The Wealth of Generations

Ageing, Pensions and Welfare from a Macroeconomic Perspective

The Wealth of Generations

Ageing, Pensions and Welfare from a Macroeconomic Perspective

Proefschrift

ter verkrijging van de graad van doctor aan de Universiteit van Tilburg, op gezag van de rector magnificus, prof. dr. F.A. van der Duyn Schouten, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op vrijdag 3 oktober 2003 om 14.15 uur

door

Bastiaan Jacobus Antonius Marinus van Groezen

geboren op 19 juli 1975 te Breda

PROMOTORES: Prof. dr. A.C. Meijdam
Prof. dr. H.A.A. Verbon

To Theo

Acknowledgements

In your hands is the result of the research I carried out as a PhD student over the last four and a half years, a period that flew by and, contrary to what I thought when I started, involved much more than only doing research. There were several ups and downs, but I can say now that I would not want to have missed this experience. Many special people who I met made it in some way or another unforgettable years and helped me in writing this thesis.

In the first place I want to thank both of my supervisors, Lex Meijdam and Harrie Verbon, for their ceaseless support, help and friendship. Doing research with them was not only instructive, but also fun. With pleasure I remember the numerous meetings that often began with an hour discussion of daily affairs, after which we wondered how we came to the subject. We also visited several conferences together, where apart from the serious work, there was always room for other enjoyments, like diving in the blue water of the Mediterranean, cycling along the hilly coastline of Cyprus and eating pancakes in Helsinki. Looking back, I realise I was lucky with them as my supervisors.

I am also grateful to Lans Bovenberg, Ben Heijdra, Pierre Pestieau, Sjak Smulders and Brigitte Unger for reading my manuscript and joining the committee.

Furthermore, I had the opportunity to visit various conferences and workshops abroad thanks to the financial support of the CentER for Economic Research, Tilburg University and the TMR project 'Savings, Pensions and Portfolio Choice'.

During the years, some people became more than colleagues. Theo Leers, with whom I shared the office for three and a half years, became a close friend. We wrote some papers together, always had a lot of fun, and traveled to conferences abroad. His constant encouragement and support helped me a lot. His sudden death just after our trip to Austria came as a great shock, which made me put things more in perspective. I will never forget his kindness, good sense of humour and joy of life. Fortunately, I can share

the good memories with many people, and Elvera in particular. Her warmth, care and support mean a lot to me: thank you!

Another person who always stands behind me is Eline van der Heijden. I am grateful for her friendship, support and willingness to be my other paranymph.

The pleasure of being a PhD student in Tilburg would not have been such without the nice colleagues of the Department of Economics and CentER. The coffeebreaks, lunches and 'aio-uitjes' gave the necessary time to relax and update the latest developments on our floor. Thanks to Jeroen, Luuk, Edwin, Daniel, Mewael, Benedikt, Marty, Astrid, Rossella, Matteo, Martin, Gijs, Marco, Henri and Richard for the good times. A special word of thanks to my last roommates. With Michèle Belot I not only shared the same office, but also some common interests. We helped each other in good and bad times; her good mood and sense of humour made it a pleasure to have her around. I am therefore sure we will go on a cruise together once! After she left, I was again lucky to have a nice person joining me. Too bad Sabine Kröger and I only shared the office for such a short while. The last, but certainly not least colleague I want to mention here is Federica "nient' affatto noiosa" Teppa. I am grateful for her friendship and being my personal consultant. While making sure that our lives would not become boring, we did some interesting field-research together, and observed remarkable similarities between spiders and particular detectives.

Apart from being at the university, I also discovered interesting places and nice people elsewhere in Tilburg. I want to thank all of them, especially my friends Geert, Mario, Mark and Martijn, for their good company and the fun we had.

Finally, I owe a lot to my family. Thanks go to my sister Marjolijn for her friendship, and naturally for making me the uncle of my cheerful niece Amy, who always manages to make me smile. But most of all, I want to deeply thank my greatest supporters, 'ons pa en ons ma', who are always there for me and encourage me throughout.

Bas van Groezen
Tilburg, July 2003

Contents

Chapter 1

Introduction

With a new century ahead of us, the ageing of the population forms one of the most important challenges that many societies have to take up. Due to low fertility rates and an ongoing increase of longevity, a vast part of the world will sooner or later be confronted with an unprecedented rise of their relative number of elderly people. Some countries even await an almost threefold increase of their old-age dependency ratio. This 'demographic time bomb' will disrupt the balance between generations and put existing social security schemes under great pressure, implying serious consequences for the welfare of different age cohorts. Not only will ageing cause direct effects at the individual level. Because of its generality and dimension, this demographic change will also have macroeconomic repercussions and thus affect for instance the return that pension funds earn on their investments, which subsequently has its effect on retirement income and consumption possibilities. Many politicians, sociologists and economists are therefore concerned about the sustainability of current arrangements that pay for and look after the elderly, such as pension and health care schemes, and substantial reforms are pressed for. This calls for a careful examination of the effects that ageing and policy reform bring about. For this purpose, the use of abstract models that describe the real world and cover the essence of the problem is indispensable. Yet caution is required when refraining from certain elements. The aim of this thesis is to shed light on some important aspects that so far have been neglected or received little attention in the economic literature on pensions and ageing.

A population that is growing older will experience a shrinking labour force, which makes capital relatively abundant. Depending on the openness of the economy, this

causes factor rewards to change. Since younger workers in general depend heavily on labour income, and elderly to a large extent on capital income, the finances of the different generations change accordingly. Worries about the economic consequences of ageing thus mainly concern the changing distribution of income over different age cohorts. This is usually analysed with models that assume an economy where only one type of commodity is produced that all individuals consume or buy as investment good. Consequently, the consumption basket of all individuals is the same. It is, however, well-known that the needs of elderly people differ from those of their children. Due to increasing dependency, the need for labour-intensive care services increases with age. Conventional models overlook this aspect. This raises the following questions: does the fact that elderly people are in greater need of services change the conclusions about the optimal design of pension schemes? And what are the effects of ageing and pension reform on growth and welfare if one takes this into account?

Another example of how spending patterns differ between generations concerns the upbringing of children. The decision how many children to raise and the resulting expenditures typically occur in the first period of life. Since the fertility rate is the main source of population growth, this microeconomic decision has macroeconomic consequences and influences the factor rewards and social security taxes and benefits. The fertility choices that people make when young thus indirectly affect their retirement income later in life, but individuals will not take these into account. This, together with the fact that fertility dropped dramatically over the last decades, raises the question whether public intervention can improve welfare. More specifically, how should the social security system, including child allowances, be designed in order to beat the ageing problem?

Finally, the international context plays an important role. Ageing is a global phenomenon and developments like the Economic and Monetary Union in Europe make that countries are increasingly linked to each other through more open capital markets. The well-being of citizens of one country may therefore be affected by demographic change and pension schemes in other countries; do people suffer or benefit from extensive unfunded pension schemes abroad, and how does ageing affect this?

The following chapters of this thesis give a theoretical-economic analysis of these questions. This chapter provides an introduction to the rest of the thesis, and is organised as follows. The next section elucidates past and prospective demographic developments. Section 1.2 discusses the various ways in which retirement income is arranged in industrial countries.

Section 1.3 brings the observations of the previous section together to analyse the general-economic consequences of population ageing, with special attention for developments on the capital markets. Next, we go into the discussion of reforming unfunded social security schemes, which is continued in Section 1.5 where several 'missing pieces' of the reform debate are briefly discussed. Section 1.6 subsequently gives an overview and summary of the thesis.

1.1 A grey future looms ahead

Broadly speaking, individuals pass through three stages of life: childhood, the working period, and old-age. Until recently, the relative size of these age groups did not change much. But demographic developments that already started half a century ago in many countries all over the world, and will continue in the coming decades, cause great and unprecedented shifts in the relative number of people in each of these age categories. The three sources of demographic change are fertility, mortality and migration. This thesis will deal with the first two sources only.

The first factor that influences the demographic composition is the rate of fertility, i.e., the average number of children raised in a household. From the late 1940s to the mid 1960s, a baby boom shut this number up to 3 in developed countries. However, with the introduction of contraceptives and improving labour force opportunities for women, fertility started to fall, gradually continued to do so hitherto, and is expected to stay at low levels in the future, as can be seen from Figure 1.1.[1] Especially in more developed countries, the average number of children dropped dramatically, and is even below the replacement level of 2.1. In more than thirty countries (notably Italy, Japan, Eastern Europe and Russia), the population will therefore eventually shrink. This has important implications for the size of the labour force that will not only have to produce consumption goods but also provide services.

The second important determinant of the demographic transition is the increasing average lifetime due to better nutrition, a higher quality and increasing accessibility of

[1] Figures 1.1-1.3 are taken from the United Nations *World Population Prospects: The 2000 Revision*. More developed regions comprise Europe, Northern America, Australia/New Zealand and Japan. Less developed regions comprise all regions of Africa, Asia (except Japan), Latin America and the Carribean. The least developed regions include 33 African and 9 Asian countries.

health care, and improving living conditions. Figure 1.2 shows the development of the life expectancy at birth.

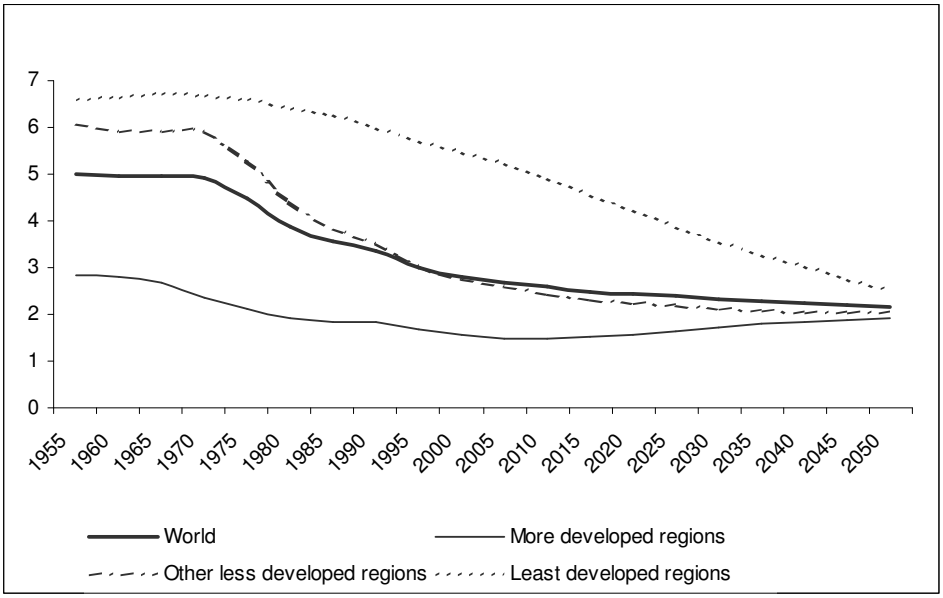

Figure 1.1 *Total fertility rate (per woman)*

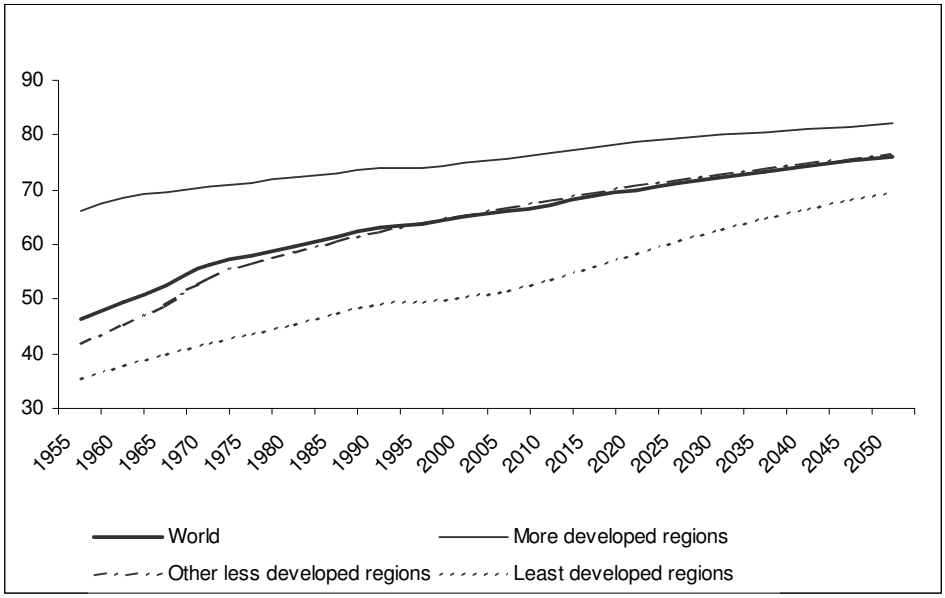

Figure 1.2 *Life expectancy at birth*

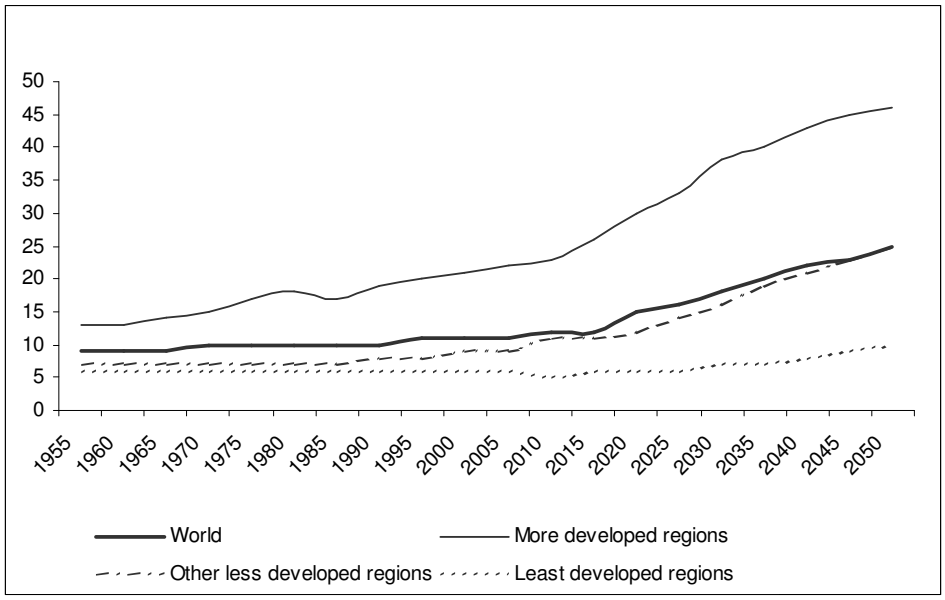

Figure 1.3 *Old-age dependency ratio*

Mortality decline particularly occurs at older ages, which contributes to the ageing of populations. In their *World Population Prospects*, the United Nations observe that currently, almost 75 percent of the newborns in the world will survive to age 60, and 33 percent to age 80. These numbers are expected to increase to 88 and over 50 respectively. So not only will more people survive to old age, they will also live longer.

These developments cause a changing balance between the different generations. Fewer young and more elderly people imply a considerable increase in the so-called old-age dependency ratio, which can be defined as the size of those aged 65 or over in proportion to the number of people aged 15 to 65. Figure 1.3 sketches the past and prospective course of this measure. As can be seen from this figure, ageing will be most pronounced in the more developed regions, where the dependency ratio in 2050 will on average be more than twice the current value. But this ratio will also increase relatively a lot in less developed regions, to almost three times of what it is today. Ageing is therefore a global phenomenon. However, the timing and extent of this demographic transition differs between countries. In the USA, for instance, there is now 1.9 retired person per young individual, which will rise to 3.5 in 2050. For Italy, this number increases from 2.7 now

to 6.8, and in Japan even to 7.2.

Since the retired are mostly consumers and not producers, this implies a growing burden for the younger generations, which have to take care of many more dependent people. The extent to which the retired can maintain a decent old-age depends on their claim on the national income. The next section therefore describes how the financing of retirement income is arranged.

1.2 Retirement arrangements

Because people in general do not participate in the labour market in the last phase of their lives, they are not able to independently acquire income. Instead, they generally depend on several sources of income, which is made up of three pillars. The first pillar is statutory social security and financed on a PAYG-basis. It thus involves a direct transfer from the working young to the retired elderly. The second pillar is composed of supplementary benefits, most of which are funded,[2] and the third pillar comprises voluntary, individual measures of old-age provision, mainly own savings (such as life insurance policies) and fully funded. Figure 1.4 shows the relative size of these pillars for two-person households in several countries. This figure clearly shows that pension systems differ greatly among countries. In Germany, nearly all pensions are financed on a PAYG-basis, whereas e.g. Dutch pensions are to a large extent funded. As for the first pillar, some part of it is provided by the government. Table 1.1 shows public pension spending (i.e., old-age cash benefits, including early-retirement benefits) for several OECD countries in 1997.

Table 1.1 *Public pension spending as % of GDP*

Australia	4.4	Japan	5.5
Canada	4.4	Netherlands	6.6
France	10.7	Spain	8.6
Germany	10.5	United Kingdom	6.8
Italy	13.2	United States	5.6

Source: OECD Social Expenditures Database 2001

[2] In Germany, these pensions are partially funded; most occupational pensions are PAYG and therefore fall under the first pillar.

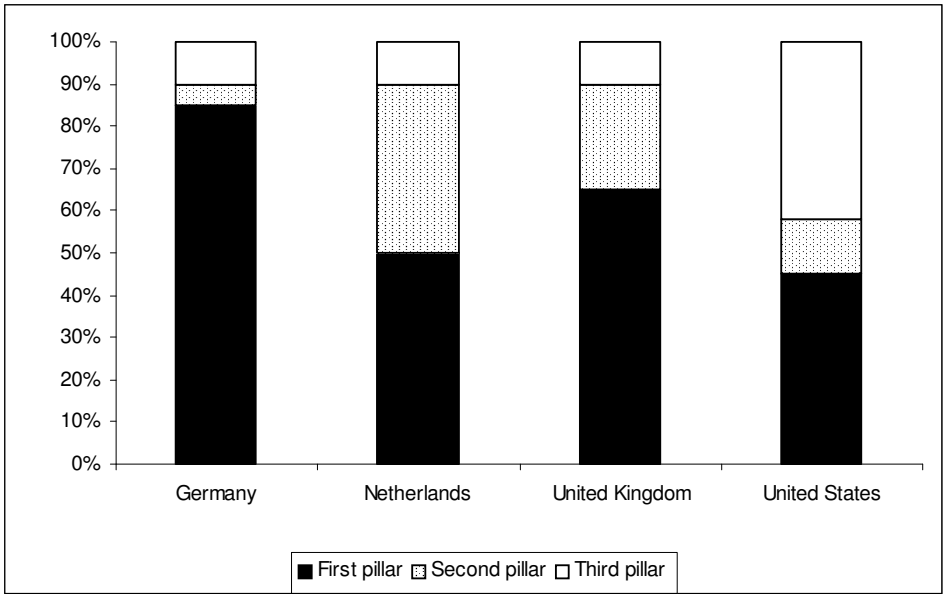

Figure 1.4 *Composition of pension schemes*
Source: Börsch-Supan et al. (1999)

As can be seen from this table, many countries (especially Italy, Germany and France) already have rather extensive public pension schemes. Since these are not funded (or only to a small extent), their sustainability particularly depends on the ratio of recipients relative to contributors, i.e., the old-age dependency ratio. In an ageing society, this would either imply a lower benefit, a rising contribution rate, or a combination of both. The upcoming demographic changes therefore urge many to reconsider the design of existing pension schemes. The call for switching to a more funded scheme is often heard as being *the* solution to beat the ageing problem. The funded component is indeed quite small in several countries. Table 1.2 presents the annual averages from 1986 through 1993 of total pension and insurance fund assets as a fraction of GDP, which can be considered an indicator for the size of funded pension schemes. In accordance with Figure 1.4, this table shows great differences in the size of funded pension schemes. Whereas people in countries like The Netherlands and the United Kingdom have a well established second and third pillar, elderly individuals in notably Italy and Spain lean strongly on the shoulders of their younger compatriots. Yet a funded system is not a safe haven either. Individual investors and pension funds rely heavily on the return they earn from investing in bonds and shares.

If these fall short, as has been the case lately with falling equity markets and low bond yields, promised retirement payments cannot be fulfilled, unless the system promises a certain benefit level. In such a Defined Benefit scheme, contributions will have to increase and the funded scheme has a PAYG-element after all.[3] These bad recent performances of pension funds may be a forerunner of what will happen on financial markets once the ageing process is getting stronger.

Table 1.2 *Assets of private pension and insurance funds as % of GDP*

Australia	35	Japan	43
Canada	48	Netherlands	108
France	20	Spain	8
Germany	33	United Kingdom	92
Italy	6	United States	67

Source: Demirgüç-Kunt and Levine (1996)

All this raises the general question whether we can afford to grow older, what the appropriate pension scheme design is, and to what extent ageing will affect the well-being of successive generations with different institutional settings. This boils down to two basic questions: how production is affected in an ageing economy, and subsequently, how it is distributed between generations. The next section goes into these questions.

1.3 Can we afford to grow older?

Studies on the economic consequences of ageing generally deal with the analysis of two effects: the dependency-ratio effect and the capital-thickening effect. The first relates to the fact that fewer young individuals produce in a society with more elderly, dependent people, whereas the latter refers to changing capital needs.

More consumers, fewer producers...

An increasing old-age dependency ratio implies that the production of relatively fewer active people has to be shared with more inactive people, which is usually referred to

[3] Defined Benefit (DB) schemes are widely observed in many countries; in Germany, Japan and Sweden nearly all private pension programs are DB.

as the *dependency-ratio effect*. The seriousness of this effect depends not only on the demographic change itself, but also on productivity growth and above all on the objectives with respect to the (relative) consumption levels of different generations (see e.g. Jackson, 1998).

If relatively more people demand commodities and fewer people produce them, this would mean that some groups in society lose part of their consumption and suffer, unless labour productivity increases sufficiently. Yet ageing is not merely a problem of the size of the cake, but rather how it is divided. Without government intervention, this boils down to the question who owns the production factors. Broadly speaking, one could say that the young generation forms the labour force, whose wage increases with the rise in productivity, and the retired own the capital stock. Although capital becomes more productive as well, and its reward increases too, this reward has to be shared with relatively more retired people. So if the dependency ratio rises relatively faster than productivity, every retired person can consume less. In order to prevent this from happening, the young will have to sacrifice part of their income. Hence, unless productivity growth is sufficiently high, ageing will lead to more redistribution from young to old.

But even if old-age consumption is thus prevented from decreasing, economic progress will cause a growing gap between the consumption of young and old individuals. If one wants to sustain *relative* consumption levels, ageing will always lead to more redistribution from young to old, no matter how fast productivity grows.

Summarising, ageing is above all a problem of intergenerational redistribution. Productivity growth, if only sufficiently large, can merely help to preserve absolute consumption levels. Relative consumption levels can only stay constant if the young pay higher taxes to the elderly.

Box 1.1 *The dependency-ratio effect of ageing*

Suppose output Y in an economy is produced according to the following production function: $Y = AK^{\alpha}L^{1-\alpha}$, where K is the capital stock, L is the working population, which is assumed equal to the number of young individuals, and A is a parameter reflecting the state of technology. If we denote the productivity of labour as $p \equiv \frac{Y}{L}$, which grows if A increases, and if all output is consumed, we can write the economy's resource constraint as $p = c^y + \mu c^o$, where c^y (c^o) is the consumption per young (old) individual, and μ is the old-age dependency ratio. From this it can easily be seen that if $dp < c^o d\mu$, the consumption of either a young or an old person has to decrease when the population grows older. That is, if due to ageing, the rise in resources demanded per head of the active population exceeds productivity growth, there will be a resources problem. Assuming that all capital is owned by the elderly, the income per inactive is equal to $\frac{\alpha p}{\mu}$ and every young individual earns $(1-\alpha)p$. If one has the objective to keep the living standards of every retired person at a constant level $\bar{c}$, the economy's resource constraint becomes $\alpha p = \bar{c}\mu$. If $\frac{dp}{p} < \frac{d\mu}{\mu}$, i.e., if the proportional growth of productivity cannot keep up with the relative increase of the dependency ratio, the consumption of each young individual will not increase as fast as his wage. Consequently, ageing leads to a redistribution from young to old, unless productivity growth is sufficiently high.[a]
But if the objective is to sustain relative consumption levels, ageing will always lead to more redistribution. For instance, suppose the consumption of a young agent relative to that of an old individual equals β, so that $c^y = \beta c^o$, and every young individual remits a proportion τ of his income to the government that pays each elderly an amount η. Balanced budget implies $\eta = \frac{\tau(1-\alpha)p}{\mu}$. The consumption levels of a young and retired person are then $c^y = (1-\alpha)p(1-\tau)$ and $c^o = \frac{\alpha p+\tau(1-\alpha)p}{\mu}$ respectively, so that $\alpha+\tau(1-\alpha) = \beta\mu(1-\alpha)(1-\tau)$. Totally differentiating this expression (and neglecting second-order effects) gives the effect of ageing on the tax, $\frac{d\tau}{d\mu} = \frac{\beta(1-\tau)}{1+\beta\mu} > 0$: ageing leads to more redistribution from young to old if relative consumption levels are to be maintained, no matter how high productivity growth is.

[a] Bütler and Kirchsteiger (1999) show that under plausible assumptions, current living standards in OECD countries can be maintained with a moderate rate of technological progress.

... but also less capital required

The previous analysis only focused on the fact that output has to be shared with more inactive people, assuming all output is consumed. However, some income is saved for investments. Because ageing is partly caused by slower population growth, this implies a decreasing labour force, so less saving is necessary to endow each worker with a certain capital stock. This so-called *capital-thickening effect* creates higher consumption possibilities and therefore constitutes a counterforce to the dependency-ratio effect. Using a Ramsey growth model, Cutler *et al.* (1990) showed that for the United States, with a moderately ageing population, the positive capital-thickening effect dominates, so that the optimal response to the demographic change would be to decrease (national) savings and thus increase current consumption.

Box 1.2 *The capital-thickening effect of population ageing*

Following Cutler et al. (1990), we can write the economy's budget constraint at time t as $L_t c_t^y + \mu L_t c_t^o = F(K_t, L_t) - K_{t+1}$, where $F(K, L)$ is a constant-returns-to-scale production function, capital depreciates fully after one period and technological progress is assumed away. If the size of the population grows at a constant rate n, such that $L_{t+1} = (1+n)L_t$, and consumption per capita is equal for every young and old individual ($c_t^y = c_t^o = c \; \forall t$), the resource constraint boils down to $c = \frac{f(k)-(1+n)k}{1+\mu}$, where $k \equiv \frac{K}{L}$ is the capital-labour ratio. Linearising this expression around the initial steady state, and setting $dk = 0$ (focusing on the long run only), we arrive at $\frac{dc}{c} = -\frac{1}{1+\mu} d\mu - \frac{k}{c(1+\mu)} dn$. The first effect is the dependency-ratio effect, as we saw before, caused by both increasing longevity and lower fertility: for a given level of the capital stock, per capita consumption decreases. The second effect (capital thickening) is of opposite sign: a lower growth rate of the labour force allows for more consumption for a given level of the capital-labour ratio.

However, in such a model, individuals are assumed to have an infinite horizon, so that the effects of ageing on redistribution between generations and savings are absent. As a matter of fact, the model does not distinguish between different generations. This is an important drawback, since in real life, extensive intergenerational redistribution schemes are present. Furthermore, unfunded pension schemes also influence savings. In order to capture this and the interaction between generations, the use of an overlapping generations (OLG) model is necessary.

Overlapping generations and savings

The two-overlapping generations model, first introduced by Samuelson (1958) and later extended by Diamond (1965) who included savings and investments, covers some important aspects: at any time, individuals of different generations are alive and may be trading with each other, and people have finite lifetimes and may therefore not take the consequences of their actions on the welfare of future generations into account. It is thus a useful tool to analyse the effects of a changing demographic environment and social security schemes on the distribution of income between different generations, and will be used in all chapters of this thesis.

The main feature of the model is the so-called life-cycle hypothesis (Modigliani and Brumberg, 1954), according to which people save in the active period of life so as to be able to consume once they are retired. Among other things, the saving rate depends on personal characteristics such as the individual rate of time preference and the intertemporal substitution elasticity (i.e., the extent to which a smooth consumption pattern over the life is desired), as well as on macroeconomic variables like the wage and the interest rate. But also institutional factors, such as the size and design of social security arrangements, play an important role.[4]

Hence, the two general effects of ageing described above will make themselves felt on the capital market, and influence the return to private savings and pension fund investments.

An upcoming age wave crisis on the capital market?

The impact of the looming demographic change on the economy depends to a large extent on the effects it will cause on the capital market. As Siegel put it,

[4]The OLG-model can also be used to analyse the effects of population ageing in the presence of a PAYG-scheme in a closed economy, as in Meijdam and Verbon (1997). There, the size of the PAYG-scheme is determined by a social planner who maximises a social welfare function. Contrary to what Cutler *et al.* found, it appears that in the long run, a lower fertility rate always decreases utility because the dependency-ratio effect dominates the capital-thickening effect. The short-run effects depend on the size of the PAYG-scheme, which has to do with the fact that it negatively influences savings. If it is large, the capital stock will be low, so that the capital-thickening effect is relatively small. The dependency-ratio effect then dominates, and ageing leads to lower utility in the short run. The reverse holds for a relatively small PAYG-scheme.

> "The next millennium reveals factors that are unlike anything we have witnessed for many generations. Population trends are likely to be the dominant force guiding the accumulation and distribution of capital into the next century." (Siegel, 1998, p. 38-39).

According to the life-cycle theory, an ageing population would imply an increase of national savings until about the year 2010, followed by a steady decrease once people start living off their investments. Although this standard life-cycle effect seems to be contradicted by microeconometric evidence that households in retirement do not appear to run down their stocks of wealth (see e.g. Poterba, 1994), one should bear in mind that from a macroeconomic point of view, the income that retired individuals receive from their pension fund is a dissaving by itself. The pension fund therefore conforms exactly to the life-cycle hypothesis, and there may well be substantial swings in savings when population ageing occurs (see e.g. Miles, 1999).

Once the age wave really sets in, babyboomers (and their pension funds) will start to dissave and liquidate their assets. This extensive sale of assets will drive the share prices down and reduce the capital return. Put differently, the high savings of previous periods have resulted in a large capital stock that is getting relatively abundant once the labour force that utilises it starts to shrink, leading to a declining reward of capital. This is the capital-thickening effect in practice, strikingly portrayed by Siegel when he writes:

> "The words "Sell? Sell to whom?" might haunt the baby boomers in the next century. Who are the buyers of the trillions of dollars of boomer assets? The [baby boomer generation] (...) has produced an 'Age Wave' that threatens to drown in financial assets. The consequences could be disastrous not only for the boomers' retirement but also for the economic health of the entire population." (Siegel, 1998, p. 41).

Although Poterba (2001) does not find much evidence of a robust empirical relationship between a changing demographic structure and asset returns for the United States, United Kingdom and Canada over the last seventy years, other studies do find substantial movements in the real price of capital. Abel (2000), for instance, concludes that a baby boom first increases the price of capital, followed by a fall in the next period, even if a bequest motive is included (see Abel, 2001). A similar result is found by Brooks (2002).

But these studies are all based on data that describe the past, whereas the real wave still has to come.

Naturally, the impact of ageing on capital markets also has a strong international dimension. In a closed economy, national savings are entirely invested domestically, and a rising dependency ratio pushes the interest rate down. But with open borders, this would incite investors to seek for more favourable investment opportunities abroad, leading to a capital outflow. Increasing integration and liberalisation of capital markets, e.g. through the introduction of a single currency in the Economic and Monetary Union, will intensify this. Hence, the openness of the economy is a crucial factor. But also the difference in timing and extent of population ageing, as well as the design of pension schemes play an important role. Empirical analyses by Higgins (1998) and Lührmann (2002) have shown that the relative difference in age structures is the most important factor that explains cross-country capital flows. Moreover, current savings and investment decisions appear to depend considerably on future demographic changes. As can be seen from Figures 1.1-1.3, these changes do not take place simultaneously and equally intense: more developed regions already see their dependency ratios rise, whereas less developed regions start ageing later and to a smaller degree. In the least developed countries, this ratio will hardly increase and stay at a rather low level in the upcoming decades. This creates opportunities for industrialised countries to invest their capital in regions with more favourable demographic circumstances. However, capital is not very mobile between those regions due to a variety of risks, capital controls, high transaction costs, asymmetric information across countries and the so-called 'home bias'.[5]

Yet differences in ageing patterns also exist within the group of developed countries. Italy and Spain, for instance, will experience a tremendous rise of its old-age dependency ratio, reaching a level of over 65 percent by the year 2050. Also Japan falls in this category, and has already started to grow old. The United States, Canada and the United Kingdom will be confronted with a much smaller increase, to about 35-40 percent. Chapter 5 of this thesis analyses the international spillover effects of different ageing patterns. These depend crucially on differences in the rate of time preference between countries, and the size of unfunded pension schemes. When a country ages more than the rest of the world,

[5] See e.g. Gordon and Bovenberg (1996). Folkerts-Landau and Ito (1995) show that pension funds and life insurance companies display a stronger home-asset preference than do mutual funds in industrial countries.

this country will eventually count fewer inhabitants. If they save relatively much due to a low rate of time preference (like in Japan), the total supply of capital per worker will decrease, leading to a higher interest rate and lower welfare. Depending on the degree of capital mobility, this spills over to regions that do not experience such a dramatic ageing.

1.4 Pay-As-You-Go social security

These days, all developed countries have a public pension scheme, which basically involves a transfer of resources from young to elderly individuals. As can be seen from Figure 1.4, this pension provision constitutes on average at least half of the retirement income. In some countries, state pensions are pretty generous and absorb already over 15% of GDP. The ageing of the population will increase this number. Table 1.3 shows an OECD-simulation of how these spending[6] and the accompanying contribution rates develop if pension benefits remain at their current levels.

Table 1.3 *Projection of pension contribution rates* and expenditures***

		2000	2030	2050
France	contribution rate	23.2	37.7	41.2
	pension expenditures	12.0	19.4	21.3
Germany	contribution rate	25.0	41.1	41.6
	pension expenditures	11.1	18.4	18.7
Italy	contribution rate	45.5	61.9	68.2
	pension expenditures	17.1	23.3	25.7
Japan	contribution rate	8.7	10.8	12.7
	pension expenditures	6.5	8.9	10.7
UK	contribution rate	6.4	6.9	5.0
	pension expenditures	4.3	4.7	3.4
USA	contribution rate	8.8	15.0	15.9
	pension expenditures	4.3	7.4	7.7

Source: Chand and Jaeger (1996)

* As percentage of the wage bill ** As percentage of GDP

[6] Due to different definitions (and years), these numbers differ somewhat from those reported in Table 1.1.

This table shows the "equilibrium contribution rate" as percentage of the average wage that has to be contributed to just cover the pension benefits if replacement rates remain at the level as they were in 1995. These tax rates will have to rise to dramatically high levels if no action is undertaken.[7] Clearly, the PAYG-scheme has become a burden for society, and reforms are virtually inevitable. Indeed, such reforms are on the political agenda, and already set in motion in countries like Germany, Italy and the United States. George W. Bush, for instance, has made partial privatisation the center piece of his solution to the "social security crisis". But what exactly are the gains of "privatising social security", as a switch to a more funded pension scheme is often referred to? And does it go without any costs?

Social security reform: long-run gains...

The plea for substantially decreasing the PAYG-scheme is usually underpinned by pointing on two beneficial effects. The first has to do with the additional returns that can be obtained. In a PAYG-pension scheme, benefits are directly paid out by current working people to the elderly. So if population grows and wages increase, the tax base grows accordingly. Hence, contributing a certain (constant) fraction of the wage bill to a PAYG-pension scheme yields a return equal to the sum of population and wage growth. Alternatively, the same amount could have been saved and invested in the capital market (either privately or through a pension fund), and thus yield a return that corresponds to the interest rate. Since nowadays, the latter generally exceeds the internal return of an unfunded scheme, a shift to a more funded system would bring about substantial increases in lifetime income.

A second argument that is put forward to support privatisation is its positive effect on savings. If the government runs a public pension scheme that provides every individual with a retirement income, the individual has a smaller incentive to save for his retirement himself. Econometric evidence (see e.g. Feldstein, 1974, and Feldstein, 1996b) confirms this negative relation. Consequently, the accumulation of capital is adversely affected by unfunded pensions, which decreases welfare in a (dynamically efficient) economy. A switch to a more funded scheme therefore stimulates investments and economic growth, leading to a higher welfare in the long run.

[7] The United Kingdom is an exception due to the assumed indexation of all flat-rate pension benefits to CPI inflation and a less pronounced ageing pattern.

Box 1.3 *PAYG-pensions and saving*

The effect of unfunded social security on the economy can be illustrated with a simple 2-overlapping generations model. Suppose individuals live for only two periods, in an economy where the interest rate is equal to r. Each period, a new generation is born. In the first period of life, individuals earn a gross wage w_t at time t, of which a part τ is taxed by the government in order to finance the public pension benefits paid out to the current elderly. If population grows at a constant rate n, and the wage growth is g, the benefit level at time $t+1$ equals $\tau w_t(1+g)(1+n)$. Net income is partly consumed (c^y) or saved (s), which yields a return r. In the second period of life, individuals are retired. They finance their consumption (c^o) with the return on their savings and pension benefits. The intertemporal budget constraint of an individual born at time t is thus given by $c_t^y + \frac{c_{t+1}^o}{1+r} = (1-\tau)w_t + \frac{\tau w_t(1+g)(1+n)}{1+r} = w_t\left[1-\tau\left(\frac{r-g-n}{1+r}\right)\right]$. The right-hand side of this expression gives lifetime income. PAYG-pensions positively affect this as long as $g+n>r$, also known as the *Aaron-condition*, or *dynamic inefficiency* in a closed economy. When public pension schemes were introduced in many developed countries, this was indeed the case.[a] Nowadays, however, the reverse holds. Privatisation implies that the amount paid in the social security scheme (τ) can be invested in the capital market, and the net advantage for the currently young and each future generation would be $\tau\left[(1+r)-(1+g)(1+n)\right]$ in the second period of life.[b] Another argument why unfunded pension scheme should be downsized is its adverse effect on savings. Suppose a person born at time t derives utility from consumption according to the following function, $U = (c_t^y)^{1-\sigma}(c_{t+1}^o)^{\sigma}$. Individual savings can be found by maximising this function subject to their budget constraint, which results in $s_t = w_t\left[\sigma - \tau + (1-\sigma)\tau\left(\frac{r-g-n}{1+r}\right)\right]$, so $\frac{ds_t}{d\tau} < 0$: savings clearly depend negatively on the social security tax for all positive values of r, g and n.

[a] Browning (1975) provides a political-economic explanation for the implementation of a PAYG-pension scheme.

[b] However, the current elderly would lose their benefits. The next section deals with this.

This incites many to promote a substantial reduction, or even a total abolition, of the PAYG-scheme (see e.g. Feldstein, 1996a, Feldstein, 1999, which analyses a system of investment-based individual Retiree Health Accounts, and Kotlikoff *et al.*, 2001). Michael Tanner, director of the Cato Institute (a public policy research foundation), writes:

"America is currently going through one of the slowest periods of economic growth

> since the Great Depression. If privatizing Social Security can increase growth, raise wages and provide more jobs – while ensuring a dignified retirement for future retirees – isn't that a nice bonus?" (taken from http://www.socialsecurity.org).

However, there is no such thing as a free lunch, and this widely accepted wisdom also applies to pension reform.

... but not without pain

The effects of a switch to a more funded scheme are not unambiguously positive. First of all, a reduction of the social security tax implies a lower pension benefit for those who are retired at that moment. If this generation is to be compensated entirely for this loss, the current young can be taxed, and government debt can be introduced so as to shift part of the burden to future generations as well. Verbon (1988) and Breyer (1989) have demonstrated that for a small open economy, characterised by exogenously given factor rewards due to perfect international capital mobility, the current value of the privatisation gains, falling to currently young and future generations, equals the amount needed to compensate the initially elderly. Put differently, no current or future generation can gain from this reform without making at least one generation worse off. An unfunded PAYG-scheme is therefore Pareto-efficient if the interest rate exceeds the rate of economic growth. In a closed economy, factor rewards are endogenously determined, and the government cannot borrow from abroad. Breyer (1989) shows that in that case, the allocation of consumption between the two periods of life is no longer optimal if there is a shift to a more funded pension scheme. This implies that at least one generation will suffer, so then again, the initial PAYG-scheme is Pareto-efficient.[8,9]

Besides the compensation issue, privatising social security also causes general-equilibrium effects, especially if many countries implement such a policy. Because privatisation boosts savings, the capital stock will grow, which is stimulated by a longer lifespan. But in an ageing society, the need for capital decreases as the labour force becomes (relatively)

[8] In this context, Paul Krugman sighs "If converting Social Security to a system of private retirement accounts is such a good idea, why can't advocates of that conversion try, just once, to make their case without insisting that $1 + 1 = 4$?" (*New York Times*, March 5, 2002).

[9] Among others, Geanakoplos *et al.* (1998), Breyer (2001) and Miron and Murphy (2001) also point at this and other drawbacks of downsizing the PAYG-scheme.

smaller. So capital will be even more abundant, which intensifies the capital-thickening effect and pushes the interest rate further down. This has a negative effect on the performance of pension funds, and thereby on their capability of paying a decent retirement income to their participants. To prevent this from happening, individuals will be inclined to increase their savings even more, not taking into account that the downward pressure on the return to their savings is aggravated. Eventually, the economy may well become dynamically inefficient, and a further reduction of the PAYG-scheme decreases welfare, not only for currently living generations, but also for future generations.

A straightforward way of reasoning which only focuses on current values of the interest, fertility and growth rate is therefore only part of the story, and overlooks negative repercussions that are caused by macroeconomic retroactions. The long-run gains are therefore not as obvious as they at first sight seem, especially because apart from the interest rate itself, also the other determinants of the Aaron-condition are not exogenous. Besides that, one should also bear in mind that the spending patterns of elderly people differ from those of the young, and the international context also plays an important role. The next section will go into these matters that are often disregarded in the debate about pension reform.

1.5 Missing pieces

International context

Naturally, the capital market effects of a switch to a more funded scheme depend in the first place on the openness of the economy. If the economy is small relative to the rest of the world, and if capital moves freely across borders, the interest rate is exogenously determined, and a complete abolition of the PAYG-scheme brings - *ceteris paribus* - the highest long-run gains. If, on the other hand, capital is not mobile, or if many countries carry through the same social security reform, the model of a closed economy is more adequate, and the scenario described in the previous section applies. In reality, economies are open and linked through international capital flows, but capital is not perfectly mobile and some countries are of such a size that they do affect the world interest rate. Factors that influence the supply and demand for capital in one country therefore act upon the direction and size of capital flows, and thus affect other economies.

Unfunded social security, for instance, depresses savings and pushes the interest rate up. This attracts capital from abroad, notably countries that save a lot for their old-age, like the United Kingdom and the Netherlands. Chapter 5 analyses whether these countries benefit from the fact that other countries, such as Germany and Italy, operate large PAYG-schemes. This turns out to depend on the relative difference in unfunded pension schemes. Countries with sizeable funded schemes will invest a lot of their savings abroad, which implies an outflow of capital that would have been invested domestically. If the domestic capital stock is already large compared to the labour force, additional capital will not be very productive, so these foregone domestic investments are not very detrimental to the economy. Instead, a high rate of return can be obtained by investing in countries where, due to a sizeable unfunded pension scheme, the capital stock is rather small. This positive effect outweighs the negative effect from a lower domestic capital stock if the differences between the size of PAYG-schemes is great, and welfare thus improves. However, if more countries start to privatise their pension systems, this will no longer be the case. But do countries have an incentive to reform in such an international environment? For both a small open economy and a closed economy, the answer is affirmative if one only focuses on long-run welfare gains and eludes the transition problems described earlier. But for a large open economy, even the long-run gains are not unambiguously positive. The reason for this is that the benefits of higher savings and investments partially spill over to other countries. Again, this depends on the relative size of the PAYG-scheme compared to that abroad. This implies that some countries will be even more reluctant to switch to a more funded pension system.

Economic structure: demand for services

So far, the focus has been on the supply side of the economy, i.e., how ageing and social security affect the availability of production factors and the distribution of income. Since only one commodity is considered, that serves as both consumption and investment (storage) good, demand effects of ageing are ignored. It is, however, well known that e.g. due to increasing dependency, elderly people have a stronger preference and need for services like housekeepers and nurses, than the young. Data from consumer expenditure surveys in the Netherlands[10] and United Kingdom[11] confirm this: 6-9 percent of the expenditures

[10] CBS Budgetonderzoek 1994.

[11] Office for National Statistics, Family Spending 2000-01.

of households where the average age (or age of head of household) is over 65 consist of very labour-intensive services; for those over 75 it is even 8-10 percent, whereas for younger households, this number equals 4-6 percent. Apart from private expenditures, a lot of spending on services is collectively financed, which consists to a large extent of labour-intensive medical care. Table 1.4 clearly shows that these expenditures substantially rise with age.

Table 1.4 *Per capita total health expenditures by age group (0-64=100)*

	65+	75+		65+	75+
France (1991)	296	373	Netherlands (1994)	442	n.a.
Germany (1994)	268	317	United Kingdom (1993)	388	559
Japan (1993)	479	573	United States (1987)	417	522

Source: OECD (1996)

To capture this difference between needs and desires of elderly and young individuals, it is necessary to consider more than one item on which households spend their income. This has important implications, because services fundamentally differ from commodities in two ways: they cannot be stored and therefore have to be provided at the moment they are demanded, and they can hardly be imported from abroad, so their provision heavily depends upon instantaneous domestic labour supply.[12] This adds another variable in the analysis: the price of services in terms of commodities. Because services are highly labour intensive, this price will strongly correlate with the current wage (in terms of commodities).

In an ageing society, labour will become scarcer because its supply decreases due to lower fertility rates, which has an upward effect on the wage. Increasing longevity will reinforce this effect through a higher demand for labour. This causes an increasing dichotomy between the different generations: the young earning higher wages and the old facing a lower interest rate and high inflation. If individuals do not easily accept such a disparity over their course of life, they will try to smooth by increasing savings. This will also happen if PAYG-pensions are replaced by a funded system. However,

[12] In a way, we can draw a parallel between services and the consumption good in the primal overlapping generations model of Samuelson (1958). In that model, individuals are not able to save as the only commodity ('chocolate') are perishable and therefore cannot be stored.

the macroeconomic consequences can make things worse: increased savings result in a growing capital stock, which is only useful in the commodity sector. Employees in that sector will thus be more productive and earn more. But the services sector cannot lag behind, so wages have to increase there too. As a result, services become more expensive, which, together with a lower interest rate, excavates the purchasing power of the retired elderly. Increased savings therefore barely translate into a better old-age, and can even imply a deterioration. Downsizing the unfunded part of social security will intensify this process. Conventional analyses on ageing and social security reform do not take this aspect of different generational demand patterns and their general-equilibrium effects into account.[13] Chapter 2 of this thesis aims at filling this gap, and shows that a PAYG-scheme of a certain size may be optimal from a long-run perspective, whereas a standard one-sector model would not conclude so.

Economic growth

Apart from the effects on the wage and interest rate, ageing and unfunded social security can also affect the economic growth rate. Various studies have investigated different theoretical channels through which this influence manifests itself, leading to mixed conclusions. The first question is where growth stems from. The economic literature basically distinguishes between two sources: the accumulation of physical capital and human capital. Focusing on the first aspect, and assuming a positive externality from capital accumulation on aggregate production, Saint-Paul (1992) and Wiedmer (1996) found that unfunded social security depresses growth since it negatively influences private savings. So reducing the PAYG-scheme stimulates growth. Furthermore, ageing would encourage savings for old-age and thereby also stimulate economic growth (as in Pecchenino and Pollard, 1997, and Futugami and Nakajima, 2001).

This rather straightforward effect does not necessarily result if one considers human capital accumulation as the engine of growth. Kemnitz (2000), Kemnitz and Wigger

[13] A recent study of the Economic Policy Committee underwrites this when analysing the impact of ageing populations on long-term care, by stating that "... increased public expenditure on health and long-term care would also translate into an increase in the size of a relatively large sector of the economy, thus affecting the sectoral structure of the economy and overall economic development. An investigation of the implications for the economy of a larger health and long-term care sector, including possible macroeconomic feedback effects, was beyond the scope of the current exercize – macroeconomic developments were assumed to be exogenous." (Economic Policy Committee, 2001, p. 33).

(2000) and Sánchez-Losada (2000), for instance, find a (possible) positive relation between PAYG-public pensions and growth, as it creates incentives for individuals to invest in education because this leads to a higher future production and pension benefits. Implementing a reform policy that decreases the size of the unfunded pension scheme may therefore reduce the rate of economic growth, and turn out to be detrimental for the welfare of especially future generations. Results are also ambiguous with respect to the effects of population ageing (see e.g. Ehrlich and Lui, 1991, and Zhang *et al.*, 2001).

In general, the prevailing view in the endogenous growth theory is that economic growth stems from productivity increases due to technological progress. This is either the result of unintended externalities that are caused by a learning process ('learning by doing'), or the consequence of intended investments in education, research and development, leading to innovations and more efficient production techniques. In both cases, the increase in labour productivity takes place in sectors where workers utilise capital to produce commodities and take advantage of economies of scale. However, as was stated earlier, national income is not solely spent on such commodities, since people also have a preference for services, in particular the elderly. This service sector is hardly characterised by technological progress, because labour is the main production factor. Nonetheless, wages in this sector will have to keep up with those in the productive sectors (especially in the long run) in order to prevent labour to move out of the service sector. Hence, the price of services will rise as labour productivity improves in other sectors of the economy, i.e., society suffers from the well-known Baumol disease (Baumol, 1967).

Ageing and pension reform can worsen this illness, by its positive effect on national savings. Chapters 3 and 4 deal with this and again emphasize the importance of the openness of the economy in this context. If the extra capital is invested in the domestic commodity sector, it may attract labour from the services sector.[14] In that case, learning spillovers increase, and productivity growth is stimulated. But if the additional savings are invested abroad, this will not happen. Instead, the economy initially becomes richer and individuals can spend more. This will increase the demand for both services and commodities. Since services cannot be imported, labour will have to be released from the commodity sector, which can be done by importing more (or exporting fewer) commodi-

[14] In an ageing society, the demand for services increases. If this effect is particularly strong, labour moves from the commodity to the services sector, despite the higher capital stock in the commodity sector.

ties. The latter sector therefore shrinks, so eventually productivity growth decreases. The long-run welfare gains of privatising social security are thus negative instead of positive.

Endogenous fertility

The discussion above shows that it is important to take account of the different items on which elderly individuals spend their income. This also holds for young individuals. One of the most important decisions of young households is how many children to raise. On the one hand, children give considerable joy and raising them increases parents' happiness. But the upbringing also entails substantial expenditures on child care, food, housing, clothes, education etcetera. Since the number of offspring is generally a free choice in modern societies, economics can be applied to analyse the development of fertility. So the expenditures of young households consist not only of consumption goods, but also of children. The rate of population growth, one of the determinants of the Aaron condition, is therefore an endogenous variable too.

Chapter 6 examines the effects of ageing and social security when fertility is indeed endogenous. A longer lifespan induces people to save more, so the expenditures when young decrease, including those on children. If parents do not want to reduce the amount spent per child, this inevitably means a lower number of children. One form of ageing thus leads to another form. Indeed, longevity has steadily risen over the last decades, and the fertility rate has gone down in many developed countries.

Apart from a longer lifespan, the existence of publicly provided old-age facilities, such as PAYG-pensions and collective health care arrangements, play a part in the decreasing family size. Parents still have to come up with a large part of the costs of raising their children, whereas later in life, these children pay for the benefits of all elderly. Obviously, when a young household decides whether or not to raise an additional child, these positive socioeconomic effects are not taken into consideration. As a result, public old-age arrangements cause a falling birth rate. One solution to turn this development would be to privatise social security. But as was described earlier, this is rather problematic and can even result in adverse side-effects. A more direct and easier policy to fight the ageing problem would be to improve the fertility rate itself, by giving the parents an appropriate financial grant that covers some of the costs of raising their children. The last chapter of this thesis analyses such a child allowance scheme.

1.6 Outline of the thesis

The rest of this thesis consists of five chapters that deal with the issues and questions raised above. This section gives a summary of each chapter.

Chapters 2, 3 and 4 consider the effects of ageing and social security in an economy where the demand for labour-intensive services varies with age. This captures the fact that elderly are more dependent and in need of services. For that purpose, a distinction is made between two sectors in the economy: a commodity sector and a service sector. Commodities serve as both consumption and investment goods that are predominantly demanded by young agents, whereas elderly spend more of their income on services. These services only require labour, whereas the production of commodities also involves the use of capital. Hence, elderly face a different price index than the young, and are more sensitive to wage increases. Because labour can freely move between the sectors, the wage paid out in both sectors of the economy is equal. So the wage in the service sector must keep pace with that of the commodity sector, and therefore also increases with the rise in labour productivity that results from e.g. a higher capital stock or technological progress. The analysis can thus be considered an extension of Baumol (1967). Furthermore, commodities can be traded internationally, while services can only be supplied domestically, so the openness of the economy will also play an important role.

In Chapter 2, we analyse the consequences of a switch from a PAYG-pension system to a more funded scheme in a closed economy. As this increases savings and investments, the capital stock rises. The resulting decrease of the interest rate and increase of the wage causes the financial position of the elderly to deteriorate: the return on their savings is lower and the services they buy are more expensive. If people prefer a smooth consumption pattern over their lifetime, this would incite them to save even more, which intensifies the negative general-equilibrium effects initiated by the social security reform. Usual analyses of such a policy do not include these additional effects as they only consider one consumption commodity, and mainly find moderate effects on the interest rate. But taking account of different age-related preferences can result in another conclusion about the desirability to privatise social security.

We show that a reduction of the unfunded part also increases long-run welfare as long as the interest rate exceeds the rate of population growth, even if it enlarges the difference between felicity when young and old. Hence, the price of services does not appear in the

Aaron-condition. However, entirely privatising the social security system will sooner lead to dynamic inefficiency than in the conventional one-sector model, especially if elderly demand services to a larger extent than the young, and if people prefer a pretty smooth consumption pattern over their life. Calibration with consumption data shows that the critical size of the PAYG-scheme, beyond which long-run welfare actually decreases if the tax rate is decreased, is roughly twice as high as the level that is applicable to an economy with only one sector, and is not considerably lower than the current value.

Chapters 3 and 4 build on the model presented in the previous chapter and include endogenous growth. Since technological progress typically occurs in capital-intensive sectors, it is assumed that only the commodity sector experiences productivity increases. This is the result of learning-by-doing spillovers that depend on the size of the sector, as measured by the number of employees active in that sector, which is influenced by, among other things, demographic factors and production in the other sector. Furthermore, the openness of the economy, reflected by the degree to which capital flows freely between countries, is an important determinant. For that matter, we distinguish between the two polar cases of a small open economy on the one hand, and a closed economy on the other.

Chapter 3 deals with the effects of population ageing on economic growth and the well-being of subsequent generations. It appears that in a small open economy, increasing longevity initially has no effect on economic growth, but in the long run negatively affects it. The reason is that a longer lifespan induces individuals to save more, in order to finance the increasing demand for services. This leads to increasing investments abroad, which translates into more imports of commodities, and a shift of domestic labour from the commodity sector to service provision. This diminishes the positive spillover effects in the capital-intensive sector and retards economic growth.

Things are quite different in a closed economy (which also applies if all countries experience the same demographic development). The crucial factor is the extent to which capital and labour are substitutable in the production of commodities. If they complement each other, a growing capital stock that is the result of a longer lifespan attracts labour from the service sector, despite the growing number of elderly who demand services. Endogenous-growth effects reinforce this, because they cause an additional increase of the wage and thereby stimulate savings even more. The commodity sector thus expands and in the long run, productivity growth is higher, contrary to the case of a small open

economy. The reverse happens if the production factors are substitutes. The rest of the chapter deals with several extensions and modifications, such as a decreasing fertility, pension benefits that are indexed to the price of services, the solution of a social planner, and productivity growth that depends on the size of the capital stock. In all cases, the results do not differ significantly from the base case.

In Chapter 4, the same model is used to investigate the effects of a shift to a more funded pension scheme on growth and welfare. The traditional view is that such a privatisation of social security increases savings and thus the capital stock, which would lead to a higher growth rate if production is characterised by positive externalities. But if productivity growth rather stems from human capital accumulation, the results may well be different. The same conclusion follows from our analysis. Again, the effects in the two-sector model crucially depend on the openness of the economy. In a small open economy, a lower social security tax will raise lifetime income if the interest rate exceeds the rate of economic growth. This will boost the demand for both services and commodities. Since services can only be supplied domestically, more commodities will be imported (or fewer exported) so as to release labour for the provision of extra services. The size of the commodity sector will therefore decrease, resulting in a lower growth. Eventually, this productivity loss will outweigh the initial gain of a higher lifetime income due to the transition to a more funded scheme. So contrary to conventional wisdom, such a policy reduces welfare in the long run, at least for a small open economy. In case of a closed economy, the effects are opposite: a higher capital stock attracts labour from the services sector to the commodity sector and thus increases growth, leading to extra welfare gains compared to the situation where no such growth effects occur. Hence, privatising social security only seems to be beneficial if the economy is closed, or if many small open economies coordinate their policies and simultaneously implement the same reform.

Chapter 5 examines the importance of international capital flows when countries have different pension schemes and ageing patterns. The 2-OLG model is therefore extended by the assumption of two regions between which capital moves freely, but labour is not mobile, as in Buiter (1981). Each region has a PAYG-scheme, but the sizes differ. As a result, also national savings differ, and thereby the autarchic interest rates. Hence, if countries become increasingly integrated, capital will flow from the region with the relatively sizeable funded scheme to the region with an extensive PAYG-scheme. The

latter country will benefit from this, since it has more capital to produce with. The other country only benefits if the higher returns on capital obtained abroad outweigh the loss of a lower domestic capital stock. This is only so if the unfunded scheme is considerably smaller than in the other country. Given the benefits that the country with a sizeable PAYG-scheme has from the capital inflow, the incentive to reduce the unfunded scheme is weakened. As a matter of fact, it turns out that privatising social security reduces long-run welfare in the particular region if initially, the unfunded component is much higher than in the other country. This can (partly) explain the reluctance to reform the pension system. An internationally coordinated policy that aims at a simultaneous switch to more funded schemes overcomes this problem.

The rest of the chapter focuses on the international spillover effects of ageing. This demographic transition has two characteristics: it is a general trend, i.e., all countries face a growing old-age dependency ratio, but some countries age sooner (like Japan) or more intense (like Italy) than others. So ageing is partially a symmetric shock, and partially an asymmetric shock. This is worked out separately in two sections. The effects of the symmetric shock strongly depend on the regional differences in the size of the PAYG-schemes. It turns out that for the country with an extensive PAYG-scheme, the effects of ageing are more favourable (or less bad) with open borders. The reverse holds for the country with a funded scheme, but to a smaller extent. If benefits are kept fixed, nearly all generations are worse off compared to the case in which contributions are constant. Strikingly, this also holds for the elderly at the time ageing first occurs, which has to do with the adverse general-equilibrium effects that are stronger if benefits are kept constant. Next, the effects of an asymmetric demographic shock are analysed. It is found that the long-run impact of population ageing on the capital-labour ratio (and thus welfare) depends on differences in individual thriftiness and on the relative generosity of the regional social security arrangements. Finally, all elements of this chapter are brought together in the last section that provides a numerical case study which distinguishes between three world regions ((Western) Europe, Japan and the United States) that differ in size, ageing pattern, PAYG-pension scheme and time preference. Especially Japan will suffer from ageing; keeping benefits rather than contributions constant mainly shifts the burden of ageing from current to future generations. Furthermore, it appears that current and future generations in the United States benefit from international capital mobility, whereas the reverse holds for Japan.

Chapter 6 analyses the effects of population ageing and social security reform in a model with endogenous fertility. The choice how many children to raise is modeled as an economic decision where individuals consider both the direct utility gains from upbringing and the costs that accompany it. However, they do not take into account that their children broaden the future tax base and raise the social security benefit of all individuals, including themselves when they have grown old. Consequently, if longevity increases, the number of children will decrease as people will save more for their old-age, causing their public pension benefit to decrease. But in a closed economy, this will have an upward effect on the capital-labour ratio, so that wages increase, which creates a counterforce to the initially negative effect on fertility. Hence, one form of ageing (increasing longevity) affects the other form of ageing (changing fertility).

The next part deals with the externalities of children that parents do not bear in mind. Comparing the outcomes of a social planner to those of an individual household, two reasons are found why the social optimum deviates from the choice of households: the dependency-ratio effect and the capital-dilution effect. The government can achieve a welfare improvement by internalising these externalities through a child allowance scheme and, in a closed economy, also by a PAYG-pension scheme. In a small open economy, the optimal child benefit equals the present value of the child's contribution to the public pension scheme, and does not change if the population grows older. In a closed economy, however, the government needs two instruments to achieve the socially optimal outcome. The optimal child allowance turns out to increase if longevity increases. So apart from reforming the public pension scheme, the government should also focus on an appropriate design of the system of child allowances, study grants, child care etcetera, especially in an ageing society.

Chapter 2

General-Equilibrium Effects of Pension Reform: A Missing Piece

2.1 Introduction

Many developed countries are and will increasingly be confronted with population ageing due to lower fertility rates and a longer life span. These upcoming demographic changes will cause the relative number of elderly individuals to increase, and thus urge many to reconsider the design of existing social security arrangements that involve transfers between generations. One of the main concerns is the sustainability of unfunded Pay-As-You-Go (PAYG) pension schemes and health care arrangements that are widely observed. Contributing a constant fraction τ of the wage to the PAYG-scheme when young yields a pension benefit of $\tau(1+n)(1+g)$ when old, where n is the rate of population growth and g denotes the rate at which wages grow. Hence, as Aaron (1966) pointed out, the internal rate of return from contributing to an unfunded social security scheme equals $(1+n)(1+g)$. But investing the same amount on the capital market yields a rate of return equal to the interest rate r. In case of dynamic inefficiency (also known as the Aaron-condition), $r < (1+n)(1+g)$ holds and individuals save too much, in the sense that the capital stock is above the level at which consumption per capita is maximal. This is due to the fact that individuals do not take into account that increasing their own savings implies a lower interest rate for all individuals and thereby reduces the return to

the savings of all.[1] Diamond (1965) showed that in that case, government intervention is desirable: by transferring resources from young to old agents, e.g. through a PAYG-pension scheme, welfare is increased.

However, most economies are characterised by dynamic efficiency, so individuals would be better off if the social security scheme were reduced. Moreover, population ageing deteriorates the internal rate of return of unfunded schemes compared to the return that can be obtained from investing in physical capital. The plea for replacing unfunded pension schemes by funded ones (often referred to as *privatising social security*) is therefore more often heard, because it would raise long-run welfare. Martin Feldstein, for instance, asserted that a complete and fast transition from unfunded to funded schemes by privatising social security was just the missing piece in social security analysis (see Feldstein 1995, 1996a). That piece was soon retreived: in the 2000 US presidential campaign, social security reform was a key issue. However, an important piece still seems to be missing in Feldstein's analysis.[2]

As was shown by e.g. Feldstein (1974), Kotlikoff and Summers (1981) and Modigliani (1988), transfers from the young to the old, such as unfunded social security, discourage private savings, which Feldstein dubbed the asset-substitution effect. A shift to a fully-funded system will thus boost savings and thereby lead to a lower interest rate. The lower interest rate induces people to substitute away from old-age consumption towards consumption when young. Although this is dictated by utility maximisation, it may still be the case that old people have rather meagre levels of consumption, while the young have abundant consumption streams. However, if people wish to have a fairly constant standard of living over their lifetime, and thus have a low elasticity of substitution between young and old-age consumption, they will decide to save even more than due to the asset-substitution effect alone so as to transfer part of the higher income when young to their old age. But these higher savings intensify the general-equilibrium effects, making individual savings less effective as a device to smooth consumption over the lifetime.

This result will be even stronger if consumption patterns differ through the lifetime

[1] Naturally, this does not hold for a small open economy that faces an exogenously given (world) interest rate. This chapter only focuses on a closed economy.

[2] Obviously, one missing piece is the negative effects on the utility of the elderly alive at the time of the transition to a privatised system. Social security reform is therefore not a Pareto-improving policy, as was demonstrated by Verbon (1988) and Breyer (1989). In this paper, we restrict ourselves to the long-run consequences of privatisation.

and inflation rates between the goods consumed by young and old vary. This has been suggested in a recent issue of *The Economist*:

> "One consequence of wide changes in relative prices is that individuals may face widely differing inflation rates, if their spending differs much from the national average. Older people, for instance, are likely to spend more on medical care and domestic services, such as those of gardeners and cleaners, that have risen sharply in price. Younger people are likely to spend more on new products, such as mobile phones or computers, which tend to fall rapidly in price during their early years. (...) In general many older people do indeed face higher inflation rates than the young. (...) Even if general inflation remains low, relative prices can shift significantly over as short a time as a decade, thanks to productivity gains brought by technological improvements, or to changes in demand; or, very often, both. And by historical standards, that has been especially true in recent years." (The Economist, "The Price of Age", Dec. 21st 2000)

So it is plausible to assume that the elderly spend relatively more on labour-intensive services,[3] both because of disability (which raises the demand for e.g. care services and housekeeping assistance) and a lower preference for (high-tech) commodities, since the varieties of this kind of commodities constantly changes due to innovation, and elderly people have more difficulties learning how to apply these new technologies and products. Budget surveys indeed show that elderly individuals spend relatively more on services: British elderly 9-10 percent compared to 6 percent of young households,[4] and Dutch elderly 6-9 percent compared to 4 percent of the young.[5] The price index of the elderly's consumption basket is then particularly sensitive to wage inflation. As noted above, such wage inflation is one of the general-equilibrium effects that accompany a transition to funded pensions. So, if old-age consumption is relatively labour intensive, the gap between young-age and old-age consumption resulting from privatisation will be even larger, and closing this gap through higher savings will be harder still. Consequently, if

[3] This was also noted by Börsch-Supan (2001), who states that "Demographic change will have a huge impact on the structure of consumption: demand will shift towards more services and products for older members of the community."

[4] See *ONS Family Spending* 2000-01.

[5] In this research, data have been used from *Budgetonderzoek 1994* of the Central Bureau of Statistics, which have been made available through the Scientific Statistical Agency (WSA).

individuals are strongly averse to such an unequal distribution of consumption over the lifetime, savings will rise and the interest rate will drop sharply so that the economy may well end up in a situation of dynamic inefficiency.

It seems that this is a missing piece in the debate on social security reform. The question this chapter therefore addresses is whether the condition of dynamic (in)efficiency is still decisive with respect to the desirability of a PAYG-public pension scheme. In other words, does it still hold in a multi-sector economy with different generational demand patterns that privatising social security raises (long-run) welfare as long as the interest rate exceeds the rate of economic growth?

It appears that this question can be answered affirmatively, at least for the case of a small reduction of the unfunded part of social security: partially privatising social security in a dynamically-efficient economy increases individuals' lifetime income, both in terms of commodities and services, and utility. However, it is likely that after having totally privatised social security, the economy will end up in a situation of dynamic inefficiency, something that may not happen in the standard one-sector economy as applied by e.g. Feldstein. In that case, the optimal PAYG-tax is strictly positive and complete privatisation of social security is not the best policy.

The remainder of this chapter is organised as follows. In Section 2.2, we present a two-sector, 2-OLG model of a closed economy with a government that runs a PAYG-public pension scheme. In one production sector, both physical capital and labour are employed to produce commodities that are bought for consumption and investment purposes. The other sector deals with the provision of services that do not use capital goods. These services are especially demanded by old individuals. In Section 2.3, we analyse whether the Aaron-condition still applies in this particular model. In Section 2.4, the optimal social security tax is calculated and compared to the one that follows from the standard one-sector model. Section 2.5 concludes.

2.2 The model

Production of commodities and services

We use a simple 2-OLG model of a closed economy. Following Baumol (1967), we distinguish between two sectors of production: a commodity and a service sector. In the *commodity sector* (labelled Y), homogeneous goods are produced that either serve as

consumption or investment good. The production process involves the employment of both physical capital and labour according to the following Cobb-Douglas production function: $Y_t = K_t^{\alpha}(L_t^Y)^{1-\alpha}$, where K_t stands for the capital stock (which is assumed to depreciate completely in one period) and L_t^Y is the number of people employed in the commodity sector.[6] Production per employee is described by k_t^{α}, with $k_t \equiv \frac{K_t}{L_t^Y}$ denoting the capital-labour ratio. Firms are fully competitive and maximise profits, so the interest and wage rate are given by $r_t = \alpha k_t^{\alpha-1}$ and $w_t = (1-\alpha)k_t^{\alpha}$ respectively.

In the *service sector* (labelled D), labour is the only factor of production.[7] One unit of labour translates into one service,[8] so the total provision of services equals total labour supply in this sector (L_t^D). The price of services in terms of commodities, p_t, is therefore equal to the wage in this sector. As labour is homogeneous and perfectly mobile across sectors, there is only one wage, so $p_t = w_t$.

Households

In each period, a young and an old generation are alive. Population grows at the rate n, such that the number of young individuals at time $t+1$ equals $N_{t+1} = (1+n)N_t$. An individual dies at the onset of old age with probability $1-\varepsilon$ and lives throughout old age with probability ε, during which (s)he is retired. In the first period of life, (s)he inelastically supplies one unit of labour and earns a wage income, which is taxed by the government. The net income is either spent on commodities and services, or saved for old-age. These savings are invested in annuities or through an actuarially fair pension fund. As only a fraction ε of young savers born at time t survives to period $t+1$, the assets of those who deceased fall to surviving contemporaries. The total return on the savings is therefore $\frac{r_{t+1}}{\varepsilon}$.

Lifetime utility of a non-altruistic representative agent born at any time t is represented by the following CIES utility function,

$$U_t = \frac{(1-\mu)(c_t^y)^{1-\theta} + \mu(d_t^y)^{1-\theta}}{1-\theta} + \gamma\varepsilon\frac{(1-\lambda)(c_{t+1}^o)^{1-\theta} + \lambda(d_{t+1}^o)^{1-\theta}}{1-\theta}, \tag{2.1}$$

[6] Allowing for a more general CES production function and another rate of depreciation does not alter the results.

[7] These are both personal services (domestic services such as gardeners, cleaners, housekeepers, butlers, and recreation and cultural services) and social services like nurses and physicians. Note that these are not services like bank services which extensively use high-tech goods like computers.

[8] Another interpretation would be that the quality of services increases with the number of employees.

where c^y (c^o) is the number of commodities consumed when young (old) and d^y (d^o) is the number of services used when young (old).[9] Furthermore, γ is the private discount factor, μ (λ) denotes the preference for services when young (old), and $\theta > 0$, so that the elasticity of substitution is the constant $\frac{1}{\theta}$.

The government runs a PAYG social security scheme financed by a proportional wage tax τ, so the benefit level at time t equals $\frac{\tau(1+n)w_t}{\varepsilon}$. Individual consumption possibilities are thus given by the following intertemporal budget constraint,

$$c_t^y + w_t d_t^y + \frac{\varepsilon c_{t+1}^o}{r_{t+1}} + \frac{\varepsilon w_{t+1} d_{t+1}^o}{r_{t+1}} = (1-\tau)w_t + \frac{\tau(1+n)w_{t+1}}{r_{t+1}}. \tag{2.2}$$

Assuming perfect foresight, maximising (2.1) subject to (2.2) results in the following first-order conditions,

$$\frac{c_t^y}{c_{t+1}^o} = \left(\frac{\gamma(1-\lambda)r_{t+1}}{1-\mu}\right)^{-\frac{1}{\theta}}, \tag{2.3.a}$$

$$\frac{c_t^y}{d_t^y} = \left(\frac{\mu}{(1-\mu)w_t}\right)^{-\frac{1}{\theta}}, \tag{2.3.b}$$

$$\frac{c_t^y}{d_{t+1}} = \left(\frac{\gamma\lambda r_{t+1}}{(1-\mu)w_{t+1}}\right)^{-\frac{1}{\theta}}. \tag{2.3.c}$$

These conditions show that the ratio of young-age commodity consumption over old-age commodity consumption depends on the interest rate r_{t+1}, whereas the ratio of young-age commodity consumption over old-age services is determined by the *real* interest rate $\frac{r_{t+1}}{w_{t+1}}$ (unless they are complements, i.e., $\theta \to \infty$). The individual demand and saving functions are

[9] Note the similarity with Samuelson (1958). In that model, individuals are not able to save at all because the goods are perishable. In our model, people can save, but when old, they do not only derive utility from the goods that they stored when young. Instead, they have to exchange some of these commodities for services by paying a price p to the young.

$$c_t^y = \frac{I_t}{1+\Omega_t+\Gamma_{t+1}}, \tag{2.4}$$

$$d_t^y = \frac{\Omega_t}{w_t}\frac{I_t}{1+\Omega_t+\Gamma_{t+1}}, \tag{2.5}$$

$$c_{t+1}^o = \left(\frac{\gamma(1-\lambda)r_{t+1}}{1-\mu}\right)^{\frac{1}{\theta}}\frac{I_t}{1+\Omega_t+\Gamma_{t+1}}, \tag{2.6}$$

$$d_{t+1}^o = \left(\frac{\gamma\lambda r_{t+1}}{w_{t+1}(1-\mu)}\right)^{\frac{1}{\theta}}\frac{I_t}{1+\Omega_t+\Gamma_{t+1}}, \tag{2.7}$$

$$s_t = \frac{\Gamma_{t+1}(1-\tau)w_t}{1+\Omega_t+\Gamma_{t+1}} - \frac{\tau(1+n)w_{t+1}(1+\Omega_t)}{r_{t+1}(1+\Omega_t+\Gamma_{t+1})}, \tag{2.8}$$

with $\Omega_t \equiv \left(\frac{\mu}{1-\mu}\right)^{\frac{1}{\theta}} w_t^{\frac{\theta-1}{\theta}}$, $\Gamma_{t+1} \equiv \varepsilon\left(\frac{\gamma(1-\lambda)}{1-\mu}\right)^{\frac{1}{\theta}} r_{t+1}^{\frac{1-\theta}{\theta}} + \varepsilon\left(\frac{\gamma\lambda}{1-\mu}\right)^{\frac{1}{\theta}}\left(\frac{r_{t+1}}{w_{t+1}}\right)^{\frac{1-\theta}{\theta}}$, and lifetime income is $I_t \equiv (1-\tau)w_t + \frac{\tau(1+n)w_{t+1}}{r_{t+1}}$.

The model presented above comprises four markets, which are all characterised by fully flexible prices and therefore simultaneously in equilibrium at each point in time.

The commodity market

Demand for commodities comes from young individuals who spend part of their after-tax wage income on commodities, both as consumption and investment, and from retired individuals. Accordingly, the equilibrium condition reads

$$Y_t = N_t w_t(1-\tau-d_t^y) + \varepsilon N_{t-1}c_{t-1}^o,$$

which in steady state[10] boils down to

$$l^Y = (1-\alpha)(1-\tau) + (1-\alpha)\left(\Omega - \frac{\varepsilon}{1+n}\left(\frac{\gamma(1-\lambda)r_{t+1}}{1-\mu}\right)^{\frac{1}{\theta}}\right)\left(\frac{r(1-\tau)+\tau(1+n)}{r(1+\Omega+\Gamma)}\right), \tag{2.9}$$

with $l_t^Y \equiv L_t^Y/N_t$. In the extreme case that elderly people only have utility from services ($\lambda = 1$) and young agents only purchase commodities ($\mu = 0$), the number of people employed in the commodity sector is constant and equal to $(1-\alpha)(1-\tau)$, also outside the steady state.

[10] Omitting the time subscript denotes the steady-state value of the variable.

The services market

As described before, one service requires the input of exactly one employee, so the total provision of services is L_t^D. Equilibrium in the services market at time t is thus given by

$$L_t^D = N_t d_t^y + \varepsilon N_{t-1} d_t^o.$$

The labour market

The total labour force at time t consists of all young individuals, who each inelastically supply one unit of labour in one of the two sectors.[11] Equilibrium in the labour market therefore implies that

$$L_t^Y + L_t^D = N_t. \tag{2.10}$$

The capital market

Since there is no scope for international lending and borrowing, aggregate savings are entirely invested domestically. If capital fully depreciates after one period, the capital market clears when $N_t s_t = K_{t+1}$ holds, or

$$s_t = (1+n) k_{t+1} l_{t+1}^Y. \tag{2.11}$$

By inserting (2.8) and (2.9), together with $w_t = (1-\alpha)k_t^\alpha$ and $r_t = \alpha k_t^{\alpha-1}$ in this expression, one can calculate the steady state capital-labour ratio.

2.3 Partially privatising social security

Privatisation of social security as proposed by Feldstein involves one big jump from an unfunded to a funded scheme. A less extreme approach would be to reduce the PAYG-tax to a permanently lower but still positive level. The option we consider in this section is a marginal decrease, so that if the economy is initially dynamically efficient, it can be assumed to remain so. For analytical convenience, we will focus on the case where $\varepsilon = 1$, $n = 0$, $\mu = 0$ and $\lambda = 1$.[12]

Then, equilibrium in both the labour, commodities and services market gives the individual number of services that each person uses when old,

$$d_t^o = \alpha + \tau - \alpha\tau. \tag{2.12}$$

[11] One could also assume that every person works part-time in both sectors.

[12] If the economy stays dynamically efficient with these parameter values, it will surely be so in more general cases in which $\varepsilon, \lambda < 1$ and $\mu > 0$. Hence, the analysis also applies to these cases.

Although the individual demand for old-age consumption will generally depend on the substitution elasticity, the services that will be supplied to a retired person depend, according to equation (2.12), only on the production elasticity α and the social security tax τ.[13]

We can write the capital accumulation function as

$$\begin{aligned} k_{t+1} &= \frac{s_t}{(1-\tau)(1-\alpha)} \\ &= k_t^{\alpha}\frac{\Gamma_{t+1}}{1+\Gamma_{t+1}} - k_{t+1}\frac{\tau}{\alpha(1-\tau)(1+\Gamma_{t+1})}. \end{aligned} \tag{2.13}$$

Linearising this expression around the initial steady state gives the following difference equation,

$$\frac{\alpha(1-\tau)(1+\theta\Gamma)+\tau}{\alpha(1-\tau)(1+\Gamma)\theta}dk_{t+1} = \frac{\alpha\Gamma k^{\alpha-1}}{1+\Gamma}dk_t - \frac{k}{\alpha(1-\tau)^2(1+\Gamma)}d\tau, \tag{2.14}$$

so the steady state is stable if $\frac{\alpha\Gamma k^{\alpha-1}}{1+\Gamma} - \frac{\alpha(1-\tau)(1+\theta\Gamma)+\tau}{\alpha(1-\tau)(1+\Gamma)\theta} < 0$. This is assumed to be the case.

From this it follows that $\frac{dk}{d\tau} = \frac{k}{\alpha(1-\tau)^2(1+\Gamma)} \Big/ \left(\frac{\alpha\Gamma k^{\alpha-1}}{1+\Gamma} - \frac{\alpha(1-\tau)(1+\theta\Gamma)+\tau}{\alpha(1-\tau)(1+\Gamma)\theta}\right) < 0$, leading to the following proposition.

Proposition 2.1 *In the two-sector model as presented above, a PAYG-system reduces the long-run capital-labour ratio. (Partially) privatising social security therefore increases k, increases the wage and decreases the interest rate.*

With a lower PAYG-tax, people have a stronger incentive to save.[14] This is the reverse asset-substitution effect. These higher national savings imply a higher capital stock. However, less labour is attracted to the services sector, leaving more employees to work with the capital stock. Because the first effect is dominant, the capital-labour ratio increases.[15]

[13] This result is due to the Cobb-Douglas specification of production. For more general specifications, the parameter θ will affect the equilibrium quantity of d_t^o as well, but the positive relation between τ and d_t^o remains.

[14] From the equilibrium condition for the capital market, $s = k(1-\alpha)(1-\tau)$, it follows that $\frac{ds}{d\tau} = (1-\alpha)(1-\tau)\frac{dk}{d\tau} - k(1-\alpha) < 0$.

[15] k would decrease if L^Y increased strongly. But then, the interest rate would increase and the wage would decrease, thus stimulating the demand for services, which contradicts the sharp rise of L^Y.

In a closed economy, the return to the funds invested on the capital market will be lower, and at the same time, employees can work with a higher number of capital goods, so their productivity and thereby their wage rate increases. In order to prevent labour moving out of the services sector, wages will rise in that sector too, making services more expensive. This is the well-known Baumol-effect. An individual is therefore confronted with higher earnings when young, enabling him to purchase more commodities which raises his utility, but when old, he faces both a lower return to his savings and a higher inflation, so the purchasing power when old could decrease. This leads to the following proposition.

Proposition 2.2 *Marginally reducing the unfunded part of pension schemes leads to a higher lifetime income, both in terms of commodities and (current) services, iff the economy is dynamically efficient. The consumption level for young individuals rises whereas the number of services enjoyed when old decreases.*

Proof Lifetime income measured in commodities is equal to $z \equiv (1-\tau)w + \frac{\tau w}{r}$, so $\frac{dz}{d\tau} = w\left(\frac{1-r}{r}\right) + \left((1-\tau)(1-\alpha)r + \tau\frac{1-\alpha}{\alpha}\right)\frac{dk}{d\tau}$, which is negative if $r > 1$. Lifetime income in terms of services is $\frac{z}{w} = 1 + \tau\left(\frac{1-r}{r}\right)$, so $\frac{d(z/w)}{d\tau} = \frac{1-r}{r} + \frac{(1-\alpha)\tau}{\alpha k^{\alpha}}\frac{dk}{d\tau}$, which is also negative if the economy is dynamically efficient. Equilibrium in the capital market can be written as $k = k^{\alpha} - \frac{c^y}{(1-\alpha)(1-\tau)}$, so $\frac{dc^y}{d\tau} = (1-\alpha)\left(k^{\alpha-1}-1\right)\left((1-\tau)\frac{dk}{d\tau} - k\right)$. Inserting $k^{\alpha-1}$ that follows from (2.12) thus gives $\frac{dc^y}{d\tau} = (1-\alpha)\left(\frac{\alpha(1-\tau)+\tau}{\alpha(1-\tau)\Gamma}\right)\left((1-\tau)\frac{dk}{d\tau} - k\right) < 0$. Furthermore, equation (2.11) immediately results in $\frac{d(d^o)}{d\tau} = 1-\alpha > 0$. ■

The change in consumption when young and services when old is due to several effects. The first is the direct positive income effect of a lower social security tax on lifetime income. First-period income increases, second-period income decreases, so that part of the rise in first-period income is saved. Hence, both c^y and d^o increase. Second, the interest rate decreases. This causes a negative income effect on c^y and d^o, a positive substitution effect on c^y and a negative substitution effect on d^o. Third, the wage rises, which implies a higher labour income and thus has a positive income effect on both c^y and d^o, but because it is also the price of services, it implies a positive substitution effect on c^y and a negative substitution effect on d^o. The total effect is a higher consumption level of commodities when young and a lower number of services when old. Table 2.1 summarises these effects.

Table 2.1 *Effects of (marginally) reducing the social security tax*

	$\tau\downarrow$	$r\downarrow$		$w\uparrow$		total
	IE	IE	SE	IE	SE	effect
c^y	+	-	+	+	+	+
d^o	+	-	-	+	-	-

The question then is whether lifetime utility increases as well if the social security system is privatised. The most obvious way to evaluate this is to compare the implicit return to individuals from contributing to the unfunded social security scheme with the explicit return that can be obtained from investing the same amount at the capital market, which in our model corresponds to the interest rate. If the latter exceeds the former, which seems to be the case in most countries, transferring an amount through a PAYG-scheme enables individuals to purchase fewer commodities when old than what they could have bought if the same amount were invested in a funded scheme. Consequently, reducing the unfunded part of social security raises long-run welfare, and the reverse holds in a dynamically-inefficient economy. However, this reasoning does not take account of general-equilibrium effects and the different spending patterns of young and old generations. In particular, contributing to the unfunded social security scheme via a proportional wage tax yields a 'return' that is linked to the wage that the next generation earns, which is also the price of the services an individual buys when old. Hence, a PAYG-scheme yields a fixed return in terms of services. On the other hand, investing at the capital mark through a funded scheme gives a return in commodities, so that the elderly are sensitive to price increases. This raises the question whether the higher consumption level when young compensates for the loss of services when old in terms of lifetime utility. That is, does a reduction of the social security tax raise lifetime utility in the two-sector model? As the following proposition asserts, the answer is affirmative.

Proposition 2.3 *(Marginally) reducing the unfunded part of a pension scheme in a dynamically-efficient two-sector economy implies a higher lifetime utility.*

Proof The effect of a marginal change of the social security tax on lifetime utility can be traced by linearising around the initial steady state. Utility changes according to $\frac{dU}{d\tau} = (c^y)^{-\theta}\,\frac{dc^y}{d\tau} + \gamma d^{-\theta}\frac{d(d^o)}{d\tau} = (1+\Gamma)\frac{dc^y}{d\tau} - \frac{c^y\Gamma}{\theta k}\frac{dk}{d\tau}$. After differentiating (2.4) we arrive at

$$\frac{dU}{d\tau} = \left[\frac{1-\alpha}{\alpha}k(1-r) + \frac{dk}{d\tau}\left(\alpha(1-\alpha)(1-\tau)k^{\alpha-1} + \frac{\tau(1-\alpha)}{\alpha} - \frac{c^y\Gamma}{k}\right)\right]. \qquad (2.15)$$

Clearly, the first term is negative if the economy is dynamically efficient, i.e., if $r > 1$, and positive in the case of dynamic inefficiency. From (2.13) it follows that $k^{\alpha-1} = \frac{1+\Gamma}{\Gamma}\left(1+\frac{\tau}{\alpha(1-\tau)(1+\Gamma)}\right)$, so dynamic efficiency is equivalent to $\alpha(1-\tau)(1+\Gamma)+\tau-(1-\tau)\Gamma > 0$. Inserting this and (2.4) into (2.15) implies that the last term of (2.15) is positive (negative) if the economy is dynamically efficient (inefficient). Knowing that $\frac{dk}{d\tau} < 0$, it follows that $\frac{dU}{d\tau} \lesseqgtr 0$ if $r \gtreqless 1$. ■

So the Aaron-condition also applies in the two-sector model. One can conclude from this that despite the fact that individuals face different prices over their lifetime, the so-called golden rule still holds: lifetime utility is maximal if the interest rate equals the rate of economic growth. If the social security tax is such that $\frac{dU}{d\tau} > 0$, people save too much in the sense that the utility of current and future generations is increased when the PAYG-scheme is (marginally) extended. Elderly then benefit from a higher social security benefit, and the current young and all future generations will have a higher lifetime income. This is so because individuals do not take into account that their own savings have a downward effect on the (future) interest rate and an upward effect on the (future) wage, which also reflects the price of the services they will enjoy when old. By creating a disincentive to save, the government internalises this negative externality and thereby increases welfare. In the opposite case (dynamic efficiency), a marginal increase of the PAYG-tax reduces the utility of currently young individuals and all future generations (although it still increases the utility of the current elderly), because their lifetime income is reduced when the unfunded scheme is extended. Therefore, the proposal to partially decrease the unfunded part of social security and simultaneously increase the funded part would still benefit current young and future generations as long as $r > 1$.

However, this conclusion is based on a small reduction of the PAYG-scheme. Some proposals encompass the entire abolition of the PAYG-scheme, for this would bring about maximum welfare gains. The following section deals with this case.

2.4 Totally privatising: When does it go wrong?

If the social security system is marginally privatised, lifetime income increases in a dynamically-efficient economy, both in terms of commodities and services. But this does not necessarily mean that an individual is better off in both periods of life. In this section, we again focus on the case in which young agents only demand commodities ($\mu = 0$) and

elderly either demand only commodities ($\lambda = 0$, which boils down to the standard one-sector model), or demand only services ($\lambda = 1$). Furthermore, $\varepsilon = 1$ and $n = 0$. Figures 2.1.a and 2.1.b show the felicity when young, $\frac{\left(c_t^y\right)^{1-\theta}}{1-\theta}$, and old, $\frac{\gamma}{1-\theta}\left((1-\lambda)\left(c_{t+1}^o\right)^{1-\theta} + \lambda\left(d_{t+1}^o\right)^{1-\theta}\right)$, as a function of the social security tax for $\lambda = 0$ and $\lambda = 1$.[16]

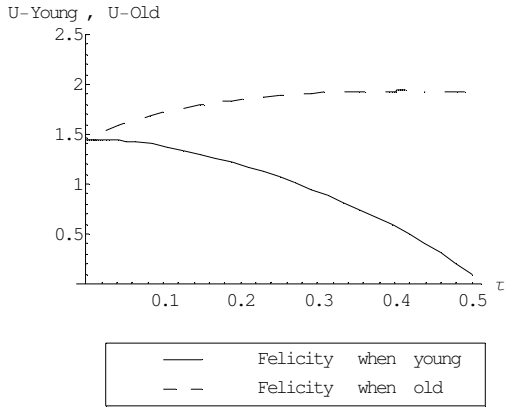

Figure 2.1.a $\mu = \lambda = 0$

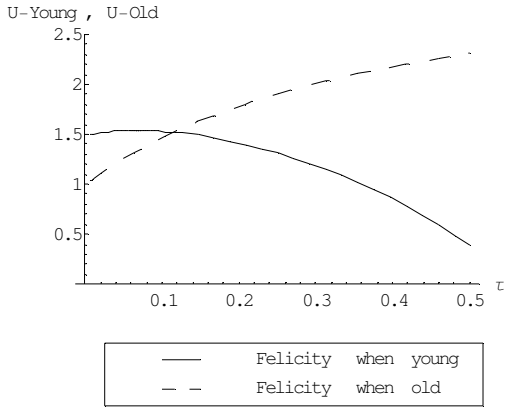

Figure 2.1.b $\mu = 0$, $\lambda = 1$

From these figures, we can see that a reduction of the social security tax lowers the felicity of an old individual, and that the decrease is stronger if λ is higher, i.e., if the elderly consume more services. Furthermore, the felicity of a young person increases when τ is reduced, unless $\lambda = 1$ and the PAYG-tax is already rather low.

If people prefer a smooth consumption pattern over their lifetime (i.e., if θ is high) they will try to carry over a large part of the net-income gain when young to their old-age by saving more. However, general-equilibrium effects will have a negative influence on old-age consumption: higher savings cause falling interest rates and wage inflation which diminishes the retirees' purchasing power. If $\lambda = 1$ and the tax rate is rather low ($\tau < 0.1$), the attempts to close the gap between felicity when young and old may

[16] The underlying production function of this picture is $Y_t = K_t^{0.3}\left(2L_t^Y\right)^{0.7}$. Furthermore, $\gamma = 0.75$. Assuming one period to be 25-30 years, this corresponds to an annual discount rate of 0.01, consistent with the empirical evidence of Hurd (1989).

Empirical studies to date find relatively large values of θ: Mankiw *et al.* (1985) find values of 2.5 and higher, whereas Blundell *et al.* (1994) find $\theta = \frac{4}{3}$, which is the value chosen for this picture, and can be considered a lower bound.

eventually even lead to lower young-age consumption.[17] Obviously, complete privatisation of social security will not be the optimal policy in that case, as the economy will become dynamically inefficient with a positive tax rate.

Of course, such a situation can also occur in a one-sector economy, as was shown by Diamond (1965). What Figures 1.a and 1.b suggest, however, is that in a two-sector economy the different spending patterns of generations create a larger bias towards dynamic inefficiency than in a one-sector economy. *Ceteris paribus*, this will call for a higher long-run optimal tax rate. Below, we demonstrate this assertion with consumption data. Before that, we illustrate the effect of the sector structure more clearly by calculating the optimal long-run tax for different values of λ.

The golden rule is attained when the social security tax is set at such a level (τ^*) that the resulting interest rate equals unity ($r = 1$), so the capital-labour ratio is equal to $\alpha^{\frac{1}{1-\alpha}}$, which determines the value of Γ at $\Gamma^{GR} = \gamma^{\frac{1}{\theta}}(1-\lambda)^{\frac{1}{\theta}} + \gamma^{\frac{1}{\theta}}\lambda^{\frac{1}{\theta}}\left((1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\right)^{\frac{\theta-1}{\theta}}$. Using the equilibrium conditions (2.9) and (2.11), we arrive at

$$\tau^* = \frac{\alpha(1+\Gamma^{GR}) + \alpha\gamma^{\frac{1}{\theta}}(1-\lambda)^{\frac{1}{\theta}} - \Gamma^{GR}}{(\alpha-1)(1+\Gamma^{GR})}.$$

Figure 2.2 shows this optimal social security tax as a function of λ and θ.[18]

The higher the extent to which elderly people use services, the smaller the commodity sector will be, so L^Y is lower. This has an upward effect on the capital-labour ratio, and the interest rate tends to be low. Saving for the future consumption of services then becomes less effective, and the optimal social security tax will be higher as a result. On the other hand, if λ exceeds a half, an increasing value of λ implies that old-age income is to a higher extent spent on one item, viz. services. The marginal utility of saving an extra dollar is then lower, and savings will actually decrease with λ. This translates into a lower capital stock, which by itself means a lower capital-labour ratio. In Figure 2.2, this last numerator effect dominates the denominator effect for $\theta = 2.5$ and values of λ exceeding 0.85, and consequently, the optimal social security tax decreases when λ rises.

For the cases shown in Figure 2.2, the economy is dynamically efficient if $\tau = 0$ only for low values of λ and θ. In all other cases, the economy is dynamically inefficient and

[17] It is evident that market forces do not allow for higher old-age consumption as in that case $d_t^o = \alpha + \tau(1-\alpha)\ \forall t$. So individuals will not be able to increase old-age services following a decrease of the tax rate τ whatever their inclination to smooth consumption.

[18] The underlying parameter values are the same as those of Figure 2.1.

savings have to be discouraged by applying a positive social security tax. Consequently, in nearly all cases complete privatisation of social security would imply long-run welfare losses.

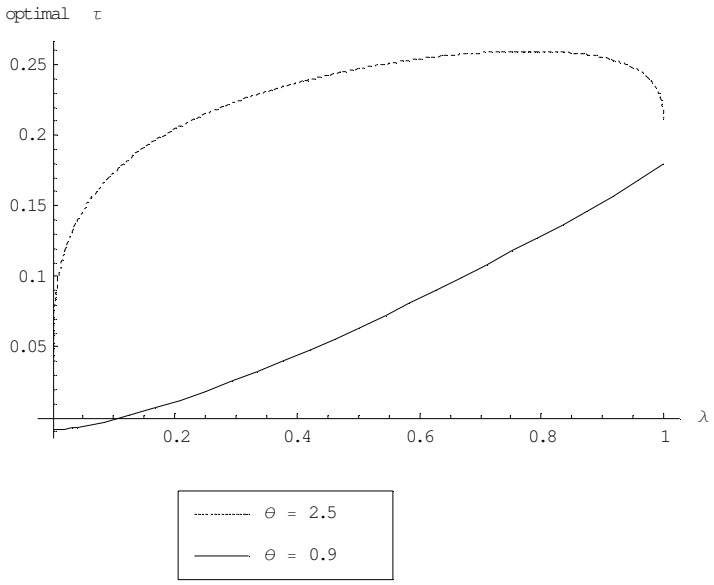

Figure 2.2 *Social security tax matching the golden rule*

Calibration

Obviously, the analysis above basically points at the theoretical possibility of a dynamically inefficient economy. The likelihood and importance of this effect is an empirical issue, on which this subsection will have a closer look.

In order to find realistic values of the parameters μ and λ, we use detailed household expenditures that can be obtained from questionnaires on family spendings. We use data from two surveys. The first is the *ONS Family Spending* 2000-01 carried out by the Office for National Statistics, which describes average weekly household expenditures on many different commodities and services, by age of the head of household in Britain. From this we can calculate the expenditures on services (that approximate the very labour-intensive services as used in our model) as the share of total household expenditures.[19] The second

[19] These services are housing repairs and maintenance, domestic help and child care, laundry, cleaning

study is the *CBS Budgetonderzoek* for the year 1994 carried out by the Central Bureau of Statistics. It gives detailed insight in the spendings on many items by Dutch households, as well as the age of the members of each household. From this we can again calculate the spendings on services and the average age in the household (to account for the fact that households with children spend a lot on e.g. childcare).[20] The findings are reported in Table 2.2.

Table 2.2 *Spendings on services as % of total spendings*

	United Kingdom	Netherlands	average
age 0-65	6.1%	4.1%	5.1%
age 65-75	8.8%	6.0%	7.3%
age 75+	10.2%	9.0%	9.6%
average 65+	9.2%	6.5%	7.9%

As can be seen from this table, elderly spend 1.5 times as much of their total expenditures on services compared to younger households.

The model used in the previous sections distinguishes between two periods of life. The individual faces a probability ε to 'survive' the first period and subsequently live throughout the entire second period, which is as long as the first period. Hence, the expected lifetime is at that moment as long as the first period. Assuming the first period to be 25-30 years, it is reasonable to suppose that an individual retires at the age of 55, because people who have reached this age are expected to live another 25-30 years.[21] The parameter ε is then the probability that someone who enters the labour market (aged around 25) reaches the age of 55, which is equal to 0.94 in 2002 and will be 0.99 in 2049.[22]

The expenditure shares of services when young (z^y) and old (z^o) are

$$z^x = \frac{pd^x}{pd^x + c^x}, \qquad x = y, o.$$

and dying, nursing fees, hairdressing, car and van repairs and servicing, and other services.

[20] These include expenditures on repair services, gardening, housework, hairdresser, beauty parlour, medical care, private health insurance, and other services.

[21] See life expectancy tables taken from CBS for 2002 and 2049.

[22] See mortality rates taken from CBS.

We seem to overestimate the length of the pension period and to underestimate the length of the working life. However, it should be mentioned here that pension entitlements can also be obtained without having a labour history in both the UK and the Netherlands. Moreover, labour participation of workers older than 55 years has decreased dramatically over the last 20 years.

From this the values of μ and λ follow, being

$$\mu = \frac{w^{1-\theta}\left(\frac{z^y}{1-z^y}\right)}{1+w^{1-\theta}\left(\frac{z^y}{1-z^y}\right)}, \quad \lambda = \frac{w^{1-\theta}\left(\frac{z^o}{1-z^o}\right)}{1+w^{1-\theta}\left(\frac{z^o}{1-z^o}\right)}.$$

Combining the equilibrium conditions for the commodity and capital market gives the social security tax at which the golden-rule steady state is attained,

$$\tau^* = \frac{(1-\alpha)\Gamma' - Q - \alpha}{(1-\alpha)(1+\Omega+\Gamma')},$$

with $\Gamma' \equiv \varepsilon\left(\frac{\gamma(1-\lambda)}{1-\mu}\right)^{\frac{1}{\theta}}(1+n)^{\frac{1-\theta}{\theta}} + \varepsilon\left(\frac{\gamma\lambda}{1-\mu}\right)^{\frac{1}{\theta}}\left(\frac{1+n}{w}\right)^{\frac{1-\theta}{\theta}}$ and $Q \equiv \frac{\alpha\varepsilon\left(\frac{\gamma(1-\lambda)(1+n)}{1-\mu}\right)^{\frac{1}{\theta}}}{1+n}$. Table 2.3 gives the values of τ^* for $\alpha = 0.3$, $\gamma = 0.75$ and different values of θ, taking $z^y = 0.051$ and $z^o = 0.067$, i.e., the average values for age categories 0-55 and 55+ respectively. These are compared to the values that apply in the one-sector model, i.e., if $z^y = z^o = 0$. We consider two steady states: a base case of an economy where demographic factors reflect those of the year 2000, with $\varepsilon = 0.94$ and $n = 0$, and an aged society as it would be in 2050, with increased longevity ($\varepsilon = 0.99$) and lower fertility ($n = -0.1$).[23]

Table 2.3 *Optimal PAYG-scheme (% of GDP)*

	base case		aged economy	
$\theta \rightarrow$	$\frac{4}{3}$	2.5	$\frac{4}{3}$	2.5
no services	0.2	1.9	1.5	3.9
services	2.1	4.0	3.5	6.1

It appears that allowing for service demand leads to an optimal PAYG-tax that is about twice the value that results from applying the conventional one-sector model. Population ageing will also considerably push this value up. Currenlty, public pension spendings are about 6.7% of GDP in the United Kingdom and the Netherlands, and 5.6% in the United States (see Table 1.1). Comparing this with the values in Table 1.2, one would indeed conclude that substantially reducing the unfunded part of social security raises long-run welfare if a model without services and population ageing is applied. But including

[23] An average number of children of 2.1 per woman corresponds to a constant population size. Many industrialised countries will have a lower number in 2049: Germany and Italy 1.6, Japan and Netherlands 1.8, UK 1.9 and only the USA will have a number of 2.1. Taking $n = -0.1$ corresponds to 1.9 children per woman.

services weakens this conclusion. Also allowing for a lower fertility rate and a longer lifespan even implies that the social security scheme in the countries mentioned above should only marginally be privatised. A substantial reform would eventually lead to a suboptimal outcome, with a rather low interest rate and high price of services.

2.5 Conclusion

Feldstein's claim that abolishing PAYG-schemes raises long-run welfare ignores an important point. If old individuals rely to some extent on labour-intensive services, extra savings, generated by privatisation of social security, create general-equilibrium effects that corrode their purchasing power. A vicious circle may arise: individuals save more in order to protect themselves against the rising cost of services that are urgently needed when old. But these additional savings lead to even higher prices of services, which erodes the purchasing power of the elderly. Maintaining some unfunded social security is then a desirable policy in order to guarantee a balanced consumption profile between young and old individuals in society.

The results found in this chapter apply to a closed economy. For a small open economy that is characterised by perfect capital mobility, the conclusion that full privatisation of social security leads to a higher long-run welfare naturally still holds, although this would nevertheless imply short-run transition costs. However, if such reforms take place simultaneously in many countries, or if capital is not very mobile, the economy rather resembles a closed economy, and the analysis presented here is more plausable.

The next two chapters include another element, viz. endogenously determined productivity growth that only occurs in the capital-intensive sector of the economy. In that case, privatising social security will also affect a small open economy in a way that is usually not taken into account. First, however, the effects of population ageing will be analysed.

Chapter 3

Population Ageing in a Two-Sector Growth Model

3.1 Introduction

Over the last centuries, mortality rates have substantially decreased, causing the average lifespan to increase. In the United States, for example, life expectancy at birth was about 40 years in 1850, rose to 68 in 1950, and is currently equal to 77. In their *World Population Prospects* (2001), the United Nations expect this to further increase and reach a level of approximately 83 in the year 2050. One of the most important causes of this spectacular rise is the higher rate of economic growth, as was emphasized by e.g. Fogel (1994). However, causality may also go in the other direction: an increase in longevity (or more generally, population ageing) can affect the rate of economic growth. This is the main topic of this chapter.

Most economic studies that deal with this issue concentrate on the accumulation of either physical or human capital as the engine of growth and, as we shall see, do not produce an unambiguous conclusion. In this chapter, we join with the view that human capital is the main source of productivity growth, but focus on another mechanism through which population ageing can affect it. We extend the two-sector model presented in the previous chapter by including endogenously determined economic growth, and concentrate on the effects of population ageing with respect to the sectoral structure of the economy, and on economic growth for different degrees of openness of the economy. Our approach closely follows Baumol (1967), who groups economic activities into two types: "technologically

progressive activities in which innovations, capital accumulation, and economies of large scale all make for a cumulative rise in output per man hour and activities which, by their very nature, permit only sporadic increases in productivity." (Baumol, 1967, p. 415-16). As to the first type of activity, "labor is primarily an instrument", whereas the second type consists of "services in which the labor is an end in itself, in which quality is judged directly in terms of amount of labor." So again, we assume two sectors: a labour-intensive service sector and a capital-intensive commodity sector. Productivity growth is endogenously determined and mainly stems from technological progress that typically takes place in the commodity (or 'high-tech') sector. As productivity grows, wages paid in this sector also increase. Because labour is assumed to be mobile between both sectors, the wage paid in the services sector will grow equally fast.

The capital-intensive sector produces homogeneous consumption and investment commodities that are demanded by young individuals, whereas elderly derive utility from labour-intensive services. Consequently, a change in the demographic composition of the economy will affect the relative aggregate demand for services and commodities. The structure of the economy, thereby the growth rate and the well-being of successive generations, are thus influenced.

Ageing is modeled as a permanently higher average lifetime. It turns out that in a small open economy, population ageing leads to a lower long-run rate of economic growth because the increase in the relative number of old implies a lower domestic production of commodities. In the short run, ageing has no effects on the labour share of the productive sector. In a closed economy, the capital-labour ratio in the commodity sector will also be influenced by domestic demographic changes. The long-run effects of ageing then crucially depend on the extent to which capital and labour are substitutable in the production process of commodities. If they are substitutes, then ageing will decrease growth, whereas the reverse holds if capital and labour are complements. Ageing has no effect on growth in the short run.

The rest of this chapter is organised as follows. In Section 3.2, we briefly discuss findings from the economic literature on the relation between population ageing and economic growth. Next, Section 3.3 describes the model we will apply. In Section 3.4, we analyse both the long-run and short-run consequences of increasing longevity in a small open economy on growth and the utility of successive generations, while Section 3.5 does the same for a closed economy. Section 3.6 deals with some extensions and modifications

of the model: population ageing as a lower fertility rate, the effects of both types of ageing if we allow for service demand by the young, and if we allow for a government that offers pension benefits that are indexed to the price of services. We also consider the decisions that would be made by a social planner who internalises the externality that causes the endogeneity of growth as we model it here, and finally how the results change if we allow for capital-induced growth. Section 3.7 concludes the chapter.

3.2 Related literature

Analyses on the relation between population ageing and growth can broadly be divided into two categories: those that focus on the accumulation of physical capital as the engine of growth, and those that concentrate on human capital accumulation.

Growth through capital accumulation

The first view on the cause of productivity growth goes back to Arrow (1962), who introduced the concept of learning-by-doing as an unintended by-product of production or investment. In his seminal paper, Romer (1986) worked this out to the well-known AK-model in which the aggregate capital stock induces positive external effects on productivity. The sheer accumulation of physical capital is therefore the source of growth. Using this framework, Pecchenino and Pollard (1997) study the impact of a longer lifespan on growth, and find that the effects depend on the extent to which total wealth (i.e., savings for old-age retirement) is annuitised. If it is fully annuitised, increasing longevity merely stimulates people to save more for their longer period of retirement, which results in a higher capital stock and thereby increases the growth rate. This was also found by Futagami and Nakajima (2001). But if wealth is not totally annuitised, and the government runs a public PAYG-pension scheme, a higher lifespan reduces unintentional bequests and decreases the marginal return to annuities, which has a negative effect on saving. On the other hand, ageing will also reduce the social security benefit if taxes are constant, and thereby stimulate savings. So in case of incomplete annuitisation, the effect of ageing on growth is ambiguous.

Growth through human capital accumulation

Another channel through which population ageing can affect economic growth is the formation of human capital,[1] as initiated by Lucas (1988) and Becker *et al.* (1990), where every producer benefits from the average level of human capital in the economy since it generates nonrival, nonexclusive knowledge. In Ehrlich and Lui (1991) parents invest in the quantity and quality (i.e., human capital) of their children in order to secure old-age insurance from their offspring. When analysing the effects of ageing, they distinct between increases in young-age and old-age longevity. The first lowers the fertility level, increases investments in human capital and growth. Increasing old-age longevity raises the parents' expected return on investment in human capital of their children, but also increases the cost to the younger generation of maintaining the old, so the increase in growth is smaller. De la Croix and Licandro (1999) investigate the effects of an increasing life span when agents invest in their own human capital. If people die later, the depreciation of aggregate human capital decreases, and the expected flow of future wages rises, so education expenditures and growth increase. On the other hand, the average age of workers increases, which has a negative impact on growth, but is not likely to dominate, so ageing increases growth.

Other studies also include altruism. In Pecchenino and Utendorf (1999), parents derive utility from educating their children. They find that increasing longevity incites people to save more by cutting back all expenditures, including those on the education of their children, which reduces growth. Applying a model with endogenous fertility, altruism and human capital formation through education, Zhang *et al.* (2001) find that the effects of a lower mortality rate crucially depends on both taste parameters and the size of the (unfunded) social security scheme. If pensions are fully funded and altruism is weak relative to people's taste for the number of children, a rise in longevity induces people to raise fewer children and invest more in the education of each child, which ultimately leads to higher economic growth. But if a PAYG-scheme is in place, a longer lifespan will imply higher tax rates, so the opportunity costs of raising children decrease, leading to more children and less education per child. On the other hand, parents would want to compensate their children for the higher tax by bequeathing more to each child, which raises the cost of a child and therefore has the opposite effect. The overall effect then depends on the size of the PAYG-scheme and the extent to which people have altruistic

[1] The important role of human capital in economic growth is emphasized in e.g. Mankiw *et al.* (1992).

feelings towards their progeny.

Kemnitz (2000) focuses on the political process. He finds that in a representative democracy, ageing deteriorates the pensioners' relative lobbying power, because the gains of a higher social security tax then become smaller. Consequently, educational expenditures are expanded, resulting in higher per capita income growth.

One can conclude that the theoretical papers so far do not provide a clearly unambiguous effect of ageing on growth. The next subsection discusses some empirical findings.

Empirical evidence

Using data for various countries between 1960 and 1985, Ehrlich and Lui (1991) find evidence that supports their theory: longevity has a significantly positive effect on the growth rate, and the effect of young-age longevity is greater than that of old-age longevity. However, important factors such as social insurance programs are not taken into account, and reverse causality may be at work. Barro and Sala-i-Martin (1995) find a strong positive effect of longevity on economic growth: an increase in life expectancy of 13 years is estimated to raise the annual growth rate by 1.4 percentage points. However, Disney (1996, Chapter 8) shows a clear negative relationship between the old-age dependency ratio and the GDP growth rate for 24 OECD countries between 1977 and 1992.

Lindh and Malmberg (1999) find that the growth pattern of GDP per worker in the OECD countries between 1950 and 1990 are to a large extent explained by age structure changes. In particular, they find a significantly positive correlation between productivity growth and the share of older workers (i.e., people aged 50-64), and a significantly negative relation between growth and the population share of retired people (i.e., aged 65 and above). This would imply that the ageing of the population eventually causes a lower economic growth.

3.3 The model

We use a two overlapping-generations model for a small open economy and a closed economy. Following Baumol (1967), we distinguish between two sectors of production. In the *commodity sector* (labelled Y), homogeneous goods are produced that either serve as consumption or investment good and can be traded internationally. The *service sector*

(labelled D) concerns the provision of labour-intensive services that are not internationally tradable. In both sectors, firms are fully competitive and maximise profits. Labour is homogeneous and perfectly mobile across sectors, so the wage in both sectors is identical.

Production of commodities

Commodities are produced with physical capital and labour according to the following standard neoclassical CRTS production function: $Y_t = F(K_t, A_t L_t^Y)$, where K_t stands for the domestic capital stock, L_t^Y is the number of people employed in the commodity sector and A_t is a productivity parameter reflecting the current state of (technological) knowledge or experience in the economy (all at time t). Production per effective unit of labour is described by $f(\kappa_t)$, with $\kappa_t \equiv \frac{K_t}{A_t L_t^Y}$ denoting the effective capital-labour ratio. The elasticity of substitution between capital and labour in this sector is constant and denoted by $\sigma \equiv -\frac{f'(\kappa_t)w_t}{f(\kappa_t)\kappa_t f''(\kappa_t)}$. The interest rate and wage are determined by the marginal productivity of capital and labour, respectively, i.e., $r_t = f'(\kappa_t)$ and $w_t = A_t\omega_t$, where $\omega_t = (f(\kappa_t) - f'(\kappa_t)\kappa_t)$.

Labour productivity A_t grows at the endogenous rate g, which is assumed to depend positively on the number of people employed in the productive sector,[2,3]

$$A_t = A_{t-1}\left(1 + g(L_t^Y)\right), \tag{3.1}$$

with $g' \equiv \frac{dg}{dL^Y} > 0$. This is a shortcut for explicitly modeling (labour-intensive) R&D activities or education. Alternatively, one could assume that part of the employees in the commodity sector devote their time to these activities, so the more people that are active in this sector of the economy, the more knowledge is created and skills are improved, which allows people to produce more final output, i.e., productivity increases (as in e.g. Romer, 1990, Grossman and Helpman, 1991, Mulligan and Sala-i-Martin, 1993).

[2]It may well be that the economy also adopts new technologies and increases its knowledge from abroad. In that case, the size of the commodity sector positively affects the *additional* productivity growth of the economy.

[3]Note that we assume A to grow instantly with the size of the sector. If learning takes time, however, one would rather want to assume some delay in the accumulation of knowledge. However, assuming that g is a function of L_{t-1}^Y instead of L_t^Y would imply that learning takes about 30 years (i.e., one period in our model), which does not seem to be very realistic.

Production of services

Production of services requires labour only and does not benefit from technological improvements. For the sake of simplicity, it is assumed that one unit of labour translates into one service. Consequently, total provision of services equals total labour supply in this sector (L_t^D) and the price of services in terms of commodities, p_t, is equal to the wage: $p_t = w_t$.

Households

The economy is inhabited by a large number of individuals (normalised to 1) who live for two periods, such that in each period, both a young and an old generation are alive.[4] Apart from age, people are identical. An agent dies at the onset of old age with probability $1 - \varepsilon$ and lives throughout old age with probability ε. We can interpret ε as the fraction of the second period of life that people on average live, i.e., average longevity. An increase of ε is thus referred to as a longer lifespan.[5]

When people grow old, the need for services increases, both because of increased disability (which raises the demand for e.g. care services and housekeeping assistance) and a higher preference for services relative to (high-tech) commodities. To simplify the analysis, we will assume old individuals to demand services only. The expected lifetime utility of a representative agent born at any time t can then be represented by the following function,

$$E_t U(c_t, d_{t+1}) = \log c_t + \gamma \varepsilon_{t+1} \log d_{t+1}, \tag{3.2}$$

where c_t stands for the consumption of commodities when young and d_{t+1} is the number of services enjoyed by the agent when old.[6]

Every young individual supplies one unit of labour inelastically and receives wage. A constant fraction τ of his labour income is taxed by the government to finance a public pension scheme. The young spend their net income on the purchase of commodities, either for consumption or investment purposes (i.e., savings, s_t). Savings are invested in

[4] In Section 3.6.1, population growth is included in order to analyse the effects of ageing as a drop in fertility.

[5] Note that ε also reflects the number of retired per young, i.e., the dependency ratio.

[6] Allowing for consumption of services when young does not qualitatively affect the results of reducing the social security tax and increasing longevity. However, the results can change if ageing is the result of a lower fertility rate. See Section 3.6.1.

annuities or through an actuarially fair pension fund. As only a fraction ε_{t+1} of young savers born at time t survives to period $t+1$, the assets of those who deceased fall to surviving contemporaries.

The total return on the savings is therefore $\frac{r_{t+1}}{\varepsilon_{t+1}}$. Furthermore, the public pension benefit that an old individual at time $t+1$ receives is equal to $\frac{\tau_{t+1}w_{t+1}}{\varepsilon_{t+1}}$. The social security tax is constant, so that the benefit level adjusts.[7] Individual consumption possibilities are thus given by the following budget constraints,

$$c_t = w_t(1-\tau) - s_t, \tag{3.3.a}$$

$$p_{t+1}d_{t+1} = \frac{r_{t+1}s_t + \tau w_{t+1}}{\varepsilon_{t+1}}. \tag{3.3.b}$$

Maximising (3.2) subject to (3.3.a) and (3.3.b) results in the following individual demand and saving functions,

$$c_t = \frac{(1-\tau)w_t}{1+\gamma\varepsilon_{t+1}} + \frac{\tau w_{t+1}}{(1+\gamma\varepsilon_{t+1})r_{t+1}}, \tag{3.4}$$

$$d_{t+1} = \frac{\gamma r_{t+1}(1-\tau)w_t}{(1+\gamma\varepsilon_{t+1})w_{t+1}} + \frac{\gamma\tau}{1+\gamma\varepsilon_{t+1}}, \tag{3.5}$$

$$s_t = \frac{\gamma\varepsilon_{t+1}(1-\tau)w_t}{1+\gamma\varepsilon_{t+1}} - \frac{\tau w_{t+1}}{(1+\gamma\varepsilon_{t+1})r_{t+1}}. \tag{3.6}$$

Note that the consumption of commodities is growing over time with the rate of economic growth. But for a given growth rate, the number of services enjoyed is constant, since the (positive) income effect of a rising wage is exactly offset by the (negative) substitution effect of higher prices of services. This is a necessary requirement for an equilibrium to exist, because the supply of labour, and thus the provision of services, is limited.[8,9]

[7] In Section 3.6.3 we examine the case of a constant benefit level.

[8] This is in line with the modelling of the labour/leisure choice in a model with economic growth, where the number of leisure hours cannot grow indefinitely. King *et al.* (1988) and Rebelo (1991) formulate the kind of utility function for which this holds (see also Barro and Sala-i-Martin, 1995, Chapter 9). The log-linear utility function applied here is a special case of this type of utility function.

[9] Instead of putting restrictions on the utility function in order to prevent corner solutions, one could also assume that the services sector experiences exogenous growth equal to $\bar{g}_S$, and growth in the commodity sector is partly exogenous, $\bar{g}_Y$, and partly endogenous, $g(L^Y)$. In that case, it must be assumed that the difference in exogenous growth rates is ultimately compensated by the endogenously determined productivity difference. The share of labour employed in the commodity sector then equals $L^Y = g^{-1}(\bar{g}_S - \bar{g}_Y)$ in the long run, in which case privatisation does not affect the growth rate in the long run.

Furthermore, for a given interest rate, a higher growth rate decreases the number of services for an old person.

Finally, the following four assumptions are made.

Assumption 3.I *A steady state exists, i.e., the function g(x) is such that* $\exists x \in (0,1) \mid 1 - \frac{r\gamma\varepsilon(1-\tau)}{(1+\gamma\varepsilon)(1+g(x))} - \frac{\gamma\varepsilon\tau}{1+\gamma\varepsilon} = x$.

Assumption 3.II *The economy converges to the steady state.*

Assumption 3.III *Endogenous growth effects are bounded by* $g' < \frac{(1+\gamma\varepsilon_t)\left(1+g(L_t^Y)\right)^2}{r\gamma\varepsilon_t(1-\tau)}$ $\forall t$.

Assumption 3.IV *The economy is always dynamically efficient, i.e.,* $r_t > 1 + g(L_t^Y)$ $\forall t$.

Equilibrium

The model comprises four markets that are simultaneously in equilibrium at each point in time. As services are not tradable, total demand for services by the old has to be met by labour supply of the young to this sector, so $L_t^D = \varepsilon_t d_t$. Labour market clearing implies that $L_t^Y = 1 - L_t^D$. These equilibrium conditions in combination with the individual first-order conditions lead to the following expression for the employment share of the commodity sector,

$$L_t^Y = 1 - \frac{r_t s_{t-1} + \tau w_t}{p_t} \tag{3.7}$$

$$= 1 - \frac{\gamma\varepsilon_t r_t \omega_{t-1}(1-\tau)}{(1+\gamma\varepsilon_t)\,(1+g(L_t^Y))\,\omega_t} - \frac{\gamma\varepsilon_t\tau}{1+\gamma\varepsilon_t}. \tag{3.8}$$

Demand for commodities, which consists of consumption, investment (K_{t+1}) and net exports (b_t), must equal aggregate domestic production, so $Y_t = c_t + K_{t+1} + b_t$. The capital market clears if $s_t + Z_{t+1} = K_{t+1}$, where Z is net foreign debt. In case of a closed economy, the interest rate is endogenous and $b_t = Z_t = 0$ $\forall t$. In the small-open-economy version of the model, the interest rate, and thus the capital-labour ratio and the marginal product of a labour efficiency unit, is exogenously fixed at the world capital market, i.e., $r_t = r$ and $\omega_t = \omega$ $\forall t$. Moreover, equilibrium in the balance of payments requires $Z_{t+1} - Z_t = rZ_t - b_t$.

3.4 A small open economy

In a small open economy, the factor rewards are exogenously given, so the current state of the economy, as implicitly summarised by (3.8), contains no variable or parameters that refer to the past and is therefore not related to its past.[10] The effects of a change in the rate of population growth can be traced by linearising (3.8) around the initial steady state. In case the small open economy is unexpectedly hit by an increase of ε, the variable L^Y will jump to another value at the time of the shock, and jumps to its new steady-state value one period later.

Long-run effects

The long-run effect of a longer life span on the employment level of the commodity sector, and thereby on the rate of economic growth, is negative, as the following proposition asserts.

Proposition 3.1 *In a small open economy, increasing longevity reduces economic growth in the long run.*

Proof The long-run impact of an increasing life span on the economy can be traced by comparative statics of (3.8), which yields the following equation,

$$\frac{dL^Y}{d\varepsilon} = -\frac{\gamma(1+g)\left[r(1-\tau)+\tau(1+g)\right]}{(1+\gamma\varepsilon)\left[(1+\gamma\varepsilon)(1+g)^2-\gamma\varepsilon r(1-\tau)g'\right]} < 0. \tag{3.9}$$

Assumption 3.III implies that the denominator of (3.9) is positive, so $\frac{dL^Y}{d\varepsilon} < 0$. ■

If people expect to live longer, they will decide to save more when they are young. If this was all invested domestically, the interest rate would drop below the world level, so these increased savings are invested abroad, yielding a return of r. With these returns, the elderly individuals buy services that are provided by their children. Because the retired use more services, the number of young people employed in that sector increases, leaving fewer people in the capital-intensive commodity sector. As a consequence, the growth rate decreases. Furthermore, the decline of domestic commodity production will

[10] If young individuals also demand services, the solution of L_t^Y is a terminal-value problem as L_t^Y is a forward-looking variable. See Section 3.6.2 and the Appendix for more details.

be compensated by higher imports of these goods from abroad (or lower exports), which is financed by the increased foreign asset returns.

Savings are influenced through three channels. First, a longer life span incites people to save more. Second, because wage grow at a lower rate, savings will do so too. The last effect stems from the PAYG-scheme. Because the future price of services decreases, the present value of the social security benefit (in terms of current commodities) decreases, which stimulates savings. Overall, savings increase. Together with the fact that future services become cheaper (i.e., the real interest rate (in terms of services) increases), each retired person can use more services.

Short-run effects

If the life span increases unexpectedly at time $t = 0$, the economy will be in its new steady state as from $t = 1$ on. At the time of the shock, however, the effects differ from the long-run consequences since the old at that time did not foresee the change in the PAYG-benefit, nor the changing rate of economic growth (which reflects the increase of the price of services) when they made their savings decision. Therefore, they still spend the same amount of money on services. Consequently, the allocation of labour over the two sectors does not change either, and growth is initially not affected by ageing.

Proposition 3.2 *In a small open economy, an unexpected increase in longevity does initially not affect economic growth.*

Proof Savings at $t = -1$ (s_{-1}) are not affected by an unanticipated increase in longevity at $t = 0$. Equation (3.7) then implies $\frac{dL_0^Y}{d\varepsilon_0} = 0$. ■

Note that this also holds if at time $t = 0$, an increase in the life span in the next period is foreseen. Then, people will start to save more in period $t = 0$, but this only implies a shift from the demand of consumption commodities to investment commodities, which does not affect the size of the commodity sector.[11]

[11] There will be effects on growth in the short run if agents also demand services when they are young, as described in Section 3.6.2.

If ageing is the result of a lower rate of population growth, productivity growth will decrease if the economy is a net lendor. If the young demand services too, the effects of such a form of ageing depend on the size of the PAYG-scheme. See Section 3.6.1 for details.

Utility

Figure 3.1 shows the consequences of population ageing on the lifetime utility of successive generations.[12] In this figure, the horizontal axis denotes the period in which a certain generation is born, and the vertical axis gives the change in utility as a fraction of the lifetime utility that those born at time $t = -1$ would have had without the shock.

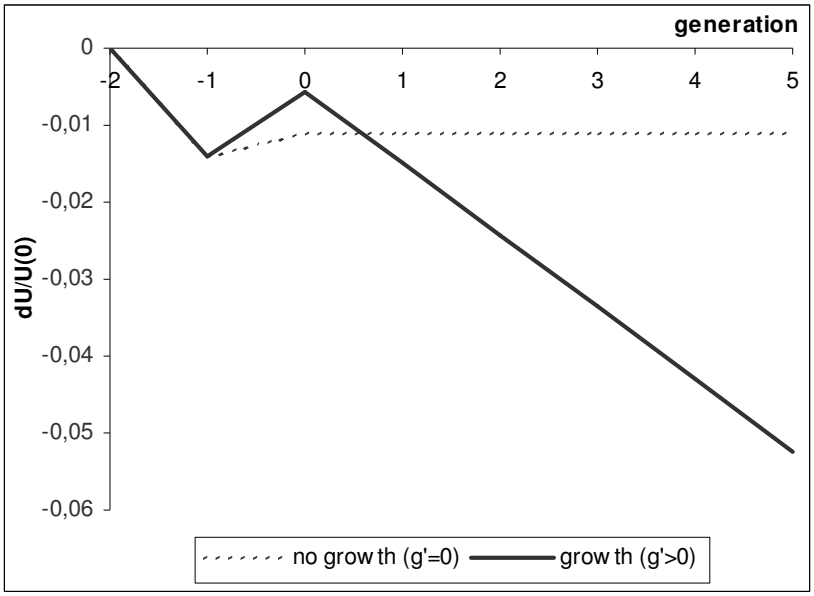

Figure 3.1 *Effect of increasing longevity on lifetime utility in a small open economy*

First consider the case without endogenous productivity changes ($g' = 0$). Because factor rewards do not change in a small open economy, ageing does not affect the real interest rate. However, as more people reach old-age, a smaller amount of assets of those who deceased fall to surviving contemporaries, so the total return on savings falls. Furthermore, the amount of social security taxes remitted by the young, which does not

[12] All simulations are based on a productivity growth that is assumed to be a linear function of the number of emplyees in the productive sector: $g_t = \bar{g} + \rho L_t^Y$, where $\bar{g}$ has such a value that the equilibrium growth rate is equal to 1 (which corresponds to an annual economywide growth rate of about 2.3%). The cases considered are $\rho \to 0$ and $\rho = 1.5$ respectively. The production function of commodities is given by $Y_t = \left[0.3K_t^{-0.25} + 0.7(A_t L_t^Y)^{-0.25}\right]^{-4}$. Furthermore, $\gamma = 0.75$ and ε increases from $\frac{2}{3}$ to 0.7. In order to compare lifetime utility before and after ageing properly, we take the value of ε constant at unity, so we compare the lifetime utility of an individual who lives both periods. Moreover, the social security tax is 15%.

change, has to be shared by more retired individuals, so the individual public pension benefits decreases. Hence, the initially old generation enjoys fewer services. The decrease in utility that results from this is larger for the generation that is old at the time of the shock than for the subsequent generations because, in contrast to the current elderly, these subsequent generations can partly compensate for the lower return by increasing savings.

With endogenous growth, the effects for the currently old are not different from the case in which $g' = 0$ because the number of employees in the commodity sector does not change at the time of the shock. The wage of the individuals born at time $t = 0$ therefore does not change either. However, fewer people will be employed in the commodity sector at time $t = 1$, so wages will then be lower. Consequently, the services that the generation born at the time of the shock will buy when they are old will be cheaper. This positive price effect alleviates the negative effect of a longer life span on utility, which is why the utility of this generation decreases less if $g' > 0$ compared to the case without endogenous growth. However, in the long run, individuals suffer from the fact that ageing reduces productivity growth .

3.5 A closed economy

Long-run effects

The long-run consequences of population ageing on the rate of economic growth depend on the extent to which capital and labour can be substituted in the production process of commodities, as stated in the following proposition.

Proposition 3.3 *In a closed economy, increasing longevity decreases (increases) long-run growth if the elasticity of substitution between capital and labour is greater (smaller) than unity. Ageing has no effect on long-run growth if this elasticity equals unity. The capital-labour ratio always increases in the long run.*

Proof See Appendix. ■

Increasing longevity incites people to save more when young so that the capital stock that is used in the commodity sector rises, stimulating labour to move from the services sector to the commodity sector. This effect will be particularly strong if capital and labour

are complements. On the other hand, the total demand for services increases, since more people will be retired, which incites labour to move in the opposite direction, i.e., away from the commodity sector. If capital and labour are substitutes in the commodity sector, this last effect will be dominant and L^Y decreases, the reverse holds in case of complementarity. In case of a Cobb-Douglas production function, capital and labour are neither complements nor substitutes, so then ageing causes no change in the division of labour and thereby in the growth rate.[13]

Short-run effects

As is the case for the small open economy, increasing longevity does not influence the allocation of labour over the two sectors in the short run, which leads to the following proposition.

Proposition 3.4 *In a closed economy, increasing longevity does initially affect neither economic growth nor the (effective) capital-labour ratio.*

Proof See Appendix. ■

Again, the first generation of elderly cannot react to the fact that more contemporaries survive, so that the initial capital stock is given. The first generation of young can and will react by increasing savings. As this merely is a shift from consumption to investment that does not affect the total demand for commodities, this will not change the allocation of labour over the two sectors and the capital-labour ratio in the commodity sector is not influenced either. Consequently, economic growth and factor rewards will initially not be affected, just as in the small open economy.

Utility

The effects of a longer life span on the lifetime utility of subsequent generations in a closed economy are displayed in Figure 3.2.[14] Because at the time of the shock the wage and

[13] This directly follows from the equilibrium condition for the commodity market, according to which the wage share does not depend on the capital-labour ratio. That is, if the production elasticity of capital is a, then $\frac{w_t A_t L_t^Y}{Y_t} = 1 - a$, so that $L_t^Y = (1 - \tau)(1 - a)$. Allowing for consumption of services by the young does not change this result.

[14] If capital and labour are complements, the elasticity of substitution is $\sigma = 0.8$, and in case of substitutability, $\sigma = 1.1$ is taken.

interest rate remain the same, the group of retired at $t = 0$ as a whole receives the same return on the savings, but because more people survived, each elderly individual gets a lower return on his savings. The initial generation of young receives the same wage as before, but will decide to save a higher fraction of it and consume fewer commodities.

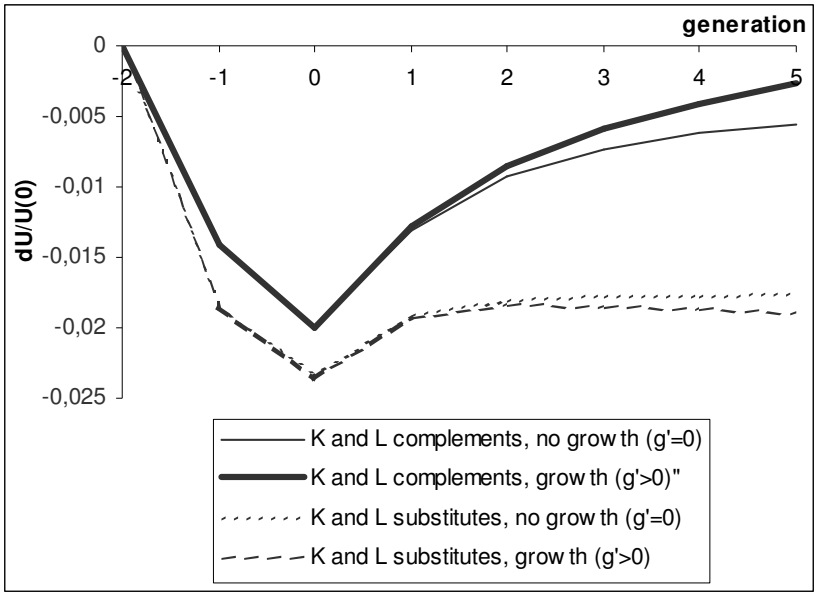

Figure 3.2 *Effect of increasing life span on lifetime utility in a closed economy*

These higher savings translate into a higher capital-labour ratio in the next period. Consequently, wages rise and services become more expensive, enabling the members of this generation to buy fewer services. The next generation earns this higher wage when young, so their savings will increase, resulting in even higher wages in the next period. Notice that this effect is much stronger if capital and labour are complements in the production process of commodities ($\sigma < 1$) than in case they are substitutes ($\sigma > 1$). In case of complementarity of capital and labour, this process is reinforced if there are endogenous growth effects ($g' > 0$), as the increase in L^Y then stimulates growth. As a result, in this case, the increase in consumption of commodities when young will eventually compensate the loss of utility that is the result of fewer services during old age. The endogenous growth effect works in the opposite direction, however, when capital and labour are substitutes. In that case, the decrease in employment in the commodity sector decreases growth and lowers utility even further.

3.6 Extensions and modifications

In this section we explore the effects of population ageing when the basic model presented above is extended or modified in several ways. First, we consider the consequences of ageing if this is due to a lower rate of population growth. Second, the assumption that only old individuals demand services is relaxed. The next section studies what changes if the pension system promises the retired a fixed amount of services. Furthermore, the basic model includes an externality due to learning-by-doing in the commodity sector. That is, individuals do not take account of the fact that consumption of commodities stimulates employment in this sector and thus economic growth, so consumption of services will be too high. This is illustrated in Section 3.6.4, where we analyse how the allocation chosen by a social planner (who internalises this externality) changes in case of an ageing population. Finally, in Section 3.6.5 we study how robust the results are if we replace the assumption that growth is due to learning-by-doing in the commodity sector by the assumption that technological development is linked to the use of capital.

3.6.1 Population ageing as a lower fertility rate

Suppose the economy is inhabited by N_t young people at time t, and population grows at rate n, such that $N_{t+1} = (1+n_t)N_t$. Population ageing takes the form of a lower value of n. Equilibrium in the labour market implies that $N_t = L_t^Y + L_t^D$. We now assume that the productivity growth rate depends on the fraction of young people employed in the productive sector: $g_t = g(l_t^Y)$, with $l_t^Y \equiv \frac{L_t^Y}{N_t}$. Furthermore, we take ε constant and equal to unity, and the social security tax τ does not change either. The PAYG-benefit that an old individual at time $t+1$ receives now equals $(1+n_{t+1})\tau w_{t+1}$. The demand and savings functions are now given by

$$c_t = \frac{(1-\tau)w_t}{1+\gamma} + \frac{(1+n_{t+1})\tau w_{t+1}}{(1+\gamma)r_{t+1}}, \tag{3.4'}$$

$$d_{t+1} = \frac{\gamma r_{t+1}(1-\tau)w_t}{(1+\gamma)w_{t+1}} + \frac{\gamma(1+n_{t+1})\tau}{1+\gamma}, \tag{3.5'}$$

$$s_t = \frac{\gamma(1-\tau)w_t}{1+\gamma} - \frac{(1+n_{t+1})\tau w_{t+1}}{(1+\gamma)r_{t+1}}. \tag{3.6'}$$

Moreover, Assumption 3.III now is $g' < \frac{(1+\gamma)\left((1+g(l_t^Y)\right)^2(1+n)}{r\gamma(1-\tau)}$ $\forall t$ and Assumption 3.IV becomes $r_t > \left(1+g(L_t^Y)\right)(1+n_t)$ $\forall t$.

Small open economy

The long-run consequences of a lower birth rate are the same as those caused by a longer life span.

Proposition 3.5 *In a small open economy, decreasing fertility reduces economic growth in the long run.*

Proof Equilibrium in both the labour and services market is given by $l_t^Y = 1 - \frac{\gamma(1-\tau)r}{(1+\gamma)(1+n_t)\left(1+g(l_t^Y)\right)} - \frac{\gamma\tau}{1+\gamma}$. Comparative statics of this equation gives

$$\frac{dl^Y}{dn} = \frac{\gamma(1-\tau)r(1+g)}{(1+n)\left[(1+\gamma)(1+n)(1+g)^2 - \gamma(1-\tau)rg'\right]} > 0, \tag{3.10}$$

which is obviously positive if Assumption 3.III holds. ■

The long-run effect of ageing on growth therefore does not depend on the type of ageing. Notice that the numerator of (3.10) decreases with τ,[15] whereas the denominator increases with τ, so $\frac{d^2l^Y}{dnd\tau} < 0$: the negative effect of a lower fertility rate on economic growth is curbed by the PAYG-scheme.

However, the short-run effects are different from those of a higher life span.

Proposition 3.6 *In a small open economy, an unanticipated decrease in fertility initially decreases (increases) growth if the country is a net lender (borrower). The economy reaches the new steady state after one period.*

Proof If the fertility rate decreases unexpectedly at time $t = 0$, the economy will again be in its new steady state as from $t = 1$ on. In this case, equation (3.7) can be written as

$$l_0^Y = 1 - \frac{rs_{-1}}{w_0(1+n_0)} - \tau,$$

where s_{-1} is take as given. Differentiating this expression we arrive at

$$\frac{dl_0^Y}{dn_0} = \frac{r\gamma(1-\tau)(1+g) - \tau(1+g)^2(1+n)}{(1+n)\left[(1+\gamma)(1+n)(1+g)^2 - r\gamma(1-\tau)g' + (1+n)(1+g)\tau g'\right]}.$$

[15] Long-run growth depends positively on the social security tax in a small open economy, as is demonstrated in the next chapter.

The denominator is negative when Assumption 3.III holds, so combining this with savings equation (3.6') gives $\frac{dl_0^Y}{dn_0} \lesseqgtr 0$ if $s \lesseqgtr 0$. ■

Ageing initially implies a relatively higher number of elderly individuals. If the country is a net lender ($s > 0$), then relatively more people will spend their income on services, and l_0^Y decreases, so productivity growth will immediately decrease. Consequently, the price of the services that these old individuals buy initially decreases, so the direct negative effect of a higher dependency ratio on the PAYG-benefit will be alleviated by a lower inflation rate, i.e., an increase in the real interest rate. The reverse holds if the economy is a net borrower ($s < 0$), which is the case if the social security tax is rather high. Then, elderly individuals who borrowed an amount of $N_{t-1}s_{t-1}$ from foreigners when they were young, have to pay off this same amount from a lower transfer they receive from their children, who have decreased in number. So, there is less money left for the purchase of services, and l_0^Y, and thereby growth, increases.

If growth decreases at time $t = 0$, the price of services that elderly buy will be lower. On the other hand, if a social security system is present, a decreasing fertility rate implies a lower pension benefit, enabling them to buy fewer services. Hence, whether or not the utility of the initially elderly decreases due to a drop in fertility depends on the size of the PAYG-scheme and the intensity of the endogenous growth effect.

If the fertility rate decreases unexpectedly at time $t = 0$, the economy will again be in its new steady state as from $t = 1$ on, characterised by a lower growth rate (if $s > 0$). So subsequent generations will be worse off after the shock.

Closed economy

In a closed economy, the long-run effects of a lower rate of fertility are the same as those of a longer lifespan. In both cases, the dependency ratio increases and people have a stronger incentive to save, causing the capital-labour ratio to increase and the (relative) number of employees in the commodity sector to increase (decrease) if capital and labour are complements (substitutes) in that sector. The same holds for the short run in case fertility (unexpectedly) decreases.

Proposition 3.7 *In a closed economy, decreasing fertility reduces (increases) growth initially as well as in the long run if the elasticity of substitution between capital and*

labour is greater (smaller) than unity. Ageing has no effect on initial and long-run growth if this elasticity equals unity. In all cases, the (effective) capital-labour ratio increases.

Proof See Appendix. ■

So contrary to increasing longevity, a lower population growth rate does affect productivity growth in the short run. This is so because a lower fertility rate immediately decreases the number of young individuals. On the one hand, this implies a shortage of labour in the services sector, which has an upward effect on the wage in that sector and stimulates labour to move away from the commodity sector. On the other hand, the capital stock is fixed in the short run. Hence, capital is becoming relatively abundant in the commodity sector, which has an upward effect on the wage of employees in that sector. If both production factors are complements, the latter effect is strong, and labour moves to the commodity sector, whereas l_0^Y decreases in case of substitutability. The effects on productivity growth are likewise.

The short-run effects of population ageing are summarised in Table 3.1.

Table 3.1 *Short-run effects of ageing if* $g' > 0$

	Decreasing fertility			Increasing longevity		
variable	SOE[1]	CLE (σ<1)	CLE (σ>1)	SOE	CLE (σ<1)	CLE (σ>1)
L_0^Y, g_0	-	+	-	0	0	0
κ_0	0	+	+	0	0	0
p_0=w_0	-	+	+	0	0	0
c_0	-	+	+	-	-	-
d_0	+/-[2]	-	-	-/+[3]	-	-
U_{-1}	+/-[2]	-	-	-	-	-
U_0	-	-	-	-/+[3]	-	-

[1] The signs in this column change if the economy is a net borrower.

[2] Negative if the PAYG-scheme is extensive.

[3] Positive if endogenous growth effects are strong.

3.6.2 Allowing for service demand by the young

In this section we investigate the effects of ageing for a small open economy in case individuals also demand services when they are young. The ageing effects in a closed

economy are qualitatively not different from the effects when services demand by the young is absent.

Expected lifetime utility of a representative agent is now given by the following function,

$$E_t U(c_t, d_t^y, d_{t+1}^o) = \log c_t + \beta \log d_t^y + \gamma\varepsilon_{t+1} \log d_{t+1}^o,$$

where d_t^y (d_{t+1}^o) is the number of services enjoyed by the agent when young (old). The budget constraints are

$$\begin{aligned} c_t + p_t d_t^y &= (1-\tau)w_t - s_t, \\ p_{t+1} d_{t+1}^o &= \frac{r s_t}{\varepsilon_{t+1}} + \frac{(1+n_{t+1})\tau w_{t+1}}{\varepsilon_{t+1}}. \end{aligned}$$

This leads to the following demand for services,

$$\begin{aligned} d_t^y &= \frac{\beta(1-\tau)}{1+\beta+\gamma\varepsilon_{t+1}} + \frac{\beta(1+n_{t+1})\tau w_{t+1}}{w_t r(1+\beta+\gamma\varepsilon_{t+1})}, \\ d_{t+1}^o &= \frac{\gamma(1-\tau)w_t r}{(1+\beta+\gamma\varepsilon_{t+1})w_{t+1}} + \frac{\gamma(1+n_{t+1})\tau}{1+\beta+\gamma\varepsilon_{t+1}}. \end{aligned}$$

Equilibrium in the services and labour market is given by

$$L_t^Y = N_t - N_t d_t^y - \varepsilon_t N_{t-1} d_t^o. \tag{3.11}$$

Note that the current state of the economy, as summarised by (3.11), is not related to its past. Instead, the solution of (3.11) is a terminal-value problem (see Appendix for details). The effects of a change in the average life span can be traced by linearising (3.11) around the initial steady state. In case the small open economy is unexpectedly hit by a shock, the forward-looking variable L^Y (or l^Y) will jump to a new value at the time of the shock, and jumps to its new steady-state value after one period, which can easily be seen from the fact that equation (3.11) only contains one parameter (ε_{t-1}) and no variable (L^Y) that refers to the past. Furthermore, Assumption 3.III now states that $g' < \frac{(1+\beta+\gamma\varepsilon)\left(1+g(l_t^Y)\right)^2(1+n)}{r^2\gamma\varepsilon(1-\tau)-\beta\tau(1+n)^2\left(1+g(l_t^Y)\right)^2}$ $\forall t$.

Increasing longevity

The long-run effects of a longer average life span can be traced by linearising equation (3.11), taking $n = 0$ and $N = 1$, which results in

$$\frac{dL^Y}{d\varepsilon} = \frac{\gamma(1+g)\left[\beta(1+g) - r(1+\beta)\right]\left[\tau(1+g) + r(1-\tau)\right]}{(1+\beta+\gamma\varepsilon)\left[r(1+\beta+\gamma\varepsilon)(1+g)^2 + \beta\tau(1+g)^2 g' - r^2\gamma(1-\tau)\varepsilon g'\right]},$$

and is clearly negative if Assumptions 3.III and 3.IV hold. Naturally, the fact that young individuals demand services too does not change the result that increasing longevity leads to lower long-run productivity growth in a small open economy.

In the short run, however, the effects are different from the case that $\beta = 0$. This can be seen by looking at the equilibrium condition for the services and labour market at time $t = 0$ (when the shock unexpectedly occurs),

$$\begin{aligned} L_0^Y &= 1 - \frac{\beta(1-\tau)}{1+\beta+\gamma\varepsilon_1} - \frac{\beta\tau\left(1+g(L_1^Y)\right)}{r(1+\beta+\gamma\varepsilon_1)} - \frac{r(1-\tau)\gamma\varepsilon}{(1+\beta+\gamma\varepsilon_1)\left(1+g(L_0^Y)\right)} \\ &\quad - \frac{(1-\beta)\tau(1+g)}{(1+\beta+\gamma\varepsilon_1)\left(1+g(L_0^Y)\right)} - \tau, \end{aligned}$$

where ε (g) refers to the initial steady state value, i.e., the one that individuals born at time $t = -1$ took into account when they made their savings decision. Differentiating this expression gives

$$\frac{dL_0^Y}{d\varepsilon} = \frac{(1+g)^2\left(\beta\gamma r(1-\tau) + \beta\gamma(1+g)\tau - (1+\beta+\gamma\varepsilon)\beta\tau g'\frac{dL^Y}{d\varepsilon}\right)}{r(1+\beta+\gamma\varepsilon)\left((1+\beta+\gamma\varepsilon)(1+g)^2 + r(1-\tau)\gamma\varepsilon g' - (1+\beta)\tau(1+g)g'\right)}.$$

Knowing that $\frac{dL^Y}{d\varepsilon} < 0$, it immediately follows that $\frac{dL_0^Y}{d\varepsilon} > 0$ if $s > 0$. That is, increasing longevity will stimulate growth in the short run if young individuals also demand services. This is so because these young individuals expect to spend a longer time in retirement and therefore decide to increase their savings. In order to do this, they will ask fewer services when young. Another reason is that future services will be cheaper, because L^Y decreases at time $t = 1$, so current services are substituted for future services. The group of elderly individuals as a whole at time $t = 0$ will buy as many services as before, because each old individual receives a proportionately lower return on his savings and social security benefit. So fewer people will be employed in the services sector, and more in the commodity sector, which stimulates productivity growth. The following proposition summarises these results.

Proposition 3.8 *If young individuals also demand services, growth will initially increase if people live longer in a small open economy. In the long run, the effects are (qualitatively) the same as before.*

Decreasing population growth

The long-run effects of a lower fertility rate can be traced by linearising equation (3.11), taking $\varepsilon = 1$, which results in

$$\frac{dl^Y}{dn} = \frac{(1+g)\left[\gamma(1-\tau)r^2 - \beta\tau(1+g)^2(1+n)^2\right]}{(1+n)\left[(1+\beta+\gamma\varepsilon)r(1+g)^2(1+n) + \beta(1+n)^2(1+g)^2\tau g' - \gamma(1-\tau)r^2 g'\right]},$$

which is positive under Assumptions 3.III and 3.IV, and a social security tax that is not too high, i.e., $\tau < \frac{\gamma}{\beta+\gamma}$. So now too, a lower birth rate will decrease the long-run rate of economic growth.

In the short run, the effects are different, both from the case in which $\beta = 0$ and from the effects of increasing longevity. Differentiating (3.11) at time $t = 0$, taking the savings of the old as given, we arrive at

$$\frac{dl_0^Y}{dn} = \frac{(1+g)\left[-\beta\tau(1+g)^2(1+n)^2 - \beta(1+n)^3(1+g)\tau g' \frac{dl^Y}{dn} + r^2(1-\tau)\gamma - (1+\beta)\tau(1+g)^2(1+n)\right]}{r(1+n)[(1+\beta+\gamma\varepsilon)(1+g)^2(1+n) + r(1-\tau)\gamma g' - (1+\beta)\tau g']}. \tag{3.12}$$

Due to Assumption 3.III, the denominator is positive. If $\tau = 0$, then $\frac{dl_0^Y}{dn} > 0$, so a drop in the fertility rate leads to lower productivity growth in the short run. If $\tau = 1$, then $\frac{dl_0^Y}{dn} < 0$ and decreasing fertility implies a higher productivity growth in the short run. This, together with the fact that (3.12) is a continuous and decreasing function of τ, implies that there is a unique $\tau^* \in (0,1)$ for which $\frac{dl_0^Y}{dn} = 0$. Consequently, for all $\tau < \tau^*$ it holds that $\frac{dl_0^Y}{dn} > 0$, so an unexpected drop in fertility leads to a lower value of l_0^Y, and the reverse holds if $\tau > \tau^*$.

Decreasing fertility has four effects on the division of labour over the two sectors at time $t = 0$. First, the relative number of old individuals who spend their savings on services increases, so l^Y decreases. If the social security tax is high, individual savings are relatively low and the pension benefit decreases considerably, so this effect will be moderate. Second, young agents at that time foresee a lower pension benefit when old, and thus decide to save more, i.e., consume fewer services, which affects l^Y positively. This is especially so if the social security tax is quite high, because then the drop in the pension benefit due to ageing will be rather sizeable. Therefore, the second effect will dominate the first effect if the PAYG-scheme is relatively generous. Economic growth will then increase in the short run due to lower fertility, whereas the reverse holds if τ is relatively low. Third, the economy is in its new steady state as from time $t = 1$ on, with a lower rate of economic growth, so current services become more expensive relative to

future services. Hence, the social security benefit the young expect to receive when old decreases in terms of current services, which decreases the current demand for services by the young, driving l^Y up, again especially if the PAYG-scheme is rather extensive. Finally, the change in the current division of labour affects the productivity growth, and thereby the current wage and price of services. If l^Y initially increases, services become more expensive, so the elderly's real interest rate is lower, urging them to demand fewer services, pushing l^Y down (the opposite holds if l_0^Y decreases). Naturally, this second-order effect does not dominate the other effects. The second and third effects disappear if young individuals do not demand any services (i.e., if $\beta = 0$).

All this leads to the following proposition.

Proposition 3.9 *In a small open economy where young individuals also demand services, decreasing fertility will imply lower productivity growth in the short run if the PAYG-scheme is relatively small, and higher growth if it is rather generous. The long run effects are qualitatively not different from the cases considered before.*

Figure 3.3 shows the consequences of a lower fertility rate on the lifetime utility of successive generations, for different values of the social security tax ($\tau = 0$ and $\tau = 0.15$ respectively).[16]

If there is no PAYG-scheme, relatively more people will be employed in the services sector at the time of the shock. This implies a lower productivity growth, so wages decline and the elderly pay a lower price for their services, allowing them to enjoy more services which increases their utility. However, the wage that the young at time $t = 0$ receive has become lower, and thereby their lifetime income. The consumption of commodities therefore decreases, whereas the number of services enjoyed when young stays the same. Because of a lower future price of services, the number of services enjoyed when old increases. Overall, utility decreases due to ageing. The same holds for all subsequent generations.

On the other hand, if the social security scheme is relatively extensive, more people will be employed in the commodity sector at the time of the shock. This increases their productivity, and thus increases the wages, so the then living elderly can enjoy fewer services. Apart from that, their utility also decreases because of the lower pension benefit

[16] Simulations are based on the same production function as applied in Figure 3.1. Furthermore, $\beta = 1$, $\gamma = 0.75$, $\varepsilon = \frac{2}{3}$, $r = 3.25$, $\rho = 1.5$ in case of endogenous growth and n is decreased from 0.2 to 0.

they receive due to ageing. As for the young generation at that time, although they earn a higher wage, the price of current services rises just as much; furthermore, the PAYG-benefit they expect to receive when old decreases in terms of current services, both due to a lower population growth and a lower future wage growth. So they decide to save more, i.e., consume fewer services and commodities when young. When they are old, they have a higher capital income at their disposal, and face lower prices of services. This allows them to enjoy more services when retired, even though the public pension benefit has decreased.

This also holds for all future generations. They suffer utility losses because the growth rate is lower, and thereby their lifetime income. However, they are less worse off when the PAYG-scheme is generous because in that case, the reduction of l^Y due to ageing is moderate.

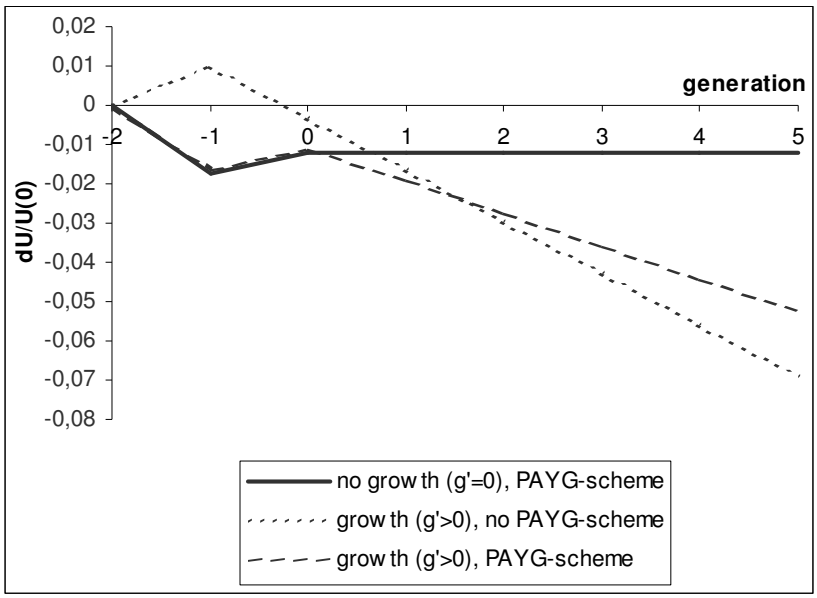

Figure 3.3 *Effect of decreasing fertility on lifetime utility in a small open economy when* $\beta > 0$

3.6.3 Fixed-benefit pensions

In the previous sections, the social security tax that young individuals pay was kept constant. So elderly individuals receive a pension benefit that decreases if the dependency

ratio rises. The retired therefore bear the full risk of adverse demographic changes. The alternative would be a social security scheme that ensures each pensioner a fixed amount of services, i.e., the pension benefit is indexed to the price of the consumption basket of an elderly individual. In that case, the PAYG-tax is flexible and responds to changes in the demographic structure and factor rewards. This section investigates the long-run effects of population ageing (i.e., increasing longevity) if such a pension scheme is in place.

If the government promises each retired person a fixed number η of services, the pension benefit at time t equals ηw_t. Balanced budget then implies a (variable) social security tax of

$$\tau_t = \eta \varepsilon_t.$$

Inserting this in the demand for services, and differentiating the equilibrium condition for the services and labour market in a *small open economy* gives the long-run change in the number of employees in the commodity sector,

$$\frac{dL^Y}{d\varepsilon} = \frac{dL^Y}{d\varepsilon}[\bar{\tau}] + \frac{(1+g)\gamma\tau(r-1-g)}{(1+\gamma\varepsilon)\left[(1+\gamma\varepsilon)(1+g)^2 - \gamma\varepsilon r(1-\tau)g'\right]},$$

where $\frac{dL^Y}{d\varepsilon}[\bar{\tau}] < 0$ is the change in the labour share of the commodity sector if the social security tax is kept constant, as given by (3.9). One can clearly see that if the economy is dynamically efficient, the decline of L^Y, and thereby of economic growth in the long run, is smaller in case of a fixed-benefit pension scheme.[17]

Increasing longevity implies that the social security tax has to rise in order to finance the higher amount of public pensions paid out to retirees. In a dynamically efficient economy, this means a decline in the lifetime income of individuals, so it decreases the demand for commodities and services and thereby offsets part of the increased demand for services that is directly caused by a higher dependency ratio.

In a *closed economy*, long-run equilibrium in the commodity market is given by $L_t^Y = (1-\eta\varepsilon_t)\left(1 - \frac{\kappa_t f'(\kappa_t)}{f(\kappa_t)}\right)$, so

$$dL_{t+1}^Y = \delta_1 d\kappa_{t+1} + \delta_2 d\varepsilon_{t+1},$$

with $\delta_1 \equiv \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}$ and $\delta_2 \equiv -\eta\left(1 - \frac{\kappa f'(\kappa)}{f(\kappa)}\right) < 0$. The labour and services markets clear if $L_{t+1}^Y = 1 - \frac{\gamma\varepsilon_{t+1} r_{t+1} w_t (1-\eta\varepsilon_{t+1})}{(1+\gamma\varepsilon_{t+1})w_{t+1}} - \frac{\gamma\eta\varepsilon_{t+1}^2}{1+\gamma\varepsilon_{t+1}}$, which results in

$$dL_{t+1}^Y = \lambda_1 d\kappa_t + \lambda_2 d\kappa_{t+1} + \lambda_3' d\varepsilon_{t+1},$$

[17] Assuming that for both pension systems, the initial steady state is the same.

where $\lambda_3' = \lambda_3 + \frac{(1+g)\gamma\tau(r-1-g)}{(1+\gamma\varepsilon)(1+g)^2-\gamma\varepsilon r(1-\tau)g'}$, and $\lambda_{1,2,3}$ are given in the Appendix. Combining these two differential equations we arrive at

$$\frac{d\kappa}{d\varepsilon} = \frac{\delta_2 - \lambda_3'}{\lambda_1 + \lambda_2 - \delta_1}.$$

Knowing that $\delta_2 = 0$ in case pension benefit are not defined, and that $\lambda_3 < \lambda_3'$, we can conclude that the capital-labour ratio rises to a smaller extent due to ageing if the social security scheme pays a fixed number of services to each retired individual. The change in the employment share of the commodity sector is

$$\frac{dL^Y}{d\varepsilon} = \delta_1 \frac{d\kappa}{d\varepsilon} + \delta_2.$$

Knowing that $\frac{d\kappa}{d\varepsilon}$ is smaller in case of a fixed-benefit scheme and $\delta_2 < 0$, it is clear that $\frac{dL^Y}{d\varepsilon}$ will be smaller if $\delta_1 \geq 0$, i.e., if $\sigma \leq 1$ (capital and labour being complements in the production of commodities), so the effects of population ageing on long-run growth are more moderate. But if $\sigma > 1$ (substitutability), such a conclusion cannot be drawn since it is not clear whether $\left|\frac{dL^Y}{d\varepsilon}\right|$ is smaller or bigger than without fixing the PAYG-benefit.

The following proposition summarises these findings.

Proposition 3.10 *In a small open economy, the negative long-run effect of ageing on growth is smaller than with fixed contributions. In a closed economy, the positive long-run effect on growth is more moderate if* $\sigma \leq 1$, *but can be smaller or larger than with fixed contributions if* $\sigma > 1$.

Hence, indexing the public pension benefit to the price of services makes that the negative long-run effect of ageing on growth in a small open economy is curbed.

3.6.4 A social planner

In the model presented above, productivity growth results from learning-by-doing spillovers in the commodity sector. This implies that the production of goods involves a positive externality, which individuals do not take into account when they make their consumption choices. In this section, we analyse this by deriving the optimal choice of a social planner who does internalise these spillover effects.

Consider a social planner at time $t = 0$ whose objective function consists of the lifetime utilities of all current and future generations (assuming ε to be constant),

$$\begin{aligned} W_0 &= \sum_{t=0}^{\infty} \theta^t U(c_{t-1}, d_t) \\ &= \sum_{t=0}^{\infty} \theta^t \left[\log(c_{t-1}) + \gamma\varepsilon \log(d_t)\right], \end{aligned}$$

where $\theta < 1$ is the social discount factor, i.e., the factor at which the planner discounts lifetime utility of future generations. Assuming full depreciation, the social planner is restricted by the following constraints,

$$\begin{aligned} K_{t+1} &= F(K_t, A_t L_t^Y) - c_t, \\ A_t &= A_{t-1}\left(1 + g(L_t^Y)\right), \\ \varepsilon d_t &= 1 - L_t^Y. \end{aligned}$$

The resulting Lagrangian is

$$\begin{aligned} \pounds_0 = \sum_{t=0}^{\infty} \theta^t \Big\{ & \log(c_t) + \tfrac{\gamma\varepsilon}{\theta} \log(d_{t+1}) \\ & + q_t \left[K_t - F(K_{t-1}, A_{t-1} L_{t-1}^Y + c_{t-1}\right] \\ & + \mu_t \left[A_t - A_{t-1}\left(1 + g(L_t^Y)\right)\right] \\ & + \lambda_t \left[\varepsilon d_t - 1 + L_t^Y\right] \Big\}, \end{aligned}$$

from which the first-order conditions follow,

$$\frac{\partial \pounds_0}{\partial c_t} = 0 \Rightarrow -\theta q_{t+1} = \frac{1}{c_t}, \tag{3.13.a}$$

$$\frac{\partial \pounds_0}{\partial L_t^Y} = 0 \Rightarrow \lambda_t = \theta q_{t+1} F_{AL} A_t + \mu_t A_{t-1} g', \tag{3.13.b}$$

$$\frac{\partial \pounds_0}{\partial d_t} = 0 \Rightarrow \lambda_t = -\frac{\gamma}{\theta d_t}, \tag{3.13.c}$$

$$\frac{\partial \pounds_0}{\partial K_t} = 0 \Rightarrow q_t = \theta q_{t+1} F_K, \tag{3.13.d}$$

$$\frac{\partial \pounds_0}{\partial A_{t-1}} = 0 \Rightarrow \mu_{t-1} = \theta q_t F_{AL} L_{t-1}^Y + \theta \mu_t (1 + g), \tag{3.13.e}$$

with $F_{AL} \equiv \frac{\partial F}{\partial (AL)} = \frac{w}{A}$ and $F_K \equiv \frac{\partial F}{\partial K} = r$.

Furthermore, the shadow prices of physical and human capital decrease each period with the rate of economic growth, so

$$\begin{aligned} q_{t+1} &= \frac{q_t}{1+g}, \\ \mu_{t+1} &= \frac{\mu_t}{1+g}, \end{aligned}$$

whereas consumption grows with this rate, i.e., $c_{t+1} = (1+g)c_t$.

Using this fact, combining equations (3.13.a)-(3.13.e) results in the following first-order condition for the social planner,

$$\begin{aligned} \left(\frac{d_t}{c_t}\right)_{Social\ Planner} &= \frac{\gamma r}{w_{t+1}} \left(\frac{(1-\theta)(1+g)}{(1-\theta)(1+g)+L^Y g'}\right) \\ &= \left(\frac{d_t}{c_t}\right)_{Individual} \left(\frac{(1-\theta)(1+g)}{(1-\theta)(1+g)+L^Y g'}\right). \end{aligned}$$

So without endogenous growth ($g' = 0$), the market solution coincides with the social optimum, but in the presence of learning spillovers, the social planner chooses a smaller number of services and a larger consumption of commodities than individuals.

However, since the structure of model is not essentially different, the consequences of ageing will not be qualitatively different. There will be quantitative differences, however. If ageing leads to an increasing consumption of services, so that growth decreases, this will be less so in case the social planner decides. If ageing leads to higher growth, a social planner will internalise the positive spillover effect, and growth will increase more. Hence, the effects on growth will be less negative or more positive.

3.6.5 Capital-induced growth

Suppose the development of labour productivity in the commodity sector not only depends on the number of employed people, but also on the capital stock they use, according to the following function,

$$A_t = A_{t-1}\left[1 + g(L_t^Y, \kappa_t)\right], \tag{3.14}$$

with $g_\kappa \equiv \frac{\partial g}{\partial \kappa} > 0$.

In a *small open economy*, the interest rate is exogenously given, so κ is also exogenous and (3.14) boils down to (3.1).

For a *closed economy*, the following proposition holds.

Proposition 3.11 *If the growth of labour productivity in a closed economy also depends positively on the capital stock, increasing longevity will lead to a higher long-run growth for a larger range of values of σ than in case growth does not depend on the capital stock.*

Proof Equation (3.8), describing equilibrium in the labour and services market, now changes to

$$L_t^Y = 1 - \frac{\gamma\varepsilon_t(1-\tau)r_t}{(1+\gamma\varepsilon_t)\left[1+g(L_t^Y,\kappa_t)\right]} - \frac{\gamma\varepsilon_t\tau}{1+\gamma\varepsilon_t}.$$

Linearising this equation gives

$$dL_{t+1}^Y = \lambda_1 d\kappa_t + \hat{\lambda}_2 d\kappa_{t+1} + \lambda_3 d\varepsilon_{t+1},$$

with $\hat{\lambda}_2 \equiv \lambda_2 + \lambda_1\left(\frac{g_\kappa\omega}{\kappa f''(\kappa)(1+g)}\right)$ and $\lambda_{1,2}$ are given in the Appendix. Notice that $\hat{\lambda}_2 > \lambda_2$, so the change in the number of individuals employed in the commodity sector, which is now given by

$$\frac{dL^Y}{d\varepsilon} = \frac{\delta\lambda_3}{\delta - \lambda_1 - \hat{\lambda}_2},$$

is (in absolute terms) smaller than before.

The change in the growth rate due to ageing is

$$\frac{dg}{d\varepsilon} = (\delta g_L + g_\kappa)\frac{\lambda_3}{\delta - \lambda_1 - \hat{\lambda}_2},$$

with $g_L \equiv \frac{\partial g}{\partial L^Y}$. This expression is negative if $\delta g_L + g_\kappa < 0 \Leftrightarrow \sigma > 1 - \frac{g_\kappa}{g_L}\frac{f(\kappa)}{\kappa f''(\kappa)} > 1$. ■

It is therefore more likely that population ageing increases economic growth in the long run if it also depends on the size of the capital stock. As we saw before, the capital-labour ratio also increases in a closed economy due to ageing. This by itself would stimulate productivity growth, and can offset the negative effect on growth of a smaller number of workers in the commodity sector if capital and labour are substitutes.

3.7 Conclusion

Elderly people typically have two sources of income at their disposal: the assets they accumulated during their working life and the transfers they receive from the current young generation. It is well known that the second source will increasingly be under pressure

with a rising dependency ratio that is due to population ageing. This does not mean, however, that the first income source is completely safe and should therefore be entirely relied upon. If many people save and thus invest, capital will become relatively abundant and the interest rate decreases, which implies a lower return on savings. As was shown in the previous chapter, this is especially so if the demand for labour-intensive services increases due to a longer life span, which makes elderly people particularly vulnerable to high inflation rates caused not only by increasing capital accumulation, but also by technological progress in the productive sector.

In this chapter, the endogeneity of economic growth was investigated. Population ageing will cause a higher demand for services, which affects growth through a change in the sectoral structure of the economy. It was shown that these effects crucially depend on the openness of the economy, and in a closed economy also on the extent to which capital and labour are substitutes in the production process. In many cases, population ageing will lead to a lower growth rate in the long run.

3.A Appendix to Section 3.4

In a closed economy, it follows from the equilibrium condition of the commodity market that at time $t+1$

$$L^Y_{t+1} = (1-\tau)\left(1 - \frac{\kappa_{t+1} f'(\kappa_{t+1})}{f(\kappa_{t+1})}\right), \tag{3A.1}$$

and from the equilibrium condition of the capital market that

$$\frac{s_t}{A_t\left(1+g(L^Y_{t+1})\right)} = \kappa_{t+1} L^Y_{t+1}. \tag{3A.2}$$

Linearising equation (3A.1) yields

$$dL^Y_{t+1} = \delta d\kappa_{t+1}, \tag{3A.3}$$

with $\delta \equiv \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}$, and linearising the equilibrium condition for the labour and services market, equation (8), at time $t+1$ gives

$$dL^Y_{t+1} = \lambda_1 d\kappa_t + \lambda_2 d\kappa_{t+1} + \lambda_3 d\varepsilon_{t+1}, \tag{3A.4}$$

where

$$\begin{aligned}
\lambda_1 &\equiv \frac{(1+g)\gamma\varepsilon(1-\tau)r\kappa f''(\kappa)}{[(1+\gamma\varepsilon)(1+g)^2 - \gamma\varepsilon(1-\tau)rg'][f(\kappa)-\kappa r]} < 0,\\
\lambda_2 &\equiv -\lambda_1 \frac{f(\kappa)}{\kappa r} > 0,\\
\lambda_3 &\equiv -\frac{\gamma(1+g)\left(r(1-\tau)+\tau(1+g)\right)}{[(1+\gamma\varepsilon)(1+g)^2 - \gamma\varepsilon(1-\tau)rg'](1+\gamma\varepsilon)} < 0.
\end{aligned}$$

Combining these two equations results in the following first-order difference equation,

$$dL^Y_{t+1} = \frac{\lambda_1}{\delta-\lambda_2} dL^Y_t + \frac{\delta\lambda_3}{\delta-\lambda_2} d\varepsilon_{t+1}. \tag{3A.5}$$

This difference equation is stable if $\left|\frac{\lambda_1}{\delta-\lambda_2}\right| < 1$. Using (3A.2) it can be shown that $\delta - \lambda_2 < 0$. This implies that the stability condition boils down to $\delta - \lambda_1 - \lambda_2 < 0$. In line with Assumption 3.II, we assume this condition to hold. Note that this is not a strong assumption. In fact, it can easily be shown that it follows from Assumption 3.III if $\sigma \geq 1$. Furthermore, simulations with a linear growth function ($g_t = \bar{g} + \rho L^Y_t$) show that the stability condition also holds for all values of σ below unity for which a steady-state equilibrium exists.

Proof Proposition 3.3 Setting $dL^Y_{t+1} = dL^Y_t = dL^Y$ and $d\varepsilon_{t+1} = d\varepsilon$ in (3A.5) gives

$$\frac{dL^Y}{d\varepsilon} = \frac{\delta\lambda_3}{\delta - \lambda_1 - \lambda_2}. \tag{3A.6}$$

Knowing that $\lambda_3 < 0$ and $\delta - \lambda_1 - \lambda_2 < 0$ (stability), it follows that $\frac{dL^Y}{d\varepsilon} \gtreqless 0$ if $\sigma \lesseqgtr 1$. Furthermore, equation (3A.3) together with (3A.6) shows that $\frac{d\kappa}{d\varepsilon} > 0$ if $\sigma \neq 1$. If $\sigma = 1$, then $\delta = 0$ so $\frac{dL^Y}{d\varepsilon} = 0$ and $\frac{d\kappa}{d\varepsilon} = -\frac{r(1-\tau)+\tau(1+g)}{(1+\gamma\varepsilon)(f(\kappa)-\kappa r)\varepsilon f''(\kappa)(1-\tau)} > 0$.

Proof Proposition 3.4 Differentiating (3A.1) at time $t = 0$ gives $\frac{dL^Y_0}{d\varepsilon} = \delta\frac{d\kappa_0}{d\varepsilon}$. Furthermore, differentiating the capital market equation (3A.2) at time $t = 0$, taking savings of the previous period as given, results in $\frac{d\kappa_0}{d\varepsilon} = -\kappa\left(\frac{g'}{1+g} + \frac{1}{L^Y}\right)\frac{dL^Y_0}{d\varepsilon}$. Combining these two expressions gives $\frac{d\kappa_0}{d\varepsilon} = \frac{dL^Y_0}{d\varepsilon} = 0$.

Short-run effect expected ageing The proof that $\frac{d\kappa_0}{d\varepsilon} = \frac{dL^Y_0}{d\varepsilon} = 0$ if the increase in ε is known at time $t = 0$ is analogous to the proof of Proposition 3.4.

3.B Appendix to Section 3.6.1

Proof Proposition 3.7 The proof of the long-run results is analogous to that of Proposition 3.3, now with

$$\begin{aligned}
\lambda_1 &\equiv \frac{(1+g)(1+n)\gamma(1-\tau)r\kappa f''(\kappa)}{[(1+\gamma)(1+n)(1+g)^2 - \gamma(1-\tau)rg']\,[f(\kappa)-\kappa r]} < 0,\\
\lambda_3 &\equiv -\frac{\gamma(1+g)r(1-\tau)}{[(1+\gamma)(1+n)(1+g)^2 - \gamma(1-\tau)rg']\,(1+n)} < 0,
\end{aligned}$$

and $d\varepsilon = dn$.

The initial effect follows from differentiating the equilibrium condition for the commodity market (equation (3A.1) with l^Y instead of L^Y) at time $t = 0$, which gives $\frac{dl^Y_0}{dn_0} = \delta\frac{d\kappa_0}{dn_0}$. Furthermore, equilibrium in the capital market is now given by $\frac{s_t}{A_t\left(1+g(l^Y_{t+1})\right)} = \kappa_{t+1}l^Y_{t+1}(1+n_{t+1})$. Taking savings as given, differentiating this equation results in

$$\frac{d\kappa_0}{dn_0} = -\kappa\left(\frac{g'}{1+g} + \frac{1}{l^Y}\right)\frac{dl^Y_0}{dn_0} - \frac{\kappa}{1+n}.$$

Combining these two expressions and rewriting gives

$$\frac{dl_0^Y}{dn} = -\left[1+\delta\kappa\left(\frac{g'}{1+g}+\frac{1}{l^Y}\right)\right]^{-1}\frac{\delta\kappa}{1+n}.$$

The last term of this expression is positive (negative) if $\sigma > (<)1$, whereas the term between brackets can numerically be shown to be always positive. Hence, $\frac{dl_0^Y}{dn} > 0$ if $\sigma > 1$, and the opposite holds if $\sigma < 1$. From this it immediately follows that $\frac{d\kappa_0}{dn} < 0$. In case of a Cobb-Douglas production function ($\sigma = 1$), $\frac{dl_0^Y}{dn} = 0$ and $\frac{d\kappa_0}{dn} = -\frac{\kappa}{1+n} < 0$.

3.C Appendix to Section 3.6.2

Linearising (3.11) around the initial steady state, we get the following expression,

$$\begin{aligned} dl_t^Y &= \frac{\gamma\varepsilon(1-\tau)rg'}{(1+\beta+\gamma\varepsilon)(1+g)^2(1+n)}dl_t^Y - \frac{\beta(1+n)\tau g'}{1+\beta+\gamma\varepsilon}dl_{t+1}^Y \\ &\quad + \frac{\gamma(1-\tau)\varepsilon r}{(1+\beta+\gamma\varepsilon)(1+g)(1+n)^2}dn_t - \frac{\beta\tau(1+g)}{r(1+\beta+\gamma\varepsilon)}dn_{t+1} \\ &\quad - \frac{(1+\beta)d^o}{(1+n)(1+\beta+\gamma\varepsilon)}d\varepsilon_t + \frac{\gamma d^y}{1+\beta+\gamma\varepsilon}d\varepsilon_{t+1}. \end{aligned}$$

In case all shocks are foreseen as of time $t-1$, we can write this expression as follows,

$$dl_{t+1}^Y = \Phi^{-1}dl_t^Y + \frac{\Delta_t}{\beta(1+n)\tau g'}, \tag{3C.1}$$

with $\Phi \equiv \frac{\gamma\varepsilon(1-\tau)r^2g' - (1+\beta+\gamma\varepsilon)r(1+g)^2(1+n)}{\beta(1+g)^2(1+n)^2\tau g'} < 0$ (due to Assumption 3.III) and $\Delta_t \equiv \gamma d^y d\varepsilon_{t+1} - \frac{(1+\beta)d^o}{1+n}d\varepsilon_t - \frac{\beta\tau(1+g)}{r}dn_{t+1} + \frac{\gamma(1-\tau)\varepsilon r}{(1+g)(1+n)^2}dn_t$. Because l^Y is a forward-looking variable and there is no backward-looking variable in this economy, the solution of (3C.1) results from iterating this equation forward. The state of the economy at time t can thus be written as a function of its future state at time $t+s$ and all shocks between $t-1$ and $t+s$,

$$dl_t^Y = \Phi^s dl_{t+1}^Y - (\beta(1+n)\tau g')^{-1}\sum_{j=0}^{s-1}\Phi^{j+1}\Delta_{t+j}.$$

Letting $s \to \infty$, the change in l_t^Y is written as a function of the change in its steady-state value and all future shocks. For the steady-state equilibrium to be well-defined, we therefore need to assume that $|\Phi| < 1$. With Assumption 3.III, the assumption that

$\beta(1+n)^2(1+g)^2\tau g' > (1+\beta+\gamma\varepsilon)r(1+g)^2(1+n) - \gamma\varepsilon(1-\tau)r^2g'$ assures that the steady-state equilibrium is asymptotically stable. Then, the change in l_t^Y can merely be written as a function of the sequence of exogenous future shocks,

$$dl_t^Y = -\left(\beta(1+n)\tau g'\right)^{-1}\sum_{j=0}^{s-1}\Phi^{j+1}\Delta_{t+j}.$$

Chapter 4

Pension Reform in a Two-Sector Growth Model

4.1 Introduction

Analyses on the sustainability of current social security arrangements are usually based on a comparison of the rates of return of both funded and unfunded schemes. This basically leads to the conclusion that as long as the economy is dynamically efficient, a public pension and health care scheme that is financed on a pay-as-you-go (PAYG)-basis is harmful for future generations, in the sense that they would gain from a switch to more funded schemes. This would even hold more if the population ages, since this deteriorates the internal rate of return of an unfunded scheme. An important aspect in this context that is often neglected is the effect of social security reform on economic growth, and thereby on the well-being of current and future generations, which is the focus of this chapter.

The traditional view on social security reform is that it stimulates savings and thereby the accumulation of capital. If this accumulation is accompanied by positive spillover effects in the production process, this would imply a higher growth rate. Focusing on human capital as the engine of growth, however, modifies this strong conclusion, as we shall see in the next section. In this chapter, we also raise the question how a switch to a more funded pension scheme would affect economic growth, by applying the same two-sector model as presented in the previous chapter, which included endogenously determined economic growth. The focus is therefore on the effects of privatisation with respect to the

sectoral structure of the economy, and on economic growth for different degrees of openness of the economy. The capital-intensive sector produces homogeneous consumption and investment commodities that are demanded by young individuals, whereas elderly derive utility from labour-intensive services. Consequently, a change in the intergenerational redistribution and demographic composition of the economy will affect the relative aggregate demand for services and commodities. The structure of the economy, thereby the growth rate and the well-being of successive generations, are thus influenced.

We find that the effects of privatisating social security crucially depend on the openness of the economy. In a small open economy that faces an exogenously given interest rate (that exceeds the rate of economic growth), the short-run effects differ from the long-run consequences. Initially, privatisation immediately decreases the demand for services by the old and thus leads to a higher concentration of labour in the productive sector, which stimulates economic growth. In the long run, however, privatising social security increases the lifetime income of generations born at or just after the time of the decrease in the PAYG-tax. This stimulates their demand for both services and commodities. As services cannot be imported, this implies that more labour will be employed in the services sector and the domestic production of commodities is reduced. Consequently, economic growth is eventually lower than it would have been without privatisation, putting future generations at a disadvantage.

On the other hand, if the economy is closed, the interest rate is endogenously determined by the interaction of savings and investments. In the short run, privatisation reduces the income of the current old and thereby increases the employment in the commodity sector, which implies a higher growth rate. In the long run, the overall demand for both commodities and services increases. Because labour is employed in both sectors, and capital only in one, labour becomes relatively more scarce so that wages increase, and thereby the price of services. This reduces the demand for services, thus stimulating productivity growth in the commodity sector. Furthermore, reducing an unfunded social security scheme increases savings and the capital stock, which attracts employees to the commodity sector as they can earn a higher wage. So also in the long run, growth is stimulated by privatisation in a closed economy.

The rest of this chapter is organised as follows. Section 4.2 gives an overview of the literature that deals with the influence of unfunded pension schemes on economic growth. In Section 4.3, we analyse both the long-run and short-run consequences of downsizing

the unfunded social security scheme in a small open economy, whereas Section 4.4 deals with the case of a closed economy. Section 4.5 concludes.

4.2 Related literature

Whether a switch to a more funded pension scheme is beneficial for growth basically boils down to the question how unfunded pensions affect growth. This question has received a lot of attention in recent economic literature. Like the analyses on the effects of population ageing, we can divide the broad literature according to the assumed source of growth: capital accumulation and human capital accumulation.

Growth through capital accumulation

Conventional studies on the relation between social security and growth focus on the impact of a PAYG-scheme on savings. As was demonstrated in e.g. Feldstein (1974, 1996b), transfers from the young to the old, such as unfunded social security, discourage private savings, resulting in less capital accumulation and a lower output level (see also e.g. Kotlikoff, 1979, and Jones and Manuelli, 1992). Laitner (1988) shows that if individuals are altruistic towards their children, unfunded schemes appear less harmful in this respect. If productivity growth is modeled by assuming an externality in production (as in Romer, 1986, and Lucas, 1988), where production is characterised by constant returns to scale at the aggregate level, unfunded social security will decrease the rate of economic growth, (see Saint-Paul, 1992, and Wiedmer, 1996).[1]

However, Zhang (1995a) and Wigger (1999) include endogenous fertility and find that unfunded social security may stimulate growth. This is also the result of Wigger (2001), which views the size of the PAYG-scheme as the outcome of a political process.

Growth through human capital accumulation

If economic growth results from the accumulation of human capital (as initiated by Uzawa, 1965, and Lucas, 1988), the effect of PAYG-pension schemes on growth may be positive.

[1] In that case, a Pareto-improving transition to a more funded scheme is possible, see Belan *et al.* (1998), Corneo and Marquardt (2000) and Gyárfás and Marquardt (2001).

Marchand *et al.* (1996) show that if population ageing occurs, unfunded social security can be welfare improving.

In Sala-i-Martin (1996), skills depreciate with age and therefore, elderly workers cause a negative externality on the productivity of their young colleagues. By introducing unfunded social security, the elderly are induced to retire, which raises aggregate output if the difference between the skills of the young and the old is large enough. In Kemnitz (2000), higher educational expenditures imply higher future wages, and thereby also higher expected pension benefits. Hence, unfunded social security creates incentives to invest in public education, and thereby stimulates growth. Kemnitz and Wigger (2000) consider an unfunded social security program where the size of the transfers to a particular old individual is tied to his own human capital. In that case, individuals are also inclined to invest more in human capital and thus implicitly take the positive external effect of their education on the productivity of future generations into account, which leads to higher economic growth.

Studies that include human capital investments with altruism of parents towards their children also find mixed results. In Pecchenino and Utendorf (1999), parents derive utility from educating their children. If the social security tax is increased and the economy is dynamically efficient, lifetime income decreases and thereby all expenditures are reduced, including those on children's education, leading to a reduced growth rate. However, Sánchez-Losada (2000) considers a trade-off between the accumulation of physical and human capital. If savings are high compared to the physical capital stock, unfunded social security can increase growth, as it leads to more investments in human capital, which offsets the decrease in physical capital due to lower savings. Kaganovich and Zilcha (1999) focus on the allocation of government resources between public education and social security, and also find a possibility for growth-enhancing social security. Lambrecht *et al.* (2001) stress the importance of liquidity constraints that prevent parents to leave bequests to their children and to invest optimally in education. Through unfunded social security, resources are transferred from the child to the parent, which allows the latter to spend more on education. If this positive effects dominates the negative effect of the PAYG-scheme on savings, growth will increase.

Finally, Zhang (1995b) analyses the effect of social security on human capital investments if people are altruistic and fertility is endogenous. In that case, individuals both derive utility from the number of children they raise and their quality, as reflected by their human capital. It appears that the effect of PAYG-pensions on growth depends on taste parameters: if the taste for the welfare of a child (driven by altruism) is not much

weaker than the taste for the number of children, unfunded social security will decrease fertility and increase human capital investments per child, thereby enhancing growth.

So as soon as we consider altruism, endogenous fertility or human capital accumulation as the engine of growth, the straightforward conclusion that unfunded social security is harmful for economic growth because it negatively affects the (physical) capital stock is not so unambiguous any more. It is therefore highly questionable whether a switch to a more funded pension scheme stimulates growth. The model presented below gives another reason to modify the alleged advantages of privatising social security.

4.3 A small open economy

The model we use is the one presented in the previous chapter, as described in Section 3.3. Since we do not focus here on the effects of ageing, we assume the dependency ratio to be constant and equal to 1, i.e., $\varepsilon_t = 1$ and $n_t = 0\ \forall t$ throughout the chapter. Furthermore, Assumptions 3.I-3.IV remain and growth depends on the number of people employed in the commodity sector.

In a small open economy, the factor rewards are exogenously determined. This implies that individual savings are given by

$$s_t = \frac{\gamma(1-\tau_t)w_t}{1+\gamma} - \frac{\tau_{t+1}w_{t+1}}{(1+\gamma)r}. \tag{4.1}$$

The labour market is in equilibrium if the number of young individuals employed in the commodity sector equals $L_t^Y = 1 - L_t^D = 1 - d_t$, where L^D (d) is the number of services supplied (demanded). Knowing that services are only bought by the elderly at a price p from the return on their savings and from their public pension benefits, the employment share of the commodity sector can be written as

$$L_t^Y = 1 - \frac{rs_{t-1} + \tau_t w_t}{p_t} \tag{4.2}$$

$$= 1 - \frac{r\gamma(1-\tau_{t-1})}{(1+\gamma)\left(1+g(L_t^Y)\right)} - \frac{\gamma\tau_t}{1+\gamma}. \tag{4.3}$$

The effects of a change in the social security tax can be traced by linearising (4.3) around the initial steady state. In case the small open economy is unexpectedly hit by a shock

(e.g. a lower value of τ), L^Y will jump to a new value at the time of the shock, and jumps to its new steady-state value after one period, which can easily be seen from the fact that equation (4.3) only contains one parameter (τ_{t-1}) and no variable (L^Y) that refers to the past.

Long-run effects

According to conventional wisdom, privatising social security would cause some short-run pain, but also bring substantial long-run gains. However, the long-run consequences of reducing the social security tax are not as straightforward as they seem. In particular, a reduction of the social security tax can change the production structure of the economy, and thereby influence the rate of economic growth. The following proposition holds for the two-sector model presented above.

Proposition 4.1 *A shift to a more funded pension scheme will eventually reduce economic growth and thus lead to a lower level of utility in a small open economy.*

Proof The long-run impact of the size of the PAYG-scheme (τ) on the economy can be traced by comparative statics of (4.3), which yields[2]

$$\frac{dL^Y}{d\tau} = \frac{\gamma(1+g)\,(r-1-g)}{(1+\gamma)(1+g)^2 - \gamma(1-\tau)rg'} > 0.$$

This expression is positive under Assumptions 3.III and 3.IV. ■

In the long run, a reduction of the PAYG-scheme has three effects on the allocation of labour over the two sectors, and thereby on the rate of economic growth. First, if $r > 1+g$, lifetime income increases when the PAYG-tax decreases, which results in a higher demand for both services and commodities. As savings increase, so will investments abroad. Consequently, net foreign assets increase, which enables the economy to finance increased imports (or decreased exports) of commodities from abroad. Domestic production of commodities will be reduced, which releases labour from this sector, and more people will be employed in the services sector. This is necessary because the increased demand for services has to be met by domestic labour. As L^Y decreases, technological progress or knowledge creation slows down. Second, since this implies a lower economic growth,

[2] Omitting time subscript denotes the (initial) steady-state value of the variable or parameter.

the internal rate of return of the PAYG-system decreases. So for a given social security tax, lifetime income decreases,[3] which leads to a lower demand for services, thus partially offsetting the initial decrease of L^Y. Third, productivity grows at a lower rate, and so do wages and thereby the price of services. Old-age consumption thus becomes cheaper. In other words, the old are confronted with a higher real interest rate, implying a higher demand for services. This also has a negative effect on the employment share of the commodity sector, and accordingly on economic growth.

Short-run effects

If the social security tax decreases unexpectedly at time $t = 0$, the economy will be in its new steady state as from $t = 1$ on.[4] At the time of the shock, however, the effects differ from its long-run consequences since the old at that time did not foresee the change in the PAYG-benefit, nor the changing rate of economic growth (which reflects the increase of the price of services) when they made their savings decision. As a matter of fact, the short-run effects are the opposite of the long-run effects, as stated in the following proposition.

Proposition 4.2 *In a small open economy, an unexpected permanent reduction of the social security scheme initially increases economic growth.*

Proof At time $t = -1$, individual savings are given by $s_{-1} = \frac{\gamma(1-\tau)w_{-1}}{1+\gamma} - \frac{\tau w_{-1}(1+g)}{(1+\gamma)r}$, where τ (g) denotes the social security tax (growth rate) at the time before the shock occurred. Because the PAYG-tax is changed unexpectedly at time $t = 0$, these savings are given, and equation (4.2) boils down to

$$L_0^Y = 1 - \frac{r\gamma(1-\tau) - \tau(1+g)}{(1+\gamma)\,(1+g(L_0^Y))} - \tau_0.$$

The short-run effects on the employment in the commodity sector can be seen from totally differentiating this equation, giving

$$\frac{dL_0^Y}{d\tau_0} = -\frac{(1+\gamma)(1+g)^2}{(1+\gamma)(1+g)^2 - r\gamma(1-\tau)g' + \tau(1+g)g'},$$

[3] Naturally, this second-order effect cannot dominate the first-order effect for a stable equilibrium.

[4] If the privatisation is announced at time $t = 0$ to take place one period later, no changes in L_0^Y will occur: young individuals decide to save more and consume less, so the total demand for commodities is unchanged. Because no shock yet occurs, the elderly do not change their demand for services. In case young individuals also demand services, L_0^Y will increase.

which is negative if Assumption 3.III holds. ■

The reason for an increase in the labour share of the commodity sector in the short run is simple: at the time the PAYG-tax is unexpectedly cut down, old individuals will have to economise the entire decrease of their pension benefits on services, so the number of employees in the commodity sector will increase.[5] This has an upward effect on the wage and thereby on the price of services that the elderly buy at time $t = 0$, whereas the interest rate does not change. So the retired will use fewer services, reinforcing the initial rise in L_0^Y.

Figure 4.1 shows the change in lifetime utility that privatising social security brings about in a small open economy.[6]

If endogenous-growth effects are absent ($g' = 0$), the elderly at the time of the shock will suffer from a reduction in their pension benefit, whereas the current young and all future generations gain equally much because the wage and interest rate do not change and accordingly, their lifetime income increases to the same extent.[7]

[5] If also young individuals demanded services, they would demand more services and commodities at the time of the shock because their lifetime income increases as long as $r > 1 + g$ holds, which would push L_0^Y down. However, the young would spread this positive income effect over their entire life, so the increase in the provision of services to the young would be smaller than the reduced demand by the current old, and L_0^Y increases anyway. Furthermore, the economy will be in its new steady state in the subsequent period, which is characterised by a lower rate of economic (and thereby wage) growth. Hence, the price of the services that the current young demand in the next period will be lower. This would induces them to substitute current services for future services, thus reinforcing the increase of L^Y at time $t = 0$.

[6] The simulations in Figures 4.1 and 4.2 are based on a productivity growth that is assumed to be a linear function of the number of employees in the productive sector: $g_t = \bar{g} + \rho L_t^Y$, where $\bar{g}$ is chosen such that the equilibrium growth rate is equal to 1 (which corresponds to an annual economywide growth rate of about 2.3%). The cases considered are $\rho \to 0$ and $\rho = 2$ respectively. The production function of commodities is given by $Y_t = \left[0.3K_t^{-0.33} + 0.7(A_t L_t^Y)^{-0.33}\right]^{-3}$, and the utility function is $U_t = \log(c_t^y) + \log(d_{t+1}^o)$. Furthermore, the interest rate in the small open economy is set equal to the equilibrium rate of the closed economy that the next section deals with. The social security tax is decreased from 20% to 10%.

[7] Compensating these elderly for this loss would require an amount equal to the present value of the gains of future generations (see Verbon, 1988, and Breyer, 1989).

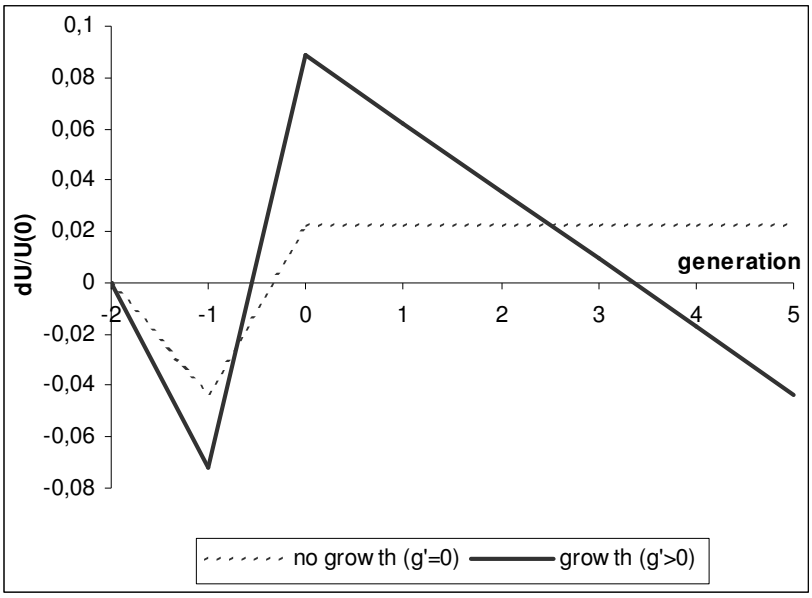

Figure 4.1 *Effect of privatisation on lifetime utility in a small open economy*

In case the production of commodities is characterised by endogenous productivity growth ($g' > 0$), the effects are different. At the time the privatisation policy is implemented ($t = 0$), more people will be employed in the commodity sector, which stimulates productivity growth. Hence, wages rise and services become more expensive. This was not foreseen by the current old when they made their savings decision, so they can demand fewer services due to both a lower pension benefit and higher prices, and are therefore worse off. Individuals born at $t = 0$ experience three effects. First, downsizing the social security scheme increases their lifetime income. Second, because L_0^Y increases, the remuneration of their labour duties increases, and third, fewer people will be employed in the commodity sector at time $t = 1$, so the services they consume when old are cheaper. All these elements substantially increase the well-being of the generation that is born at the time of the social security reform. All future generations also benefit from a higher lifetime income that is directly caused by a lower social security tax, as well as from the higher labour productivity they 'inherit' from generation $t = 0$. The number of services they enjoy increases, which has a positive effect on lifetime utility. However, these future generations face a lower value of L^Y, so their own labour productivity grows at a smaller rate. After some time, this effect will become dominant, and the consumption of

commodities by the young will be lower than before the reduction of the public pension scheme. How long it takes before individual well-being actually deteriorates depends on both individual preferences and economic factors. First, if agents have a rather strong preference for services (i.e., if γ is high), the utility gain of more services is significant, and the utility loss of fewer commodities is small. It then takes quite a long time before the utility loss of fewer commodities, due to a lower growth rate, outweighs the higher utility from more services. Furthermore, if g' takes a high value, the fall in productivity growth is high and the decrease in utility will sooner occur. Third, if the interest rate is relatively low or the initial rate of economic growth high (and the economy is close to the golden rule), a decrease in the social security tax causes a relatively moderate rise in lifetime income. This implies a modest contraction of the commodity sector, and thereby a small decrease of productivity growth, so that utility decreases are delayed.

We can thus conclude that in a small open economy, endogenous-growth effects make the variance of the gains and losses caused by privatisation higher: those who are retired at the time the reform is implemented suffer more, whereas those who are young gain more. Moreover, future generations will not benefit but suffer from such privatisation in the long run.

4.4 A closed economy

A decrease of the unfunded part of social security creates an incentive for individuals to save more for their old age. In a closed economy, this will result in a higher physical capital stock and consequently affect factor rewards. This section focuses on the effects of privatisation in such an economy.[8]

As in a small open economy, the labour and service markets both clear if

$$L_t^Y = 1 - d_t. \tag{4.4}$$

However, no international trade of commodities is possible in a closed economy, so the total domestic demand for commodities, which consists of consumption and investment goods, must equal the aggregate domestic production, i.e., $Y_t = c_t + s_t = w_t(1 - \tau_t)$, or

$$L_t^Y = (1 - \tau_t)\left(1 - \frac{\kappa_t f'(\kappa_t)}{f(\kappa_t)}\right). \tag{4.5}$$

[8]Note that this framework is also applicable to the case of many (small) open economies that simultaneously implement the same privatisation policy.

Finally, since there is no scope for international lending and borrowing, aggregate savings are entirely invested domestically, and the interest rate is determined endogenously. If capital fully depreciates in one period, this implies that the capital market clears when $s_t = K_{t+1}$ holds, or

$$\frac{s_t}{A_t\left(1+g(L^Y_{t+1})\right)} = \kappa_{t+1}L^Y_{t+1}. \tag{4.6}$$

Combining these equations gives a two-dimensional non-linear system in L^Y_t and κ_t. Because the capital stock is determined by savings in the previous period, we get a saddle-point stable system with one forward-looking (or jump) variable, L^Y, and one backward-looking (or state) variable, κ. The effects of a change in the social security tax are analysed by linearising the equilibrium conditions around the initial steady state. We assume this steady state to be saddle-point stable.

Long-run effects

In the previous section it was shown that for a small open economy characterised by perfect capital mobility, reducing the PAYG-scheme has a negative impact on the rate of economic growth in the long run, because it increases the demand for services and import goods from abroad. Thus, fewer people will be employed in the productive sector and growth effects are smaller. In a closed economy, however, international lending and borrowing is not possible, and the capital stock is entirely determined by domestic savings. As a consequence, the long-run effects of privatising social security on the rate of economic growth are quite different, as stated in the following proposition.

Proposition 4.3 *A shift to a more funded scheme will eventually stimulate economic growth and thus lead to a higher level of utility in a closed economy. The (effective) capital-labour ratio rises in the long run.*

Proof See Appendix. ■

A lower social security tax increases the lifetime income of individuals. For a given price level and interest rate, this implies a higher demand for both services and commodities. Because capital is only employed in one sector, and labour in both, labour will become relatively scarcer, which pushes the wage up. As this implies an increase in the price of services, the relative demand for services will decrease, so the employment share of the

commodity sector grows and productivity growth increases. Furthermore, a lower social security tax also increases savings and thereby stimulates the accumulation of physical capital, which is only used in the commodity sector. This increase in the capital stock outweighs the rise in employment in the commodity sector so that the capital-labour ratio rises. This makes labour more productive in the commodity sector and consequently, wages rise and the services sector contracts.

Note that the decrease in the provision of services implies that individuals can enjoy fewer services when they are old, despite the higher lifetime income and increased savings. This is due to the strong decrease in the real rate of interest $\left(\frac{r_t}{p_t}\right)$. The higher lifetime utility that privatisation causes in the long run is therefore solely due to the increased consumption of commodities when young. Consequently, this rise in lifetime utility goes along with a decrease in equity between the old and the young living in the same period: the young profit from the higher savings of the previous generation, while this old generation itself effectively gets poorer.

Short-run consequences of lower transfers

A sudden change in the social security tax at time $t = 0$ initially only affects the economy through a change in the division of labour over the two sectors since the size of the capital stock is determined by the savings of the previous period.[9] In the subsequent periods savings and thus the capital stock gradually move to the new steady-state level, and so does the number of employees in the commodity sector.

Proposition 4.4 *In a closed economy, an unexpected switch to a more funded pension scheme causes more people to be employed in the commodity sector and thus stimulates growth in the short run. Consequently, the (effective) capital-labour ratio decreases.*

Proof See Appendix. ■

Privatising social security thus immediately increases the growth rate because the purchasing power of the initial elderly is reduced, implying a smaller services sector. On the other hand, κ initially decreases because both A_0 and L_0^Y increase, and the physical capital stock is fixed in the short run. This implies that wages may well fall in the short run if τ is reduced, making services cheaper. This will be so if growth effects are not

[9] In case of an expected shock, no change in L_0^Y nor κ_0 occurs (see Appendix).

very substantial and if labour and capital are complements in the production process of commodities (i.e., if the elasticity of substitution is low).

The decrease in τ leads to higher savings, thus to a higher capital-labour ratio and higher wages in the next period. The higher wages raise savings further so that the capital-labour ratio in the subsequent period is again higher. This process of a rising capital stock also implies that more people are attracted to be employed in the commodity sector, so also L^Y steadily rises. Therefore, the number of services enjoyed by elderly individuals decreases, whereas savings gradually rise. That is, although people save more, this does not allow them to consume more services when they have grown old. The reason for this is that the only way to transfer purchasing power to old age is through investment in capital that is only useful for the production of commodities, something the elderly do not want to consume. Instead, they demand more services, which increases the price of services, and together with a lower interest rate offsets the effect of the increase in savings. Figure 4.2 shows the effects of a shift to more funded pensions on lifetime utility in a closed economy (the effects for a small open economy, as shown in Figure 4.1, are also displayed).

First consider the case without growth ($g' = 0$). Because at time $t = 0$ the capital-labour ratio decreases, the elderly receive a higher interest on their savings, and wages decrease which makes their services cheaper. This partly offsets the decrease of their public pension benefit, so they are less worse off than they would be in a small open economy. Individuals born at the time of the privatisation ($t = 0$), however, are confronted with a lower wage when they are young, and also save more, resulting in a growing capital stock. This causes the capital-labour ratio to be higher at the time they have become old, so wages and thereby the price of future services will be higher. Overall, these individuals suffer from the social security reform, contrary to their contemporaries in a small open economy.

Later generations will experience increasingly higher wages though, and are therefore better off as long as the economy stays dynamically efficient. The consumption of commodities when young increases, but due to a lower PAYG-benefit, a higher wage and a lower interest rate, the number of services received when old decreases. So the utility gap between the young and old increases. If individuals care a lot about an equal distribution of well-being over their lifetime, or if there is some minimum number of services a person needs when old, they will try to prevent the increase of this utility gap from happening

by saving more. However, this will increase the capital-labour ratio further, so wages rise even more and the interest decreases to a greater extent, which only increases the gap further. The economy will thus soon end up in a situation of dynamic inefficiency, and the government can increase welfare by running a PAYG-scheme. Yet in Figure 4.2, the economy stays dynamically efficient (by assumption). Because the capital-labour ratio increases, it moves closer to the golden rule level. This does not happen in a small open economy, so future generations eventually gain more from privatisation in a closed economy as compared to a small open economy.

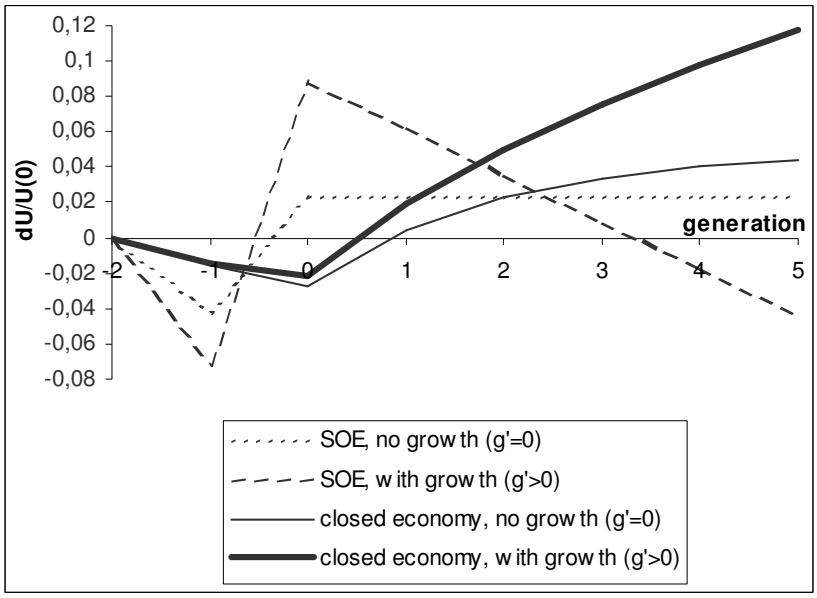

Figure 4.2 *Effect of privatisation on lifetime utility*

Figure 4.2 also shows the change in utility in case there is endogenous growth. The higher it is, the more wages rise at time $t = 0$ due to the initial increase of L^Y in a closed economy. At the same time, however, it makes the capital stock that the elderly at time $t = 0$ own more productive, as the employees apply it more productively, and this raises the interest rate. The difference with the case in which $g' = 0$ is therefore very small for the retired at time $t = 0$. The young at the time of the transition, however, are better off (or less worse off) if g' is higher, and naturally the same holds for all future generations since L^Y increases in the long run. Therefore, the welfare effects of privatising social security depend rather crucially on the openness of the economy if growth effects in

the productive sector are quite important. Again, the consumption of commodities when young increases, whereas the number of services enjoyed when old decreases. Endogenous growth adds to this growing utility gap between young and old.

4.5 Conclusion

In this chapter we showed that the welfare effects of reforming social security are not as clear as is often suggested. Because individuals' tastes for tradable commodities and non-tradable services differ with age, a change in the intergenerational redistribution causes a change in the production structure of the economy. Depending on the openness of the economy, this affects international trade and the extent to which the economy specialises in the provision of services. As the accumulation of (technological) knowledge typically takes place in the capital-intensive sector, the rate at which productivity grows subsequently changes and thereby the welfare of future generations.

In particular, in a small open economy with endogenous growth, privatising social security as advocated by Feldstein will lead to a dilemma: lifetime income will initially rise more than with exogenous technological progress, but this rise in wealth will lead to a kind of "Dutch disease": the import of commodities rises and the country specialises more in the production of non-tradable low-tech services, which lowers growth.[10] That is, the larger welfare gains for the current young come at the cost of lower welfare for generations born in the (distant) future. Moreover, endogenous-growth effects increase the welfare loss for the current old caused by the privatisation. So in an open economy, introducing endogenous growth intensifies the difference in position between the two "parties" that have to decide on Feldstein's proposal in the political arena, i.e., the current young and the current old. At the same time, however, it may weaken the position of the young, as it deprives them of the argument that privatisation will unambiguously benefit future generations.

In case of a closed economy, the effects are quite different. Increased savings now raise the capital stock and thus increase wages, i.e., the price of services. As a consequence, privatisation does not lead to an increase in the relative importance of the services sector

[10] A similar shift of labour from tradables production to non-tradable services and a subsequent productivity loss is found in Smulders and Van de Klundert (2001) who analyse the effects of the initial distribution of financial wealth across countries on long-run growth and cross-country productivity levels.

but causes this sector to shrink, thus stimulating growth. As a result, it favours future generations. However, this comes at the cost of a decrease in welfare of both current generations: not only the old but also the young will lose. Endogenous-growth effects reduce the welfare loss for the current young and reinforce the gains for future generations without harming the current elderly. This will not affect decision making in a representative democracy where decisions are made by current generations that do not take the welfare effects of future generations into account. It only increases the costs of myopic decision making in terms of the welfare of future generations.

Another conclusion we can draw is that international policy coordination with respect to privatising social security is beneficial, a point that also Pemberton (1999) made. A small open economy acting on its own would suffer in the long run if endogenous-growth effects are present. But if all economies implement the same social security reform, the effects of a closed economy are applicable and long-run utility rises.

4.A Appendix to Section 4.4

Linearising the equilibrium condition for the commodity market at time $t+1$, equation (4.5), yields

$$dL^Y_{t+1} = \delta_1 d\kappa_{t+1} + \delta_2 d\tau_{t+1}, \tag{4A.1}$$

with $\delta_1 \equiv \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)}$ and $\delta_2 \equiv \frac{\kappa r - f(\kappa)}{f(\kappa)}$. Linearising (4.4), which describes equilibrium in both the labour and services market, at time $t+1$ around the initial steady state gives

$$dL^Y_{t+1} = \lambda_1 d\kappa_t + \lambda_2 d\kappa_{t+1} + \lambda_3 d\tau_t + \lambda_4 d\tau_{t+1}, \tag{4A.2}$$

where $\lambda_1 \equiv \frac{(1+g)\gamma(1-\tau)r\kappa f''(\kappa)}{[(1+\gamma)(1+g)^2-\gamma(1-\tau)rg'][f(\kappa)-\kappa r]} < 0$, $\lambda_2 \equiv -\lambda_1 \frac{f(\kappa)}{\kappa r} > 0$,
$\lambda_3 \equiv \frac{(1+g)\gamma r}{(1+\gamma)(1+g)^2-\gamma(1-\tau)rg'} > 0$ and $\lambda_4 \equiv -\lambda_3 \frac{1+g}{r} < 0$.

Combining these two equations yields the following first-order difference equation,

$$dL^Y_{t+1} = \frac{\lambda_1}{\delta_1 - \lambda_2} dL^Y_t + \frac{\delta_1\lambda_4 - \delta_2\lambda_2}{\delta_1 - \lambda_2} d\tau_{t+1} + \frac{\delta_1\lambda_3 - \delta_2\lambda_1}{\delta_1 - \lambda_2} d\tau_t. \tag{4A.3}$$

This difference equation is stable if $\left|\frac{\lambda_1}{\delta_1 - \lambda_2}\right| < 1$. Using (4.6) it can be shown that $\delta_1 - \lambda_2 < 0$. This implies that the stability condition boils down to $\delta_1 - \lambda_1 - \lambda_2 < 0$. In line with Assumption 3.II, we assume this condition to hold. Note that this is not a strong assumption. In fact, it can easily be shown that it follows from Assumption 3.III if $\sigma \geq 1$. Furthermore, simulations with a linear growth function ($g_t = \bar{g} + \rho L^Y_t$) show that the stability condition also holds for all values of σ below unity for which a steady-state equilibrium exists.

Proof Proposition 4.3 Setting $dL^Y_{t+1} = dL^Y_t = dL^Y$ and $d\tau_{t+1} = d\tau_t = d\tau$ in (4A.3) gives

$$\frac{dL^Y}{d\tau} = \frac{\delta_1\lambda_3 - \delta_2\lambda_1 + \delta_1\lambda_4 - \delta_2\lambda_2}{\delta_1 - \lambda_1 - \lambda_2}. \tag{4A.4}$$

The denominator of this equation is negative. Simulations with a linear growth function ($g_t = \bar{g} + \rho L^Y_t$) show that the nominator of (4A.4) is always positive, so $\frac{dL^Y}{d\tau} < 0$.

Furthermore, equation (4A.2) becomes in steady state $\left(\lambda_1\left(1 - \frac{f(\kappa)}{\kappa r}\right)\right)\frac{d\kappa}{d\tau} = \frac{dL^Y}{d\tau} - \lambda_3\left(1 - \frac{1+g}{r}\right)$. Knowing that $\lambda_1, \frac{dL^Y}{d\tau} < 0$, $\lambda_3 > 0$ and $1 + g < r$, it follows that $\frac{d\kappa}{d\tau} < 0$.

Proof Proposition 4.4 Differentiating the commodity market equation (4.5) at time $t = 0$ gives $\frac{dL_0^Y}{d\tau} = \frac{(1-\tau)(\sigma-1)\kappa f''(\kappa)}{f(\kappa)} \frac{d\kappa_0}{d\tau} - \frac{f(\kappa)-\kappa r}{f(\kappa)}$. Furthermore, differentiating the capital market equation (4.6) at time $t = 0$, taking savings of the previous period as given, results in $\frac{d\kappa_0}{d\tau} = -\kappa \left(\frac{g'}{1+g} + \frac{1}{L^Y} \right) \frac{dL_0^Y}{d\tau}$. Combining these two expressions and rewriting gives

$$\frac{dL_0^Y}{d\tau} \left[1 + \frac{(1-\tau)(\sigma-1)\kappa^2 f''(\kappa)}{f(\kappa)} \left(\frac{g'}{1+g} + \frac{1}{L^Y} \right) \right] = -\frac{f(\kappa) - \kappa r}{f(\kappa)}.$$

The RHS of this expression is always negative, whereas the LHS can numerically be shown to be always positive. Hence, $\frac{dL_0^Y}{d\tau} < 0$. From this it immediately follows that $\frac{d\kappa_0}{d\tau} > 0$.

Short-run effect expected privatisation If the reduction in τ is announced at time $t = 0$ to take place at $t = 1$, the short-run change in the capital-labour ratio can be found by again differentiating (4.6), taking savings as given, which gives $\frac{d\kappa_0}{d\tau} = -\kappa \left(\frac{g'}{1+g} + \frac{1}{L^Y} \right) \frac{dL_0^Y}{d\tau}$. Furthermore, (4A.1) boils down to $\frac{dL_0^Y}{d\tau} = \delta_1 \frac{d\kappa_0}{d\tau}$. This implies that $\frac{d\kappa_0}{d\tau} = \frac{dL_0^Y}{d\tau} = 0$. As in a small open economy, young individuals decide to save more, but because they do not demand services, this merely implies a lower consumption demand for commodities and an equally higher investment demand for commodities, leaving the labour division unchanged.

Chapter 5

International Spillover Effects of Unfunded Pensions and Population Ageing

5.1 Introduction

It is widely recognised that increasing dependency ratios will cause serious economic effects in the next decades. The size and design of national intergenerational transfer schemes, such as public pensions, will play an important role in the extent to which the welfare of different generations is going to be affected. Pay-As-You-Go (PAYG) pension schemes, for instance, are vulnerable to population ageing, as it will either have to lead to increasing contribution rates or to lower benefits paid out to the elderly. But also funded schemes will are vulnerable: since population ageing implies that capital becomes relatively abundant, the rate of return on which these funds so heavily depend, decreases. Furthermore, unfunded social security has a negative impact on savings (see Feldstein, 1974 and 1996b, for evidence), and thereby on the capital stock, which subsequently influences factor rewards in a closed economy.

However, since capital markets are increasingly liberalised and integrated, such a closed economy does not seem to be realistic for a single country. Consequently, countries will also be affected by demographic developments and pension policies implemented abroad. A good example is the European Union (EU), where the member states run different pension schemes. In countries like Germany, Italy and France, pensions are mostly financed

on a PAYG-basis, whereas the United Kingdom and The Netherlands have substantially accumulated assets to finance their pensions, and are therefore particularly sensitive to interest rate changes. Because capital moves across borders, national policies that affect savings behaviour and factor rewards in one country also have implications for other countries. This, together with the prospect of an increasing share of elderly in the population, raises concerns about the vulnerability of nations with extensive funded schemes to developments in other regions.

On the other hand, population ageing in not a phenomenon that occurs simultaneously and on an equal scale. In the industrialised countries, for instance, ageing will be most severe in Japan, Italy and Spain, and much less intense in the United States. The aim of this chapter is to investigate to what extent differences in both the size of unfunded pension schemes and population ageing cause international spillover effects and accordingly affect the well-being of current and future generations in various countries. The main question we focus on is whether or not a country benefits from the fact that its capital market is integrated with that of another country, which has another way of financing its pensions and (possibly) experiences a different demographic development. Because countries vary in various ways from each other, we first disentangle these features and describe theoretically the effects of the each aspect separately (Sections 5.3-5.5). Subsequently, all aspects are brought together in a numerical example in the last section.

The rest of this chapter is organised as follows. Section 5.2 describes and discusses some empirical facts and related (theoretical) literature about the international (spillover) effects of ageing and social security schemes. In Section 5.3, we present a two-overlapping-generations model with two regions. It is shown that perfect capital mobility does not necessarily benefit all regions. In particular, an economy of which the size of its PAYG-scheme is not much smaller than that of the other region, would be better off in the long run if it was a closed economy. Furthermore, this section focuses on the question whether it is always beneficial from a long-run perspective for a country to (partly) switch to a more funded pension scheme if the interest rate exceeds the rate of economic growth. It turns out that such a reform is indeed increasing the utility of future generations in the region with the most sizeable unfunded scheme. However, this does not necessarily hold for the other region because part of the gains of more capital accumulation will flow abroad.

Sections 5.4 and 5.5 are devoted to the theoretical analysis of population ageing, a

development that has two aspects. First, it is a general trend with which nearly all countries are confronted, so part of this shock happens simultaneously and can therefore be considered a symmetric shock. This is what Section 5.4 focuses on, where we distinguish between two regions that only differ in the size of the PAYG-scheme. Two political-economic settings are discussed: one in which the social security tax is kept constant, so that the burden of ageing lies with the elderly, and one in which the pension benefit is kept constant, so that the young bear the burden of ageing by paying a higher tax. Another aspect of ageing is that some countries experience it earlier or more intensely than other countries. So some countries age relative to others, which can be modeled as an asymmetric demographic shock. This is analysed in Section 5.5. We first investigate the effects of a regional shock to the rate of population growth when individual thriftiness differs across regions and social security is absent. We find that the capital-labour ratio increases in the short run, but the long-run effects depend on the relative patience of the ageing country. If the population of the relatively patient country grows old, the capital-labour ratio eventually falls. Next, we introduce unfunded social security and find that the long-run impact of population ageing on the capital-labour ratio depends on the relative generosity of the PAYG-scheme, as private savings are inversely related to the size of unfunded pensions. In this respect, a generous social security scheme corresponds with a high rate of time preference.

In Section 5.6, we numerically explore the magnitude of the effects of ageing in three regions: Europe, Japan, and the United States. Section 5.7 concludes.

5.2 Background

Population ageing: a global phenomenon with regional differences

Since the 1960s, birth rates have been falling in nearly all industrialised countries. This, together with an increasing life span, causes the relative number of elderly to increase substantially in the upcoming decades. Figure 5.1 shows the expected development of the elderly dependency ratio in several EU-countries.[1]

This figure clearly shows that major European countries will experience an ageing population. Especially Italy and Spain are confronted with substantially higher old-age

[1] Figures 5.1 and 5.2 are based on United Nations (2001).

dependency ratios, but also France and the United Kingdom will have almost twice as many elderly per young individual in the middle of the 21st century.

Outside Europe, populations will grow older too. Figure 5.2 illustrates this for other major OECD-countries.[2]

The United States will face a rather moderate rise in the relative number of elderly. Notice that the demographic development in the US lags approximately 25 years behind that of Japan and Europe. In Japan, the population has already started to grow old. In 2020, this ageing process will level off, but after 2040 the dependency ratio will start rising again to one of the highest levels in the world. In a sense, one could therefore say that this country experiences a double demographic shock.

One can conclude that all over the world, the population grows older. Unless the retirement age rises substantially, this will imply a shrinking labour force, making capital relatively abundant which puts a downward pressure on the interest rate. If such developments do not take place elsewhere, or to a smaller extent, this will cause capital to move out of the country. Hence, differences in the timing of demographic change across countries and regions affect international capital flows. Empirical studies confirm this. Drawing on time-series and cross-section data for 100 countries, Higgins (1998) shows that this was indeed empirically the case for the period 1950-1990. Lane and Milesi-Ferretti (2001) find that long-term capital movements are explained by a small set of fundamentals, among which demographic factors. Lührmann (2001) even demonstrates that demography is the most important determinant of cross-border capital movements.

[2] In Europe, all countries have about the same ageing pattern; the dependency ratio in some countries rises eventually to a higher level than that of Germany (e.g. Italy and Spain), whereas in other countries it will be below the German level (e.g. France and the UK). Germany can therefore be thought of as the average European development.

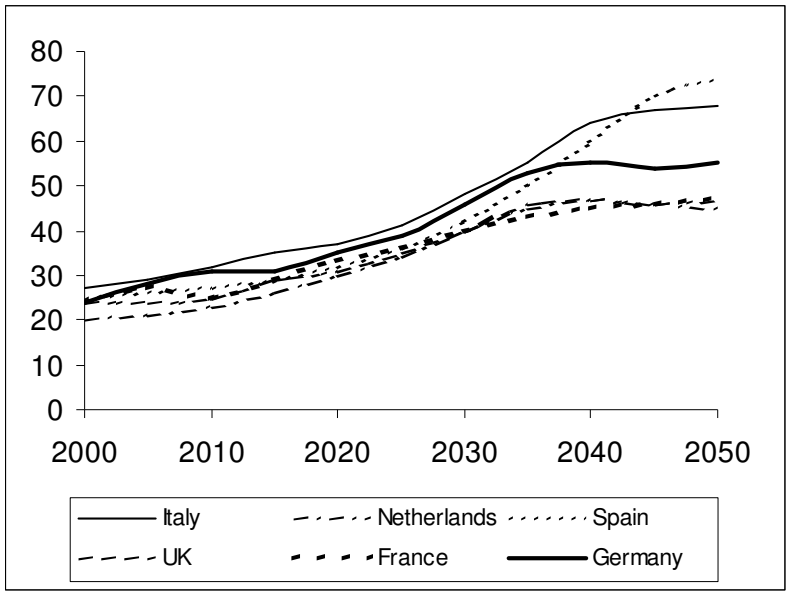

Figure 5.1 *Old-age dependency ratios in Europe*

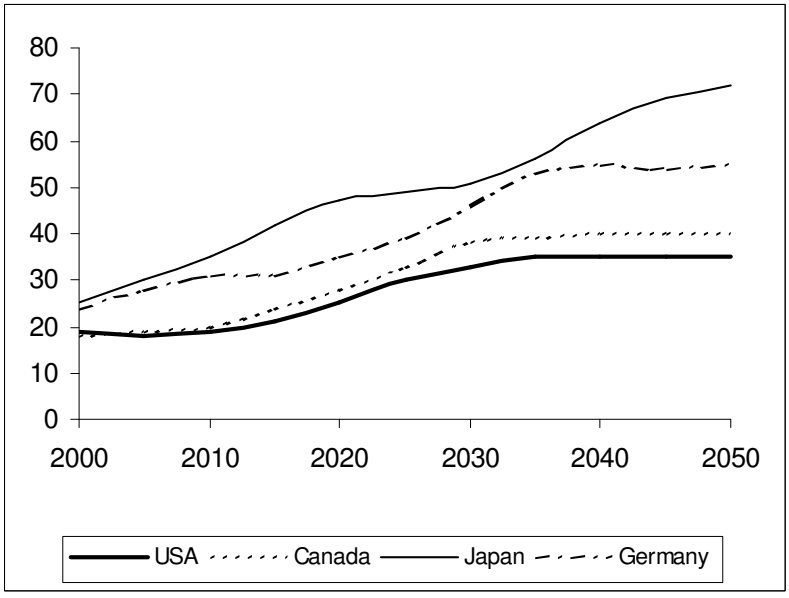

Figure 5.2 *Old-age dependency ratios some OECD-countries*

Several studies explored this topic using simulations. Cutler *et al.* (1990), for instance, consider the implications of ageing in an integrated world capital market. In a two-country Ramsey growth model, they show that the pattern of demographic change can have a large effect on the optimal consumption response to ageing in the US. The rest of the world will export capital to the US, which increases US consumption and reduces saving in the short run. However, in such a model agents live infinitely long. Brooks (2000) applies an overlapping-generations framework to simulate the effects of demographic change and concludes that it significantly affects savings behaviour across countries. The returns of investments in stocks and bonds to baby boomers will be substantially below the returns to earlier generations. This, however, is a model of a closed economy. In a more recent study, Börsch-Supan *et al.* (2001) also apply an OLG-model to calculate the effects of ageing for Germany as an open economy, taking account of the different timing of ageing in industrialised and less developed countries. This study indicates that capital flows from fast-ageing industrialised countries to the rest of the world will be substantial. The decrease in the interest rate that is the result of ageing can be significantly reduced by international diversification. This conclusion is not shared by other studies (such as Turner *et al.*, 1998, MacKellar and Reisen, 1998, Holzmann, 2000, and Reisen, 2000), which find that capital flows to slowly ageing regions can only slightly attenuate (but certainly not reverse) the consequences of ageing on the returns to capital. Furthermore, even small countries with relatively young populations are potentially vulnerable to the effects of ageing in large economies, as was shown by Kenc and Sayan (2001) to be the case for Turkey and the EU.

Pensions: PAYG and funding

The way old-age pensions are financed differs considerably among countries. In a recent study, Holzmann *et al.* (2001) reviews some cross-country estimates that have been made for the implicit pension debt that is inherent in unfunded public pension liabilities. Table 5.1 shows these estimates, together with data on the national pension assets and gross saving rates.

The second column shows calculated unfunded pension liabilities of seven countries in 1990, and the numbers in the third column come from an IMF-study of 1996. The numbers in the next two columns are taken from a study by Kuné, who used two different

definitions for the implicit public debt for twelve EU members in 1990.[3]

Table 5.1

	Implicit PAYG-debt				Pension	Gross
Country	**OECD**	**IMF**	**Kuné**	**Kuné**	**assets***	**national saving****
Belgium	-	-	101	75	-	26.2
Canada	121	94	-	-	0.48	23.7
Denmark	-	-	117	87	0.54	23.3
France	216	265	112	83	0.20	21.8
(West-) Germany	157	221	186	138	0.33	21.3
Greece	-	-	245	185	-	18.2
Ireland	-	-	78	55	-	24.1
Italy	242	357	207	157	0.06	20.2
Japan	162	166	-	-	0.43	27.7
Netherlands	-	-	144	103	1.08	27.6
Portugal	-	-	128	93	-	2.7
Spain	-	-	129	93	0.08	22.3
Sweden	-	131	-	-	0.56	21.6
UK	156	117	92	68	0.92	15.7
USA	113	106	-	-	0.67	17.7

Source: Holzmann et al. (2001), Demirgüç-Kunt and Levine (1996) and OECD (2002)

*Assets of private pension and insurance funds to GDP (annual averages, 1986-1993)

**In 2000, as a percentage of GDP

This table clearly shows that among the OECD-countries, Italy, France and Germany have relatively sizeable unfunded pension schemes, whereas those of the Anglo-Saxon countries are relatively small. Within the European Union, Greece, Italy and Germany are the countries with the most extensive PAYG-schemes, and the unfunded schemes of the United Kingdom and Ireland are smallest.

[3]The second and third columns of this table are based on a so-called Projected Benefit Obligation, taking into account future salary levels but not future increases due to post-retirement indexation, whereas the first and last columns are based on Accumulated Benefit Obligation, which ignores future salary increases.

The size of funded pension schemes also differs greatly among countries. The Netherlands, the United Kingdom and the United States are the countries with extensive pension and insurance funds, and Italy, Spain and France perform badly in this respect, which are also the countries with a relatively extensive unfunded component. The gross saving rate is low in Portugal, the United States and the United Kingdom, whereas it is high in Japan and The Netherlands. There does not seem to be an unambiguous relation between these numbers and the size of (un)funded pension schemes, so it is likely that other factors, such as a different rate of time preference, are also important in explaining the varying savings.

A number of papers deals with the theoretical aspects of internationally different PAYG-schemes. Geide-Stevenson (1998) analyses the connection between social security policy and international factor movements in a world where one country runs a PAYG-public pension scheme and the other a fully funded private pension scheme. Among other things it is shown that the introduction of capital mobility is never Pareto-optimal because the young generation of the country with the social security scheme always loses, whereas the long-run welfare effects are ambiguous. Casarico (1999) also investigates the effects of the integration of two large economies, running different pension schemes, in a world of perfect capital mobility. She too finds that it is not trivial and straightforward to establish who gains and who loses. This is due to the short-run effects of this integration being different from the long-run implications. The impact of the integration of capital markets on PAYG-transfers is quantified in MacKellar *et al.* (2001), which finds that globalisation causes significant (but modest) effects. The poor, who depend on labour income and public pension benefits, suffer, while those who derive income from capital benefit. Pemberton (1999, 2000) deals with the international externalities of social security systems in a world consisting of many small countries. A change in the savings of an individual country, e.g. caused by a more generous social security scheme, hardly affects the world interest rate and the capital-labour ratio in the country. Because local governments do not take these spillover effects into account when deciding on the size of intergenerational redistribution, the PAYG-scheme that is implemented is too large. Consequently, international policy coordination could improve welfare.[4]

[4]However, Beltrametti and Bonatti (forthcoming) show that this only holds if the government only cares about the current well-being of the living generations. Under more plausible assumptions, international coordination of pension policies does not improve welfare.

5.3 Social security in a two-region model

In this section, we present the model that will be used throughout the chapter. Furthermore, we focus on the question which country actually benefits from the fact that capital is fully mobile across borders, and to what extent a reduction of the unfunded part of social security improves long-run welfare.

5.3.1 The model

Following Buiter (1981), we distinguish between two regions, A and B. Both are identical, except for the size of the public pension scheme. Production per young individual is described by a standard neoclassical constant-returns-to-scale production function, $f(k_t^i)$, k_t^i being the amount of capital per young individual in region i in period t.[5] Perfect competition among producers gives the usual equilibrium conditions for the regional production factor markets, $r_t^i = f'(k_t^i) - \delta$ and $w_t^i = f(k_t^i) - f'(k_t^i)k_t^i$, where r_t^i is the interest rate, w_t^i denotes the corresponding wage rate in period t in the respective region, and δ is the rate at which capital depreciates. Capital is perfectly mobile across regions, but labour is immobile. This implies that both regions face the same interest rate, i.e., $r_t^A = r_t^B = r_t$ $\forall t$, and consequently also the same capital-labour ratio and wage.

Both economies are populated by an equally large number of non-altruistic, identical individuals with perfect foresight who live for two periods, such that in each period, both a young and an old generation are alive. Population grows at the same exogenous rate n_t at time t.[6] Normalising the initial number of young at unity, we can thus write the number of young at time t as $N_t = \prod_{j=0}^{t}(1 + n_j)$.

Lifetime utility of a representative individual in region i is given by the following utility function,[7]

$$U_t = U(c_t^{y,i}, c_{t+1}^{o,i}) = \log c_t^{y,i} + \gamma^i \log c_{t+1}^{o,i},$$

[5] The production function f is assumed to be twice continuously differentiable; $f(0) = 0$; $f'(k) \equiv \frac{\partial f}{\partial k} > 0$; $f''(k) \equiv \frac{\partial^2 f}{\partial k^2} < 0$; $\lim_{k\to 0} f'(k) = \infty$; $\lim_{k\to\infty} f'(k) = 0$.

[6] This rate of population growth is assumed to be below the interest rate in an integrated economy, so $r^i > n_t^i$ $\forall i, t$, i.e., the world economy is dynamically efficient.

[7] We consider an additively separable utility function with logarithmic felicity functions for expositional clarity only. Our main results still hold if we only assume U to be twice continuously differentiable, strictly quasi-concave and increasing in its arguments. Furthermore, the Inada-conditions should apply.

where γ^i is the private discount factor of individuals living in region i, $c_t^{y,i}$ is consumption when young, and $c_{t+1}^{o,i}$ is consumption in the second period of life. When young, the individual inelastically supplies one unit of labour.

In both regions, the government runs a public PAYG-pension scheme. Young individuals born at time t pay a PAYG-tax equal to $\tau_t^i w_t$. When old, they receive a public pension benefit of $\tau_{t+1}^i w_{t+1}(1+n_{t+1})$. Their lifetime budget constraint is thus

$$c_t^{y,i} + \frac{c_{t+1}^{o,i}}{1+r_{t+1}} = (1-\tau_t^i)w_t + \frac{\tau_{t+1}^i w_{t+1}(1+n_{t+1})}{1+r_{t+1}},$$

and the individual consumption and savings are given by respectively

$$\begin{aligned} c_t^{y,i} &= \frac{(1-\tau_t^i)w_t}{1+\gamma^i} + \frac{\tau_{t+1}^i w_{t+1}(1+n_{t+1})}{(1+\gamma^i)(1+r_{t+1})}, \\ c_{t+1}^{o,i} &= \frac{\gamma^i(1-\tau_t^i)w_t(1+r_{t+1})}{1+\gamma^i} + \frac{\tau_{t+1}^i w_{t+1}(1+n_{t+1})}{1+\gamma^i}, \qquad (5.1) \\ s_t^i &= \frac{\gamma^i(1-\tau_t^i)w_t}{1+\gamma^i} - \frac{\tau_{t+1}^i w_{t+1}(1+n_{t+1})}{(1+\gamma^i)(1+r_{t+1})}. \qquad (5.2) \end{aligned}$$

A higher social security benefit influences savings negatively for two reasons: first, it decreases net income when young, and second, it reduces the incentive to save. Furthermore, a higher interest rate implies a lower net present value of the pension benefit and thereby higher savings.[8]

People invest their savings either in their own country or abroad. Because they do not suffer from a so-called home bias, and capital flows are not impeded by capital controls, transaction costs etc., their portfolios will be composed such that interest rates are equalised. Equilibrium in the international capital market is given by

$$N_t\left(s_t^A + s_t^B\right) = 2N_{t+1}k_{t+1}. \qquad (5.3)$$

Assuming an initially constant population size in both region ($n^A = n^B = 0$), this equation boils down to $s^A + s^B = 2k$ in steady state.

The balance of trade for region i at time t equals $BT_t^i = N_t f(k_t^i) - N_t c_t^{y,i} - N_{t-1}c_t^{o,i} - (K_{t+1}^i - K_t^i)$, where K_t^i is the capital stock of region i at time t. In per capita terms, the balance of trade can be written as

$$bt_t^i \equiv \frac{BT_t^i}{N_t} = f(k_t^i) - c_t^{y,i} - \frac{c_t^{o,i}}{1+n_t} - \left[(1+n_{t+1})k_{t+1} - k_t\right].$$

[8] Due to our specification of the utility function, the income and substitution effect of a change in the interest rate exactly cancel out.

The current account is defined as the balance of trade plus the returns on investments abroad, i.e., $CUA_t^i = BT_t^i + r_t(S_{t-1}^i - K_t^i)$, where S_{t-1}^i are the aggregate savings of region i at time $t-1$. The per capita current account is given by $cua_t^i \equiv CUA_t^i/N_t = bt_t^i + r_t\left(\frac{s_{t-1}^i}{1+n_t} - k_t\right)$. Using the fact that due to constant returns to scale, $f(k_t) = w_t + r_t k_t$, implies

$$cua_t^i = s_t^i - \frac{s_{t-1}^i}{1+n_t} - \left[(1+n_{t+1})k_{t+1} - k_t\right].$$

Naturally, since we consider only two regions, it must hold that $cua_t^A + cua_t^B = 0$.

5.3.2 The effect of capital mobility

In this subsection we focus on the question whether perfect capital mobility is beneficial for both regions in the long run if they only differ in the size of their social security schemes (so $\gamma^i = \gamma$ for both regions). In order to answer this question, we will compare the long-run utility level of two different economies: one with closed borders, so that all capital is invested domestically, and one with open borders, as described above.

Let A be the region with a relatively funded pension scheme, and B the region with an extensive social security scheme ($\tau^A < \tau^B$). Consequently, savings in region A will be higher than in B. This implies that if region B was closed, the capital-labour ratio would be low and consequently, the interest rate high, whereas the reverse holds for the other region. Integration of capital markets then leads to a capital flow from A to B until interest rates are both equal to r, and a capital-labour ratio that lies between the autarkic values: $k^A > k > k^B$. This leads to the following proposition.

Proposition 5.1 *The region with the most extensive unfunded social security scheme benefits in the long run from perfect capital mobility.*

Proof As we assumed that the interest rate in an integrated world exceeds the rate of population growth, it automatically holds that region B would be dynamically efficient in autarky, i.e., $r^B > r > n$. This implies that in region B, capital flows in and the capital-labour ratio moves closer to the golden rule after integration with region A. Because inhabitants of this region do not invest abroad, they are not harmed by the lower interest rate and consequently, the long-run utility is higher than it would be in autarky. ■

For the region with more funded pensions, such an unambiguous conclusion cannot be drawn.

Proposition 5.2 *The region with the relatively small PAYG-scheme will either benefit or suffer in the long run from perfect capital mobility. If the size of its PAYG-scheme differs much from the other region, the economy benefits from capital mobility, but if its social security scheme is also rather extensive, long-run utility in this region would be highest in case of closed borders.*

By being integrated with the other region, the welfare of region A is affected in two different ways. On the one hand, the capital-labour ratio will be lower than in autarky because part of domestic savings will be invested abroad. Individuals in region A will thus work with a smaller capital stock, implying a lower wage, which lowers their utility. On the other hand, region A will export part of its capital to the other region where it can be invested more profitably, so the interest rate with perfect capital mobility is higher than in autarky. This has a positive effect on welfare. Figure 5.3 illustrates this. The horizontal axis gives the social security tax in region A, whereas the tax in B is fixed at 25%.[9] For all values of τ^A shown, the integrated economy as a whole is dynamically efficient. If τ^A is high, savings are low in region A, so the marginal product of capital is high, i.e., extra savings due to a marginal decrease of τ^A can be invested rather productively in that region and the wage increases substantially. If capital is internationally mobile, some of the savings of region A will flow abroad and the resulting wage decrease cannot compensate the increase in the interest rate, so utility is lower. But if τ^A is relatively low, the reverse holds. Savings are then higher, so the region benefits a lot from a higher interest rate in case of open borders. The loss due to an outflow of capital is not great, since it was already rather affluent. Note that this situation already applies when the closed region A is still dynamically efficient. For even lower values of τ^A, savings in region A would so high that the economy would be dynamically inefficient if it were closed, so that integration with the region that has a large PAYG-scheme corrects this inefficiency and improves welfare in the long run.

So region A does not necessarily suffer from the fact that B runs a more generous unfunded pension scheme, which can also be seen from the fact that the curve representing region A as a closed economy also shows steady-state utility if both economies have the same PAYG-tax ($\tau^A = \tau^B$).

[9] The figure is based on simulations with the following production function, $f(k_t) = k_t^{0.25}$. Furthermore, $\gamma = 1$, $\delta = 1$, and $n = 0$.

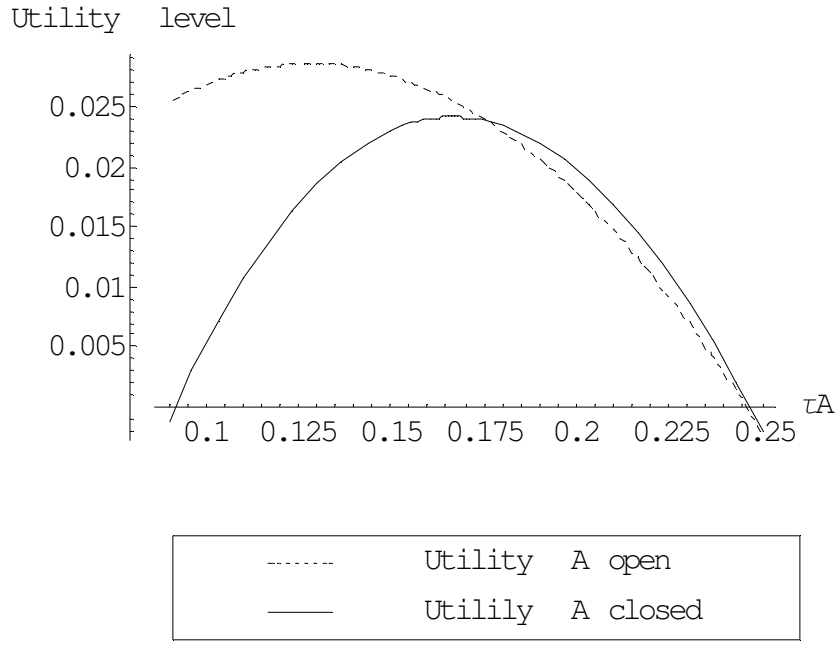

Figure 5.3 *Long-run lifetime utility as a function of the PAYG-tax*

5.3.3 Privatising social security

Reducing the unfunded part of social security is a frequently proposed policy to raise long-run welfare. The main argument is that nowadays, interest rates in most economies exceed the rate at which the economy grows, so the return to a funded scheme is higher than the implicit return of a PAYG-scheme. Indeed it is true that if this condition holds, long-run welfare is maximized if the PAYG-scheme were abolished in a small open economy, as the interest rate would not change. For a closed economy, it also holds that (marginally) reducing the PAYG-scheme increases long-run utility as long as the economy is dynamically efficient. But can such a conclusion also be drawn in case of a large open economy, i.e., if the open-economy aspects are taken into account?

First consider region A, with the relatively moderate-sized social security scheme. If this country partially privatises, long-run utility will either increase or decrease, depending on how high τ^A initially is compared to τ^B. This can be seen from Figure 5.3. A

reduction of the PAYG-tax to approximately 12.5 percent would raise the utility of future generations, but a further reduction is not beneficial, even though the capital-labour ratio increases and the interest rate stays remains higher than the rate of population growth. The reason for this is that such a policy has two effects. First, it increases the capital-labour ratio and thereby reduces the interest rate, which by itself has a negative impact on utility since all individuals save. On the other hand, the capital stock with which the individuals work when young increases, so the wage is higher, implying a higher utility. In a closed economy, this effect is strong because all savings are invested domestically, and these two effects exactly cancel out when $r = n$. However, with open borders, part of the increased savings will be invested abroad, so the positive effect of privatisation on the wage is shared with the other region, whereas the negative interest rate effect is not different between the case of a closed and an open economy. Hence, the 'turning point' for region A already occurs when $r > n$ still holds. The lower τ^A, the more region A saves, so the more sensitive it is to lower interest rates. Furthermore, if τ^B is higher, inhabitants of region A will invest a larger part of their savings in B, so the positive effect of additional savings on their wage will be smaller.

For region B, the opposite holds: all savings are invested domestically, and (partial) privatisation of the social security system in region A will raise long-run utility, because foreigners will invest more capital. This leads to the following proposition.

Proposition 5.3 *A small reduction of the social security tax in region A will raise long-run utility in that region if this tax initially does not differ much from that in the other region, but it will lead to a lower long-run utility level if the taxes differ relatively much, even if the interest rate still exceeds the fertility rate. Long-run utility in region B increases.*

Proof See Appendix. ■

Alternatively, region B, with the relatively extensive social security scheme, can decide to reduce it. As argued before, the effects for this region itself will be positive in the long run, because the capital-labour ratio moves closer to the golden rule and all savings are (still) invested domestically (as long as $\tau^B > \tau^A$). Region A can either benefit or suffer from this policy in the long-run, which again can be seen from Figure 5.3. As a matter of fact, a reduction of τ^B makes region B more similar to region A, so the dashed curve, representing long-run utility for region A, moves closer to the solid curve. If the

social security tax in A does not differ too much from that in B, this will imply a higher utility for future generations in region A, but if it is rather low, the reverse holds, and the integrated economy as a whole may even become dynamically inefficient. This is summarised by the following proposition.

Proposition 5.4 *Partial privatisation of social security in region B will benefit future generations in that region. Long-run utility of individuals living in region A will increase too if the size of its PAYG-scheme does not differ much from that of region B, whereas it will decrease if its PAYG-scheme is much smaller than in region B, even if the interest rate still exceeds the fertility rate.*

Proof See Appendix. ■

So the conclusion that it is beneficial for current young and future generations to at least marginally privatise social security as long as the interest rate exceeds the rate of population growth, which does hold for both a closed and small open economy, does not necessarily hold for two equally large economies with integrated capital markets. If it adversely affects utility in the long run, such a policy would unambiguously decrease overall welfare in that region. But even if long-run utility increases, the positive effects will be shared with the other region and therefore cannot outweigh, in present value terms, the costs that the region incurs due to the lower benefits of the currently retired. That is, domestic losers cannot be compensated by (future) domestic winners without making at least one generation worse off. A proposal to privatise social security would thus be politically infeasible.[10]

The remainder of this chapter focuses on the effects of population ageing, assuming either the social security tax or benefit to remain constant.

5.4 Symmetric demographic shocks

This section focuses on the effects of a symmetric demographic shock, i.e., the general trend that ageing occurs in all countries. The only difference between the countries that we focus on in this section is the size and design of the PAYG-public pension scheme.

[10] See also Pemberton (1999, 2000), who analyses the gains from international coordination of social security reform in a model with positive production externalities.

For analytical convenience and without loss of generality, we will assume that region A does not have a PAYG-scheme ($\tau^A = 0$). Countries belonging to this region can therefore be considered the ones that have extensive funded schemes, like the Unites States, the United Kingdom, The Netherlands, Switzerland and Norway. The other region, B, does have unfunded pensions, so one could think of France, Italy, Germany and Spain. The main question is how this difference in (un)funded schemes influences the (spillover) effects of population ageing affecting both regions, and whether the design of the social security scheme matters.

5.4.1 Population ageing with fixed contribution rates (*FC*)

In this subsection, we analyse the consequences of population ageing in case the PAYG-scheme is characterised by a constant social security tax rate, i.e., $\tau_t^B = \tau \; \forall t$. This implies that the burden of ageing is entirely with the elderly, since a higher old-age dependency ratio goes along with a lower public pension benefit.

Population ageing is modeled as a permanent decrease in the fertility rate occurring simultaneously and to the same extent in both regions. One period before the economies are actually hit by the shock, individuals already foresee that ageing will occur and adapt their decisions so as to anticipate optimally to the economic consequences of the shock.[11]

Let the fertility rate at time t be given by $n_t = \pi h_t$, where h_t describes the time pattern of a shock and π its magnitude. Hence, ageing is reflected by a negative value of h. The effects of a marginal change in the rate of population growth can be traced by the method of comparative dynamics, i.e., by linearising the capital accumulation function (5.3) with respect to n around the initial steady state. This gives

$$\frac{\partial s_t^A}{\partial \pi} + \frac{\partial s_t^B}{\partial \pi} + \left(s^A + s^B - 2k\right) \sum_{j=0}^{t} h_j = 2\frac{\partial k_{t+1}}{\partial \pi} + 2k h_{t+1},$$

which results in the following first-order difference equation,

$$\frac{\partial k_{t+1}}{\partial \pi} = -\frac{\gamma k f''(k)(2-\tau)(1+r)^2}{\Psi}\frac{\partial k_t}{\partial \pi} - \frac{2k(1+\gamma)(1+r)^2 + \tau w(1+r)}{\Psi} h_{t+1}, \qquad (5.4)$$

with $\Psi \equiv 2(1+\gamma)(1+r)^2 - \tau k f''(k)(1+r) - \tau w f''(k) > 0$. The steady-state equilibrium is locally stable if $-\gamma k f''(k)(2-\tau)(1+r)^2 < \Psi$, which implies an (implicit) under bound

[11] An unexpected demographic shock would not change the main results.

for the initial generosity of the social security scheme.[12] Furthermore, we will assume that $\left|\frac{\partial s_t^i}{\partial k_t}\right| > \left|\frac{\partial s_t^i}{\partial k_{t+1}}\right|$, i.e., savings react more heavily to a change in the current capital-labour ratio than to a change in the future capital-labour ratio. This seems plausible, since for our specification of the utility function, a change in the future capital-labour ratio only affects savings because of a changing net present value of the PAYG-benefit.[13] This therefore implies a maximum value for the social security tax rate. Moreover, with this assumption, there is a positive relation between private savings and the capital-labour ratio in the long run $\left(\frac{\partial s^i}{\partial k} > 0\right)$. Altogether, this boils down to the following assumption.

Assumption 5.I *The social security tax is bounded:*
$\tau \in \left(\frac{2(1+r)^2[1+\gamma+\gamma k f''(k)]}{f''(k)[\gamma k(1+r)^2+k(1+r)+w]}, \frac{\gamma k(1+r)^2}{\gamma k(1+r)^2+k(1+r)+w}\right)$.

Suppose the demographic shock first takes place at time $t = 0$. People born before that period know that and foresee all the economic consequences. Those born at $t = -2$ and before will not change their savings, so until period $t = -1$, the capital-labour ratio will remain constant. However, individuals born one period before the shock occurs will adapt their savings.[14] Consequently, the change in the capital-labour ratio at time $t = 0$ is given by

$$\begin{aligned}\frac{\partial k_0}{\partial \pi} &= \frac{1}{2}\frac{\partial s_{-1}^B}{\partial \pi} - kh_0 \\ &= -\frac{[w\tau + 2k(1+\gamma)(1+r)]\,(1+r)}{\Psi}h_0 > 0. \qquad (5.5)\end{aligned}$$

A decrease in the rate of population growth at time $t = 0$ reduces the number of young people, so labour becomes relatively scarce and capital relatively abundant. This *capital-thickening effect* (the last term in (5.5)) puts downward pressure on the interest rate,

[12] This is given by $\tau > \frac{2(1+r)^2[1+\gamma+\gamma k f''(k)]}{f''(k)[\gamma k(1+r)^2+k(1+r)+w]}$. Galor and Ryder (1989) show that, given our assumptions with respect to the production function and our specification of the utility function, $-\frac{1}{2}kf''(k) \in (0,1)$, which implies that the underbound for the social security tax is nearly always negative, so that the stability condition is automatically fulfilled.

[13] $\frac{\partial s_t^i}{\partial k_{t+1}} = 0$ whenever $\tau_{t+1} = 0$ due to the logarithmic felicity functions.

[14] Note that if labour supply is endogenous, these individuals will also change the number of hours they work, which immediately implies a different capital-labour ratio and changing factor rewards. This, in turn, will be foreseen by the previous generation, which consequently also adapts its labour supply, so that the capital-labour ratio changes already at that time. Hence, in case of elastic labour supply, the capital-labour ratio will change as soon as it becomes publicly known that a demographic shock will take place in the future.

which is foreseen by young individuals living in both regions at time $t = -1$. Citizens of region A do not respond to this because their savings do not depend on the interest rate and their wage does not change. Inhabitants of region B, on the other hand, decide to increase their savings because they will be confronted with lower public pension benefits when they are retired (the so-called *dependency-ratio effect*, the first term in (5.5)). This direct positive effect on savings is partly reduced by the negative effect of a lower interest rate and a higher wage in the next period, which increases the present value of the public pension benefit. Still, savings increase in region B, which intensifies the initial positive effect of ageing on the capital-labour ratio and consequently, the negative effect on the interest rate. Naturally, this effect would be absent if region B did not have a PAYG-scheme, and the capital-labour ratio would only rise because of the capital-thickening effect, implying a smaller decrease of the interest rate in the subsequent period.

The higher capital-labour ratio causes wages to rise, leading to higher savings in both regions. So when times passes by, the capital-labour ratio increases further, as is summarised by (5.4). In the long run, the increase of the capital-labour ratio is equal to

$$\frac{\partial k}{\partial \pi} = -\frac{2k(1+\gamma)(1+r)^2 + \tau w(1+r)}{\Psi + \gamma k f''(k)(2-\tau)(1+r)^2} h > 0.$$

Figures 5.4 and 5.5 show the change of the capital-labour ratio and the interest rate due to ageing for two cases: with and without capital mobility, denoted as respectively 'open' and 'closed'.[15]

The first picture shows that for region A, international capital mobility implies a smaller increase in the capital-labour ratio. However, as can be seen from the second figure, the drop in the interest rate is greater. The reason for the first effect is that the capital stock in this region would be higher with closed borders, so the capital-thickening effect of ageing would be stronger, implying a higher increase of the capital-labour ratio. This in turn would cause wages to be higher, and consequently also savings, making the subsequent rise of k even stronger. The second effect stems from the specification of the production function, so that the extent to which the interest rate decreases due to a marginal increase of k depends negatively on the initial value of the capital-labour ratio.

[15] This figure is based on simulations with the following production function, $f(k_t) = k_t^{0.3}$. Furthermore, $\gamma = 1$, $\delta = 0$ and $\tau = 0.2$.

Note that the case of region A being a closed economy corresponds to the case of perfect capital mobility and $\tau = 0$.

Naturally, the reverse holds for the region with a sizeable social security scheme.

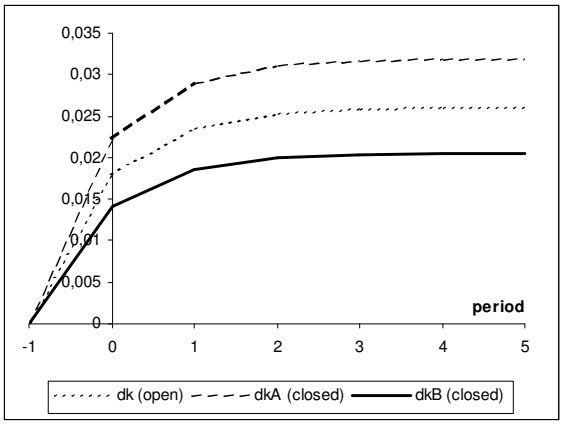

Figure 5.4

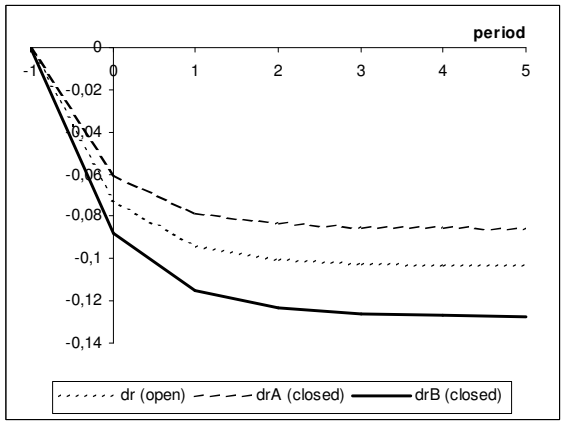

Figure 5.5

The relative change of consumption when young and old in the two regions is displayed in Figure 5.6 and 5.7, again for the cases with and without open borders.

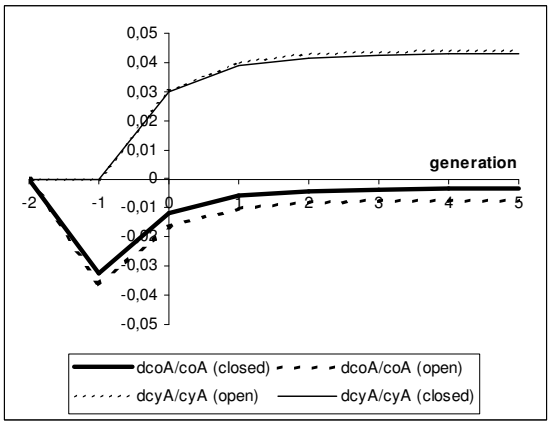

Figure 5.6

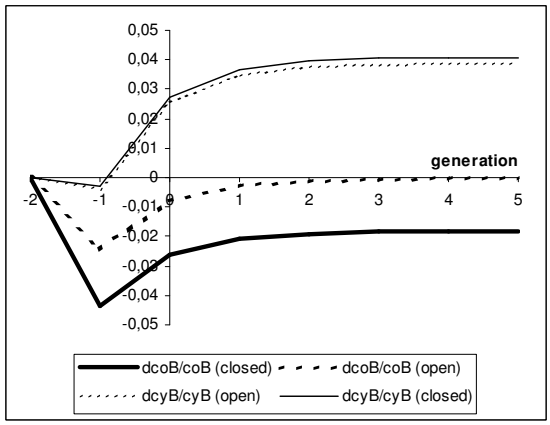

Figure 5.7

Persons living in region A and born one period before the shock suffer from the lower rate of return that they earn on their savings, so their consumption when old decreases. A PAYG-scheme in the other region slightly intensifies this. The same holds for the next generations, though to a smaller extent: because they earn a higher wage when young, their savings will also be higher, which reduces the decrease in their old-age consumption. In the long run, the positive effect of a higher wage on savings, and thereby on old-age consumption, is not sufficient to offset the negative effect of a lower interest rate. However, population ageing makes consumption when young rise. An unfunded social

security scheme abroad will hardly affect this. Eventually, lifetime utility will rise due to population ageing, as can be seen from Figure 5.8.

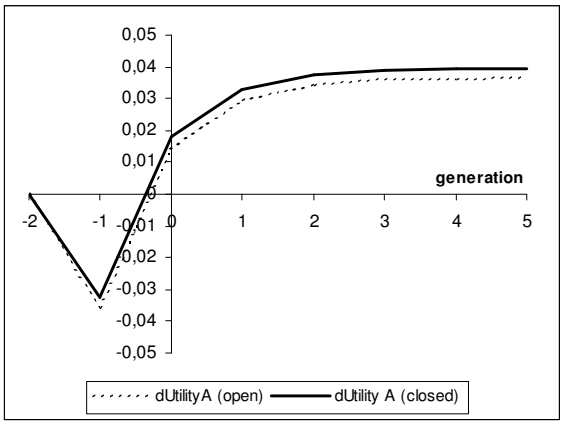

Figure 5.8

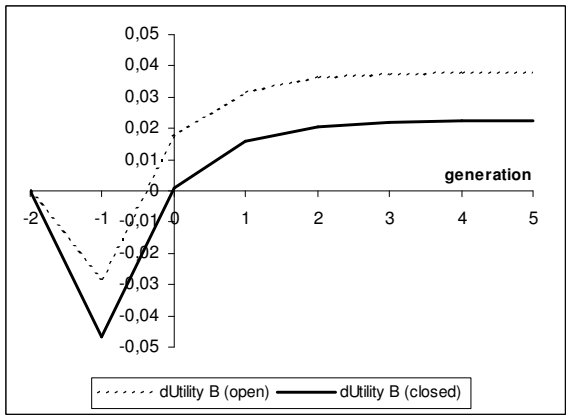

Figure 5.9

In the absence of a PAYG-scheme in region B, or with closed borders, ageing would affect all generations in region A more positively. Put differently, citizens of countries with a sizeable funded pension scheme suffer from extensive PAYG-programmes abroad. The extent, however, is rather small.

Individuals residing in region B and born at time $t = -1$ save more in response to the lower public pension benefit they will receive when old, so they will consume less when young. Also their old-age consumption decreases, so they unambiguously suffer from the demographic shock, which would be much less so if their pensions were mostly funded. The next generation (those born at $t = 0$) is also confronted with a lower public pension benefit, but at the same time, their wage increases due to a higher capital-labour ratio. Since this last effect dominates, they consume more when young. Because the presence of a PAYG-scheme implies an extra incentive to save more, the capital-labour ratio increases more, and thereby the wage. In the long run, this means that consumption is relatively higher. At the same time, the interest rate is decreasing more due to unfunded social security, which, together with a lower benefit, causes old-age consumption in region B to be lower if a PAYG-scheme is in place. As can be seen from Figure 5.9, all generations of the region with the extensive PAYG-scheme are better off with open borders, i.e., they benefit from the fact that pensions abroad are funded.

International trade and capital flows

Because individuals in region A do not receive PAYG-pensions, their savings will be higher than in region B. Consequently, they will invest part of their savings abroad and receive interest payments with which they can pay their net imports of commodities. Hence, capital flows from A to B and goods move in the opposite direction, so $BT^A = -BT^B < 0$ and $CUA^A = -CUA^B = 0$. A demographic shock will influence this. The change in the per capita trade balance is given by

$$\frac{\partial bt_t^i}{\partial \pi} = [1 + f'(k)] \frac{\partial k_t}{\partial \pi} - \frac{\partial c_t^{y,i}}{\partial \pi} - \frac{\partial c_t^{o,i}}{\partial \pi} + c^{o,i} h_t - \frac{\partial k_{t+1}}{\partial \pi} - k h_{t+1},$$

and the current account changes as follows,

$$\frac{\partial cua_t^i}{\partial \pi} = \frac{\partial s_t^i}{\partial \pi} - \frac{\partial s_{t-1}^i}{\partial \pi} + s^i h_t - \frac{\partial k_{t+1}}{\partial \pi} - k h_{t+1} + \frac{\partial k_t}{\partial \pi}.$$

Figure 5.10 shows the development of the per capita trade balance and current account in region A when ageing simultaneously occurs in both regions.[16]

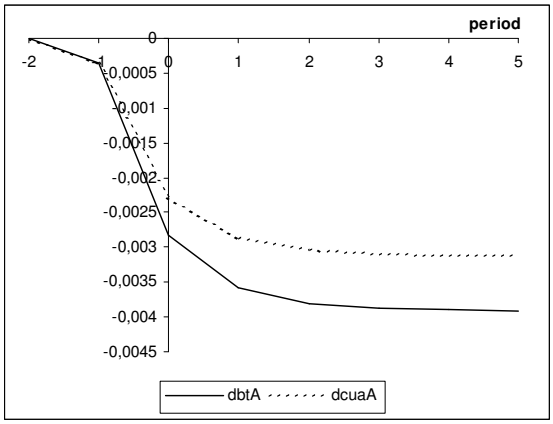

Figure 5.10

At time $t = -1$, individuals in region B decide to save more because they will be confronted with a lower public pension benefit. The discrepancy with the savings of the other region then becomes smaller, so the initial foreign investments of region A decrease. Hence, commodities move to region A at that time, resulting in a trade deficit. This process will be reinforced after the capital-labour ratio starts to rise.

[16] Since we only distinguish two regions that experience the same demographic developments, the balance of trade and current account of region B move in the opposite direction of those of region A.

5.4.2 Population ageing with fixed benefits (*FB*)

In the previous subsection it was assumed that the social security tax always remained at its initial level, no matter how high the share of elderly in society would be. This supposes that population ageing is reflected in a lower pension benefit, which the retired take for granted. Of course, this does not seem to be realistic, since the generosity of the social security system is the outcome of a political process, in which also the old participate and will become an increasingly important party if their number grows relatively. This subsection therefore considers the other case, namely fixed PAYG-benefits. Accordingly, the social security tax is adjusted to balance the government budget, so the burden of ageing now totally lies with the young.

Suppose the public pension benefit level in region B is fixed at some amount η. This implies that the social security tax is equal to $\tau_t = \frac{\eta}{(1+n_t)w_t}$, which, bearing in mind that $n = 0$ in steady state, changes due to ageing according to

$$\frac{\partial \tau_t}{\partial \pi} = -\tau h_t + \frac{\tau k f''(k)}{w}\frac{\partial k_t}{\partial \pi}.$$

In that case, the first-order condition describing the evolution of the capital-labour ratio boils down to

$$\frac{\partial k_{t+1}}{\partial \pi} = -\frac{2\gamma k f''(k)(1+r)^2}{\Delta}\frac{\partial k_t}{\partial \pi} + \frac{\gamma\eta(1+r)^2}{\Delta}h_t - \frac{2k(1+\gamma)(1+r)^2}{\Delta}h_{t+1},$$

with $\Delta \equiv 2(1+\gamma)(1+r)^2 - \eta f''(k) > 0$, and stability of the steady state equilibrium requires $-2\gamma k f''(k)(1+r)^2 < \Delta$. This, together with the assumption that savings react more heavily to a change in the current rather than the future capital-labour ratio, gives the equivalent of Assumption 5.I for *FB*-pensions.

Assumption 5.I' *The social security tax is bounded:* $\tau \in \left(\frac{2(1+r)^2[\gamma k f''(k)+1+\gamma]}{w f''(k)}, \frac{\gamma k(1+r)^2}{w}\right)$.

The generation born in region B before the shock occurs will adapt its savings to the lower interest rate at time $t = 0$ that results from the capital-thickening effect. Because the PAYG-benefit does not decrease, a lower interest rate implies lower savings by this generation: $\frac{\partial s^B_{-1}}{\partial \pi} = \frac{\eta f''(k)}{(1+\gamma)(1+r)^2}\frac{\partial k_0}{\partial \pi}$, so the initial change of the capital-labour ratio is

$$\frac{\partial k_0^{FB}}{\partial \pi} = -\frac{2k(1+\gamma)(1+r)^2}{\Delta}h_0.$$

The long-run change of k is given by[17]

$$\frac{\partial k^{FB}}{\partial \pi} = \frac{\gamma\eta(1+r)^2 - 2k(1+\gamma)(1+r)^2}{\Delta + 2\gamma k f''(k)(1+r)^2} h.$$

Population ageing has two effects. The capital-thickening effect always occurs: a lower relative number of workers increases the capital-labour ratio. The dependency-ratio-effect, however, depends on the type of pension scheme. With a fixed social security tax, this effect implies higher savings, which intensifies the first effect, so the capital-labour ratio unambiguously rises. But if the PAYG-benefit is fixed, contributions will increase due to ageing, which has a negative impact on savings and thus on k. However, the capital-thickening effect is dominant. Figure 5.11 shows the development of the capital-labour ratio in case of a fixed contribution rate (as discussed in the previous subsection) and a fixed benefit.

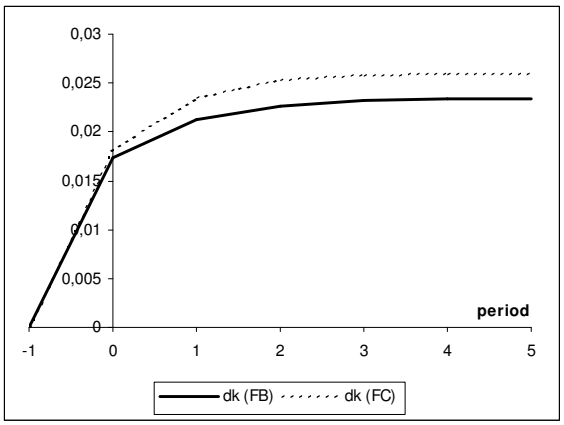

Figure 5.11

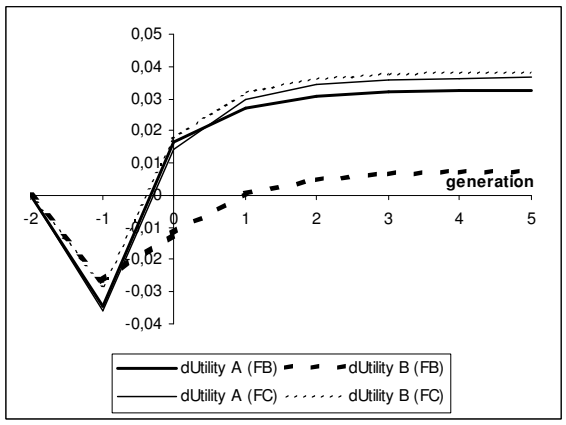

Figure 5.12

If the public pension benefit is fixed, the incentive to save more when the population ages is not great for inhabitants of region B. The increase in the capital-labour ratio is therefore less than in case of fixed contributions. Consequently, the increase of the utility of future generations in region A is smaller, which also holds for region B. The change of consumption when young and old in region A does not differ much between the case

[17] The denominator of this expression is always positive due to the stability assumption. The nominator can be of either sign. For values of τ exceeding $\frac{2k(1+\gamma)}{\gamma w}$, the capital-labour ratio decreases if ageing occurs. However, simulations show that for a wide range of parameter values, these taxes would be unrealistically high (more than 50% of the wage), so we abstract from this possibility.

of a fixed contribution and a fixed benefit (see Figure 5.13). The differences in the region with the social security scheme are bigger, as can be seen from Figure 5.14.

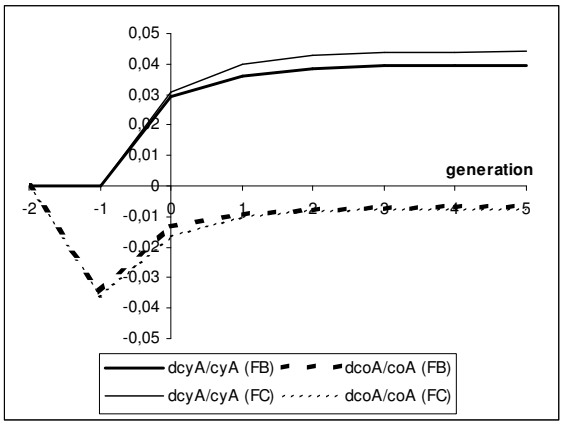

Figure 5.13

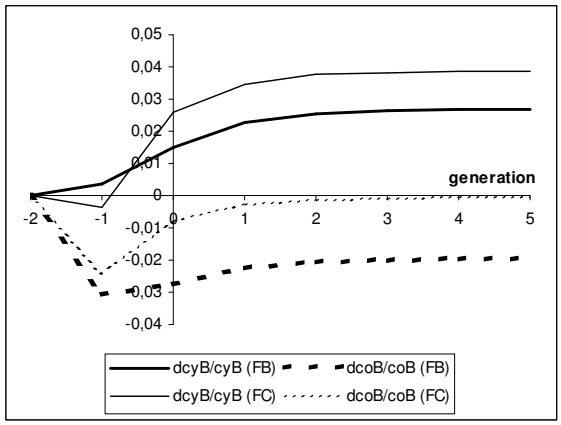

Figure 5.14

Note that the utility of all generations in region B decreases if a FB-scheme is in operation, including the generation born before the shock. Because the social security benefit is kept constant and ageing implies a lower interest rate, the present value of this benefit increases. Consequently, people born at time $t = -1$ decide to save less. This, together with the lower rate of return on their savings, causes their consumption when old to be lower than in case of a FC-scheme. In the latter case, there is an incentive to save extra, as ageing implies a lower pension benefit. So promising a fixed public pension benefit actually leads to a *lower* utility of the elderly alive at the time of the shock.

The generation born at time $t = 0$ in region B is confronted with both a higher tax rate and a higher wage. Overall, the net wage increases, enabling them to save more. Still, they are not able to consume more once retired because of the fall of the interest rate. In later periods, the capital-labour ratio rises further, implying a growing wage and therefore higher consumption when young. This positive utility effect dominates the negative effect of a lower consumption level when old. If a FC-scheme had been in place, there would have been a stronger incentive to increase savings, which subsequently would have raised the capital-labour ratio substantially, and thereby the wage, so that consumption when young would have increased much more, consumption when old decreased less, and utility increased more.

5.5 Asymmetric demographic shocks[18]

In general, many countries will experience a demographic shock. However, the timing and extent differ among countries. Part of the ageing process that will take place can therefore be considered an asymmetric demographic shock, which is what this section focuses on. In other words, one region ages relative to the other. In the model, this is translated into an anticipated, temporary decrease in the fertility rate at time $t = 0$ in region A.[19] So this region can for instance be Japan, as compared to the United States, or within Europe, Italy as compared to Germany (see Figures 5.1 and 5.2). We will analyse the effects of this shock for two cases: one in which the rate of time preference differs between regions, in the absence of any intergenerational redistribution ($\tau^A = \tau^B = 0$), and one in which preferences are the same but the size of the PAYG-scheme differs.[20]

5.5.1 Differences in individual thriftiness

Abstracting from intergenerational redistribution, private savings are completely determined by the wage and the pure rate of time preference,

$$s_t^i = \frac{\gamma^i w_t}{1+\gamma^i}. \tag{5.6}$$

As wages in both regions are equal, differences in the level of savings are completely determined by differences in time preference. In particular, this difference appears to be responsible for the long-run effects of ageing on the capital-labour ratio.

Substituting (5.6) into (5.3) gives the following capital-accumulation equation,

$$\left(\prod_{j=0}^{t}(1+\pi h_j)\frac{\gamma^A}{1+\gamma^A}+\frac{\gamma^B}{1+\gamma^B}\right)w_t = \left(\prod_{j=0}^{t+1}(1+\pi h_j)+1\right)k_{t+1}. \tag{5.7}$$

[18] This section is an adapted part of Groezen and Leers (2000).

[19] We only allow for temporary demographic shocks to avoid problems that will occur if one region grows infinitely large or small. We restrict ourselves to one-period shocks for expositional simplicity only; incorporation of multi-period shocks is straightforward. Note that a temporary decline in the rate of population growth does not imply that once the shock has come to an end, a baby boom occurs following the baby bust. It only means fertility regains its original level.

The effects of an unanticipated asymmetric shock are analysed in Groezen and Leers (2000).

[20] We only consider different time preferences in this section, as it does not matter if the shock is symmetric and no PAYG-schemes are in place.

Linearising this expression around the initial steady state, the evolution of the capital-labour ratio is described by the following first-order difference equation,

$$\frac{\partial k_{t+1}}{\partial \pi} = -\tfrac{1}{2} f''(k) k \left(\frac{\gamma^A}{1+\gamma^A} + \frac{\gamma^B}{1+\gamma^B} \right) \frac{\partial k_t}{\partial \pi} + \tfrac{1}{4}(s^A - s^B) H_t - \tfrac{1}{2} k h_{t+1}, \tag{5.8}$$

with $H_t \equiv \sum_{j=0}^{t} h_j$.

At the moment the shock occurs, the capital-labour ratio increases ($\frac{\partial k_0}{\partial \pi} = -\frac{1}{2} k h_0$) due to the declined labour force of the world economy. Before that time, no changes occur because the current savings decision is not based on the next-period's rate of interest.[22]

The term $\frac{1}{4}(s^A - s^B)$, which directly follows from (5.8), determines the (sign of the) long-run effect of a temporary demographic shock. Therefore, the long-run impact of population ageing on the capital-labour ratio depends on the difference in individual thriftiness of both regions' inhabitants. The following proposition summarises these effects.

Proposition 5.5 *In the long run, the change of the capital-labour ratio depends on the difference in savings between the regions. It holds that*

$$\begin{aligned} \frac{\partial k}{\partial \pi} &> 0 \quad \textit{if} \quad s^A < s^B \Leftrightarrow \gamma^A < \gamma^B, \\ \frac{\partial k}{\partial \pi} &< 0 \quad \textit{if} \quad s^A > s^B \Leftrightarrow \gamma^A > \gamma^B, \\ \frac{\partial k}{\partial \pi} &= 0 \quad \textit{if} \quad s^A = s^B \Leftrightarrow \gamma^A = \gamma^B. \end{aligned}$$

Proof In the long run, $\frac{\partial k}{\partial \pi}$ is determined by the fixed point of difference equation (5.8), which is given by $\frac{\partial k}{\partial \pi} = \frac{s^A - s^B}{4 + 2 f''(k) k \left(\frac{\gamma^A}{1+\gamma^A} + \frac{\gamma^B}{1+\gamma^B} \right)} \sum_{j=0}^{\infty} h_j$. Furthermore, it follows from (5.6) that $\frac{\partial s^i}{\partial \gamma^i} > 0$, which completes the proof. ■

Proposition 5.5 can be interpreted as follows. If inhabitants of region A are relatively impatient ($\gamma^A < \gamma^B$), savings will be relatively low ($s^A < s^B$). Ageing in that region then implies that eventually, the population share of the impatient region in the world economy decreases. So the world is on average populated by relatively more patient individuals. This implies that average savings per individual increase, and therefore, because of perfect capital mobility, the capital-labour ratio increases. The reverse holds if the inhabitants of

[22] The short-run effect of an anticipated shock is therefore equal to the effect of an unexpected shock.

region A are relatively patient. Ergo, a temporary demographic shock can have persistent effects on the world economy. If individuals are equally patient in both regions, average savings do not change in the long run, and the capital-labour ratio is not affected.

5.5.2 Different unfunded schemes

This subsection analyses the effects of an asymmetric demographic shock if the regions run different PAYG-schemes, whereas the individuals' rate of time preference is equal. We only consider the case of fixed contribution rates.[23]

Linearising equation (5.8) around the initial steady state gives

$$\frac{\partial s_t^A}{\partial \pi} + \frac{\partial s_t^B}{\partial \pi} + \frac{1}{2}\left(s^A - s^B\right)\sum_{j=0}^{t} h_j = 2\frac{\partial k_{t+1}}{\partial \pi} + k h_{t+1},$$

which results in the following difference equation,

$$\begin{aligned}\frac{\partial k_{t+1}}{\partial \pi} &= -\frac{\gamma k f''(k)(2-\tau^A-\tau^B)(1+r)^2}{\Phi}\frac{\partial k_t}{\partial \pi} - \frac{w\tau^A(1+r)+k(1+\gamma)(1+r)}{\Phi}h_{t+1} \\ &\quad + \frac{(\tau^B-\tau^A)w\left[1+\gamma(1+r)\right]}{2\Phi}\sum_{j=0}^{t} h_j, \end{aligned} \tag{5.9}$$

with $\Phi \equiv 2(1+\gamma)(1+r)^2 - (\tau^A+\tau^B)f''(k)\left[k(1+r)+w\right] > 0$. Assuming the steady-state equilibrium to be locally stable boils down to $-\gamma k f''(k)(2-\tau^A-\tau^B)(1+r)^2 < \Phi$.

The demographic shock in region A takes place at time $t = 0$ but is foreseen one period earlier. The initial change in the capital-labour ratio is therefore given by

$$\frac{\partial k_0}{\partial \pi} = -\frac{\left[w\tau^A - k(1+\gamma)(1+r)\right](1+r)}{\Phi}h_0 > 0,$$

which is almost similar to the case of a symmetric demographic shock (see Section 5.4).

The long-run effect of this temporary asymmetric shock is described by the following proposition.

[23] As is shown in Groezen and Leers (2000), it does not matter for the long-run effects of such a temporary shock whether the contribution rate (τ) or the benefit (η) is fixed. The short-run effects differ only quantitatively.

Proposition 5.6 *In the long run, the sign of* $\frac{\partial k}{\partial \pi}$ *is determined by the sign of* $(\tau^B - \tau^A)$*:*

$$\begin{aligned} \frac{\partial k}{\partial \pi} &> 0 \quad \textit{if} \quad \tau^B < \tau^A, \\ \frac{\partial k}{\partial \pi} &< 0 \quad \textit{if} \quad \tau^B > \tau^A, \\ \frac{\partial k}{\partial \pi} &= 0 \quad \textit{if} \quad \tau^B = \tau^A. \end{aligned}$$

Proof In the long run, $\frac{\partial k}{\partial \pi}$ is determined by the fixed point of difference equation (5.9), which is given by $\frac{\partial k}{\partial \pi} = \frac{(\tau^B - \tau^A) w[1+\gamma(1+r)](1+r)}{2\Phi + 2\gamma(2-\tau^A-\tau^B)kf''(k)(1+r)^2} \sum_{j=0}^{\infty} h_j$. The denominator of this expression is positive (due to the stability assumption), so the relation between $\frac{\partial k}{\partial \pi}$ and $(\tau^B - \tau^A)$ immediately follows. ■

The sign of the long-run change of the capital-labour ratio depends on the relative generosity of the local social security arrangements. Note the similarity between differences in individual thriftiness and the size of the PAYG-scheme: both a high rate of time preference and a high social security tax negatively affect private savings.

Proposition 5.6 can be interpreted as follows. If the government of region A runs a relatively generous public pension scheme ($\tau^A > \tau^B$), savings will be relatively low ($s^A < s^B$). Population ageing in region A then implies that eventually, the population share of that region in the world economy decreases. So individual savings increase on average, and thereby, because of perfect capital mobility, the capital-labour ratio. This implies a reduction of the interest rate and an increase of the (gross) wage. A lower interest rate causes savings to decline, while a higher wage stimulates savings. As by assumption the latter effect dominates the former, savings eventually increase.

If savings increase, then there is a shift from the unfunded social security arrangements to a funded pension system, which yields a higher internal return. Together with the increased wage this will have a positive effect on overall long-run welfare, irrespective of the institutional setting.

The reverse holds if $\tau^A < \tau^B$. Finally, if $\tau^A = \tau^B$, a temporary demographic shock will not affect the average long-run level of private savings, so the capital-labour ratio will remain unaltered and lifetime utility of inhabitants of both regions will eventually not be affected either.

Figures 5.15 and 5.16 display the evolution over time of the capital-labour ratio, interest rate and lifetime utility for the case of $\tau^A < \tau^B$ (which is comparable to Section

5.4). The figure also shows the development if capital is not internationally mobile. For region B, this would imply no changes at all, since it is not hit by a shock. If $\tau^A \geq \tau^B$, the figures would be similarly shaped, since the parameters of difference equation (5.9) do not change sign.

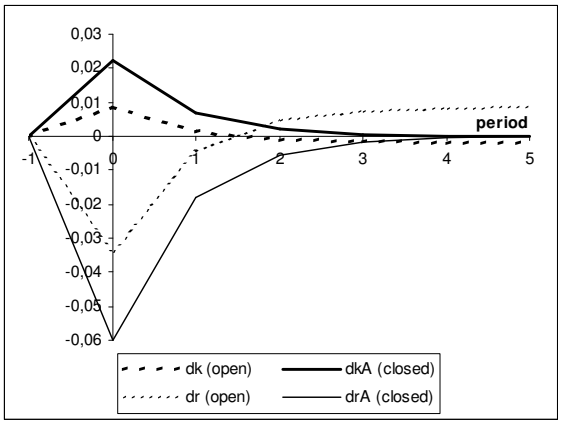

Figure 5.15

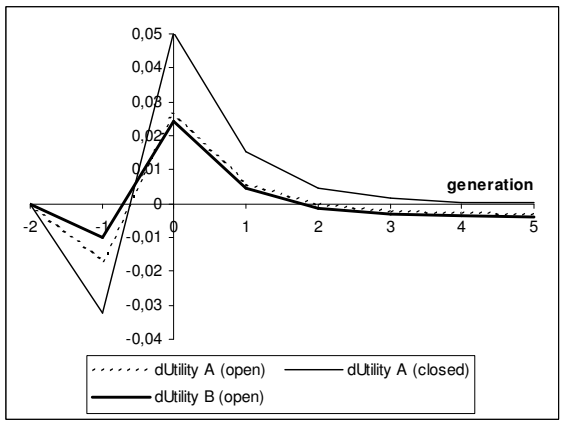

Figure 5.16

At the time of the shock, the capital-labour ratio increases. Because capital becomes relatively abundant in region A, a higher fraction of its savings will be invested in region B, which would not have been possible if it were a closed economy. This causes the capital-labour ratio to increase in both regions. The generation that is retired at the time of the shock will therefore suffer, since it faces a lower return on its savings. This loss is shared with the retired in region B. This retired generation will especially suffer from ageing in region A if $\tau^A > \tau^B$, because then, savings in B will be relatively high, making them very sensitive to changes in the interest rate.[24] The reverse holds for the generation born at the time of the shock: they benefit from the higher wage. This is translated into higher individual savings, so the capital-labour ratio also rises at time $t = 1$. Eventually, though, k decreases, which is due to the fact that region A, which is characterised by higher per capita savings, will be smaller in the long run. Note that if capital is not internationally mobile, the utility in both regions will not change in the long run. For region B this is obvious, as it does not experience a shock. Long-run utility in region A will not change either if capital is not mobile, since the effects of the temporary shock gradually fade away. But with open borders, its utility decreases.

[24] This can be considered e.g. the case of Italy (being region A with a relatively sizeable PAYG-scheme and severe population ageing) and The Netherlands (being region B with a large funded part).

Because individual savings are higher in region A, it is exporting capital to the other region. Since the number of inhabitants of region A decreases, investing the same amount in region B would imply a dramatic decrease of the capital-labour ratio in A. Therefore, people decide to export less capital abroad, so that the capital-labour ratio decreases to the same extent in both regions. Consequently, utility decreases (although it is at a higher level than without open borders).

5.6 Case study: Ageing in the OECD

The previous sections described the effects of ageing, taking the different aspects of this demographic development and region-specific characteristics separately into account. In this section, we will numerically explore the size and magnitude of the combined effects of ageing in the coming century. We consider the case of the OECD as a whole, subdivided in three regions. The first region is denoted "Europe" and consists of the four major economies in Western Europe: Germany, France, Italy and the United Kingdom. The second region is Japan, the third is the USA. It is assumed that individuals and institutional features within each region are identical, and that capital moves freely between the regions. Following on the previous sections, the regions differ in three ways from each other: the size of their public PAYG-scheme, the rate of time preference, and demography (i.e., the initial size and pattern of population ageing). Table 5.2 shows the values of different region-specific parameters used in the simulations.

Table 5.2

		Europe	**Japan**	**USA**
initial population size	N_{1975}^{young}	266 713	49 542	80 135
old-age dependency ratio	1975	0.52	0.36	0.54
	2000	0.58	0.71	0.48
	2025	0.89	1.15	0.80
	2050	1.31	1.55	0.91
population growth	1975-2000	0.18	0.09	0.55
	2000-2025	-0.12	-0.17	0.08
	2025-2050	-0.28	-0.25	0.08
social security tax rate	τ	0.13	0.0975	0.065
private discount factor	γ	0.70	0.93	0.48

Size of the PAYG-scheme

The PAYG-tax is chosen such that the social security contributions in the USA are 4.5% of GDP, which is the amount spent on Old-Age&Survivors Insurance and Disability Insurance as reported in the OASDI Trustees Report of 2002. From Table 5.1 we know that the implicit unfunded pension debt in Japan is (averaged over the four studies mentioned) 1.5 times higher than in the USA, and that of the region called Europe about twice as high as the US level. So the social security tax rates of Europe and Japan are chosen accordingly.

Individual rate of time preference

The values of γ are chosen differently for each country, such that the resulting savings for period $t = 1$ as a percentage of GDP, given the value chosen for the social security tax, equal the gross national savings in 2000 to GDP as reported in the last column of Table 5.1.[25]

Demography

Population numbers are taken from the medium variant of the United Nations *World Population Prospects: The 2000 Revision*. One period is equal to 25 years. Individuals are assumed to enter the labour market at the age of 25 and retire when they are 55. So the number of people denoted "young" in the model is set equal to the total number of

[25] For the four European countries, the average, weighted with relative population size equals 19.9%.

people aged 25 to 55, whereas "old" refers to those aged 55 and above. We consider the generations that enter(ed) the labour market in 1975 (denoted $t = 0$), 2000 ($t = 1$), 2025 ($t = 2$) and 2050 ($t = 3$). After that, the demographic structure is assumed not to change. Except when calculating the steady state, a distinction should now be made between the (inverse of) the dependency ratio (which is defined as the number of old divided by the number of young) and the growth rate of the (young) population. The first replaces the $(1+n)$-term in the savings equation, whereas the latter is used for calculating equilibrium in the capital market.

As can be seen from Table 5.2, ageing already takes place in Japan in the year 2000, followed by the United States and Europe one period later.

The steady state is defined as the (fictitious) state of the economy if the demographic structure remained as it was in 1975.[26] As for the production technology, we choose the following production function: $f(k_t) = k_t^{0.3}$, and assume full depreciation after one period ($\delta = 1$).[27] This specification holds for all regions.

Figures 5.16 and 5.17 give the change in utility as a result of ageing (compared to the steady state) in case of fixed contributions and fixed benefits, respectively, when capital moves freely between the regions.[28]

The capital-labour ratio is steadily rising, especially between the years 2000 and 2025, and will only decrease a bit after 2050. This implies a significant drop of the interest rate, from which Japan suffers the most as it saves a lot. Furthermore, this country is also experiencing a dramatic rise in the old-age dependency ratio, so in case of fixed contributions, the unfunded pension benefits will decrease. All this causes old-age consumption to be much lower than it would be if the population did not age. Lifetime utility decreases. Although Europe has a more sizeable PAYG-scheme, its population ages to a smaller extent. Besides, savings are lower, so the drop in the interest rate is not felt as much as

[26] This is a non-existing steady state, as the young individuals of 1975 already foresee the (consequences of) ageing in particularly Japan in the year 2000.

Note that for calculating this steady state, the value of $1+n$ should equal the inverse of the dependency ratio. The value taken is the average of the growth rate of young individuals between 1950 and 1975, and the ratio of young over old individuals in 1975.

[27] Another value of δ would not change the results significantly.

[28] The results in this section are based on calculated equilibria for the different time periods, and not on the linearised version that was presented in previous sections.

in Japan (also because the rate of time preference is higher). The United States suffer the least from ageing. There, savings are rather low, so relatively insensitive to the lower interest rate, whereas this country benefits from the capital flows from abroad and its own population is not ageing much.

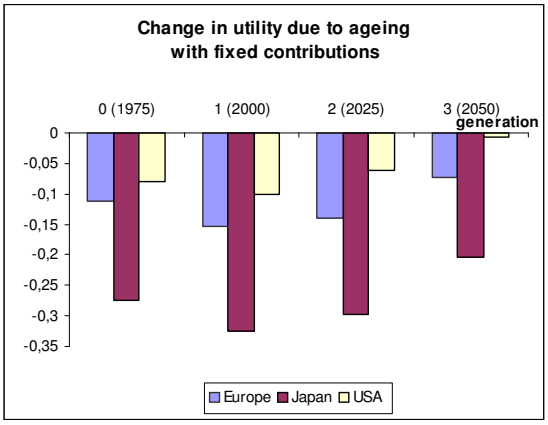

Figure 5.17

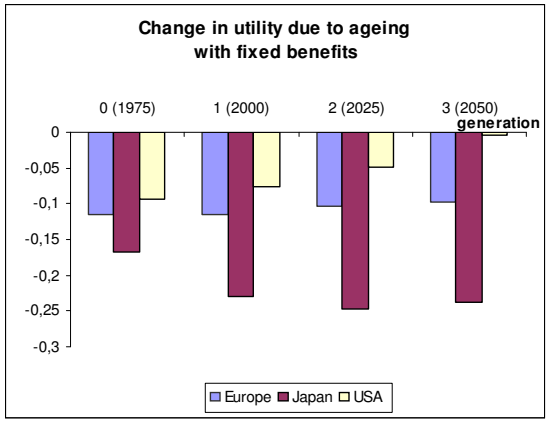

Figure 5.18

In all regions, the utility of generation 2000 decreases the most. This is because the capital-labour ratio only rises fast in 2025 (when Europe and the USA are also hit by ageing). Comparing Figure 5.17 to Figure 5.18, it appears that it matters especially for Japan how the PAYG-scheme is designed. If benefits are kept fixed, the negative effect of an increasing dependency ratio on old-age consumption is alleviated, for which the next generation bears the cost. So in that case, generation 2025 is suffering most. Furthermore, since ageing causes the contribution rate to rise, savings are negatively affected, which partly offsets the positive effect of a higher capital-labour ratio and thereby a higher wage on savings. The capital-labour ratio therefore does not rise as much, so future generations ($t = 2050$) will be worse off with fixed benefits. However, both in Europe and the USA, generation 1975 is better off with fixed contributions. Contrary to Japan, Europe's population hardly grows older between 1975 and 2000, whereas the relative number of young in the US even increases. This implies that the gains from having fixed benefits are small (or for the US, even negative). Furthermore, although the Japanese generation 1975 saves less with fixed benefits compared to fixed contributions, the reverse holds for the other regions. This means that the capital-labour ratio will be higher in 2000 if benefits are fixed, and consequently, the interest rate is lower. Hence, old-age consumption of the first generation in Europe and the USA is lower if benefits are fixed,

and so is utility. Generation 2000 benefits from the higher capital-labour ratio, and is also confronted with an ageing population in all regions, so fixed benefits yields them a higher utility. In the long-run, however, the capital-labour ratio will be lower, and utility too. Hence, the negative effects of ageing and social security are more evenly spread over the generations if contributions adjust so as to keep the benefit levels constant. This is at the expense of the first generations in Europe and the United States, and future generations in all regions.

In order to analyse the magnitude of the international spillover effects caused by population ageing, we can compare the utility levels for each region in the fully integrated economy to what it would be if the economies were all closed, i.e., without capital mobility. Figures 5.19-5.24 show this again for the case of fixed contributions and fixed benefits.

First consider the (fictitious) steady states. It holds for all regions that the interest rate exceeds the population growth, both in a closed and in an open economy. For Europe, the capital-labour ratio is higher if it were closed, and thereby also the wage. Open borders thus implies a lower wage, which by itself negatively affects utility in a dynamically efficient economy. Still, capital mobility is beneficial, because the savings can be invested more profitably abroad, implying a higher interest rate which for this region compensates the lower wage. Hence, utility is higher with open borders in Europe. The same holds for Japan, but for a different reason. Although the Japanese save most, the steady-state population growth is also high, which means a low capital-labour ratio. With open borders, capital flows from Europe to Japan, so the capital-labour ratio, the wage and utility increase. As for the United States, savings are lowest due to the high rate of time preference. This implies that if capital is mobile, Europeans will invest part of their savings in the USA, so this region benefits from open borders as well.

For Europe, being integrated with other OECD countries does not make much of a difference (for all generations considered), no matter how the public pension scheme is designed.

In Japan, however, capital mobility does matter. Apart from the first generation, all individuals would be better off if borders were closed, for both types of pension scheme. This is so because the low rate of time preference causes savings to be high in Japan. If capital were not mobile internationally, these savings would be entirely invested domestically. Together with the sharp decrease in the number of young, this would imply a great increase of the capital-labour ratio, so current young and future generations would

benefit, whereas the current old would face a lower interest rate on their savings. With open borders, this is be less so, which is why generation 1975 is better off in an open economy. The utility of future generations is higher in a closed economy because it is closer to the golden rule. The same conclusions can be drawn from the case in which benefits are kept at a constant level, although the differences are smaller.

This does not apply to the United States. All generations except generation 1975 benefit from international capital mobility. The increased savings in Europe and Japan are partly invested in the USA, whereas its population is not ageing as dramatically. This means a substantially higher capital-labour ratio than would result if the United States were a closed economy. Hence, this region benefits from the demographic developments in the rest of the world, both in case of fixed benefits and fixed contributions.

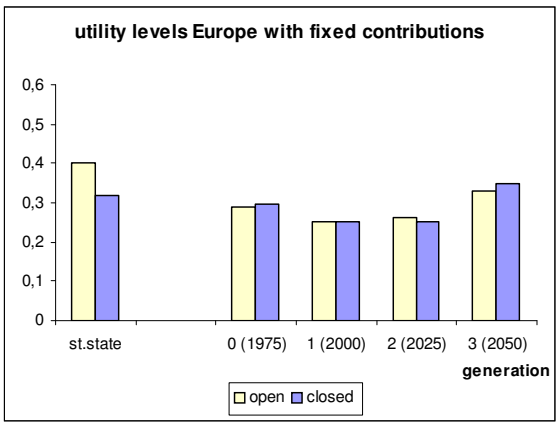

Figure 5.19

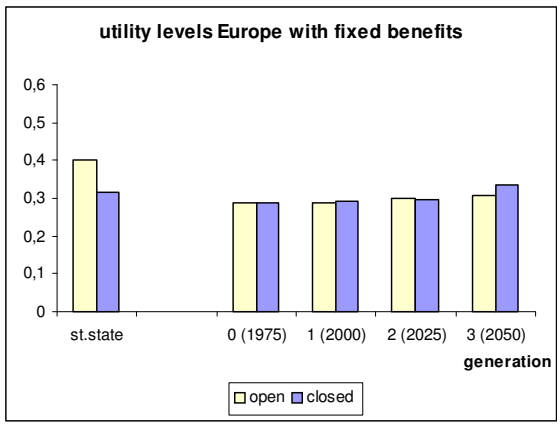

Figure 5.20

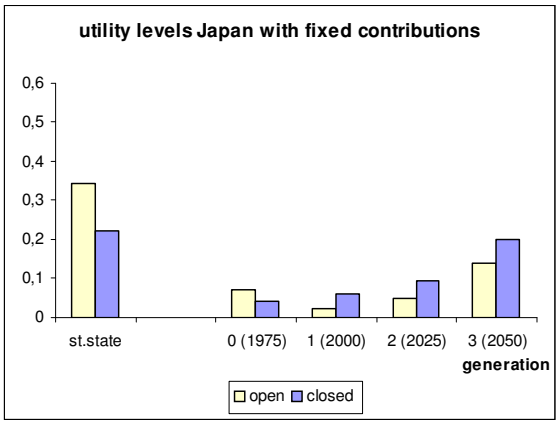

Figure 5.21

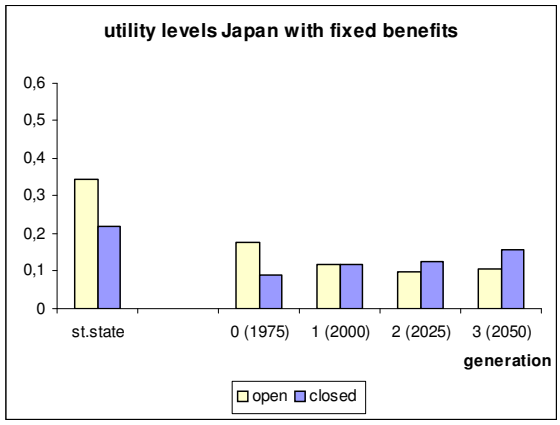

Figure 5.22

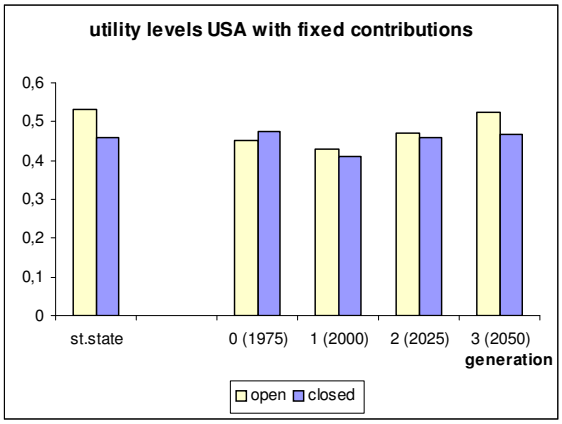

Figure 5.23

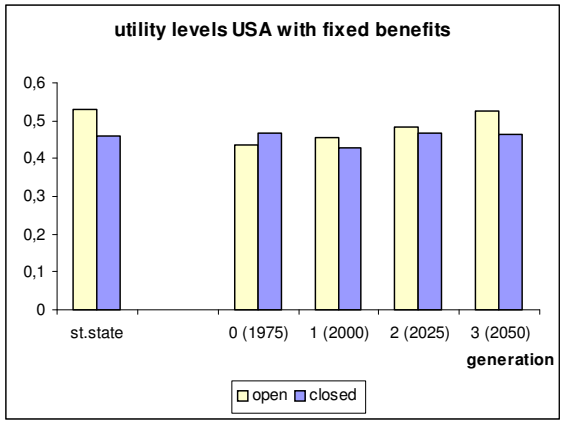

Figure 5.24

5.7 Conclusion

In the last decades, the capital markets in especially developed countries have become more closely interwoven. This implies that capital moves rather freely to regions where it can be invested most profitably. Put differently, the national investments in a particular country have become more dependent on factors that influence the reward to capital, viz. the interest rate. Consequently, the capital stock, wages, consumption and ultimately the well-being of different generations are affected, not only by domestic factors and developments, but also by circumstances abroad. This chapter focused on several important factors that influence savings and investment, and that will become even more important in the coming decades when most countries' populations grow older. Because countries differ in the size of unfunded pension schemes, demographic development and individual thriftiness, the effects of ageing and international capital mobility on the welfare of current and future generations can be substantial. Whether or not a country benefits from the mobility of capital depends crucially on the relative size of its PAYG-scheme. This has also repercussions for the effects of social security reform, because part of the benefits that result from a switch to a more funded scheme can flow abroad, and thus deprive a country from the incentive to implement such a policy.

Population ageing will also cause international spillover effects, even if the demographic shock is equally large and occurs simultaneously in all countries. Furthermore, for the quantitative effects, it matters a lot whether or not the social security tax is adjusted so as to keep the public pension benefits at their current level. A numerical example shows that especially the Japanese economy is negatively affected by ageing, and that due to capital mobility, these effects are reinforced, whereas the United States benefit from capital flows induced by ageing in other parts of the world.

In this chapter, we only considered ageing in developed countries. In less developed countries, the dependency ratio will not rise to levels as high as in developed countries (except for China, whose population will grow considerably older in the coming decades). Still, the relative increase is approximately the same, though it occurs later than in Europe and the US. By investing in those regions, developed countries could take advantage of the more favourable demographic situation and earn a higher return on their capital, which would offset part of the negative effects that we found. However, several studies (Turner *et al.*, 1998, MacKellar and Reisen, 1998, and Holzmann, 2000), indicate that those effects are moderate. Furthermore, the assumption of perfect capital mobility is highly

questionable, considering the high transaction costs and risks associated with investing in less developed countries. An interesting extension would therefore be to also include such countries and capital rigidities.

5.A Appendix to Section 5.3.3

Proof Proposition 5.3

The long-run change of consumption when young and old in region i is

$$dc^{y,i} = (1-\tau^i)dw - wd\tau^i - ds^i, \tag{5A.1.a}$$
$$dc^{o,i} = s^i dr + (1+r)ds^i + (1+n)wd\tau^i + (1+n)\tau^i dw. \tag{5A.1.b}$$

Using the first-order condition $z^i = \gamma(1+r)c^i$ gives the change in long-run utility in region i,

$$dU^i = \frac{1}{c^{y,i}}\left(dc^{y,i} + \frac{dc^{o,i}}{1+r}\right).$$

Inserting (5A.1.a) and (5A.1.b) gives

$$dU^i = \frac{f''(k)}{c^{y,i}(1+r)}\left[s^i - k(1+r) - \tau^i k(n-r)\right]dk + \frac{w(n-r)}{c^{y,i}(1+r)}d\tau^i.$$

The marginal change of long-run utility in region A due to a marginal change in the PAYG-tax of that region is

$$\frac{dU^A}{d\tau^A} = \frac{f''(k)}{c^{y,A}(1+r)}\left[s^A - k(1+r) - \tau^A k(n-r)\right]\frac{dk}{d\tau^A} + \frac{w(n-r)}{c^{y,A}(1+r)}. \tag{5A.2}$$

From the fact that $\tau^A < \tau^B$ we know that $s^A > k(1+n)$. Suppose that τ^A is so low that $r = n$. In that case, (5A.2) boils down to $\frac{f''(k)}{c^{y,A}(1+r)}\left[s^A - k(1+n)\right]\frac{dk}{d\tau^A}$. Knowing that $f''(k), \frac{dk}{d\tau^i} < 0$, this is positive. That is, if $r = n$, there is an incentive for region A to increase its PAYG-scheme: both the currently old benefit and the currently young and future generations.

On the other hand, if $\tau^A = \tau^B$, no international capital flows occur and $s^A = k(1+n)$, so that (5A.2) becomes $\frac{dU^A}{d\tau^A} = \frac{f''(k)}{c^{y,A}(1+r)}(n-r)(1-\tau^A)\frac{dk}{d\tau^A} + \frac{w(n-r)}{c^{y,A}(1+r)}$, which is negative if $r > n$. Hence, there is a value of $\tau^A \in [0, \tau^B)$ for which it holds that both $\frac{dU^A}{d\tau^A} = 0$ and $r > n$.

For the other region, it holds that

$$\frac{dU^B}{d\tau^A} = \frac{f''(k)}{c^{y,B}(1+r)}\left[s^B - k(1+n) + k(1-\tau^B)(n-r)\right]\frac{dk}{d\tau^A},$$

which is clearly negative if $r \geq n$, since $s^B < k(1+n)$ as long as $\tau^A > \tau^B$.

Proof Proposition 5.4

Likewise, we can write the change in long-run utility of region B as a consequence of a change in the PAYG-tax in that region as

$$\frac{dU^B}{d\tau^B} = \frac{f''(k)}{c^{y,B}(1+r)}\left[s^B - k(1+n) + k(1-\tau^B)(n-r)\right]\frac{dk}{d\tau^B} + \frac{w(n-r)}{c^{y,B}(1+r)}.$$

Knowing that $\tau^B > \tau^A$ and $r \geq n$, it follows that $s^B < k(1+n)$, so $\frac{dU^B}{d\tau^B} < 0$.

For region A we have

$$\frac{dU^A}{d\tau^B} = \frac{f''(k)}{1+r}\left[s^A - k(1+n) + k(1-\tau^A)(n-r)\right]\frac{dk}{d\tau^B}.$$

If $\tau^A < \tau^B$ such that $r = n$, this expression is positive, and it is negative if $\tau^A = \tau^B$. So there exists a value of $\tau^A \in [0, \tau^B)$ for which it holds that both $\frac{dU^A}{d\tau^B} = 0$ and $r > n$.

Chapter 6

Ageing and Social Security Policy when Fertility is Endogenous

6.1 Introduction[1]

All over the world, countries are and will increasingly be confronted with rising dependency ratios, urging many to reconsider the design of existing social security programs that are financed on a PAYG-basis. The most often heard answer to the 'demographic time bomb' is to reduce the unfunded part of pensions and replace it by savings, as they earn a higher return than the implicit return of a PAYG-scheme. The latter consists of the growth rate of wages and population, and is mostly assumed to be exogenous. In previous chapters, we focused on the impact of social security reform and population ageing itself on the interest rate and economic growth. It was shown that an easy way of reasoning passes over some important economic effects that can significantly weaken or even reverse the usual conclusions about the appropriate policy actions. In this chapter, we will concentrate on the fertility rate, which is actually the result of a choice that individuals make on how many children to raise. This endogeneity of fertility has important implications for the economy and thus for the suitable design of the social security system.

In many western countries, people are nowadays to a large extent able to choose how many children they want to raise. Since rearing children generally increases utility but also involves substantial costs, economic theory can be applied when analysing fertility

[1]This chapter is a combination and extension of Groezen, Leers and Meijdam (2002) and Groezen, Leers and Meijdam (2003).

changes. As Razin and Sadka (1995, p. 3) put it, "noneconomic factors play a crucial role in shaping parental preferences. But once preferences are shaped [...], economics comes into play." The economic approach to fertility was introduced by Leibenstein (1957) and Becker (1961), who postulated that the number of children directly enters the utility function, just like consumption goods do.[2] So one can apply the standard consumer demand theory to fertility choice (see Birdsall, 1988, for a summary). But apart from giving instantaneous utility, children can also enhance the parents' well-being when they are retired by providing services to them, the so-called old-age security hypothesis. Offspring is therefore also comparable to a capital good, that enhances utility later in life by serving as an insurance against the risk of old-age dependency. However, in the second half of the last century, old-age care has been institutionalised in many developed countries through the installation and extension of public pension and health care arrangements. This is one of the reasons for the dramatic drop in the fertility rate: raising a child does not 'pay out' to the individual parent herself any more, but to society as a whole.[3] This social externality is not taken into account when fertility choices are made, so this theory hardly provides an explanation any more for the number of children in a household (see e.g. Cigno, 1993, and Folbre, 1994, who typifies offspring as a public rather than a private 'capital good'). A PAYG-pension scheme thus creates an incentive for an individual to free ride on the public system by raising few children, while still being entitled to a retirement benefit (see e.g. Sinn, 1997 and 1998). This will be especially so if the parents have to bear most of the (substantial) costs of raising their children themselves.

This chapter therefore follows the consumer-demand approach to explain fertility. The aim is twofold. First, we raise the question how one form of ageing, viz. increasing longevity, affects the rate of fertility. Second, we investigate how the decisions of an individual deviate from the social optimum, in what way a government can intervene to improve welfare, and how this depends on both the size of an already existing PAYG-scheme and on the average lifespan. As in previous chapters, we analyse this for the case of a small open economy with perfect capital mobility, and a closed economy. The rest

[2] According to Zhang and Zhang (1998), this can be viewed as a weak form of altruism towards children. Alternatively, much as in e.g. Razin and Ben-Zion (1975) and Zhang (1995b), parents could also derive direct utility from the utility of their offspring, which can be considered complete or strong altruism.

[3] Another reason is that women got better labour force opportunities and (real) wages increased, so that the opportunity costs of raising children increased. See e.g. Becker and Barro (1988) and Galor and Weil (1996).

of the chapter is organised as follows. In Section 6.2, the model is presented. Section 6.3 deals with the effects of increasing longevity on the fertility rate. In Section 6.4, we describe the command optimum and the social externalities of progeny, while Section 6.5 analyses the optimal child allowances for both types of economy. Section 6.6 concludes.

6.2 The model

Consider an economy where production is described by a standard neoclassical constant-returns-to-scale production function, $f(k_t)$, with k_t the amount of capital per young individual in period t. The interest rate and wage are given by, respectively, $r_t = f'(k_t) \equiv \frac{df(k_t)}{dk_t}$ and $w_t = f(k_t) - k_t f'(k_t)$. The economy is populated by a large number of individuals who live for at most two periods, such that in each period, both a young and an old generation are alive.[4] Apart from age, individuals are identical. Every young individual faces a probability ε of growing old, so $1 - \varepsilon$ is the fraction of young that die after one period of life.[5]

Every young agent inelastically supplies one unit of labour. The government imposes a proportional tax τ on labour income so as to finance the public pension benefit of the current elderly. The remainder of first-period income is either spent on consumption (both material consumption (c^y) and offspring (n)[6]) or is saved for old age (s). That is, we assume children to be viewed as normal goods. A parent only wants to raise children with a certain level of well-being, that is, a child is only joyful for its parents if it is assured to receive a particular number of commodities and services like education, food, clothes, holidays etcetera during its upbringing. This is reflected in the price p of such a 'quality-child', which is constant and equal for all children.[7] So first-period consumption

[4] As a matter of fact, people live for three periods. In childhood, they are raised (from which their parents derive utility), and make no economic decisions. The subsequent period, which is the first period in our model, is their industrious life, and during the last period they are retired.

[5] One could also interpret ε as the average time spent in retirement.

[6] For simplicity we neglect gender differences as well as the difference between an individual and a household. Furthermore, we assume that every individual is able to freely choose a number of children from the set of nonnegative real numbers.

[7] See e.g. Praag and Warnaar (1997) for a survey on the measurement of the costs of raising children. These costs include food, clothing, housing, child care and education. Longman (1998) estimated that for the United States these costs can amount to $200,000 (corrected for inflation and not taking the foregone wage income into account). For another estimation, see e.g. Bourguignon (1999).

is restricted by the following individual budget constraint,

$$c_t^y + pn_t = w_t(1-\tau) - s_t. \tag{6.1.a}$$

In the second period of life, the individual is retired and entitled to a public pension benefit. She derives utility from old-age (material) consumption (c^o)[8], which is financed from the return on first-period savings and the pension benefit (η). Savings are invested in annuities or through an actuarially fair pension fund. As only a fraction ε of young savers survive to the next period, the assets of those who passed away fall to surviving contemporaries. The total return on the savings is therefore $\frac{1+r}{\varepsilon}$, so the second-period budget constraint can be written as

$$c_{t+1}^o = \frac{1+r_{t+1}}{\varepsilon} s_t + \eta_{t+1}. \tag{6.1.b}$$

We assume agents to have perfect foresight with respect to the level of the future public pension benefit.[9]

Young people derive utility from the number of children they raise (n_t) and material consumption (c_t^y), whereas the retired value material consumption (c_{t+1}^o) only, as mentioned above. For analytical convenience, we assume the utility function of a representative individual to be additively separable and the felicity functions to be logarithmic,

$$U(c_t^y, n_t, c_{t+1}^o) = \log(c_t^y) + \gamma \log(n_t) + \beta\varepsilon \log(c_{t+1}^o), \tag{6.2}$$

where β is the private discount rate and γ the utility weight of raising children relative to material consumption.[10]

[8] If she derived utility from children too, the main results would still hold as long as the marginal utility from children in the second period of life is not greater than in the first period of life. This is a reasonable assumption since the utility one derives from children depends to a great extent on the time spent together with them, which is most during the childhood years, i.e., when parents are young.

[9] Of course, in reality agents' financial position during the rest of their lives is always uncertain. In particular, the generosity of the social security system once one is retired is uncertain as a PAYG-scheme inherently implies dependency on other generations. Groezen, Leers and Meijdam (2002) show that, in a model with endogenous fertility, this uncertainty makes society vulnerable to, for example, pessimistic beliefs and changes in life expectancy.

[10] This is a conventional way of modelling endogenous fertility. See e.g. Eckstein and Wolpin (1985) and Galor and Weil (1996).

The government runs the public pension system as a PAYG-scheme with, at any time t, the following (balanced) budget constraint,

$$\varepsilon\eta_t = n_{t-1}\tau w_t. \tag{6.3}$$

Individual agents act atomistically and do not take this budget constraint into account when they choose how many children to raise. So we abstract from the possibility that an individual household considers offspring as an 'investment good' that yields a return in the form of a transfer when old. This 'old-age security hypothesis' played an important role in traditional economies (and still does in many developing countries) where the transfers from the young to the old took place within the family (see e.g. Cigno, 1992). However, in modern western economies, intergenerational transfers for old age are institutionalised in public pension systems. This socialises the investment aspect of children and introduces the possibility for an individual household to free ride on the system by rearing few children (or none at all) and still being entitled to a full pension benefit (see e.g. Cigno, 1993, and Folbre, 1994). Consequently, the only reason that people choose for a certain family size is the utility that they directly derive from raising their offspring.[11]

Individuals maximise lifetime utility (6.2) subject to their single-period budget constraints (6.1.a) and (6.1.b). Given the specification of the utility function, the optimal levels of material consumption and offspring are uniquely determined by the following first-order conditions,

$$c^o_{t+1} = (1 + r_{t+1})\beta c^y_t, \tag{6.4.a}$$

$$pn_t = \gamma c^y_t, \tag{6.4.b}$$

where $c^o_{t+1} = \frac{1+r_{t+1}}{\varepsilon}s_t + \eta_{t+1}$ with $s_t = w_t(1-\tau) - c^y_t - pn_t$. Equation (6.4.a) is the standard condition that households equate the marginal rate of substitution between current and future consumption to the rate of interest. According to equation (6.4.b), households choose the number of children and consumption in their first period of life such that the marginal rate of substitution between a child and current consumption equals the marginal cost of rearing an extra child.

[11] To assure a determinate steady state, we assume that $p > \frac{\tau w}{1+r}$, that is, the costs of raising children exceed the (implicit social) return on a child, which equals the present value of her contribution to the PAYG-scheme. If this were not the case, the number of children would grow infinitely large and the steady state of the model would be indeterminate.

From (6.1.a), (6.1.b), (6.3), (6.4.a) and (6.4.b) we can derive the steady state,[12]

$$c^y = \frac{w(1-\tau)p(1+r)}{(1+\gamma+\beta\varepsilon)p(1+r)-\gamma\tau w}, \qquad n = \frac{\gamma w(1-\tau)(1+r)}{(1+\gamma+\beta\varepsilon)p(1+r)-\gamma\tau w}. \tag{6.5}$$

6.3 Increasing longevity

The ageing of the population that is expected in the next decades for most industrialised countries is not only brought about by a decrease in the fertility rate, but also by an increasing life span. Because people are expected to live longer and the retirement age does not rise (or even decreases), a higher fraction of lifetime consumption will have to be backed by pension income, such as savings and PAYG-benefits.[13] This section explores the relation between increasing longevity, the number of children and the size of the PAYG-pension scheme in the economy described in Section 6.2, assuming perfect foresight. We distinguish between two polar cases: a small open economy with perfect capital mobility and a closed economy.

6.3.1 Ageing in a small open economy

In a small open economy, the interest rate is exogenously determined on the world capital market. Assuming no major shocks occur abroad, this rate is constant and equal to $\bar{r}$ at all times. Consequently, both the capital-labour ratio and the wage are constant as well, at $\bar{k}$ and $\bar{w}$ respectively. The effects of a longer lifespan then immediately follow from (6.5), as verbalised in the following proposition.

Proposition 6.1 *If the (expected) life span increases in a small open economy, savings increase and the number of children declines. This causes the dependency ratio to rise even further. Consequently, the PAYG-pension scheme deteriorates both in the short and in the long run.*

Proof Differentiating (6.5) gives $\frac{\partial c^y}{\partial \varepsilon} < 0$ and $\frac{\partial n}{\partial \varepsilon} < 0$, so $\frac{\partial s}{\partial \varepsilon} > 0$. This also holds for a more general utility function (see Appendix). ■

[12] Whenever time subscripts are omitted, we refer to the steady-state value of the respective variable.

[13] Assuming the retirement age does not increase. If the length of the industrious period of life and the period of retirement increase equiproportionally, then the fraction of lifetime consumption that is backed by pension income does not change.

As people live longer, both the return on assets and the PAYG-benefit decrease, so old-age consumption decreases as well. This incites the young to save more and to economise on children,[14] so that both consumption when young and the number of offspring decrease instantaneously to the new steady-state values.[15] This reduces the PAYG-benefit, which intensifies the increase in savings. A longer lifespan therefore implies that a larger part of old-age income is financed through a funded scheme. The problem of ageing is thus exacerbated by individual fertility decisions. Agents apparently take decisions that are not optimal for society as a whole, thus eventually putting themselves at a disadvantage. The next sections therefore explore the socially optimal number of children and what the appropriate government action might be.

6.3.2 Ageing in a closed economy

In a closed economy, savings are entirely invested domestically. Equilibrium in the capital market is therefore given by

$$s_t = n_t k_{t+1}. \tag{6.6}$$

So the capital-labour ratio is the result of the individual savings and fertility choices. This in turn determines the wage and interest rate on which economic agents base their decisions. As the following proposition shows, the effects of an increasing lifespan may differ in a closed economy from those in a small open economy.

Proposition 6.2 *If the (expected) life span increases in a closed economy, the long-run capital-labour ratio increases. The presence of a PAYG-scheme reduces this increase. Furthermore, the number of children may increase or decrease in the long run.*

Proof Inserting (6.5) into (6.1.a) and combining with (6.6) gives the implicit solution of k,

$$\beta\varepsilon p(1+r) - \gamma\tau w = \gamma(1+r)k.$$

[14] Due to increasing longevity, old-age consumption will get a higher weight in the decision on the intertemporal allocation of lifetime income. At the same time, however, the return on annuities decreases if ε gets higher, which makes saving less attractive. As can be seen from equation (6.4.a), these opposite effects exactly cancel out, so that there is no direct effect of a change in ε on savings.

[15] Due to the small-open-economy assumption factor prices are fixed and the economy converges to its new steady state within one period.

Comparative statics gives $\frac{\partial k}{\partial \varepsilon} = \frac{\beta p(1+r)^2}{\gamma[(1+r)^2 - \tau k f''(k)(1+r) - \tau w f''(k)]} > 0$, which clearly is a negative function of the social security tax. From this it follows that $\frac{\partial r}{\partial \varepsilon} < 0$ and $\frac{\partial w}{\partial \varepsilon} > 0$.

The change in the number of children due to a longer life span is given by

$$\frac{\partial n}{\partial \varepsilon} = \frac{n}{\Delta}\left[-\beta p(1+r) + (1+\gamma+\beta\varepsilon)p(1+r)\frac{\partial w/\partial \varepsilon}{w} - \gamma\tau w\frac{\partial r/\partial \varepsilon}{1+r}\right],$$

with $\Delta \equiv (1+\gamma+\beta\varepsilon)p(1+r) - \gamma\tau w > 0$. The first term between brackets is negative (the effect that occurs in a small open economy), whereas the last two terms reflect the positive general-equilibrium effects on fertility. Hence, the number of children can decrease or increase due to a longer lifespan. ■

Increasing longevity urges people to save more, which initially implies lower expenses on consumption when young and also on the number of children. So in the short run, fertility decreases. The higher savings are invested domestically, so that in subsequent periods, the capital stock increases. This, together with a smaller population growth, implies a higher capital-labour ratio. A PAYG-pension scheme reduces the incentive to save (extra), so it dampens these effects. Furthermore, the wage increases and the interest rate decreases. This causes a positive income effect on the number of children,[16] and may eventually dominate the direct negative effect, depending on how strong the factor prices change. For instance, if the capital-labour ratio is initially quite low (e.g. due to an extensive unfunded pension scheme), the marginal product of capital will be high. Consequently, a growing capital stock will induce a substantial wage increase, boosting the fertility rate. The same applies if capital and labour are strong complements in the production process.

6.4 The first-best solution

In the model of endogenous fertility presented above, the (only) reason for people to raise children is the direct utility they get from it. The fact that their children will also participate in the labour market one period later and consequently increase production is thus neglected. This section therefore explores the externalities that a child brings about for society as a whole, by comparing the choices that individuals make in a market setting to the choice that a social planner would make.

[16] A lower interest rate increases the present value of the public pension benefit (that also rises if wages grow), so that individuals save less and raise more children.

Consider a social planner at time t whose objective function consists of the lifetime utilities of all current and future generations,

$$W_t = \sum_{i=t}^{\infty} \delta^{i-t} U(c_{i-1}^y, n_{i-1}, c_i^o), \tag{6.7}$$

where $\delta < 1$ is the social discount factor, i.e., the factor at which the planner discounts lifetime utility of future generations.[17]

Assuming that capital does not depreciate,[18] the social planner is each period restricted by the economy's resource constraint,

$$f(k_t) + (n_t d_{t+1} - d_t) - r_t d_t = c_t^y + p n_t + \frac{\varepsilon c_t^o}{n_{t-1}} + (n_t k_{t+1} - k_t), \tag{6.8}$$

with d_t denoting the per capita foreign debt of the country. This equation says that total resources, consisting of production and debt creation minus interest payments, are allocated to consumption of the young and old, the costs of rearing children and domestic investments.[19]

We can again distinguish between a small open economy and a closed economy. In case of a small open economy, the capital-labour ratio is fixed and equal to $\bar{k}$, and d_t is one of the social planner's instruments. If the economy is closed, it holds that $b_t = d_t \equiv 0$ and k_t replaces d_t as one of the social planner's instruments.

Maximising (6.7) at time t subject to the resource constraint (6.8) results in the

[17] Notice that we assume the weight of a generation's utility in the social welfare function to be independent of the size of this generation, i.e., we apply a Millian rather than a Benthamite welfare criterion. In contrast to a Millian SWF, utilitarianism in Benthamite form, i.e., $W_t = \sum_{i=t}^{\infty} \delta^{i-t} \prod_{j=0}^{i} n_{j-2} U(c_{i-1}^y, n_{i-1}, c_i^o)$, would imply that in the absence of intergenerational redistribution, the command optimum does not coincide with the Pareto-efficient market outcome. Therefore, we employ the Millian welfare concept.

[18] Any rate of depreciation could be assumed, but would not affect the results.

[19] An alternative way of writing the resource constraint is $b_t - r_t d_t \equiv f(k_t) - c_t^y - \frac{c_t^o}{n_{t-1}} - p n_t - (n_t k_{t+1} - k_t) - r d_t = -(n_t d_{t+1} - d_t)$, where b_t denotes (per capita) net exports. The (per capita) current account can be written as $b_t - r_t d_t$ and the (per capita) capital account is given by $n_t d_{t+1} - d_t$. Equilibrium of the balance of payments requires these two to sum up to zero, implying $n_t d_{t+1} = (1 + r_t) d_t - b_t$.

following first-order conditions for the command optimum,

$$\frac{c_{t+1}^{y*}}{c_t^{y*}} = \frac{\delta(1+r_{t+1})}{n_t^*}, \tag{6.9.a}$$

$$\frac{c_{t+1}^{o*}}{c_t^{y*}} = \beta(1+r_{t+1}), \tag{6.9.b}$$

$$\frac{c_t^{y*}}{n_t^{**}} = \frac{p+\bar{k}-d_{t+1}^*}{\gamma+\beta\varepsilon}, \tag{6.9.c [open economy]}$$

$$\frac{c_t^{y*}}{n_t^*} = \frac{p+k_{t+1}^*}{\gamma+\beta\varepsilon}, \tag{6.9.c [closed economy]}$$

where x^* denotes the socially optimal value of variable x. Conditions (6.9.a) and (6.9.b) describe the optimal inter- and intragenerational allocation, respectively. In the steady state, condition (6.9.a) boils down to $n^* = \delta(1 + r_{t+1})$, which says that the socially optimal number of children is uniquely determined by the social discount factor and the interest rate. Furthermore, condition (6.9.b) is equivalent to equation (6.4.a), implying that the social planner respects the individual's savings decision. Condition (6.9.c) gives the optimal relationship between the number of children and young-age consumption. This equation differs from the one the private sector uses to choose family size. Comparing (6.9.c) with (6.4.b) indicates the two reasons why the choice of individual households may deviate from the social optimum:

- the *dependency-ratio effect*, as captured by the $\beta\varepsilon$-term in (6.9.c) which the private sector does not take into account: a higher number of children implies that in the future, total output increases, which is shared with the same number of pensioners; future consumption enters the private agents' utility function with the parameters $\beta\varepsilon$; note that this effect gains importance if the average lifespan increases: if people live longer, they do not take into account that they can enjoy the additional output of an extra child for a longer time;

- the *capital-dilution effect*: the private sector does not consider the term $k_{t+1} - d_{t+1}$ in equation (6.9.c). In a small open economy, the capital-labour ratio is given at $\bar{k}$. Part of the capital stock is financed by national savings (s), the remainder by foreigners (d). In a closed economy, the whole capital stock is financed with national savings. If the number of children increases, a higher capital stock is required in order to keep per capita production at its former level. This means that savings

should be higher, unless the total capital stock is entirely financed with foreign debt (i.e., $\bar{k} = d$). However, individual non-altruistic households do not take into account that an extra child should induce them to save more. In fact, the true price of an extra child is not just p, but also the extra capital stock that the child needs to be endowed with in order to be as productive (in the future) as the other children. From this point of view, people decide to have too many children, because they do not realise that an extra child implies - *ceteris paribus* - a lower capital-labour ratio, so less production per capita.

So in principle, the number of children in a market economy may be too low or too high as compared to the command optimum, depending on whether the dependency-ratio effect dominates or is dominated by the capital-dilution effect. Fertility is optimal if the dependency-ratio effect happens to exactly offset the capital-dilution effect. The next section discusses how the government can achieve the social optimum.

6.5 Child allowances

It is well-known that in case of exogenous fertility, the government is able to replicate the first-best solution in a market setting provided it has a system of lump-sum intergenerational transfers at its disposal (see Blanchard and Fischer, 1989, Chapter 3). However, the analysis above shows that in general, the government needs an additional instrument in order to be able to do so when the number of offspring is also a choice variable. If the dependency-ratio effect exceeds the capital-dilution effect, too few children are born and the government is in want of an additional instrument in order to stimulate households to have a larger number of offspring. Obviously, this could be accomplished by a child allowance scheme. Since offspring can be considered a normal good, a decrease of its price due to a subsidy per child would induce people to have more children.[20] In the opposite case it would need a tax on children so that the number of children that parents decide to raise decreases.

Suppose the government provides a child allowance of φ per child, which cannot only be thought of as a (direct) child benefit, but also involves e.g. subsidizing child care, public

[20] Empirical evidence suggests that child allowances indeed have a significant (though moderate) positive effect on fertility choice (see e.g. Blanchet and Ekert-Jaffé, 1994, and Vallés Giménez and Marco, 2002, for Spain).

education and student grants. These allowances are financed by a proportional wage tax of θ.[21] The individual's first-period budget constraint then becomes

$$c_t^y + (p - \varphi)n_t = (1 - \tau - \theta_t)w_t - s_t, \tag{6.1.a'}$$

and the government's budget constraint for the child allowance scheme is

$$\theta_t w_t = n_t \varphi,$$

which agents do not take into account when deciding on the number of children due to their atomistic behaviour. First-order condition (6.4.b) becomes

$$\gamma c_t^y = (p - \varphi)n_t. \tag{6.4.b'}$$

The resulting consumption when young and fertility are

$$c^y = \frac{w(1-\tau)(p-\varphi)(1+r)}{(1+\gamma+\beta\varepsilon)(p-\varphi)(1+r)+\gamma\varphi(1+r)-\gamma\tau w} \tag{6.10}$$

$$n = \frac{\gamma w(1-\tau)(1+r)}{(1+\gamma+\beta\varepsilon)(p-\varphi)(1+r)+\gamma\varphi(1+r)-\gamma\tau w}. \tag{6.11}$$

6.5.1 Child allowances in a small open economy

From (6.8) and (6.9.a)-(6.9.c) we can derive the following expressions for young-age consumption and fertility in the command optimum in a small open economy,

$$c^{y*} = \frac{\bar{w} - (1+\bar{r})p}{1 + \beta\varepsilon + \gamma - \frac{\gamma}{\delta}}, \qquad n^* = \delta(1+\bar{r}). \tag{6.12}$$

The next proposition shows that the government can realise the first-best solution in a market economy if it is able to use a combination of lump-sum intergenerational transfers and an instrument to effectively alter the number of offspring chosen by households.

[21] Notice that we allow for the possibility that $\varphi < 0$, in which case offspring is taxed and the revenues of this tax are rebated to the young. To assure a determinate steady state, we assume that $p - \varphi > \frac{\tau w}{1+r}$, that is, the net private costs of raising children are larger than the (implicit social) returns, which equal the present value of their contribution to the PAYG-scheme.

Proposition 6.3 *The government can realise the first-best outcome in a market*

(i) without intervention iff $\delta = \delta_0 \equiv \frac{\gamma\bar{w}}{(1+\beta\varepsilon+\gamma)(1+\bar{r})p}$,

(ii) using both child allowances and PAYG-transfers to the old iff $\delta > \delta_0$,

(iii) using both a tax on children and lump-sum transfers from the old to the young iff $\delta < \delta_0$.

Proof Substituting $\theta = \tau = 0$ in (6.10) and (6.11) gives $c^{y*} = \frac{\bar{w}}{1+\beta\varepsilon+\gamma}$, $n^* = \frac{\gamma\bar{w}}{(1+\beta\varepsilon+\gamma)p}$. Inserting $\delta = \delta_0 \equiv \frac{\gamma\bar{w}}{(1+\beta\varepsilon+\gamma)(1+\bar{r})p}$ in (6.12) shows that $c^y = c^{y*}$ and $n = n^*$ which proves *(i)*.

When we compare the conditions (6.4.a) and (6.4.b') to the first-order conditions for the command optimum (6.9.a)-(6.9.c), we see that these coincide if $\theta = \theta^* \equiv \frac{pn^* - \gamma c^{y*}}{\bar{w}}$ and $\tau = \tau^* \equiv 1 - \frac{(1+\beta\varepsilon+\gamma)c^{y*}}{\bar{w}}$. Notice that, in accordance with *(i)*, $\theta^* = \tau^* = 0$ if $\delta = \delta_0$. Moreover, equation (6.12) implies that $\frac{\partial c^{y*}}{\partial\delta} < 0$ and $\frac{\partial n^*}{\partial\delta} > 0$, from which it follows that $\frac{\partial\theta^*}{\partial\delta} > 0$ and $\frac{\partial\tau^*}{\partial\delta} > 0$. This proves *(ii)* and *(iii)*. ■

The first part of Proposition 6.3 defines the government discount factor for which the market solution without any form of government intervention coincides with the command optimum. So for this level of the social discount factor the government does not redistribute between generations. Moreover, the dependency-ratio effect and the capital-dilution effect exactly cancel out, implying that the government does not need an instrument to affect the number of offspring. Notice that it follows from this result that the market outcome without any form of government intervention is Pareto-efficient.

But if the government of a small open economy wants to redistribute from the young to the old through an unfunded pension scheme, this should be accompanied by a positive child allowance in order to realise a first-best outcome, while it is optimal to levy a tax on children if the government redistributes from the old to the young. So it appears that the net external effect of children is positive, that is, the number of offspring without child allowances is too low, if a PAYG-pension scheme is in place. The reason for this is that such a pension system implies that part of the social benefits of having a child reveals itself in a growing tax base and are imperceptible to the individual parent. If, on the other hand, the government redistributes from the older to younger generations,

the capital-dilution effect dominates the dependency-ratio effect, so that it is optimal to introduce a tax on offspring.

The following proposition provides a precise measure for the size of the external effect caused by intergenerational redistribution, and thus a yardstick for the Pigouvian subsidy (tax), i.e., the child allowance (tax on offspring) necessary to correct it.

Proposition 6.4 *The optimal child allowance in a small open economy equals* $\varphi^* = \frac{\tau\bar{w}}{1+\bar{r}}$, *and therefore does not depend on the average lifespan.*

Proof The welfare of the old generation in period t is not affected by the level of the child allowance in that period, as the child allowance scheme is financed by a lump-sum tax θ on the young generation. Therefore, for a given value of the PAYG-tax τ, the social welfare function (6.7) is maximised if the level of the child allowance maximises lifetime utility of the young in each period. Using (6.3.b), (6.4.a) and (6.6) we can write the young's life-time utility in period t as

$$U_t = \log\left(\left[\frac{(p-\varphi)(1+\bar{r})\bar{w}(1-\tau)}{p(1+\bar{r})(1+\beta\varepsilon+\gamma)-\varphi(1+\beta\varepsilon)(1+\bar{r})-\gamma\tau\bar{w}}\right]^{1+\beta\varepsilon+\gamma}\left[\frac{\gamma}{p-\varphi}\right]^{\gamma}(1+\bar{r})^{\beta\varepsilon}\beta^{\beta\varepsilon}\right).$$

Setting $\frac{\partial U_t}{\partial\varphi} = 0$ then gives $\varphi^* = \frac{\tau\bar{w}}{1+\bar{r}}$. ■

The optimal subsidy to (tax on) the parents is equal to the present value of a child's contribution (costs) to the intergenerational redistribution scheme during his working life (i.e., next period).[22]

Notice that the optimal child allowance does not depend on the average lifespan, although the dependency-ratio effect is influenced by it. The reason is that the (future) contribution of a child to society does not depend on ε, i.e., the PAYG-benefit that every individual receives when old decreases if longevity increases.

6.5.2 Child allowances in a closed economy

In a closed economy, a child allowance scheme will not only increase the number of children, but also have an effect on the capital-labour ratio, since both savings and the size

[22] In Groezen, Leers and Meijdam (2003) it is shown that there exists a child allowance $\frac{\tau}{1+\bar{r}} < \hat{\varphi} < p$ such that introducing a child allowance level $0 < \varphi < \hat{\varphi}$ in a small open economy with a PAYG-pension scheme is Pareto-improving.

of the labour force change. This implies that the conclusion on the optimal size of the child allowance scheme is quite different from the case of a small open economy.

As the following proposition asserts, a child allowance scheme influences the capital-labour ratio, and thereby the factor rewards.

Proposition 6.5 *In a closed economy, a child allowance scheme pushes the capital-labour ratio down.*

Proof Capital market equilibrium is given by $s = nk$. Using (6.1.a'), (6.10) and (6.11), the steady-state level of k is implicitly given by $\beta\varepsilon(p-\varphi)\left[1+r(k)\right] = \gamma\left[1+r(k)\right]k + \gamma\tau w(k)$. Comparative statics of this expression gives $\frac{\partial k}{\partial \varphi} = -\frac{\beta\varepsilon(1+r)}{\gamma(1+r)-f''(k)[\gamma\tau k+\beta\varepsilon(p-\varphi)-\gamma k]} < 0$. ■

A child allowance lowers the marginal costs of raising a child, so people will decide to have more children and save less; both effects decrease the capital-labour ratio. This implies that future generations have a smaller capital stock to produce with, putting them at a disadvantage. The government therefore cannot confine to only one instrument, but needs an extra tool to achieve the socially optimal outcome. Since an unfunded social security scheme directly affects private savings (and thus the future capital stock), it serves as the additional instrument. This is in contrast to a small open economy, where the optimal child allowance follows from a given value of the PAYG-tax.

In order to analyse the optimal child allowance and how it depends on longevity, we will assume a Cobb-Douglas production function, $f(k) = k^{\alpha}$, and full depreciation of capital after one period, which implies that $1 + r = \alpha k^{\alpha-1}$. The equilibrium capital-labour ratio that results in a market setting when individuals are confronted with a child allowance φ and social security tax τ is then given by

$$k = \frac{\alpha\beta\varepsilon(p-\varphi)}{\gamma(\alpha+\tau-\alpha\tau)}. \tag{6.13}$$

In equilibrium, per capita consumption will be constant. According to (6.9.a), the socially optimal number of offspring is then given by $n_t = \delta(1+r_{t+1})$. The socially optimal level of the capital-labour ratio can be found by substituting this, (6.9.b) and (6.9.c) in (6.8) (setting $d_t = d_{t+1} = 0$), which gives

$$k^* = \frac{\alpha p\left[\delta+\beta\varepsilon+\delta(\gamma+\beta\varepsilon)\right]}{(1-\alpha\delta)(\gamma+\beta\varepsilon)-\alpha(\delta+\beta\varepsilon)}. \tag{6.14}$$

Combining (6.13) and (6.14), we find that τ and φ should be such that

$$\gamma(\alpha + \tau - \alpha\tau)k^* = \alpha\beta\varepsilon(p - \varphi).$$

Furthermore, the individual choice between material consumption and offspring, as given by (6.4.b') must coincide with that of the social planner, described by (6.9.c). This gives the following relation between τ and φ,

$$\frac{\alpha\beta\varepsilon(p-\varphi)}{(p-\varphi)(\gamma+\beta\varepsilon)-\gamma p} = \alpha + \tau - \alpha\tau.$$

The optimal child allowance follows from these two expressions, being

$$\varphi^* = \frac{p\beta\varepsilon - \gamma k^*}{\gamma + \beta\varepsilon}. \tag{6.15}$$

This leads to the next proposition.

Proposition 6.6 *In a closed economy, increasing longevity implies a higher optimal child allowance if* $\alpha \leq 0.5$.

Proof Substituting (6.14) in (6.15) gives $\varphi^* = p\left(1 - \frac{\gamma}{(1-\alpha\delta)(\gamma+\beta\varepsilon)-\alpha(\delta+\beta\varepsilon)}\right)$, so $\frac{\partial\varphi^*}{\partial\varepsilon} = \frac{\beta\gamma p}{[(1-\alpha\delta)(\gamma+\beta\varepsilon)-\alpha(\delta+\beta\varepsilon)]^2}(1 - \delta\alpha - \alpha)$. Knowing that $\delta < 1$, it directly follows that $\alpha \leq 0.5$ is a sufficient condition for $\frac{\partial\varphi^*}{\partial\varepsilon} > 0$. ■

So for realistic values of the production parameter α, population ageing (caused by a longer lifespan) means that the optimal child allowance increases.

If longevity increases, people live longer in retirement, so production has to rise in order to prevent a decrease in old-age consumption. Individuals decide to save more, but contrary to a small open economy, these additional savings cannot be invested abroad. Instead, domestic investments increase, so the capital stock grows. This capital can only be deployed productively if there are sufficient (young) people to work with. However, when parents make their fertility decision, they do not take account of this by raising more children. Instead, they are even likely to raise fewer children, as was stated in Proposition 6.2. So the optimal child allowance has to increase.

One can therefore conclude that only focusing on reforming the pension scheme is not sufficient; the need for an appropriate child allowance scheme becomes more urgent as many societies will experience serious population ageing.

6.6 Conclusion

Dealing with the (adverse) consequences of population ageing involves more than merely reforming the public pension scheme. As a matter of fact, the most direct way would be to battle one of the causes of ageing itself: a lower fertility rate, which is clearly the result of individual deliberation. This chapter had a closer look at fertility as the endogenous source of population growth, and thereby of the implicit return of a PAYG-financed social security scheme. It was found that one form of ageing, increasing longevity, may incite a lower number of children, thus aggravating the ageing problem, depending on the openness of the economy. Furthermore, offspring causes two externalities on society that individuals do not take into account. A government can internalise this by implementing programs that effectively lower the individual costs of rearing children, such as a system of child allowances, study grants and child care, so as to bring the fertility rate to its optimal level. This is an effective, relatively easy and politically achievable way of dismantling the demographic time bomb. The effect of a longer lifespan on the optimal level of child allowances depends on the openness of the economy. In a small open economy, the optimal allowance is not affected by increasing longevity, but it increases in a closed economy where people grow older. Neglecting this part of social security will thus result in greater welfare losses.

6.7 Appendix to Section 6.3.1

This appendix describes the effects of increasing longevity for a small open economy and a more general utility function, which is given by

$$U(c_t^y, n_t, c_{t+1}^o) = u(c_t^y, n_t) + \beta\varepsilon v(c_{t+1}^o),$$

where u and v stand for the strictly concave and twice continuously differentiable felicity functions when young and old, respectively.

Individuals maximise this lifetime utility subject to their single-period budget constraints (6.1.a) and (6.1.b). Given the properties of the felicity functions, the optimal levels of material consumption ($\hat{c}_t^y$, $\hat{c}_{t+1}^o$) and offspring ($\hat{n}_t$) are uniquely determined by the following first-order conditions,

$$\begin{aligned} u_c(\hat{c}_t^y, \hat{n}_t) &= \beta(1+\bar{r})v_c(\hat{c}_{t+1}^o), \\ u_n(\hat{c}_t^y, \hat{n}_t) &= p\, u_c(\hat{c}_t^y, \hat{n}_t), \end{aligned}$$

where $u_c \equiv \frac{\partial u(\cdot)}{\partial c^y}$, $u_n \equiv \frac{\partial u(\cdot)}{\partial n}$, $v_c \equiv \frac{\partial v(\cdot)}{\partial c^o}$.

Taking the government budget constraint into account, comparative statics of the first-order conditions (6.4.a) and (6.4.b) with respect to ε yield

$$\begin{bmatrix} \frac{\partial \hat{c}^y}{\partial \varepsilon} \\ \\ \frac{\partial \hat{n}}{\partial \varepsilon} \end{bmatrix} = \frac{A}{|A|} \begin{bmatrix} \pi \\ \\ 0 \end{bmatrix},$$

where the matrix $A = \{a_{ij}\}$, $i, j = 1, 2$, with

$$\begin{aligned} a_{11} &= u_{nn} - pu_{cn}, \\ a_{12} &= -\frac{\beta(1+\bar{r})^2}{\varepsilon} v_{cc}\left(p - \frac{\tau\bar{w}}{1+\bar{r}}\right) - u_{cn}, \\ a_{21} &= pu_{cc} - u_{nc}, \\ a_{22} &= u_{cc} + \frac{\beta(1+\bar{r})^2}{\varepsilon} v_{cc}, \end{aligned}$$

$v_{cc} \equiv \frac{\partial^2 v(\hat{c}^o)}{\partial (c^o)^2}$ and $\pi \equiv -\frac{\beta(1+\bar{r})\hat{c}^o}{\varepsilon} v_{cc} > 0$.

If $p > \frac{\tau\bar{w}}{1+\bar{r}}$ and the felicity functions are strictly concave and additively separable ($u_{cn} = u_{nc} = 0$), then a_{11}, a_{21}, $a_{22} < 0$ and $a_{12} > 0$, so $|A| > 0$. Consequently, $\frac{\partial \hat{c}^y}{\partial \varepsilon}$, $\frac{\partial \hat{n}}{\partial \varepsilon} < 0$.

References

Aaron, H. (1966) "The Social Insurance Paradox" *Canadian Journal of Economics* **2**: 371-379.

Abel, A.B. (2000) "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security" *ILE Research Paper 01-15,* University of Pennsylvania Law School.

Abel, A.B. (2001) "Will Bequests Attenuate the Predicted Meltdown in Stock Prices when Baby Boomers Retire?" *Review of Economics and Statistics* **83**: 589-595.

Arrow, K.J. (1962) "The Economic Implications of Learning by Doing" *Review of Economic Studies* **29**: 155-173.

Barro, R.J. and X. Sala-i-Martin (1995) *Economic Growth* New York: McGraw-Hill.

Baumol, W.J. (1967) "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis" *American Economic Review* **57**: 415-426.

Becker, G.S. (1961) "An Economic Analysis of Fertility" in R. Easterlin (ed.) *Demographic and Economic Change in Developed Countries* Princeton: Princeton University Press.

Becker, G.S. and R.J. Barro (1988) "A Reformulation of the Economic Theory of Fertility" *Quarterly Journal of Economics* **103**: 1-25.

Becker, G.S., K.M. Murphy and R. Tamura (1990) "Human Capital, Fertility, and Economic Growth" *Journal of Political Economy* **98**: S12-S37.

Belan, P., M. Philippe and P. Pestieau (1998) "Pareto-Improving Social Security Reform" *Geneva Papers on Risk and Insurance Theory* **23**: 119-125.

Beltrametti, L. and L. Bonatti (forthcoming) "Does International Coordination of Pension Policies Boost Capital Accumulation?" *Journal of Public Economics*.

Birdsall, N. (1988) "Economic Approaches to Population Growth" in H. Chenery and T.N. Srinivasan (eds.) *Handbook of Development Economics* Amsterdam: Elsevier Science.

Blanchard, O.J. and S. Fischer (1989) *Lectures on Macroeconomics* Cambridge MA: MIT Press.

Blanchet, D. and O. Ekert-Jaffé (1994) "The Demographic Impact of Family Benefits: Evidence from a Micro-Model and from Macro-Data" in J. Ermisch and N. Ogawa (eds.) *The Family, the Market and the State in Ageing Societies* Oxford: Clarendon Press.

Blundell, R., M. Browning and C. Meghir (1994) "Consumer Demand and the Life-Cycle Allocation of Household Expenditure" *Review of Economic Studies* **61**: 57-80.

Börsch-Supan, A. (2001) "Labor Market Effects of Population Aging" *NBER Working Paper* 8640.

Börsch-Supan, A., M. Miegel, V. Brombacher Steiner, A.L. Bovenberg, L. Meijdam, R. Disney, D.A. Wise and K. Schmidt-Hebbel (1999) *Reforming the Pension-System. What Germany Can Learn from Other Countries* Köln: Deutsches Institut für Altersvorsorge.

Börsch-Supan, A., A. Ludwig and J. Winter (2001) "Aging and International Capital Flows" *NBER Working Paper* 8553.

Bourguignon, F. (1999) "The Cost of Children: May the Collective Approach to Household Behavior Help?" *Journal of Population Economics* **12**: 503-521.

Breyer, F. (1989) "On the Intergenerational Pareto Efficiency of Pay-as-You-Go Financed Pension Systems" *Journal of Institutional and Theoretical Economics* **145**: 643-658.

Breyer, F. (2001) "Why Funding is not a Solution to the "Social Security Crisis"" *IZA Discussion Paper* 328.

Brooks, R. (2000) "What Will Happen to Financial Markets when the Baby Boomers Retire?" *IMF Working Paper* 18.

Brooks, R. (2002) "Asset-Market Effects of the Baby Boom and Social-Security Reform" *American Economic Review* **92**: 402-406.

Browning, E.K. (1975) "Why the Social Insurance Budget Is Too Large in a Democracy" *Economic Inquiry* **13**: 373-388.

Buiter, W.H. (1981) "Time Preference and International Lending and Borrowing in an Overlapping-Generations Model" *Journal of Political Economy* **89**: 769-797.

Bütler, M. and G. Kirchsteiger (1999) "Aging Anxiety: Much Ado about Nothing?" *CentER Discussion Paper* 9937, Tilburg University.

Casarico, A. (2001) "Pension Systems in Integrated Capital Markets" *Topics in Economic Analysis and Policy* **1**: 1-17.

Chand, S.K. and A. Jaeger (1996) "Aging Populations and Public Pension Schemes" *IMF Occasional Paper* 147.

Cigno, A. (1992) "Children and Pensions" *Journal of Population Economics* **5**: 175-183.

Cigno, A. (1993) "Intergenerational Transfers without Altruism. Family, Market and State" *European Journal of Political Economy* **9**: 505-518.

Corneo, G. and M. Marquardt (2000) "Public Pensions, Unemployment Insurance, and Growth" *Journal of Public Economics* **75**: 293-311.

Cutler, D.M., J.M. Poterba, L.M. Sheiner and L.H. Summers (1990) "An Aging Society: Opportunity or Challenge?" *Brookings Papers on Economic Activity* **1**: 1-73.

De la Croix, D. and O. Licandro (1999) "Life Expectancy and Endogenous Growth" *Economics Letters* **65**: 255-263.

Demirgüç-Kunt, A. and R. Levine (1996) "Stock Market Development and Financial Intermediaries: Stylised Facts" *The World Bank Economic Review* **10**.

Diamond, P.A. (1965) "National Debt in a Neoclassical Growth Model" *American Economic Review* **55**: 1126-1150.

Disney, R. (1996) *Can We Afford to Grow Older?* Cambridge MA: MIT Press.

Eckstein, Z. and K.I. Wolpin (1985) "Endogenous Fertility and Optimal Population Size" *Journal of Public Economics* **27**: 93-106.

Economic Policy Committee (2001) "Budgetary Challenges Posed by Ageing Populations" EPC/ECFIN/655/01-EN final, October.

Ehrlich, I. and F.T. Lui (1991) "Intergenerational Trade, Longevity, and Economic Growth" *Journal of Political Economy* **99**: 1029-1059.

Feldstein, M. (1974) "Social Security, Induced Retirement and Aggregate Capital Accumulation" *Journal of Political Economy* **82**: 905-926.

Feldstein, M. (1995) "Would Privatizing Social Security Raise Economic Welfare?" *NBER Working Paper* 5281.

Feldstein, M. (1996a) "The Missing Piece in Policy Analysis: Social Security Reform" *American Economic Review, Papers and Proceedings.*

Feldstein, M. (1996b) "Social Security and Saving: New Time Series Evidence" *National Tax Journal* **49**: 151-164.

Feldstein, M. (1999) "Prefunding Medicare" *NBER Working Paper* 6917.

Fogel, R.W. (1994) "Economic Growth, Population Theory, and Physiology: The Bearing of Long-Term Processes on the Making of Economic Policy" *American Economic Review* **84**: 369-395.

Folbre, N. (1994) "Children as Public Goods" *American Economic Review* **84**: 86-90.

Folkerts-Landau, D. and T. Ito (1995) *International Capital Markets: Developments, Prospects and Policy Issues* Washington DC: IMF.

Futagami, K. and T. Nakajima (2001) "Population Ageing and Economic Growth" *Journal of Macroeconomics* **23**: 31-44.

Galor, O. and H.E. Ryder (1989) "Existence, Uniqueness, and Stability of Equilibrium in an Overlapping-Generations Model with Productive Capital" *Journal of Economic Theory* **49**: 360-375.

Galor, O. and D.N. Weil (1996) "The Gender Gap, Fertility, and Growth" *American Economic Review* **86**: 374-387.

Geanakoplos, J., O.S. Mitchell and S.P. Zeldes (1998) "Would a Privatized Social Security System Really Pay a Higher Rate of Return?" *NBER Working Paper* 6713.

Geide-Stevenson, D. (1998) "Social Security and International Labor and Capital Mobility" *Review of International Economics* **6**: 407-406.

Gordon, R.H. and A.L. Bovenberg (1996) "Why Is Capital So Immobile Internationally? Possible Explanations and Implications for Capital Income Taxation" *American Economic Review* **86**: 1057-1075.

Groezen, B. van and T. Leers (2000) "The Effects of Asymmetric Demographic Shocks with Perfect Capital Mobility *CentER Discussion Paper* 2000-88, Tilburg University.

Groezen, B. van, T. Leers and L. Meijdam (2002) "The Vulnerability of Social Security When Fertility is Endogenous" *Journal of Institutional and Theoretical Economics* **158**: 715-730.

Groezen, B. van, T. Leers and L. Meijdam (2003) "Social Security and Endogenous Fertility: Pensions and Child Allowances as Siamese Twins" *Journal of Public Economics* **87**: 233-251.

Groezen, B. van, L. Meijdam and H. Verbon (2002) "General-Equilibrium Effects of Privatisation: The Missing Piece in Social Security Reform" *CentER Discussion Paper* 2002-24, Tilburg University.

Grossman, G.M. and E. Helpman (1991) *Innovation and Growth in the Global Economy* Cambridge MA: MIT Press.

Gyárfás, G. and M. Marquardt (2001) "Pareto Improving Transition from a Pay-As-You-Go to Fully Funded Pension System in a Model of Endogenous Growth" *Journal of Population Economics* **14**: 445-453.

Higgins, M. (1998) "Demography, National Savings, and International Capital Flows" *International Economic Review* **39**: 343-369.

Holzmann, R. (2000) "Can Investments in Emerging Markets Help to Solve the Aging Problem?" *CESifo Working Paper* 304.

Holzmann, R., R. Palacios and A. Zviniene (2001) "Implicit Pension Debt: Issues, Measurement and Scope in International Perspective" paper presented at the IIPF conference 2001.

Hurd, M. (1989) "Mortality Risks and Bequests" *Econometrica* **57**: 779-813.

Jackson, W.A. (1998) *The Political Economy of Population Ageing* Cheltenham: Edward Elgar Publishing.

Jones, L.E. and R.E. Manuelli (1992) "Finite Lifetimes and Growth" *Journal of Economic Theory* **58**: 171-197.

Kaganovich, M. and I. Zilcha (1999) "Education, Social Security, and Growth" *Journal of Public Economics* **71**: 289-309.

Kemnitz, A. (2000) "Social Security, Public Education, and Growth in a Representative Democracy" *Journal of Population Economics* **13**: 443-462.

Kemnitz, A. and B.U. Wigger (2000) "Growth and Social Security: The Role of Human Capital" *European Journal of Political Economy* **16**: 673-683.

Kenc, T. and S. Sayan (2001) "Demographic Transmission from Large to Small Countries. An Overlapping Generations CGE Analysis" *Journal of Policy Modelling* **23**: 677-702.

King, R.G., C.I. Plosser and S. Rebelo (1988) "Production, Growth and Business Cycles: I. The Basic Neoclassical Model" *Journal of Monetary Economics* **21**: 195-232.

King, R.G. and S. Rebelo (1990) "Public Policy and Economic Growth: Developing Neoclassical Implications" *Journal of Political Economy* **98**: S126-S150.

Kotlikoff, L.S. (1979) "Social Security and Equilibrium Capital Intensity" *Quarterly Journal of Economics* **43**: 233-254.

Kotlikoff, L.S. and L.H. Summers (1981) "The Role of Intergenerational Transfers in Aggregate Capital Accumulation" *Journal of Political Economy* **89**: 706-732.

Kotlikoff, L.J., K.A. Smetters and J. Walliser (2001) "Finding a Way Out of America's Demographic Dilemma" *NBER Working Paper* 8258.

Krugman, P. (1987) "The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Thatcher" *Journal of Development Economics* **27**: 41-55.

Krugman, P. (2002) "Breaking the Contract" *The New York Times* March 5th.

Laitner, J. (1988) "Bequests, Gifts, and Social Security" *Review of Economic Studies* **55**: 275-299.

Lambrecht, S., P. Michel and J.P. Vidal (2001) "Public Pensions and Growth" *European Central Bank Working Paper* 90.

Lane, P.R. and G.M. Milesi-Ferretti (2001) "Long-Term Capital Movements" *NBER Working Paper* 8366.

Leibenstein, H.M. (1957) *Economic Backwardness and Economic Growth* New York: Wiley.

Lindh, T. and B. Malmberg (1999) "Age Structure Effects and Growth in the OECD, 1950-1990" *Journal of Population Economics* **12**: 431-450.

Longman, P.J. (1998) "The Cost of Children" *U.S. News* March 30th.

Lucas, R. (1988) "On the Mechanics of Industrial Development" *Journal of Monetary Economics* **22**: 3-48.

Lührmann, M. (2001) "The Role of Demographic Change in Explaining International Capital Flows" mimeo, University of Mannheim.

MacKellar, L. and H. Reisen (1998) "A Simulation Model of Global Pension Investment" *OECD Technical Paper* 137.

MacKellar, L.,T. Ermolieva and H. Reisen (2001) "Globalization, Social Security, and Intergenerational Transfers" mimeo, International Institute for Applied Systems Analysis.

Mankiw, G., J. Rotemberg and L. Summers (1985) "Intertemporal Substitution in Macroeconomics" *Quarterly Journal of Economics* **100**: 225-251.

Mankiw, N.G., D. Romer and D.N. Weil (1992) "A Contribution to the Empirics of Economic Growth" *Quarterly Journal of Economics* **107**: 407-437.

Marchand, M., P. Michel and P. Pestieau (1996) "Intergenerational Transfers in an Endogenous Growth Model with Fertility Changes" *European Journal of Political Economy* **12**: 33-48.

Matsuyama, K. (1992) "Agricultural Productivity, Comparative Advantage, and Economic Growth" *Journal of Economic Theory* **58**: 317-334.

Meijdam, L. and H.A.A. Verbon (1997) "Aging and Public Pensions in an Overlapping-Generations Model" *Oxford Economic Papers* **49**: 29-42.

Miles, D. (1999) "Modelling the Impact of Demographic Change upon the Economy" *Economic Journal* **109**: 1-36.

Miron, J.A. and K.M. Murphy (2001) "The False Promise of Social Security Privatization" *Bastiat Institute Discussion Paper* 2.

Modigliani, F. (1986) "Life Cycle, Individual Thrift, and the Wealth of Nations" *American Economic Review* **76**: 297-313.

Modigliani, F. (1988) "The Role of Intergenerational Transfers and Life Cycle Saving in the Accumulation of Wealth" *Journal of Economic Perspectives* **2**: 15-40.

Modigliani, F. and R. Brumberg (1954) "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data" in K.K. Kurihara (ed.), *Post-Keynesian Economics*, New Brunswick: Rutgers University Press.

Mulligan, C.S. and X. Sala-i-Martin (1993) "Transitional Dynamics in Two-Sector Models of Endogenous Growth" *Quarterly Journal of Economics* **108**: 737-773.

OECD (1996) *Ageing in OECD Countries: A Critical Policy Challenge* Paris.

OECD (2002) *Economic Outlook* **71** Paris.

Pecchenino, R.A. and P.S. Pollard (1997) "The Effects of Annuities, Bequests, and Aging in an Overlapping Generations Model of Endogenous Growth" *Economic Journal* **107**: 26-46.

Pecchenino, R.A. and K.R. Utendorf (1999) "Social Security, Social Welfare and the Ageing Population" *Journal of Population Economics* **12**: 607-623.

Pemberton, J. (1999) "Social Security: National Policies with International Implications" *Economic Journal* **109**: 492-508.

Pemberton, J. (2000) "National and International Privatisation of Pensions" *European Economic Review* **44**: 1873-1896.

Poterba, J.M. (1994) (ed.) *International Comparisons of Household Saving* Chicago: Chicago University Press.

Poterba, J.M. (2001) "Demographic Structure and Asset Returns" *Review of Economics and Statistics* **83**: 565-584.

Praag, B. van and M. Warnaar (1997) "The Cost of Children and the Use of Demographic Variables in Consumer Demand" in M. Rosenzweig and O. Stark (eds.) *Handbook of Population and Family Economics* Amsterdam: Elsevier Science.

Razin, A. and U. Ben-Zion (1975) "An Intergenerational Model of Population Growth" *American Economic Review* **65**: 923-933.

Razin, A. and E. Sadka (1995) *Population Economics* Cambridge MA: MIT Press.

Rebelo, S. (1991) "Long-Run Policy Analysis and Long-Run Growth" *Journal of Political Economy* **99**: 500-521.

Reisen, H. (2000) *Pensions, Savings and Capital Flows. From Ageing to Emerging Markets* Cheltenham: Edward Elgar.

Romer, P.M. (1986) "Increasing Returns and Long-Run Growth" *Journal of Political Economy* **94**: 1002-1037.

Romer, P.M. (1990) "Endogenous Technological Change" *Journal of Political Economy* **98**: S71-S102.

Saint-Paul, G. (1992) "Fiscal Policy in an Endogenous Growth Model" *Quarterly Journal of Economics* **106**: 1243-1259.

Sala-i-Martin, X. (1996) "A Positive Theory of Social Security" *Journal of Economic Growth* **1**: 277-304.

Samuelson, P.A. (1958) "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money" *Journal of Political Economy* **66**: 467-482.

Samuelson, P.A. (1975) "Optimum Social Security in a Life-Cycle Growth Model" *International Economic Review* **16**: 531-538.

Sánchez-Losada, F. (2000) "Growth Effects of an Unfunded Social Security System When There Is Altruism and Human Capital" *Economics Letters* **69**: 95-99.

Siegel, J.J. (1998) *Stocks for the Long Run. The Definitive Guide to Financial Market Returns and Long-Term Investment Strategies* New York: McGraw-Hill.

Sinn, H.W. (1997) "The Value of Children and Immigrants in a Pay-As-You-Go Pension System: A Proposal For a Partial Transition to a Funded System" *CEPR Discussion Paper* 1734.

Sinn, H.W. (1998) "The Pay-As-You-Go Pension System as a Fertility Insurance and Enforcement Device" *CEPR Discussion Paper* 2023.

Smulders, S. and T. van de Klundert (2001) "Loss of Technological Leadership of Rentier Economies" *Journal of International Economics* **54**: 211-231.

Torvik, R. (2001) "Learning by Doing and the Dutch Disease" *European Economic Review* **45**: 285-306.

Turner, D., C. Giorno, A. De Serres, A. Vourc'h and P. Richardson (1998) "The Macroeconomic Implications of Ageing in a Global Context" *OECD Working Paper* 193.

United Nations (2001) *World Population Prospects: The 2000 Revision* New York.

Uzawa, H (1965) "Optimal Technical Change in an Aggregative Model of Economic Growth" *International Economic Review* **6**: 18-31.

Vallés Giménez, J. and A. Zárate Marco (2002) "Do Child Tax Allowances and Social Benefits Affect the Decision to Have Children? An Empirical Study in Spain" *Estudios sobre la Economia Española* 149.

Verbon, H.A.A. (1988) "Conversion Policies for Public Pensions Plans in a Small Open Economy" in B. Gustafsson and N. Anders Klevmarken (eds.) *The Political Economy of Social Security* Amsterdam: Elsevier Science.

Wiedmer, T. (1996) "Growth and Social Security" *Journal of Institutional and Theoretical Economics* **152**: 531-539.

Wigger, B.U. (1999) "Pay-As-You-Go Financed Public Pensions in a Model of Endogenous Growth and Fertility" *Journal of Population Economics* **12**: 625-640.

Wigger, B.U. (2001) "Productivity Growth and the Political Economy of Social Security" *Public Choice* **106**: 53-76.

Young, A. (1991) "Learning by Doing and the Dynamic Effects of International Trade" *Quarterly Journal of Economics* **106**: 369-405.

Zhang, J. (1995a) "Does Unfunded Social Security Also Depress Output Growth?" *Economics Letters* **49**: 307-312.

Zhang, J. (1995b) "Social Security and Endogenous Growth" *Journal of Public Economics* **58**: 185-213.

Zhang, J. and J. Zhang (1998) "Social Security, Intergenerational Transfers, and Endogenous Growth" *Canadian Journal of Economics* **31**: 1225-1241.

Zhang, J., J. Zhang and R. Lee (2001) "Mortality Decline and Long-Run Growth" *Journal of Public Economics* **80**: 485-507.

Samenvatting (Summary in Dutch)

In de komende decennia krijgen veel landen te maken met een vergrijzende bevolking. Dalende geboortecijfers en een almaar toenemende levensduur zorgen ervoor dat er relatief meer ouderen zullen zijn en het actieve deel van de bevolking kleiner wordt. Bijgevolg worden meer personen afhankelijk van minder werkenden: de zogenaamde afhankelijkheidsratio zal over 30 jaar in de meeste landen verdubbelen, in sommige zelfs bijna driemaal zo groot worden. Dit heeft ongetwijfeld grote gevolgen voor de welvaartspositie van jongeren en ouderen, waarbij het cruciaal is hoe de productie tussen de generaties wordt verdeeld. Een belangrijk aspect is de financiering van pensioenen. In veel landen wordt gebruik gemaakt van een (publiek) omslagstelsel, waarbij de huidige beroepsbevolking betaalt voor de voorzieningen van de huidige gepensioneerden. Dit betreft dus een directe inkomensoverdracht tussen generaties. Het 'rendement' van zo'n omslagstelsel hangt af van de groei van de premiebasis, wat neerkomt op bevolkingstoename en loongroei. Dit geldt ook voor verschillende socialezekerheidsregelingen en zorgvoorzieningen, waarvan ouderen relatief meer gebruik maken. In een vergrijzende samenleving zal zo'n stelsel onder druk komen te staan aangezien premies zullen moeten stijgen of pensioenuitkeringen dalen.

Het alternatief voor een omslagstelsel is een op kapitaaldekking gebaseerd stelsel. Hierbij sparen individuen zelf of via een pensioenfonds voor hun oude dag. Het rendement dat op de kapitaalmarkt wordt behaald is dan van doorslaggevend belang voor de hoogte van de pensioenuitkering. Recente ontwikkelingen bij pensioenfondsen laten zien dat ook deze financieringsvorm niet zonder risico's is: de reële waarde van de pensioenuitkeringen daalt door achterblijvende indexatie, tenzij premies stijgen zodat er ook bij pensioenfondsen een omslagelement te bespeuren valt. Desalniettemin wordt er alom gepleit voor een (gedeeltelijke) omschakeling van omslagstelsel naar kapitaaldekkingsstelsel, zowel in Europa als in de Verenigde Staten. De voornaamste redenering hierbij is dat het rendement

dat op de kapitaalmarkt kan worden behaald hoger is dan het indirecte rendement van een omslagstelsel, zodat een dergelijke pensioenhervorming - zeker op lange termijn - substantiële welvaartswinsten oplevert. Bovendien stimuleert een kapitaaldekkingsstelsel de besparingen en daarmee de investeringen, zodat meer kapitaal in de economie aanwezig is waarmee meer geproduceerd kan worden en de welvaart toeneemt. Deze redenering laat een aantal belangrijke aspecten echter buiten beschouwing. Allereerst is het probleem van vergrijzing bovenal een verdelingskwestie. Als er relatief meer gepensioneerden komen zal de jonge, actieve generatie meer van hetgeen zij in het productieproces voortbrengt moeten afstaan om het relatieve consumptieniveau van ouderen niet te laten dalen. Bovendien ontstaan er op korte termijn transitiekosten. Eén generatie zal immers zowel de pensioenen van de huidige gepensioneerden moeten betalen als moeten sparen voor de eigen oude dag, hetgeen een dubbele last impliceert. Verbon (1988) en Breyer (1989) toonden aan dat een dergelijke pensioenhervorming daarom geen Pareto-verbetering kan zijn.

In dit proefschrift worden de vermeende langetermijnvoordelen van een overgang naar een meer op kapitaaldekking gebaseerd pensioenstelsel onder de loep genomen en de macro-economische effecten van vergrijzing bestudeerd. Hierbij wordt een aantal aspecten benadrukt die in conventionele economische analyses niet of nauwelijks aandacht krijgen. Achtereenvolgens zijn dat de looninflatie als gevolg van vergrijzing en toenemende kapitaalaccumulatie, waarmee vooral ouderen geconfronteerd worden, de internationale consequenties van verschillen in pensioenstelsels en vergrijzing, en tenslotte de mogelijkheden om de welvaart te vergroten door een geschikte politiek van geboortestimulering. Variabelen als de rentevoet, economische groei en bevolkingsgroei, die binnen de pensioendiscussie een belangrijke rol spelen en veelal als exogeen worden beschouwd, zijn in de afzonderlijke hoofdstukken van deze dissertatie juist endogeen.

In elk hoofdstuk wordt gebruik gemaakt van een model van overlappende generaties zoals dat is ontwikkeld door Samuelson (1958) en Diamond (1965). Hierin leven individuen (maximaal) twee perioden. In de eerste periode is men actief op de arbeidsmarkt; een deel van het arbeidsinkomen wordt in de vorm van premies afgedragen aan de overheid die daarmee een publiek pensioenstelsel financiert. Van het netto inkomen wordt gespaard en geconsumeerd. In de tweede periode is men gepensioneerd; consumptie wordt gefinancierd uit de opbrengst van de besparingen en de publieke pensioenuitkering.

Het eerste deel van dit proefschrift richt zich op de toenemende behoefte van ouderen

aan arbeidsintensieve dienstverlening. Zoals blijkt uit budgetonderzoeken geven gepensioneerden relatief meer uit aan diensten als zorg, huishoudelijke hulp, kappers et cetera dan jongeren. Daarnaast lopen de publiek gefinancierde zorguitgaven aanzienlijk op naarmate men ouder wordt. Deze diensten verschillen in een aantal opzichten fundamenteel van andere producten. Allereerst maken zij voornamelijk gebruik van de productiefactor arbeid. Dit betekent dat de prijs van dienstverlening in grote mate afhangt van het loon. De koopkracht van ouderen wordt daarom negatief beïnvloed door loonstijgingen. Bovendien kunnen diensten in tegenstelling tot goederen niet bewaard of geïmporteerd worden, maar moeten op het moment van aanschaf terstond worden geleverd door binnenlandse arbeidskrachten. Tenslotte kan de arbeidsproductiviteit bij deze soort van dienstverlening nauwelijks worden verhoogd door technologische vooruitgang. In tegenstelling tot gebruikelijke modellen van overlappende generaties nemen de hoofdstukken 2, 3 en 4 deze aspecten expliciet in beschouwing en analyseren in hoeverre conclusies ten aanzien van vergrijzing en de vormgeving van het pensioenstelsel hierdoor veranderen.

De basis van deze hoofdstukken is een model waarin twee sectoren worden onderscheiden, een goederensector en een dienstensector. In de goederensector worden zowel kapitaalgoederen als arbeid ingezet om producten voort te brengen die dienen als consumptie- en investeringsgoed. Deze goederen worden voornamelijk door jongeren gekocht. De dienstensector betreft dienstverlening die gebruik maakt van arbeid als productiefactor en niet of nauwelijks van kapitaalgoederen. Deze diensten worden hoofdzakelijk aan ouderen verleend die daarvoor een prijs betalen, die sterk aan het loon gerelateerd is. Omdat arbeid homogeen verondersteld wordt en er voor werknemers geen belemmeringen zijn om van sector te wisselen, zal het loon in beide sectoren gelijk zijn. Dit betekent dat productiviteitsstijgingen in de goederensector ook leiden tot een hoger loon in de dienstensector en daarmee de prijs van diensten opdrijven.

Hoofdstuk 2 analyseert de lange-termijn algemeen-evenwichtseffecten van een overgang naar meer kapitaalgedekte pensioenen voor een gesloten economie. Een verkleining van het omslagstelsel zal leiden tot hogere besparingen. Dit impliceert een vergroting van de kapitaalgoederenvoorraad zodat de arbeidsproductiviteit in de goederensector toeneemt en de lonen zullen stijgen. Hierdoor worden diensten duurder. Bovendien zal het rendement van besparingen dalen aangezien kapitaal relatief overvloedig wordt. De koopkracht van ouderen neemt daardoor af. Tegenover toenemende consumptiemogelijkheden gedurende de actieve levensperiode (als gevolg van de loonstijging) staan dan verminderde

consumptiemogelijkheden na pensionering. De vraagstelling van dit hoofdstuk is daarom of hierdoor de langetermijnvoordelen van zo'n pensioenhervorming teniet worden gedaan en zelfs tot een lagere welvaart kunnen leiden.

Het blijkt dat een marginale reductie van het omslagstelsel ook in een tweesectorenmodel het individuele nut (althans op lange termijn) zal doen stijgen zolang de economie dynamisch efficiënt is (dat wil zeggen, een rente heeft die hoger is dan de som van bevolkings- en productiviteitsgroei), ook als de dienstverlening in de tweede levensperiode afneemt. Echter, als er sprake is van een substantiële hervorming van het pensioenstelsel (zoals het volledig afschaffen van het omslagstelsel), dan zullen individuen hun besparingen aanzienlijk verhogen om de lagere toekomstige pensioenuitkering te compenseren. De algemeen-evenwichtseffecten zijn dan sterk, waardoor er een groot verschil ontstaat tussen de consumptiemogelijkheden van jongeren en ouderen. Individuen zullen over het algemeen een dergelijke onevenwichtigheid tussen beide levensperioden niet accepteren en besluiten hun besparingen verder te verhogen teneinde het voordeel in de eerste levensperiode gedeeltelijk over te brengen naar de tweede periode. Zij houden echter geen rekening met de algemeen-evenwichtseffecten die dit vervolgens veroorzaakt. Uiteindelijk wordt er teveel gespaard in de economie en is er sprake van dynamische inefficiëntie, waarbij sparen voor de oude dag leidt tot een lagere welvaart. Deze situatie doet zich sneller voor in een tweesectoreneconomie vanwege de gevoeligheid van oudedagsconsumptie voor loonstijgingen. Dit aspect wordt volledig genegeerd in éénsectormodellen, die ten onrechte tot de conclusie kunnen leiden dat een aanzienlijke reductie van het omslagstelsel de welvaart op lange termijn zal verhogen. Tentatieve berekeningen aan de hand van data uit budgetonderzoeken laten zien dat dit verschil tussen beide modellen niet irreëel is, vooral in een vergrijzende samenleving waarin individuen een tamelijk vlak consumptiepatroon over hun leven wensen.

De hoofdstukken 3 en 4 sluiten aan bij het vorige hoofdstuk en richten zich specifiek op groeiverschillen tussen de onderscheiden sectoren. De centrale vragen zijn hoe vergrijzing en hervorming van het pensioenstelsel de groei op korte en lange termijn beïnvloedt. In de economische literatuur worden globaal twee oorzaken van endogene groei onderscheiden: positieve spillovereffecten als gevolg van kapitaalaccumulatie en de vorming van 'human capital' door investeringen in zowel onderwijs als onderzoek en ontwikkeling, hetgeen ook met positieve externaliteiten gepaard kan gaan. In hoofdstuk 3 en 4 worden beide benaderingswijzen geïntegreerd door aan te nemen dat productiviteitsgroei alleen plaatsvindt in

de sector waar kapitaal wordt ingezet (de goederensector) en veroorzaakt wordt door spillovers die het gevolg zijn van leereffecten ('learning-by-doing') en uitgaven aan onderzoek en ontwikkeling. Deze groei wordt impliciet gemodelleerd door de veronderstelling dat de arbeidsproductiviteit toeneemt naarmate er meer personen in de productieve sector werkzaam zijn. Dientengevolge zal het loon toenemen. Marktwerking zorgt ervoor dat het loon in de dienstensector even hard stijgt, waardoor diensten duurder worden.

Hoofdstuk 3 behandelt de vraag welk effect vergrijzing op de economische groei heeft in een dergelijke economie met twee sectoren. Van cruciaal belang hierbij is de grootte en openheid van de economie. In een kleine open economie zullen de toegenomen besparingen die het gevolg zijn van vergrijzing in het buitenland worden geïnvesteerd zonder dat dit effect heeft op de rentevoet. Er zullen meer ouderen zijn die hun inkomen voor een relatief groot deel aan diensten besteden. Aangezien deze diensten niet geïmporteerd kunnen worden, zal er een verschuiving van arbeid van de goederensector naar de dienstensector plaatsvinden. De binnenlandse productie van goederen vermindert en er zullen meer goederen ingevoerd worden (of minder geëxporteerd). Als gevolg hiervan zullen de positieve spillovereffecten en daarmee de groei op lange termijn afnemen. Aldus ontstaat een rentenierseconomie die lijdt aan de zogenaamde 'Dutch disease'.

De effecten zijn anders in een gesloten economie. De toegenomen besparingen resulteren in extra investeringen, zodat de kapitaalgoederenvoorraad groeit. Dit additioneel kapitaal wordt ingezet in de goederensector, hetgeen arbeid in deze sector productiever maakt en uit de dienstensector aantrekt. Anderzijds zal er een toenemende behoefte aan diensten zijn in een ouder wordende bevolking. Welke kracht uiteindelijk overheerst hangt af van de mate waarin arbeid in de goederensector productiever wordt door kapitaalaccumulatie, wat afhangt van de substitueerbaarheid van beide productiefactoren in deze sector. Als arbeid en kapitaal complementair zijn, zal het eerste effect domineren en vindt er een herallocatie van arbeid plaats ten gunste van de goederensector, met een toenemende groei als gevolg; het tegenovergestelde gebeurt wanneer arbeid en kapitaal substituten zijn.

Het laatste deel van dit hoofdstuk is gewijd aan enkele uitbreidingen en modificaties van het model. Het behandelt de effecten van vergrijzing als gevolg van een lager geboortecijfer en de kortetermijneffecten van beide soorten vergrijzing als ook jongeren diensten vragen. Vervolgens wordt de aanname van een constante pensioenbelasting losgelaten. Het blijkt dat het negatieve effect van vergrijzing op de groei in een kleine open economie

geringer is indien de publieke pensioenuitkering constant is en de premies stijgen. Voor een gesloten economie geldt dat evenzo ingeval van complementariteit van de productiefactoren; bij substitueerbaarheid kan het verschil met vaste belastingen zowel positief als negatief zijn. Vervolgens wordt het resultaat van een markteconomie vergeleken met dat van een sociale planner die de spillovereffecten internaliseert. Het blijkt dat de consumptie van diensten dan lager is dan in een markteconomie, maar omdat de structuur van het model feitelijk niet verandert zullen de conclusies ten aanzien van vergrijzing blijven gelden. Tenslotte wordt de aanname dat groei louter afhangt van het aantal werknemers in de goederensector losgelaten door ook rekening te houden met groei als direct gevolg van kapitaalaccumulatie. Dit verandert de conclusies niet wezenlijk. Alleen in het geval van een gesloten economie zal er voor meer waarden van de substitutie-elasticiteit gelden dat vergrijzing op lange termijn tot een hogere groei leidt.

In hoofdstuk 4 wordt hetzelfde model gebruikt om de effecten op economische groei te analyseren van een overgang naar meer kapitaaldekking bij het pensioenstelsel. Ook nu blijkt het type economie van doorslaggevende betekenis. In een kleine open economie zal een terugdringing van het publieke omslagstelsel leiden tot een toename van de besparingen. Uitgaande van een dynamisch efficiënte economie zal zo'n hervorming van het pensioenstelsel aanvankelijk het inkomen van de dan levende jongeren vergroten, aangezien zij een hoger rendement op de kapitaalmarkt kunnen behalen in plaats van het indirecte rendement van het omslagstelsel. Dit heeft een stijgende vraag naar zowel goederen als diensten tot gevolg, zodat het nut op korte termijn toeneemt. De toegenomen vraag naar diensten betekent echter dat meer arbeidskrachten in de laagproductieve dienstensector werkzaam zullen zijn. De productiviteitsgroei in de economie neemt daardoor gestaag af. Uiteindelijk zal dit effect het aanvankelijk positieve, directe effect op het nut van de overgang naar meer kapitaaldekking overheersen en de welvaart dalen. In tegenstelling tot de conclusie die volgt uit een standaard éénsectormodel, zullen toekomstige generaties dus geen voordeel hebben van een terugdringing van het omslagstelsel, zelfs als de economie dynamisch efficiënt blijft.

Bovenstaande conclusie geldt niet voor een gesloten economie. In dat geval zullen de toegenomen besparingen resulteren in een grotere kapitaalgoederenvoorraad, hetgeen arbeid naar de goederensector trekt, zodat de productiviteitsgroei toeneemt. Op lange termijn zal de welvaart in een gesloten economie daarom extra toenemen in een tweesectorenmodel. Indien vele landen nagenoeg dezelfde hervorming van hun pensioenstelsel

doorvoeren is het model van een gesloten economie meer van toepassing. Vanuit dit oogpunt impliceert dat dat internationale beleidscoördinatie wenselijk is.

De afgelopen decennia zijn internationale kapitaalmarkten in toenemende mate geliberaliseerd en geïntegreerd. De vorming van de Economische en Monetaire Unie is daarvan een goed voorbeeld. Dit heeft ervoor gezorgd dat financieel kapitaal steeds mobieler is geworden en derhalve gevoeliger voor internationale verschillen van de rentevoet. Deze kunnen optreden door een verschillende ontwikkeling van nationale besparingen en kapitaalbehoeften als gevolg van demografische veranderingen. Ook de omvang van het kapitaaldekkingsstelsel speelt hierbij een belangrijke rol, aangezien dit een omvangrijke component van de nationale besparingen vormt. Vergrijzing en de vormgeving van pensioenstelsels zullen daarom een beduidend effect hebben op internationale kapitaalstromen en zo ook de welvaart in andere landen beïnvloeden. Hoofdstuk 5 bestudeert deze internationale spillovereffecten. Meer specifiek richt het zich op de volgende vragen: heeft een land met een omvangrijk kapitaaldekkingsstelsel last van kapitaalmobiliteit naar een ander land met een omvangrijk omslagstelsel? Bestaan er vanuit dit opzicht prikkels om het omslagstelsel te reduceren, of vloeien veel van de voordelen naar het buitenland? Welke effecten heeft vergrijzing op de welvaart van huidige en toekomstige generaties als landen verschillende pensioenstelsels en demografische ontwikkelingen hebben?

Met behulp van een tweelandenmodel kunnen de effecten van kapitaalmobiliteit worden nagegaan. Het blijkt dat het land met een omvangrijk omslagstelsel baat heeft van een geïntegreerde kapitaalmarkt. Dit wordt veroorzaakt door het feit dat de besparingen relatief laag zijn, zodat de rente hoog is en kapitaal uit het buitenland wordt aangetrokken. Landen met een omvangrijk kapitaaldekkingsstelsel hoeven hier echter geen voordeel van te hebben. Enerzijds kan in het land met een groot omslagstelsel een hoger rendement worden behaald, zodat renteopbrengsten naar het land vloeien. Anderzijds wordt er minder in het land zelf geïnvesteerd, hetgeen de binnenlandse productie en daarmee de beloning van arbeid doet dalen. Welk effect domineert hangt af van het verschil in pensioenstelsel tussen beide landen. Als de verschillen groot zijn is internationale kapitaalmobiliteit ook gunstig voor het land met veel kapitaaldekking, als de verschillen klein zijn geldt het omgekeerde en zou dit land beter af zijn in autarkie.

Ook de langetermijnvoordelen van een overgang naar een kleiner omslagstelsel en meer kapitaaldekking hangen af van de mate waarin beide omslagstelsels verschillen. Als het land met een relatief klein omslagstelsel dit nog verder reduceert, kan het nut op lange ter-

mijn erdoor dalen. Dit geldt ook als de andere regio besluit het omslagstelsel te verkleinen.

De komende vergrijzing treft nagenoeg alle landen van de OESO, maar het tempo en de omvang verschillen. Sommige landen, zoals de Verenigde Staten, vergrijzen in mindere mate dan veel Europese landen, en een land als Japan vergrijst eerder. Deze demografische ontwikkelingen kunnen daarom worden gesplitst in een algemene trend, die overal gelijktijdig en in dezelfde mate plaatsvindt, en een asymmetrische ontwikkeling die één enkel land treft. Allereerst komt de symmetrische vergrijzing aan bod. Omdat hierdoor de kapitaalarbeidsverhouding toeneemt op termijn zal ook de welvaart stijgen. In het land met een groot kapitaaldekkingsstelsel zou deze stijging echter groter zijn als het niet was geïntegreerd met een ander land dat een omvangrijk omslagstelsel heeft. Op korte termijn zijn de effecten anders. Door de daling van de rente en pensioenuitkeringen hebben reeds gepensioneerde individuen minder te besteden en dus een lager nut als gevolg van vergrijzing.

De effecten van asymmetrische vergrijzing blijken cruciaal van twee factoren af te hangen: internationale verschillen in tijdsvoorkeur en in de omvang van het omslagstelsel.

De laatste paragraaf van dit hoofdstuk brengt alle losse elementen van de vorige paragrafen samen in een tentatieve simulatie van vergrijzing in de OESO. Daartoe worden drie regio's onderscheiden: Europa, Japan en de Verenigde Staten. Gebruik makend van demografische projecties en schattingen van verschillende tijdsvoorkeurvoeten en de omvang van publieke pensioenstelsels, blijkt dat met name Japan hard getroffen wordt door vergrijzing, gevolgd door de Europese regio. Vooral huidige en toekomstige generaties in de Verenigde Staten hebben profijt van internationale kapitaalmobiliteit, terwijl het omgekeerde geldt voor Japan. Indien premies stijgen om de pensioenuitkeringen constant te houden zullen vooral toekomstige generaties hierdoor benadeeld worden.

Het laatste hoofdstuk van deze dissertatie behandelt het geboortecijfer als endogene variabele. Tegenwoordig is het kindertal over het algemeen in de westerse landen een keuze die individuen zelf maken. Zoals bij veel beslissingen spelen hierbij onder andere economische factoren een rol. Eén van de oorzaken van vergrijzing (een dalend geboortecijfer) is derhalve het gevolg van individuele economische beslissingen. In hoofdstuk 6 wordt de vraag gesteld hoe een verlenging van de levensduur uitwerkt op de bevolkingsgroei, of de individuele keuzes afwijken van wat sociaal optimaal is en hoe de overheid daar in een vergrijzende samenleving het beste op in kan spelen. Zoals in andere hoofdstukken wordt ook nu een onderscheid tussen een gesloten en een kleine open economie gemaakt.

Endogene fertiliteit wordt benaderd vanuit de zogenaamde consumptievraagbenadering. Hierbij hangt het individuele nut direct en positief af van het aantal kinderen dat men opvoedt. Een groot deel van de aanzienlijke kosten wordt door de ouders zelf gedragen, waarbij te denken valt aan kinderopvang dan wel gederfde arbeidsinkomsten, huisvesting, studiekosten et cetera. Als individuen langer leven zullen zij besluiten meer te sparen; dit gaat ten koste van de bestedingen in de eerste levensfase. Aannemende dat men niet bezuinigt op de uitgaven per kind, zal dit een dalend geboortecijfer tot gevolg hebben. De ene soort vergrijzing roept daardoor de andere soort op. In een gesloten economie is er echter nog een tweede effect dat loopt via de stijging van de kapitaalarbeidsverhouding, waardoor het looninkomen toeneemt. Als dit algemeen-evenwichtseffect erg sterk is, zal de fertiliteit uiteindelijk toenemen.

De bevolkingsgroei die het gevolg is van individuele besluitvorming heeft consequenties voor de toekomstige productie en financiële houdbaarheid van het omslagstelsel. Dit zijn typisch macro-economische gevolgen waarmee een individu geen rekening houdt. Daarom zal over het algemeen het sociaal-optimale kindertal afwijken van het feitelijke geboortecijfer. Zoals uit dit hoofdstuk blijkt wordt dit veroorzaakt door twee tegengestelde externaliteiten. Als er een omslagstelsel is zal men bij de beslissing over het aantal kinderen geen rekening houden met het feit dat deze kinderen de toekomstige premiebetalers zijn. Het aantal kinderen is dan vanuit sociaal opzicht te laag. In dat geval dient de overheid ervoor te zorgen dat de ouders in de kosten van de opvoeding tegemoet worden gekomen, bijvoorbeeld door het subsidiëren van kinderopvang, studiefinanciering en kinderbijslag. Aldus wordt een hoger welvaartsniveau bereikt. In een kleine open economie heeft vergrijzing in de vorm van een langere levensduur geen effect op de omvang van zo'n stelsel. In een gesloten economie daarentegen blijkt de optimale tegemoetkoming in de opvoedingskosten toe te nemen naarmate de bevolking vergrijst.

Center for Economic Research, Tilburg University, The Netherlands
Dissertation Series

No.	Author	Title	Published
1	P.J.J. Herings	Static and Dynamic Aspects of General Disequilibrium Theory; ISBN 90 5668 001 3	June 1995
2[1]	Erwin van der Krabben	Urban Dynamics: A Real Estate Perspective - An institutional analysis of the production of the built environment; ISBN 90 5170 390 2	August 1995
3	Arjan Lejour	Integrating or Desintegrating Welfare States? - a qualitative study to the consequences of economic integration on social insurance; ISBN 90 5668 003 X	September 1995
4	Bas J.M. Werker	Statistical Methods in Financial Econometrics; ISBN 90 5668 002 1	September 1995
5	Rudy Douven	Policy Coordination and Convergence in the EU; ISBN 90 5668 004 8	September 1995
6	Arie J.T.M. Weeren	Coordination in Hierarchical Control; ISBN 90 5668 006 4	September 1995
7	Herbert Hamers	Sequencing and Delivery Situations: a Game Theoretic Approach; ISBN 90 5668 005 6	September 1995
8	Annemarie ter Veer	Strategic Decision Making in Politics; ISBN 90 5668 007 2	October 1995
9	Zaifu Yang	Simplicial Fixed Point Algorithms and Applications; ISBN 90 5668 008 0	January 1996
10	William Verkooijen	Neural Networks in Economic Modelling - An Empirical Study; ISBN 90 5668 010 2	February 1996
11	Henny Romijn	Acquisition of Technological Capability in Small Firms in Developing Countries; ISBN 90 5668 009 9	March 1996
12	W.B. van den Hout	The Power-Series Algorithm - A Numerical Approach to Markov Processes; ISBN 90 5668 011 0	March 1996
13	Paul W.J. de Bijl	Essays in Industrial Organization and Management Strategy; ISBN 90 5668 012 9	April 1996

No.	Author	Title	Published
14	Martijn van de Ven	Intergenerational Redistribution in Representative Democracies; ISBN 90 5668 013 7	May 1996
15	Eline van der Heijden	Altruism, Fairness and Public Pensions: An Investigation of Survey and Experimental Data; ISBN 90 5668 014 5	May 1996
16	H.M. Webers	Competition in Spatial Location Models; ISBN 90 5668 015 3	June 1996
17	Jan Bouckaert	Essays in Competition with Product Differentiation and Bargaining in Markets; ISBN 90 5668 016 1	June 1996
18	Zafar Iqbal	Three-Gap Analysis of Structural Adjustment in Pakistan; ISBN 90 5668 017 X	September 1996
19	Jimmy Miller	A Treatise on Labour: A Matching-Model Analysis of Labour-Market Programmes; ISBN 90 5668 018 8	September 1996
20	Edwin van Dam	Graphs with Few Eigenvalues - An interplay between combinatorics and algebra; ISBN 90 5668 019 6	October 1996
21	Henk Oosterhout	Takeover Barriers: the good, the bad, and the ugly; ISBN 90 5668 020 X	November 1996
22	Jan Lemmen	Financial Integration in the European Union: Measurement and Determination; ISBN 90 5668 021 8	December 1996
23	Chris van Raalte	Market Formation and Market Selection; ISBN 90 5668 022 6	December 1996
24	Bas van Aarle	Essays on Monetary and Fiscal Policy Interaction: Applications to EMU and Eastern Europe; ISBN 90 5668 023 4	December 1996
25	Francis Y. Kumah	Common Stochastic Trends and Policy Shocks in the Open Economy: Empirical Essays in International Finance and Monetary Policy; ISBN 90 5668 024 2	May 1997
26	Erik Canton	Economic Growth and Business Cycles; ISBN 90 5668 025 0	September 1997

No.	Author	Title	Published
27	Jeroen Hoppenbrouwers	Conceptual Modeling and the Lexicon; ISBN 90 5668 027 7	October 1997
28	Paul Smit	Numerical Analysis of Eigenvalue Algorithms Based on Subspace Iterations; ISBN 90 5668 026 9	October 1997
29	Uri Gneezy	Essays in Behavioral Economics; ISBN 90 5668 028 5	October 1997
30	Erwin Charlier	Limited Dependent Variable Models for Panel Data; ISBN 90 5668 029 3	November 1997
31	Rob Euwals	Empirical Studies on Individual Labour Market Behaviour; ISBN 90 5668 030 7	December 1997
32	Anurag N. Banerjee	The Sensitivity of Estimates, Inferences, and Forecasts of Linear Models; ISBN 90 5668 031 5	December 1997
33	Frans A. de Roon	Essays on Testing for Spanning and on Modeling Futures Risk Premia; ISBN 90 5668 032 3	December 1997
34	Xiangzhu Han	Product Differentiation, Collusion and Standardization; ISBN 90 5668 033 1	January 1998
35	Marcel Das	On Income Expectations and Other Subjective Data: A Micro-Econometric Analysis; ISBN 90 5668 034 X	January 1998
36	Jeroen Suijs	Cooperative Decision Making in a Stochastic Environment; ISBN 90 5668 035 8	March 1998
37	Talitha Feenstra	Environmental Policy Instruments and International Rivalry: A Dynamic Analysis; ISBN 90 5668 036 6	May 1998
38	Jan Bouwens	The Use of Management Accounting Systems in Functionally Differentiated Organizations; ISBN 90 5668 037 4	June 1998
39	Stefan Hochguertel	Households' Portfolio Choices; ISBN 90 5668 038 2	June 1998
40	Henk van Houtum	The Development of Cross-Border Economic Relations; ISBN 90 5668 039 0	July 1998
41	Jorg Jansen	Service and Inventory Models subject to a Delay-Limit; ISBN 90 5668 040 4	September 1998

No.	Author	Title	Published
42	F.B.S.L.P. Janssen	Inventory Management Systems: control and information issues; ISBN 90 5668 041 2	September 1998
43	Henri L.F. de Groot	Economic Growth, Sectoral Structure and Unemployment; ISBN 90 5668 042 0	October 1998
44	Jenke R. ter Horst	Longitudinal Analysis of Mutual Fund Performance; ISBN 90 5668 043 9	November 1998
45	Marco Hoeberichts	The Design of Monetary Institutions; ISBN 90 5668 044 7	December 1998
46	Adriaan Kalwij	Household Consumption, Female Employment and Fertility Decisions: A microeconometric analysis; ISBN 90 5668 045 5	February 1999
47	Ursula Glunk	Realizing High Performance on Multiple Stakeholder Domains: A Resource-based Analysis of Professional Service Firms in the Netherlands and Germany; ISBN 90 5668 046 3	April 1999
48	Freek Vermeulen	Shifting Ground: Studies on the Intersection of Organizational Expansion, Internationalization, and Learning; ISBN 90 5668 047 1	February 1999
49	Haoran Pan	Competitive Pressures on Income Distribution in China; ISBN 90 5668 048 X	March 1999
50	Laurence van Lent	Incomplete Contracting Theory in Empirical Accounting Research; ISBN 90 5668 049 8	April 1999
51	Rob Aalbers	On the Implications of Thresholds for Economic Science and Environmental Policy; ISBN 90 5668 050 1	May 1999
52	Abe de Jong	An Empirical Analysis of Capital Structure Decisions in Dutch Firms; ISBN 90 5668 051 X	June 1999
53	Trea Aldershof	Female Labor Supply and Housing Decisions; ISBN 90 5668 052 8	June 1999
54	Jan Fidrmuc	The Political Economy of Reforms in Central and Eastern Europe; ISBN 90 5668 053 6	June 1999
55	Michael Kosfeld	Individual Decision-Making and Social Interaction; ISBN 90 5668 054 4	June 1999

No.	Author	Title	Published
56	Aldo de Moor	Empowering Communities - A method for the legitimate user-driven specification of network information systems; ISBN 90 5668 055 2	October 1999
57	Yohane A. Khamfula	Essays on Exchange Rate Policy in Developing Countries; ISBN 90 5668 056 0	September 1999
58	Maurice Koster	Cost Sharing in Production Situations and Network Exploitation; ISBN 90 5668 057 9	December 1999
59	Miguel Rosellón Cifuentes	Essays on Financial Policy, Liquidation Values and Product Markets; ISBN 90 5668 058 7	December 1999
60	Sharon Schalk	Equilibrium Theory: A salient approach; ISBN 90 5668 059 5	December 1999
61	Mark Voorneveld	Potential Games and Interactive Decisions with Multiple Criteria; ISBN 90 5668 060 9	December 1999
62	Edward Droste	Adaptive Behavior in Economic and Social Environments; ISBN 90 5668 061 7	December 1999
63	Jos Jansen	Essays on Incentives in Regulation and Innovation; ISBN 90 5668 062 5	January 2000
64	Franc J.G.M. Klaassen	Exchange Rates and their Effects on International Trade; ISBN 90 5668 063 3	January 2000
65	Radislav Semenov	Cross-country Differences in Economic Governance: Culture as a major explanatory factor; ISBN 90 5668 065 X	February 2000
66	Alexandre Possajennikov	Learning and Evolution in Games and Oligopoly Models; ISBN 90 5668 064 1	March 2000
67	Eric Gaury	Designing Pull Production Control Systems: Customization and Robustness; ISBN 90 5668 066 8	March 2000
68	Marco Slikker	Decision Making and Cooperation Restrictions; ISBN 90 5668 067 6	April 2000
69	Ruud Brekelmans	Stochastic Models in Risk Theory and Management Accounting; ISBN 90 5668 068 4	June 2000
70	Flip Klijn	A Game Theoretic Approach to Assignment Problems; ISBN 90 5668 069 2	June 2000

No.	Author	Title	Published
71	Maria Montero	Endogenous Coalition Formation and Bargaining; ISBN 90 5668 070 6	June 2000
72	Bas Donkers	Subjective Information in Economic Decision Making; ISBN 90 5668 071 4	June 2000
73	Richard Nahuis	Knowledge and Economic Growth; ISBN 90 5668 072 2	October 2000
74	Kuno Huisman	Technology Investment: A Game Theoretic Real Options Approach; ISBN 90 5668 073 0	October 2000
75	Gijsbert van Lomwel	Essays on Labour Economics; ISBN 90 5668 074 9	November 2000
76	Tina Girndt	Cultural Diversity and Work-Group Performance: Detecting the Rules; ISBN 90 5668 075 7	December 2000
77	Teye A. Marra	The Influence of Proprietary Disclosure Costs on the Decision to Go Public: Theory and Evidence; ISBN 90 5668 076 5	January 2001
78	Xiaodong Gong	Empirical Studies on the Labour Market and on Consumer Demand; ISBN 90 5668 078 1	February 2001
79	Kanat Çamlibel	Complementarity Methods in the Analysis of Piecewise Linear Dynamical Systems; ISBN 90 5668 079 X	May 2001
80	Joost Driessen	Empirical Studies on the Pricing of Bonds and Interest Rate Derivatives; ISBN 90 5668 080 3	June 2001
81	Bram van den Broek	Uncertainty in Differential Games; ISBN 90 5668 081 1	June 2001
82	Ricardo Recht	Organisational Culture and Privatisation: A Case Study of the Argentinean Railway Sector; ISBN 90 5668 082 X	June 2001
83	Willem Verhagen	Inflation Targeting and Interest Rate Policy; ISBN 90 5668 083 8	June 2001
84	Vincent Verouden	Essays in Antitrust Economics; ISBN: 90 5668 086 2	June 2001
85	Rosalia Vazquez Alvarez	A Nonparametric Approach to the Sample Selection Problem in Survey Data; ISBN 90 5668 087 0	June 2001

No.	Author	Title	Published
86	Theo Leers	Public Pensions and Population Ageing: An Economic Analysis of Fertility, Migration, and Social-Security Policy; ISBN 90 5668 084 6	June 2001
87	Bernard Conlon	Consumer Rationality in Choice; ISBN 90 5668 085 4	June 2001
88	Judith Timmer	Cooperative behaviour, uncertainty and operations research; ISBN 90 5668 088 9	September 2001
89	Enrico Diecidue	Nonexpected Utility and Coherence; ISBN 90 5668 089 7	November 2001
90	Alexander Konovalov	Essays in General Equilibrium Theory; ISBN 90 5668 091 9	November 2001
91	Stefan Stremersch	Essays on Marketing Strategy in Technology-Intensive Markets; ISBN 90 5668 090 0	December 2001
92	Rian Drogendijk	Expansion Patterns of Dutch Firms in Central and Eastern Europe: Learning to Internationalize; ISBN 90 5668 092 7	December 2001
93	Han Donker	Takeovers. An Empirical Study of Ownership Structure and the Wealth to Shareholders in Dutch Takeovers; ISBN 90 5668 093 5	December 2001
94	Erica van Herpen	Perceptions and Evaluations of Assortment Variety; ISBN 90 5668 094 3	December 2001
95	Jana Vyrastekova	Cheap Talk and Spiteful Preferences in Ultimatum Games: Experiments and Evolutionary Rationale; ISBN 90 5668 095 1	March 2002
96	Tao Lin	Statistics of Extremes in the Space of Continuous Functions; ISBN 90 5668 096 X	May 2002
97	Sander Onderstal	Papers in Auction Theory; ISBN 90 5668 097 8	May 2002
98	Wolf Wagner	Risk Sharing under Incentive Constraints; ISBN 90 5668 098 6	June 2002
99	Willem-Jan van den Heuvel	Integrating Modern Business Applications with Objectified Legacy Systems; ISBN 90 5668 099 4	June 2002
100	Jacco Wielhouwer	Optimal Tax Depreciation and its Effects on Optimal Firm Investments; ISBN 90 5668 100 1	June 2002
101	Peter Roosenboom	Corporate Governance Mechanisms in IPO Firms; ISBN 90 5668 101 X	September 2002

No.	Author	Title	Published
102	Victoria Shestalova	Essays in Productivity and Efficiency; ISBN 90 5668 103 6	September 2002
103	Paula van Veen-Dirks	Flexibility and Control: An Empirical Study Relating Production Flexibility to the Design of Performance Management Systems; ISBN 90 5668 102 8	September 2002
104	Michal Matějka	Management Accounting in Organizational Design: Three Essays; ISBN 90 5668 105 2	November 2002
105	Maarten Pronk	Market Liquidity around Earnings Announcements; ISBN 90 5668 106 0	November 2002
106	Nancy Kamp-Roelands	Towards a framework for auditing environmental reports; ISBN 90 5668 108 7	November 2002
107	Ana Ferreira	Statistics of extremes: estimation and optimality; ISBN 90 5668 107 9	November 2002
108	Alexei Goriaev	On the behaviour of Mutual Fund Investors and Managers; ISBN 90 5668 110 9	November 2002
109	Paul Ingenbleek	Money for Value Pricing from a Resource-Advantage Perspective; ISBN 90 5668 109 5	December 2002
110	Frank Wijen	Stakeholder influence and organizational learning in environmental management; ISBN 90 5668 111 7	December 2002
111	Michele Belot	Labor market institutions in OECD countries: Origins and consequences; ISBN 90 5668 113 3	January 2003
112	Federica Teppa	Estimating preference parameters in individual financial decision making; ISBN 90 5668 112 5	January 2003
113	Dmitry Danilov	The effects of pretesting in econometrics with applications in finance; ISBN 90 5668 104 4	February 2003
114	Jacco Thijssen	Investment under uncertainty, market evolution and coalition spillovers in a game theoretic perspective; ISBN 90 5668 114 1	May 2003
115	Laura Spierdijk	Empirical Studies of Market Microstructure; ISBN 90 5668 115 X	June 2003
116	Nicole van Beeck	A New Decision Support Method for Local Energy Planning in Developing Countries; ISBN 90 5668 116 8	June 2003
117	Grzegorz Pawlina	Corporate Investment under Uncertainty and Competition: A Real Options Approach; ISBN 90 5668 118 4	June 2003

No.	Author	Title	Published
118	Jana P. Fidrmuc	The Impact of Blockholders on Information Signalling, Productivity, and Managerial Disciplining; ISBN 90 5668 117 6	June 2003
119	Sjoerd Beugelsdijk	Culture and Economic Development in Europe; ISBN 90 5668 121 4	September 2003
120	Bas van Groezen	The Wealth of Generations: Ageing, Pensions and Welfare from a Macroeconomic Perspective; ISBN 90 5668 120 6	October 2003

[1] Copies can be ordered from Thela Thesis, Prinseneiland 305, 1013 LP Amsterdam, The Netherlands, phone: + 31 20 6255429; fax: +31 20 6203395; e-mail: office@thelathesis.nl