for students with math learning difficulties. Mercer, Jordan, and Miller (1994) give a starting point with their description of an "explicit-to-implicit continuum of constructivism". They state that students with learning difficulties clearly need the help of the teacher to become active and self-regulated students. Therefore, a more explicit instruction is needed, but this can nevertheless be designed in accordance with constructivist principles. However, very few studies have examined the possibilities and effectiveness of such interventions (e.g., Woodward & Baxter, 1997), and more research is certainly necessary.

The central question in the present study is therefore whether RME is also appropriate for the instruction of students with learning difficulties and whether or not low performers can benefit from the same type of math instruction as their average performing peers. One characteristic of RME will be examined in particular, namely the contributions of the students themselves. More specifically, it will be examined whether an intervention calling for student's own findings and contributions in connection with guided instruction is more effective than more structured intervention requiring students to apply strategies and procedures modeled by the teacher. In other words, we will attempt to answer the question of whether guided instruction is suitable for teaching children with special learning needs, or if structured instruction is always needed for these children, instead of, or in addition to guided instruction.

3.2 Method

3.2.1 Procedure and design

A quasi-experimental design containing two experimental and one control condition was conducted. Pre-, post-, and follow-up tests were conducted to measure achievement in mathematics ability, automaticity, and transfer. The post-test was administered directly after the training period; the follow-up test was administered three months later, after the summer vacation. The follow-up test was administered to only 58 of the 75 participating students due to a number of students changing schools.

The experimental groups received 30 half-hour multiplication lessons across a period of four months. Research assistants trained and coached by the experimenter conducted the intervention. By means of video observations it was checked if the research assistants implemented the programs as instructed. The lessons were conducted twice weekly in small groups of four to six students each, at the time that the children would normally receive math instruction. On the other three days of the week, the children in the experimental condition followed the regular math curriculum (i.e., the same as their classmates in the control condition), except for the instruction in multiplication. The children in the control condition also followed the regular curriculum and also received a minimum of multiplication instruction twice a week. The regular curriculum included a mixture of different instructional procedures. The instruction that the control children received differed across the different schools but was always somewhere in between guided and structured instruction.

3.2.2 Participants

Seventy-five students from schools for regular and special education participated in the experiment. The students were selected on the basis of low math performance in the opinion of the teacher and as indicated by their scores on general mathematics tests. Only students scoring below the 25th percentile on the national criterion-based tests were selected for inclusion in the study. The students also had to meet the following criteria: they had to be able to count and add to 100 (in order to learn multiplication on the basis of repeated addition) and they had to score below the 50% level on a test of multiplication facts up to ten times ten.

Within each participating school, students were selected according to the above-mentioned criteria. The students were then assigned randomly to an experimental or the control condition. In each school, only one experimental condition was implemented, with one or two groups of five students. In general, the learning difficulties of the students in special education were found to be more severe and complex than the learning problems of those students with special needs in general education.

A total of 75 students from 7 schools were selected: 30 for Guided Instruction (GI), 20 for Structured Instruction (SI), and 25 for the control group (C). As can be seen from Table 3.1, the study included 35 boys and 40 girls, with a mean age of 9.4 years (*sd* = 1.4) and a mean IQ of 88.5 (*sd* = 11.0). Univariate and multivariate analyses showed no differences across the three conditions for age, IQ, or months of multiplication instruction.

However, the children from the regular schools and the children from the special schools were found to differ significantly from each other with regard to age and IQ. As can be seen from Table 3.2, the children in the schools for special education were older (p = .000) and had a lower IQ (p = .000) on average when compared to the children in the regular schools.

Table 3.1
Comparison of groups

Condition	N	Sex		Mean IQ (sd)	Education		Mean	Experience
		male	female		regular	special	age (sd)	in months*
Guided	30	16	14	89.7 (10.1)	20	10	9.5 (1.3)	8.2 (4.9)
Structured	20	8	12	84.1 (11.9)	10	10	9.7 (1.6)	6.5 (6.1)

Control	25	11	14	90.6 (10.8)	18	7	9.1 (1.3)	6.1 (5.9)
Total	75	35	40	88.5 (11.0)	48	27	9.4 (1.4)	7.0 (5.6)

*Experience in months: the time children had received instruction in multiplication before the intervention period

Table 3.2
Comparison of schools

Education	N	Mean IQ (sd)	Mean age (sd)	Mean experience in months (sd)
Regular	48	92.7 (9.7)	8.7 (0.9)	8.0 (5.3)
Special	27	81.0 (9.1)	10.7 (1.3)	5.4 (5.8)

3.2.3 Materials

At pre- and post-test, two multiplication tests were administered: an ability test and an automaticity test (see Appendix 3.1). The ability test contains 20 items with multiplication problems up to 10 x 20. The test consists of ten items in the form of 6 x 7 = ___, and ten in the form of a short story. The children are also asked to write down their solution procedure. The answers are scored as right or wrong, and the solution strategies used by each child are recorded. The ability test includes a total of 10 items beyond 10 x 10. As the students were taught only tasks below 10 x 10, these more difficult items constituted a transfer measure. Transfer refers to the effects of prior learning on later learning in practice (Schopman, 1998).

The automaticity test contains 40 multiplication items of ascending difficulty; 25 of the items are multiplication problems up to 10 x 10; the other 15 items consist of problems with one number below 10 and the other between 10 and 50. The students are asked to solve as many problems as possible within a one-minute period. The answers are then scored as right or wrong.

Instruction program

In this study, two interventions are compared: guided versus structured instruction. To delineate an area within the broad domain of mathematics to be taught within a few months, the study was restricted to the teaching of multiplication problems. For the two experimental interventions, adjustments were made to the MASTER Training Program (Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999), which is a remedial program for multiplication and division. In this study, only the multiplication part was used. The program originally consisted of 23 multiplication lessons; in the adjusted versions, two lessons were added. The new program contains three series of lessons: (1) 8 lessons on basic procedures; (2) 11 lessons on multiplication tables; and (3) 6 lessons on 'easy' problems above 10 x 10. In each lesson, a new kind of task is introduced. Each of the series teaches new steps for the problem solving related to specific tasks. The steps between the lessons are small. In order to study the effects of the content of the intervention, it is tried to keep the structure of the two instruction programs as much the same as possible.

Structured instruction. The lessons in the structured condition are always built up in the same pattern. After a repetition of what was done in the preceding lesson, the teacher provides an introduction to the topic of the current lesson. The teacher explains how to solve the task in question, when necessary with the help of special materials like the number-line, blocks or other objects. After the introduction, several tasks are practiced in the group, guided by the teacher, who asks the questions and gives feedback. The students then practice on their own, when necessary with the help of the teacher. All of the tasks are discussed after their completion. During the practice phase, the children familiarize themselves with the kinds of tasks and establish connections to the mental solution of the problems. The goals of the final phase are control, shortening, automatization, and generalization.

The teacher always tells the children how and when to apply a new strategy. The children are then instructed to follow the example of the teacher. A concrete to abstract teaching sequence is followed in teaching new learning steps. The program also promotes the use of the self-instruction method and modeling, by means of a strategy decision sheet, which contains the following questions: "What is the multiplication problem?", "Do I know the answer directly?", "Do I know the answer if I reverse the problem?", "Do I know the answer if I say the multiplication table aloud?", and "Do I know the answer if I do long addition?". Later on, the sheet will be expanded to include the following strategies: splitting by five or ten, using a neighbor problem, and doubling. The strategy decision sheet helps students learn to use different strategies.

To make a clear distinction between both instructions, it was decided that in the structured instruction there would be no room for contributions from the children themselves that are not in line with the procedures the teacher teaches them. It is asked from the children that they contribute to the lessons by answering the teacher's questions and applying the strategies that were taught by the teacher. When a child applies a strategy that has not been taught, the teacher may state that the particular strategy is a possible strategy for solving the problem in question but that they are being taught a different strategy and then request application of the strategy being taught.

Guided instruction. The lessons in the guided instruction condition also start with a review of the previous lesson. What the students do and say in this phase is then taken as the starting point for the current lesson. The teacher states what the topic of the lesson will be (e.g., "we will now practice with the table of 6"). However, the discussion is then centered on the contributions of the children themselves, which means that other topics may sometimes be discussed. Just as in the structured instruction condition, the guided instruction condition contains an introductory phase, a group practice phase, and an individual practice phase. However, in the guided instruction condition, greater attention is devoted to the discussion of possible solution procedures and strategies than in the structured instruction condition.

In the guided instruction condition, much more space is provided for the individual contributions of the students. The main idea is that the teacher presents a problem and the children actively search for a possible solution. The students work together on the solution of the problems, and are given the opportunity to demonstrate their own strategies. The teacher can encourage the discovery of new strategies by offering additional and/or more difficult problems. The teacher supports the learning process by asking questions and promoting discussion between the students. The teacher never demonstrates the use of a particular strategy. As a consequence, when the children do not discover a particular strategy, the strategy will not be discussed within the group. The teacher does, however, structure the discussions during the lessons by helping the students classify various strategies and posing questions about the usefulness of particular strategies, for example.

Control condition. The students in the control condition received instruction as based on the regular curriculum method. Because the students in the control condition are also students with math difficulties, their teachers give generally more attention and extra instruction to them, as compared to their peers. It is important to notice that the control condition is not a 'no intervention' condition, but much more a condition with 'the usual instruction', implemented as optimal as possible. The experimental conditions thus have to compete with a control condition in which the given instruction has already proved to be a good instruction.

Much variance is found in the way the lessons in the control condition were given, because of the variety in teaching methods in the participating schools. The majority of the participating schools used a realistic mathematics teaching method. However, many teachers said that they adapted their instruction to the specific needs of the students by making the instruction less guided and more directed. The teachers also spent more attention to the automatization of the multiplication tables than to strategy use. The regular lessons contain as well instruction as practicing phases. Multiplication is part of the regular program, and in almost every lesson attention is given to multiplication skills. In general, the amount of time spent on multiplication is about one hour a week. The group size in the control condition varied from little groups of three to five students working at the same level to class wide instruction for groups of 25 to 30 students.

The experimental students received the same instruction as their classmates on the days that they did not receive the experimental instruction, except for the multiplication instruction. The teachers were asked to try to give multiplication instruction only at the times that the experimental students received their own instruction, and otherwise the experimental students received worksheets, so that they would not join in the class multiplication instruction.

3.3 Results

In this study, the effects of two mathematics interventions were investigated: guided instruction (GI) and structured instruction (SI). Tables 3.3 and 3.4 show the results of the three conditions for each school type. The effects of instruction in small groups were first compared to the effects of regular instruction (i.e., the two experimental conditions together were compared to the control condition). In addition to this, whether children with math learning difficulties can benefit from a teaching method that requires them to contribute actively to lessons was examined (i.e., the two experimental conditions were compared).

3.3.1 Experimental versus control conditions

First, the difference is studied between both experimental groups together versus the control group. No significant differences were found between the control and experimental groups at pre-test. The results show both the experimental and control groups to improve on both the ability and the automaticity test; however, the amount of improvement differed substantially for the ability test with the experimental groups improving significantly more than the control group, $t(1, 73) = 3.56$, $p = .001$. No significant differences between the experimental and the control group were found for automaticity and transfer. The training thus affected the children's use of strategies the most, which allows them to solve the given problems adequately. The three month follow-up tests showed that the experimental and control groups continued to perform at the same level.

Table 3.3

Means, standarddeviations and effect sizes for ability test (max. score: 20)

condition	school	pretest	posttest	effect size[1]	follow-up
GI	regular	7.4 (4.3)	14.3 (4.1)*	1.67	15.6 (4.4)
	special	4.0 (4.2)	13.8 (5.1)*	2.10	11.6 (3.0)
	both	6.2 (4.5)	14.1 (4.4)*	1.79	14.7 (4.4)
SI	regular	2.8 (2.7)	5.8 (4.3)*	0.83	10.5 (5.2)*
	special	5.0 (4.4)	13.2 (4.9)*	1.75	10.1 (4.3)
	both	3.9 (3.8)	9.5 (5.9)*	1.14	10.3 (4.6)
Control	regular	7.3 (6.5)	9.7 (5.7)*	0.39	13.8 (4.3)*
	special	8.1 (6.8)	13.9 (3.9)*	1.03	13.0 (4.6)
	both	7.5 (6.5)	10.8 (5.5)*	0.55	13.6 (4.3)*

[1] $d = (M_{post}-M_{pre})/\sigma_{pooled}$

* significant improvement compared with previous measurement ($\alpha \leq .05$)

Table 3.4

Means, standarddeviations and effect sizes for automaticity test

condition	school	pretest	posttest	effect size	follow-up
GI	regular	11.8 (2.7)	17.7 (4.4)*	1.61	17.8 (3.9)
	special	8.9 (1.5)	10.7 (1.7)*	1.12	14.2 (2.6)
	both	10.8 (2.7)	15.3 (5.0)*	1.13	17.0 (4.0)
SI	regular	7.6 (2.6)	9.8 (2.9)	0.81	16.1 (6.5)*
	special	4.9 (4.6)	15.2 (4.6)*	2.25	13.7 (4.3)
	both	6.3 (3.9)	12.5 (4.6)*	1.47	14.9 (5.4)
Control	regular	10.6 (5.6)	13.3 (5.7)*	0.69	16.0 (2.7)
	special	7.4 (5.2)	15.6 (2.2)*	1.91	15.6 (1.5)
	both	9.7 (5.6)	14.0 (5.0)*	0.79	15.9 (2.4)

* significant improvement compared with previous measurement ($\alpha \leq .05$)

3.3.2 Structured versus guided instruction

The most important question in this study concerns the potential differences between the two experimental conditions as the purpose of the study was to determine which form of instruction is most effective for low performing children. Before we address this question, some basic differences between the two groups of children in the experimental condition should be mentioned. The children were not randomly assigned to one of the experimental conditions. While the schools themselves were randomly assigned to one of the study conditions, the children within a particular school were assigned to either the experimental condition of the school or the control condition. In other words, differences between schools may contribute to differences between the study conditions. With regard to the pre-test scores, a difference was found on the automaticity test. The children in the SI condition showed lower scores on automaticity than the children in the GI condition, $t(1, 48) = 4.617$, $p = .000$. The ability test showed no differences at pre-test.

Due to the differences between the students in the schools for special education and those in the schools for regular education, a 2 x 2 MANOVA was used to analyze the effects of school type (regular versus special) and condition (guided versus structured). The automaticity pre-test scores were included as a covariate in the analyses. A significant main effect of condition was found. Univariate analyses showed a significant effect of condition on the ability post-test, $F(1, 48) = 11.640$, $p = .001$. The GI group showed the highest scores. No differences were found for the automaticity post-test.

No main effect was found for school type. Interaction effects were found for the ability post-test ($F(1,48) = 9.047$, $p = .004$) and the automaticity post-test ($F(1,47) = 43.469$, $p = .000$). With regard to the ability test, the SI group in regular education showed little improvement while the other groups appeared to clearly benefit from the training. With regard

Table 3.5

Means, standarddeviations and effect sizes for transfer

condition	school	pretest	posttest	effect size	follow-up
GI	regular	2.2 (1.9)	5.7 (3.3)*	1.35	5.9 (2.6)
	special	1.5 (2.5)	6.4 (3.3)*	1.69	3.4 (1.3)*
	both	1.9 (2.0)	5.9 (3.3)*	1.51	5.4 (2.6)
SI	regular	0.5 (1.0)	0.9 (1.1)	0.38	3.4 (2.3)*
	special	1.3 (1.6)	4.9 (3.4)*	1.44	2.3 (2.8)
	both	0.9 (1.3)	2.9 (3.2)*	0.89	2.9 (2.5)
control	regular	2.5 (3.4)	3.1 (2.8)	0.19	5.4 (3.6)*
	special	3.3 (4.1)	5.7 (3.9)*	0.60	5.4 (2.5)
	both	2.7 (3.6)	3.8 (3.3)*	0.32	5.4 (3.3)*

* significant improvement compared with previous measurement ($\alpha \leq .05$)

to the automaticity test, an interesting interaction effect was visible: the children in special education benefited most from the SI condition while the children in regular education benefited most from the GI condition.

After a period of three months, the differences between the conditions appeared to fade somewhat but the acquired knowledge was nevertheless still present for all groups. The SI and control groups in regular education even showed improvement. The strategy use of the children was also studied. However, no significant differences were found. The use of strategies was more or less the same in the different conditions. Although the experimental children were better able to talk about and reflect on their strategies, they did not use alternative or additional strategies when compared to the control children.

A final result concerns the transfer of learned tasks. As can be seen from Table 3.5, the post-test results show significant main effects for condition (p = .003) and school type (p = .002). The students in the GI condition outperformed the students in the other conditions although a significant difference was not found at follow-up. With regard to the effects of school type, it appears that the children in special education improved more on the transfer test than the children in regular education. At follow-up, however, the scores of the children in regular education had improved while those of the children in special education had declined.

In sum, all of the groups improved significantly on the ability and transfer tests although the children receiving guided instruction performed higher than the children receiving structured instruction in both the regular and special schools. The SI group in regular education showed the lowest scores. No main effects of school or instruction were found for the automaticity test although an interaction effect was found to suggest that guided instruction may be better for children in regular education while structured instruction may be better for children in special education.

3.4 Conclusions and discussion

In this study, the effects of guided and structured instruction for the teaching of multiplication skills to low math performers were explored. Seventy-five students participated in the study. Because of this relatively small amount of participants, the results should be interpreted with caution. Further research is necessary before the conclusions could be generalized. In this study, the effects of intervention in small groups versus no intervention at all were first tested. Both of the experimental groups and the control group improved significantly on ability and automaticity. The two experimental groups together showed greater improvement than the control group on the ability test, however. No significant differences were found for improvement in automaticity or transfer. In other words, the two interventions appear to be more effective than regular math instruction. However, this may be partly explained by the fact that the experimental students received the instruction in small groups, which is, in general, more effective than class instruction.

The lack of a difference between the experimental and control groups for automaticity can be partly explained by the fact that the students in the control condition received more instruction in automaticity than the students in the experimental conditions. The accent in the intervention was on insight and strategy use while the teachers in special education in particular tend to emphasize automaticity. It is also worth noting that the experimental students received two times a week special instruction, but the other three days they received the same math instruction as their classmates, except for multiplication instruction. It is plausible that the general math instruction also influences the students' learning of multiplication, which would decrease the effects of a special intervention. Therefore the focus of this study is on the differences between the two experimental conditions, in which the influence of general instruction will be more or less the same for both conditions.

The two experimental groups were found to differ significantly from each other on the ability test with the GI group showing greater improvement than the SI group. This difference was particularly apparent in the schools for regular education. The GI group also outperformed the SI group on the transfer test. The ability test requires students to adequately use and apply various problem-solving strategies. Because the students in the GI condition have learned to actively think and talk about these strategies, it is not surprising that they performed particularly well on this test. The students in the SI condition have learned to apply strategies taught by the teacher and therefore apply these strategies less flexibly in the test. The automaticity test shows a different picture. In regular education, the GI group shows greatest improvement; in special education, the SI group shows greatest improvement. It thus appears that significant differences exist between the needs of children in different types of schools. It seems that guided instruction appears to be particularly well-suited to students in regular education and structured instruction to students in special education. However, this study is limited by the fact that the students were not randomly assigned to one of the experimental conditions, although the schools were. In fact, some small non-significant differences were found between the students of both the conditions. It appeared that the students in the structured condition had a slightly lower IQ, that they had received less multiplication instruction before the training, and showed a lower starting level. Although these differences are not significant, it may have influenced the results.

A remarkable difference between the different types of schools was found for the transfer test. The children in special education showed significantly greater improvement than their peers in regular education during the training period. However, at follow-up measurement, a small decline in the transfer scores for the children in special education was found while the children in regular education showed some slight improvement. It thus appears that the students in special education have learned adequately to apply strategies to new tasks; however, the same skills disappear when intensive training is no longer provided. Conversely, the students in regular education appear to acquire the same skills but also generalize these skills to non-trained problems after completion of intensive training. It should be mentioned here that the follow-up test was administered to only 58 out of the 75 students, and therefore may be less representative for the whole group.

In the period after training, regular math instruction continued, which means that the low-achieving students also received the same instruction as their normally-achieving peers. In keeping with this observation, the follow-up tests also show the differences observed at post-test to have diminished. In other words, the impact of instruction appears to be sufficient to undo the differences produced by intervention.

In conclusion, the results show both forms of math intervention to be more effective than regular math instruction for low performing students. The students in regular education were found to benefit most from guided instruction while the students in special education benefited most from structured instruction. As the learning difficulties of the students in regular education were less severe than the difficulties of the students in special education, however, we should examine the differences between these two groups of low math performers in greater detail.

The children with mild learning difficulties can benefit from the same instruction as their peers: that is, realistic mathematics education appears to be effective provided they are taught at their own level. The children in the present study also received instruction in small groups with the same math level. This means that the teachers were able to monitor the children's understanding of the instruction. However, when these same low performers receive instruction in regular classes, the level of instruction may be too high for them, they may not understand what is happening during instruction, and they may not be able to follow the steps in the learning process, and the extent to which teachers are actually aware of the problems may widely vary. In practice, low performers often receive remedial math instruction, which is usually based on structured instruction. However, the results of the present study show guided instruction to be quite effective for the instruction of such low performing students.

In contrast to the above, the students in special education appeared to benefit most from clearly structured, direct instruction, which is in line with the findings of other studies. However, this finding may also be due to the fact that most students in special education receive only very structured, direct instruction. The duration of the training in this study was four months and perhaps too short for this group of students to get accustomed to a different kind of instruction. Future research involving longer interventions is therefore needed to draw more firm conclusions.

The results of this study show that guided instruction can be quite effective for low performing students. It therefore seems legitimate and very interesting to continue research on realistic math education for such students. In future research, greater attention should be paid to the specific characteristics of the students such as seriousness of the learning difficulties, intelligence, motivation, and strategy use. Greater research should provide greater insight into the effects of the different aspects of realistic math education, and the involvement of a larger number of children, teachers, and schools should make the results more powerful.

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Appendix 3.1
Examples of test items

Automaticity test (40 items)

1 x 4 = ___	6 x 5 = ___
2 x 9 = ___	12 x 4 = ___
6 x 3 = ___	7 x 12 = ___
5 x 9 = ___	2 x 36 = ___

Ability test (20 items)

In a box of chocolates are 4 rows of 7 chocolates each.
How many chocolates does the box contain?

8 x 5 = ___

There are 40 apartments in one building.
How many apartments are in 3 buildings together?

6 x 40 = ___

An accordion has 4 rows of 16 buttons each.
How many buttons does an accordion have?

4 x 15 = ___