Chapter 1
Introduction and theoretical background

1.1 Introduction

This thesis is concerned with mathematics instruction for students having difficulties with the learning of mathematics in general and multiplication in particular. The focus lies on the kind of instruction that these students need to adequately master the mathematics skills required by the elementary school curriculum. One of the most important questions in this light is whether students with difficulties learning mathematics actually benefit from the instruction used for their normal-achieving peers or perhaps require another form of instruction specifically adapted to their needs. In this thesis, the results of a research project devoted to this question are reported. More specifically, the question of whether guided instruction (generally viewed as adequate for use with normal-achieving students) is just as effective as the use of directed instruction (generally advised for low achievers) was investigated.

The research project reported on here is part of a national research program (Van Lieshout, 1997) aimed at optimizing the teaching and learning of arithmetic knowledge. More specifically, the study was initiated to investigate the possibilities and to optimize the use of Realistic Mathematics Education (RME), which is a form of instruction that has introduced a number of changes in the mathematics curriculum in The Netherlands, such as a more important role for the students' own contributions (Streefland, 1990; Treffers, 1991). One of the central questions in this research program is whether mathematics instruction based on students' own contributions (i.e., guided instruction), is always better than clearly structured instruction (i.e., directed instruction). This question is particularly important when it comes to the education of students with limited cognitive capacities. Given that such students often have special needs, a form of instruction other than the form of instruction used with normal-performing students may be called for (Carnine, 1997; Mercer, 1997; Miller & Mercer, 1997). That is, findings showing RME to benefit the student population in general cannot be automatically generalized to special populations.

The research reported in this thesis addresses the effectiveness of changing forms of mathematics instruction for use with a special population of students, namely those students who have difficulties learning mathematics. Recent changes have occurred in the mathematics curriculum, and it is therefore important to know what the effects of these changes are on the learning and performance of all students, including such special populations as students with difficulties learning mathematics. The effects of these changes are investigated in an intervention study concerned with the effects of the use of two different instructional principles for the teaching of multiplication skills.

In the present chapter, the relevant theoretical background will first be briefly reviewed. Thereafter, the specific research questions guiding the research project will be presented and the contributions of the various studies described in the different chapters to answering the research questions briefly outlined.

1.2 Theoretical background

In this section, the theoretical background to the present research project will be briefly reviewed. An overview of the most important theories of learning and instruction will first be provided. This overview may provide insight into the selection of instructional forms for examination in the present research project. Thereafter, the population of students participating in the present project will be described, namely students encountering difficulties with the learning of mathematics. Finally, the current practices with regard to mathematics education in the Netherlands will be described in order to understand how students are expected to learn the various aspects of mathematics and basic multiplication in particular and just how these current practices relate to the learning of students encountering difficulties with learning mathematics.

1.2.1 Learning and instruction

One of the first issues calling for review is the question of just how students learn and what the role of instruction is in this process. Several theories of learning have been put forth over the years in the psychological and educational research literature, and any consideration of instruction cannot ignore such theories because assumptions about how stu-
students learn will always influence the manner in which they are taught and, conversely, just how students are taught will always influence the manner in which they learn. For this reason, an overview of the most important learning theories will be presented below.

The different theories of learning distinguish themselves on the basis of the assumptions made about the most important factors in learning. Behavioral theories emphasize the role of the environment and just how events in the environment influence behavior. Cognitivist theories emphasize the role of mental processes in learning and perception. Interactionist perspectives take both the environment and mental processes as important. Interactionists assume that the environment, mental processes, and behavior all interact during the process of learning (Gredler, 2001). The behavioral and cognitive frameworks constitute the major paradigms for studying the phenomenon of human learning (Mercer, 1997).

**The behavioral approach**

The behavioral approach is based on the premise that the environment greatly influences behavior (Mercer, 1997). According to behaviorists, moreover, the goal of science is to discover the lawful relations between environmental events and behavior (Bredo, 1997). In this approach, learning is also viewed as the shaping of behavioral repertoires. For learning in the classroom, carefully selected stimuli and reinforcers are therefore called for. And because the teacher cannot provide the individual reinforcement needed for all students, computers and other modern technologies are seen as essential.

In the behaviorist approach, the two most important factors in education are student behavior (learner characteristics) and those stimuli that lead to behavioral change (instructional characteristics). Important learner characteristics are individual differences, readiness for learning, and motivation. Behaviorists recognize the existence of several different stages of learning (Mercer, 1997): acquisition, proficiency, maintenance, generalization, and adaptation. Given the behaviorist’s emphasis on the environment as a critical factor for learning, considerable emphasis is also placed on the teacher’s arrangement of the classroom for optimal learning. And a failure to arrange the classroom in an optimal manner is considered a major cause of learning difficulties. In other words, both learner characteristics and teacher characteristics are considered responsible for the learning process within the behavioral approach.

One of the essential components of the behavioral approach to learning is direct instruction. The key principle underlying direct instruction is that both the curriculum materials and the teacher presentation of these materials must be very clear and unambiguous (Mercer, 1997). This includes an explicit step-by-step strategy, development of mastery at each step in the learning process, strategy corrections for students errors, gradual fading of teacher-directed activities and increased independent work, use of systematic practice with an adequate range of examples, and cumulative review of newly learned concepts. Task analysis is used to determine the sequence of skills to be taught.

**The cognitive approach**

The cognitive approach involves the study of the human mind, and developed a model for how people receive, process, and recall information (Bredo, 1997). At present, numerous cognitive theories and considerable research are concerned with the process of human problem solving and strategies for the management of learning. The main theory within the cognitive approach is the information-processing theory, which construes learning as the process of obtaining, coding, and remembering information (Gredler, 2001). Although different perspectives on the exact nature of human memory exist, a multilevel conceptualization is currently the predominant perspective. Based on a computer metaphor, it is assumed that information is processed in sequential stages and that these sequential stages are linked to different memory systems: the various sensory registers, short-term memory, working memory, and long-term memory. The according processes that take place are perception, encoding, storage, and retrieval.
Within the cognitive approach, two important concepts are metacognition and problem solving. Metacognition involves thinking about thinking. The main components are knowledge of cognition and regulation of cognition. Knowledge of cognition involves knowledge and awareness of one's own thinking along with knowledge of when and where to use specific strategies. Regulation of cognition involves the planning, evaluation, and monitoring of cognition. Problem solving is a broad term but generally defined as the handling of new and unfamiliar tasks when the relevant solution methods are not known (Bottge, 2001).

The focus of most cognitive approaches to learning is on instruction consistent with how students actually think during specific learning tasks. What happens to the learner internally during the learning process is considered just as important as what happens to the learner externally (Mercer, 1997). The teacher and teaching materials are only considered important insofar as they help students construct new meanings. In keeping with the main information-processing approach, two general instructional paradigms can be distinguished: reductionism and constructivism (although constructivism is also sometimes described as a third approach to learning in addition to the behavioral and cognitive approaches; Bredo, 1997). Reductionism is based on the premise that new concepts must be analyzed and reduced to simpler, smaller, and more understandable components in order to be learned. An example of reductionism is cognitive behavior modification and the self-instructional program. The idea is that cognitions influence behavior and that behavior can thus be changed by modifying cognitions. The constructivist approach to learning constitutes a more purely cognitive approach.

In the past two decades, increasingly greater emphasis has been placed on the active role of the learner. Constructivism views knowledge as a human construction, and thus subjective as opposed to part of an objective, external reality (Cobb, 1994). Constructivism also underlies an instructional ideology that assumes students to be naturally active learners who are constantly constructing new and highly personalized knowledge with the establishment of links between their prior knowledge and new information (Mercer, Jordan, & Miller, 1996). A distinction can be made between radical constructivism and social constructivism. Radical constructivism is based on Piaget’s theory of cognitive development with the exception that 1) knowledge is viewed as a human construction and not, as in Piagetian theory, as an external reality and 2) the focus of radical constructivism is specifically on the learning of school tasks and not the development of logical thinking in general. Radical constructivism construes learning as an internal process occurring in the mind of the individual, and radical constructivists thus call for problem-based learning as opposed to teacher-directed instruction (Gredler, 2001). In keeping with Vygotsky's discussions of knowledge as constructed by societies or cultures, social constructivists construe the classroom as a community that develops its own knowledge (Gravemeijer, 1997).

Constructivists do not agree on the nature of teacher-student interactions. A distinction can be made between endogenous and exogenous constructivism with a continuum of positions occurring in between. The endogenous constructivists think that instruction should be structured to help students discover new knowledge without explicit instruction (Mercer, 1997). Exogenous constructivists think that teachers should engage students by providing explicit instruction via the provision of descriptions, explanations, modeling, and guided practice with feedback. An expanded interpretation of constructivism to include both explicit and implicit instruction is receiving growing acceptance, particularly for the instruction of students with learning difficulties (Mercer et al., 1996).

In this section on learning and instruction, two main approaches were reviewed: behaviorism and cognitivism. The current mathematics curricula appear to be based on largely cognitivist views. Information processing theory provides a good framework for the understanding of learning. And the majority of the recent mathematics instruction programs and textbooks are based on constructivist principles. However, the application of distinct theories within the educational setting is not always clear. Many psychologists and educators have merged elements from the different theories to create new educational programs. For example, cognitive-based instruction may be supplemented with direct-instruction elements. Or a teacher utilizing a basically constructivist curriculum may consider the use of a reductionist self-instruction approach to be particularly effective for the acquisition of a specific part of the curriculum with a particular group of students. In addition, it is increasingly being recognized that no single instructional approach is suited for the education of all students. Given that learning clearly depends on a number of different factors including the individual learner and his or her needs but also such factors as the time and place of learning, instruction should always be adapted.
to the particular learner and particular situation. Different approaches could thus be adopted and different elements combined to meet the particular needs of the students involved in a particular learning situation.

1.2.2 Students with difficulties learning mathematics

Given that the present thesis is concerned with students having difficulties with the learning of mathematics, the population will be described more thoroughly at this point. Before the presentation of some general characteristics of the population, a few remarks will first be made about the definition of the group. Students with serious learning problems are often referred to as students with learning disabilities or difficulties (ld) or students with specific learning disabilities (sld) in the case of students with specific spelling, reading, and/or mathematics problems. However, considerable controversy surrounds the definition and diagnosis of ld. One of the controversies concerns the role of intelligence. Disagreement exists about the students who perform very low in reading or mathematics and also have low IQ scores. When ld is defined as a discrepancy between a child’s cognitive abilities and his actual performance, such students are not diagnosed as ld but as mild mentally retarded (mmr). Some have therefore argued that it is simply more practical to define ld as performing below a particular level for a particular subject (i.e., students who score below the 25th percentile on a criterion-based math test are diagnosed as having a math learning disability; Siegel, 1999). Exclusion criteria such as serious attention or behavioral disorders should still be included, however (Kaufman & Kaufman, 2001).

The focus of the present thesis is on all students encountering difficulties with the learning of mathematics, which thus encompasses students with learning disabilities but also students with mild mental retardation and students performing below the norm without a specific disability. Although the group is quite varied, the students all have low mathematics performance and thus a need for special instruction in common (Kavale & Forness, 1992). The students in this group will therefore be referred to as students with difficulties learning math or low-performing students and defined as students performing below the 25th percentile on a criterion-based math test. The terms learning disability and mild mental retardation will only be used to refer to those students explicitly diagnosed as such by professionals using the official criteria.

General characteristics

The group of students with difficulties learning math is very heterogeneous. Although the students may have difficulties with only mathematics, many of them have other difficulties as well. It is also possible that difficulties with reading, language, and writing may negatively influence the students’ math performance (Mercer, 1997). Despite this diversity, it is nevertheless possible to present some general characteristics of students who have difficulties learning math.

First, students who have difficulties learning math often show memory deficits (Rivera, 1997) and particularly problems with the storage of information in long-term memory and the retrieval of such information (Geary, Brown, & Samaranayake, 1991). These same students show greater difficulties than their peers with the automatized mastery of such basic facts as addition up to 20 or the multiplication tables. As a result, they tend to make more mistakes on tests of basic skills and often have to calculate the answers that others know directly (Pellegrino & Goldman, 1987). Given that clear mastery of such basic facts is needed for further math performance, deficits in this area can certainly influence students’ later math performance and their mastery of the remainder of the math curriculum.

A second characteristic is that students with difficulties learning math often show inadequate use of strategies to compute answers or solve word problems. This can be explained at least in part by the aforementioned memory deficits, which produce slower development of the relevant strategies than in normal achieving students (Rivera, 1997). Moreover, students with such delays may also fail to apply the strategies they have learned in an adequate manner. Inadequate strategy use may also be caused by metacognitive deficits (Goldman, 1989). Metacognitive knowledge and skills are clearly necessary for the identification, selection, and application of appropriate problem-solving strategies.

A third general characteristic of students with difficulties learning math is deficits in other metacognitive regulation processes such as the organization, monitoring, and evaluation of information (Mercer, 1997). As a result of these deficits, the students often produce mistakes showing the incorrect application of solution. Such students also remain unaware of their mistakes because they do not attempt to evaluate the procedures they apply as good problem solvers have been found to. An additional characteristic is that such students show deficits in the generalization and transfer of information and/or problem-solving strategies. They have difficulties with connecting different tasks and with the application of already acquired knowledge and skills to new or different tasks (Goldman, 1989).
In addition to these general (meta-)cognitive characteristics of students with difficulties learning math, they frequently have other problems such as attention deficits or motivational problems. Various aspects of motivation can be distinguished as particularly important for learning. One aspect is the role of attributions or the explanations that students provide for their successes and failures. Students with an internal locus of control tend to explain the outcomes of particular actions on the basis of their own abilities and effort. In contrast, students with an external locus of control tend to think that factors outside their control (such as luck or task difficulty) determine their results (Mercer, 1997). Students with learning difficulties are more likely than normally achieving students to attribute their successes to external factors. Students with difficulties learning math obviously have a history of academic failure, which may also result in a lack of confidence with regard to their intellectual abilities and doubts about anything that might help them perform better. This situation can lead to marked passivity in the domain of math and possibly other domains of learning. A second aspect of motivation that has been found to be important for learning is the type of goal orientation. A distinction can be made between goals focused on mastery of a task (i.e., a task orientation) and goals focused on performance (i.e., an ego orientation; Nicholls, 1984). Research has shown task-oriented students to generally do more their best than ego-oriented students (Ames & Ames, 1989). A third important aspect of motivation for learning is the self-concept of the individual involved. Just how a student perceives his or her abilities can influence his or her learning. Students with a poor self-concept may lack sufficient self-confidence and resist academic work due to a fear of failure (Mercer & Mercer, 1998).

Instructional needs

Students with difficulties learning math require special attention and instruction adapted to their specific needs. Given the heterogeneity of this group of students, their educational needs are likely to be quite diverse. Nevertheless, many educators argue that most of the students with mathematical difficulties (including students with learning disabilities and mild mental retardation) have more or less the same educational needs as their learning patterns do not differ qualitatively from each other (Kavale & Forness, 1992; Van Lieshout, Jaspers, & Landewé, 1994). It is thus recognized here that students may differ in their educational needs but still have a lot in common. In keeping with the general characteristics described above, a number of general educational needs can thus be identified and seen to reflect those areas in which the students encounter the most difficulties: automaticity, strategy use, and metacognitive skills (Rivera, 1997). Nevertheless, when planning the instruction for a student with special needs, one should always take the specific individual needs of the student into consideration as well.

In addition to the possible heterogeneity of instructional needs highlighted above, there is no complete agreement on the type of instruction that students with difficulties learning math may need. One’s vision of learning and instruction basically determines the type of instruction one considers suitable. For instance, a behaviorist will focus on the student’s environment and probably attempt to change the behavior of the teacher or the instructional program when a student does not perform adequately. A cognitivist, in contrast, will concentrate on the information-processing capacities of the student and probably attempt to improve the student's strategy use.

Nevertheless, the general characteristics of the students with difficulties learning math point to the type of instruction needed. One of the major problems confronting these students is attaining automaticity. Special attention should therefore be devoted to the automatization of basic facts. Automaticity can be attained by practicing the skill in question. This means that such students will need extra time and possibilities to practice. In addition, such students must learn how to proceed when they do not know an answer directly or, in other words, to apply backup strategies (Lemaire & Siegler, 1995). For instance, when a student does not automatically know the answer to 6 x 4, it is useful for him or her to know that one simply has to add 4 to his or her knowledge that 5 x 4 = 20 or that one can double 3 x 4 = 12 to attain the answer.

A second major problem confronting students with difficulties learning math is that many of them show deficits in the adequate use of strategies. For adequate strategy use, students must have an adequate repertoire of strategies (strategy acquisition) and also know just how and when to apply the various strategies (strategy application). In general, elementary school students with difficulties learning math rely more heavily on counting strategies than normally achieving students (Pellegrino & Goldman, 1987). An adequate repertoire of math strategies can be built in several ways; it should be noted, however, that students with difficulties learning math do not have an exhaustive repertoire of strategies and that teaching them only a limited (but effective) number of strategies may be sufficient (Jones, Wilson, &
Bhojwani, 1997). The acquisition of many different strategies may only lead to confusion. It is often recommended to make the relevant strategies explicit and clear and that the different steps in the strategy must be made as overt and explicit as possible (Carnine, 1997). Such instruction gives students the opportunity to learn the different strategies on a step-by-step basis. However, it is also possible to use more cognitively-oriented instruction, such as self-instruction procedures. Although positive results have been found with self-instruction procedures, many theorists think that increased teacher assistance and special forms of instructional scaffolding are needed for students with difficulties learning math (Carnine, 1997). In scaffolded instruction, the teacher gradually withdraws as the student becomes more competent and confident. Such scaffolding can be combined with the Concrete-Semiconcrete-Abstract (CSA) sequence, and many researchers and teachers believe that this is an excellent manner of instruction for students with problems understanding math concepts, operations, and applications (Mercer & Mercer, 1998). From a constructivist perspective, a repertoire of strategies can be built via exposure to and practice with different problems; the students are not told how to solve the problems and must therefore discover which strategies to use in discussion with other students. In such a manner, students learn from their own experiences. Such instruction is rarely recommended for students with difficulties learning math, however (Kroesbergen & Van Luit, in press).

In order to select and apply different strategies, students must have considerable metacognitive knowledge and skills. Students with difficulties learning math often need extra help with the selection of the appropriate strategies, monitoring the application of a particular strategy, and evaluation of the effectiveness of the chosen strategy. Given that they typically do not do this on their own, explicit instruction is often recommended to teach students effective strategy use (Jones et al., 1997). According to this view, students with learning difficulties can also be expected to generalize knowledge and skills to situations outside the instructional setting less effectively and less frequently than other students when not explicitly instructed. This observation may be one of the reasons for the increased popularity of the constructivist approach to learning as this approach encourages students to participate in instructional activities that enable them to construct their own knowledge of various concepts, skills, and hierarchies of such. There is still too little research along these lines to state what the effects of such an instructional approach may be for students with difficulties (Kroesbergen & Van Luit, in press).

To conclude, students with difficulties learning math need special attention for the acquisition and automatization of basic math facts and the mastery of various cognitive and metacognitive strategies. Direct instruction is one of the most popular methods of instruction employed with these students (Bottge, 2001). And, indeed, research consistently shows carefully constructed explicit instruction to be very effective for students with math learning difficulties (Carnine, 1997). The majority of constructivists also appear to be shifting towards the explicit-implicit instruction continuum for students with difficulties learning math (Mercer & Mercer, 1998). Of course, adequate learning only takes place when the students are sufficiently motivated to learn. Attitudes, beliefs, and motivation thus play an important role in the learning of mathematics (Mercer, 1997), and fostering motivation and a positive attitude towards mathematics is just as important as the other instructional procedures reviewed above.

1.2.3 Learning mathematics
In this section, how mathematics is currently taught and particularly how multiplication is taught will be briefly reviewed. First, the mathematics curriculum will be described along with the place of multiplication instruction within the curriculum. Second, the current teaching methods will be briefly described. And third, a few remarks will be made with regard to low performers and special mathematics instruction.

The mathematics curriculum
The mathematics curriculum starts with the development of number sense in kindergarten and first grade. At this time, a basic understanding of the arithmetic operations is to be established (Correa, Nunes, & Bryant, 1999). Formal math instruction usually begins with addition and subtraction, and then proceeds to multiplication and division before such more advanced skills as fractions, decimals, and percentages are taught. Mathematics has a logical structure, which means that the mastery of lower-level math skills is essential for learning higher-order skills. The concept of learning readiness is thus important for math instruction as well (Mercer, 1997).
When formal math instruction begins, students must master the different operations and basic axioms in order to acquire basic computational and problem-solving skills. The most important axioms are the commutative property of addition, the commutative property of multiplication, the associative property of addition and multiplication, the distributive property of multiplication over addition, and the inverse operations for addition and multiplication (Mercer, 1997). Two different goals can be distinguished within the mathematics curriculum: the automatization of basic skills and the development of sufficient flexibility and adaptability for adequate problem solving (Goldman, 1989). In the present study, the focus lies on the learning of multiplication skills requiring both automaticity and problem solving. As mentioned above, multiplication instruction can start once students have mastered the basic addition and subtraction skills. Normally, multiplication instruction is initiated in second grade.

The basis for multiplication is understanding that it involves repeated addition. When students understand the basic axioms for multiplication, they can move on to other strategies that can then improve their multiplication skills. The ultimate goal of basic multiplication instruction is for the students to have memorized the multiplication facts up to 10 x 10. The instruction typically begins with the simpler tables, such as the tables for 1, 2, 5, and 10. The tables for 5 and 10 are particularly important because they are often used to solve other problems. Thereafter, students must learn to apply the different multiplication strategies to solve unknown multiplication problems.

According to Vygotsky’s cultural-historical theory, the learning process develops in several stages from material to mental actions (Bredo, 1997). For this reason, it is generally recommended that early math instruction follow three sequential stages: concrete (e.g., sticks, blocks), semi-concrete (e.g., pictures, number lines), and abstract (e.g., numerals, symbols). This sequence can also be followed to help students acquire an understanding of basic multiplication. For instance, a teacher can demonstrate using concrete materials (e.g., blocks or chair legs) that 3 x 4 is the same as 4 + 4 + 4. In keeping with a constructivist approach to learning, concrete materials can help students discover this math relation on their own. In other words, students should be encouraged to explore, discover, and summarize their own representations of problems. In fact, such self-invented models clearly fit within the semi-concrete stage of instruction. When students understand the basic axioms of multiplication, they can continue with the automatization and discovery of different solution strategies.

Strategy instruction can also be provided in several different ways and thus in keeping with the different theories of learning. However, it is always very difficult to know how people decide what to do (i.e., which strategy to use; Siegler, 1988). Siegler has proposed a model, the distributions of associations model, based on the assumption that people make at least some strategy choices without reference to explicit knowledge. Within this model, individual problems and their answers are represented in an associative network. In order to find the answer to a problem, then, three sequential phases can be followed: retrieval, elaboration of the representation, and application of an algorithm. Retrieval appears to be innate to human beings. When students do not know the answer to a problem directly, they must then rely on elaborations of the representations and the application of solution algorithms that appear to be acquired largely via direct instruction (Siegler, 1988). Nevertheless, it is also possible to offer students problems and have them invent their own solution procedures, as suggested by constructivists.

**Current practices**

Most of the schools in the Netherlands today use a teaching method based on the principles of realistic mathematics education. This practice is in line with the changes that have occurred in other countries, for instance with the reforms in mathematics in the United States. Realistic mathematics education is based upon constructivist theories (Cobb, 1994; Klein, Beishuizen, & Treffers, 1998). Learning mathematics is conceptualized as familiarizing oneself with the ideas and methods of the mathematical community (Gravemeijer, 1997). The classroom is a community that develops its own mathematics. The children in the classroom jointly acquire knowledge. Students work and learn together; discuss possible solutions with each other; and must explain and justify their answers to each other. Using realistic situations or context problems, students’ mathematical reasoning is stimulated. Research suggests that even very young students can invent and acquire their own arithmetic routines for the solution of computational problems that, in some cases, can lead to deep understanding (Singer, Kohn, & Resnick, 1997). However, students allowed to proceed entirely on their own could establish misconceptions with regard to certain math concepts and/or algorithms (Bottge, 2001).

Although instruction based on the principles of realistic mathematics education has shown promising results (Cobb et al., 1991; Gravemeijer et al., 1993; Klein et al., 1998), its beneficial value for students with learning difficulties is
highly doubted (Klein et al., 1998; Van Zoelen, Houtveen, & Booij, 1997; Woodward & Baxter, 1997). As described above, children with difficulties learning math appear to need more directed instruction than provided within the framework of realistic mathematics education. Special educators thus tend to employ instructional methods based on cognitive behavior modification principles or direct instruction principles. When students appear to need special instruction, moreover, they are often entered into a special program specifically designed to remedy their problems and instruct those who do not appear to learn sufficiently from the regular teaching program. However, the choice for a special program or instructional method is often very difficult and always depends on the student, the task, and the teacher providing the special instruction. In the next section, such special programs will be considered in greater detail.

Low performers and special instruction

Given the special instructional needs of students with difficulties learning mathematics (see section 1.2.2 above), special programs have indeed been developed to help these students where the regular instruction fails. A first step in the remediation of mathematics problems is diagnosing the problem and mapping the specific needs of the student in question. Special instruction can take a number of forms from a few minutes of extra instruction each lesson to a special program specifically designed to replace regular instruction. The form selected for use depends on the needs of the student, the task to be learned, and the resources of the teacher.

Although a variety of recommendations with regard to the instruction of low performers exist, most of them have in common that the provision of clear structure is recommended (e.g., Archer et al., 1989; Carnine, 1997; Mercer & Mercer, 1998; Ruijssenaars, 1992). Both the form and content of the lessons should be clearly structured. With regard to the form of the lessons, it is recommended that the lessons always be built up using the same pattern including an opening phase with reflection on the previous lesson, a brief presentation of the material to be learned, a practice phase with both guided and individual practice, and a closing phase (e.g., Archer et al., 1989; Veenman, 1993). The instructional principles recommended for use with low performers include the modeling of explicit strategies, cumulative introduction of information, isolation of independent pieces of information, separation of confusing elements and terminology, use of a concrete/semi-concrete/abstract sequence, use of explicit-implicit math instruction, emphasis on relations, explicit generalization instruction, building retention, and instructional completeness (e.g., Carnine, 1989; Mercer & Mercer, 1998; Ruijssenaars, 1992). The main difference between regular instruction and special instruction is that nothing is left to chance in the latter (Ruijssenaars, 1992).

In the final part of this section, the MAthematics Strategy Training for Educational Remediation program (MASTER; Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999) will be described. This is an example of a remediation program employing many of the aforementioned instructional principles. The program was also used for the experimental intervention undertaken as part of this research project. The MASTER program was specifically designed to encourage strategy utilization for the solution of multiplication and division problems. The program is largely based on information processing theory although elements from other theories of learning have been incorporated where suited. Behavioral elements underlie the conduct of a task analysis and the use of direct instruction, for example. Cognitive elements underlie the information-processing characterizations of the students’ learning difficulties, the use of cognitive behavior modification and self-instruction principles to deal with the problems, and the following of a concrete/semi-concrete/abstract sequence of instruction.

The goals of the MASTER program are to increase student’s:

1. understanding of multiplication as repeated addition;
2. understanding of the number system and the premises of such strategies as reversibility, association, and doubling;
3. memorization of the basic multiplication facts up to 100;
4. understanding of division as repeated subtraction;
5. understanding of the relation between division and multiplication;
6. memorization of the basic division facts up to 100;
7. application of multiplication and division facts in both real and imaginable situations.

The program contains 23 lessons in multiplication and 19 lessons in division and is divided into 6 series of lessons. Each series teaches the steps needed to solve the problem or problems related to specific tasks. A series of lessons starts
with an orientation phase in which the child is taught to solve the task with the help of materials. In the next phase, the connection is made to a mental solution in keeping with the concrete/semi-concrete/abstract sequence of instruction. Thereafter, the phases of control, shortening, automatization, and generalization are undertaken. The teacher tries to realize the various phases of instruction with the use of self-instruction. For students who need greater assistance, explicit suggestions or repetition of a particular problem-solving strategy by the teacher may be called for. For example, explicit modeling is included in the program. The intensity of the instruction is clearly adapted to the needs of the student and/or the particular task: a brief explanation of the task to be solved is less intense than explicit modeling of the solution of the task, for example. A small repertoire of multiplication and division strategies is explicitly taught to the students as part of the MASTER program. However, the students are also given the opportunity to develop their own strategies and solutions to various problems. The MASTER program has been found to be very effective for the instruction of both students with learning disabilities and students with mild mental retardation (Van Luit & Naglieri, 1999).

1.3 Research questions

The focus of the present research project is on mathematics instruction for low performing students. The central question concerns the type of instruction required by such students. A distinction can be made between instruction that encourages the students to contribute to their own learning (guided instruction) and instruction that is largely directed by the teacher and thus leaves little or no space for students to contribute to their own learning (directed instruction). This distinction reflects the different instructional theories currently being promoted for mathematics education (constructivist-based instruction) and instruction promoted by special educators (behavioral and/or cognitive instruction; see sections 1.2.1 and 1.2.3 above). In the present research, it is attempted to determine which method of instruction appears to be most suited for use with low-performing students. This was done by designing two different types of intervention for the teaching of multiplication to low-performing students. The two interventions involved adaptation of the multiplication part of the MASTER remediation program described in section 1.2.3 above to the different instructional principles. The effectiveness of the interventions was measured using both multiplication and motivation tests. For multiplication, the areas of automaticity, multiplication ability, and strategy use were investigated as these areas have been found to be clear problem areas for low performers (see section 1.2.2 above). The following motivational aspects were measured: goal orientation, beliefs, attributions, and self-concept. The effects of the interventions were also compared to the effects of regular instruction. In addition, the potential effects of such child characteristics as sex, age, ethnicity, and cognitive abilities, were also studied. Specific attention was devoted to the relation between mathematics knowledge and cognitive abilities.

The following research questions were then addressed.
1. Which instructional principle is more effective: guided or directed instruction?
2. How do students acquire strategies during both interventions (guided and directed)?
3. Does special intervention aimed at strategy use appear to be more effective than regular instruction?
4. What are the effects of different child characteristics (sex, age, ethnicity, IQ, math level, special needs) on the child’s ability to learn multiplication?
5. What is the relation between mathematics knowledge and cognitive abilities?

The above five questions will be answered in the different chapters of this thesis.

1.4 Outline of this thesis

This thesis reports on the effectiveness of two different mathematics interventions for use with low performing students. Before the effectiveness of the interventions is described, the results of a meta-analysis of the current research literature on mathematics interventions are reported in Chapter 2. Articles examining a particular type of mathematics intervention were sought and just which variables appear to make certain interventions particularly effective was then investigated in the meta-analysis.
In Chapter 3, the results of an initial pilot study are presented. In this study, the effects of guided versus directed instruction were examined (questions 1 and 3). The aim of the pilot study was to examine the suitability of the research design, the adequateness of the intervention programs, and the appropriateness of the test materials. On the base of the findings and experiences attained in the pilot study, the testing materials were adapted and expanded for further use. In Chapters 4 and 5, the results of the main study are reported. In Chapter 4, the effects of guided versus directed instruction on automaticity, ability, and motivation in the domain of multiplication are described (question 1). The effects of such special instruction are also compared to the effects of regular instruction (question 3). In this chapter, the effects of different child variables are also considered (question 4). In Chapter 5, the processes that appear to underlie the learning fostered by the different forms of intervention are considered in greater depth and detail. It is shown how the students in both of the experimental conditions acquired new strategies and just what the effects of the different forms of instruction on the students’ strategy use appeared to be (question 2). In Chapter 6, the issue of how students’ cognitive abilities relate to their math learning problems is examined (question 5). And finally, the results of the present research project are integrated and further discussed in Chapter 7.

Given that the present thesis is a compilation of research articles, some overlap between the different chapters is unavoidable. More specifically, the research reported in Chapters 3, 4, and 5 of this thesis involves intervention studies following more or less the same design, which means that the method sections in these chapters and the descriptions of the experimental conditions are highly similar. Another consequence of this form is that the sample sizes in the different chapters could lead to confusion. Although the total sample of the main study consisted of 283 students, the studies described in Chapters 4 and 5 report on 265 participants and Chapter 6 reports on a study with 267 participants. Differences in sample sizes (both of the total sample and of the three conditions separately) are due to missing data. In the analyses for Chapter 4, cases with one or more missing values in one of the four multiplication tests have been deleted. Chapter 5 reports on all students except for those who did not fill in their strategy use. The analyses for Chapter 6 have been conducted with the data of only those students from who the CAS data were completely available.

References


