Reader’s Guide

Theorems, lemmas, propositions, and corollaries are numbered consecutively in the form c.s.n where c is the chapter and s is the section. Displayed equations are numbered consecutively within each chapter in the form (c.n).

References [n] are to books and papers listed beginning on page 101. The end (or absence) of a proof is signalled by the symbol ‘☐’.

Structure of the Thesis

The layout of the thesis is illustrated by the following reading scheme.

1. Introduction
2. Getting Started
3. Invariant Theory
4. Klein forms
5. General Properties
6. Lifting in Rings
7. Galois Cohomology
8. Finite Fields
9. p-adic Integers
10. Hermite Reduction
11. Rational Integers

Chapter 1 is an introduction.

Chapters 2–6 develop the theory of parameterizations in a general setting. This requires Classical Invariant Theory and a knowledge of Klein forms. A characterization of these forms is given that allows us to consider parameterizations as an algebraic set of dimension 3 inside the space of all binary forms of the correct degree.

The thesis then forks. Readers who are only interested in parameterizations for the rational integers, may skip chapters 7–9 and only need to read chapters 10 and 11. Finite Field people and p-adic people, on the other hand, can concentrate on chapters 7–9.

There are four appendices. Appendix A contains the explicit equations that define the space of parameterizations C(r, d). Appendix B contains the
proofs of various propositions that require special arguments when the field has low non-zero characteristic. Appendix C describes the theory of Twisted Conjugacy Classes. Appendix D lists full sets of parameterizations to the equations $X^2 + Y^3 = \pm Z^r$, $r \in \{3, 4, 5\}$, that specialize to all co-prime rational integer solutions.