Masses and Magnetic Moments of Baryons
in a QCD-string Model
Cover illustration: an artistic view of three quarks connected by the QCD-string, John Bosma, 2004.
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Masses and Magnetic Moments of Baryons
in a QCD-string Model

Massa’s en MagnetischeMomenten van Baryonen
in een QCD-snaarmodel

(proefschrift)

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Chapter 1

Introduction

The history of physics shows an ongoing search for the most elementary constituents of matter and the rules that govern their behavior. In the beginning of the twentieth century, the structure of the atom consisting of a positively charged nucleus surrounded by negatively charged electrons was established. The nucleus appeared to be made up of protons and neutrons which were tightly bound by the so-called strong force. Yukawa’s meson hypothesis from 1935 [87] and the discovery of pi-mesons by the group of Powell in 1947 [46] can be marked as the beginning of an era in which a rapid increasing number of subnuclear particles was found. Since that time, an astonishing amount of experimental data has been accumulated on this ‘Particle Zoo’, as the situation with the many subnuclear particles was sometimes called. Many of their properties, such as their masses, decay widths and channels, resonances, magnetic moments, etc., have been determined.

The analysis of the experiments led to a classification of the subnuclear particles into a number of easily identifiable categories. According to their properties and behavior, the particles are listed as leptons, mesons, baryons and gauge bosons. Examples are electrons, pions, protons and photons. Baryons and mesons are called hadrons. They mainly interact by means of the strong force, the same force that binds together the protons and neutrons of the nucleus. Some properties of the baryons, which are important to this thesis, are listed in Table 1.1. The baryons can be ordered in an isospin-strangeness chart as is shown in Figure 1.1. Note that the charge is conserved along diagonal lines, \( Q/e = I_3 + (1 + S)/2 \). It appeared to be possible to summarize the behavior of these particles using some empirical rules. From these rules a fundamental theory of the strongly interacting particles could be obtained.

In 1964 Murray Gell-Mann [26] and, independently, George Zweig [88] realized that the experimental results showed evidence for a repeated simple structure of three constituents which come in three flavors, called the ‘up’ (u), ‘down’ (d) and ‘strange’ (s) quark. All hadrons are composed of quarks. Protons and neutrons are made of up and down quarks, while strange quarks play a role in particles as the K-mesons and some other short-lived hadrons. According to the \( SU(3) \)-flavor group, these quarks
Table 1.1: Some properties and quantum numbers of the baryon octet and decuplet representation of the SU(3)-flavor group are listed below [32]. The symbols J, I, P, S and Y represent spin, isospin, parity, strangeness and hypercharge. The mass is given in units of MeV/c², the magnetic moment µ_B in units of the nuclear magneton.

<table>
<thead>
<tr>
<th>N</th>
<th>I(J^P)</th>
<th>S</th>
<th>Y</th>
<th>quark content</th>
<th>mass</th>
<th>µ_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>1/2(1/2+)</td>
<td>0</td>
<td>1</td>
<td>uud</td>
<td>938</td>
<td>2.79</td>
</tr>
<tr>
<td>n</td>
<td>1/2(1/2+)</td>
<td>0</td>
<td>1</td>
<td>udd</td>
<td>940</td>
<td>-1.91</td>
</tr>
<tr>
<td>Λ</td>
<td>0(1/2+)</td>
<td>-1</td>
<td>0</td>
<td>uds</td>
<td>1116</td>
<td>-0.61</td>
</tr>
<tr>
<td>Σ^+</td>
<td>1(1/2+)</td>
<td>-1</td>
<td>0</td>
<td>uus</td>
<td>1189</td>
<td>2.46</td>
</tr>
<tr>
<td>Σ^0</td>
<td>1(1/2+)</td>
<td>-1</td>
<td>0</td>
<td>uds</td>
<td>1193</td>
<td></td>
</tr>
<tr>
<td>Σ^-</td>
<td>1(1/2+)</td>
<td>-1</td>
<td>0</td>
<td>dds</td>
<td>1197</td>
<td>-1.16</td>
</tr>
<tr>
<td>Ξ^0</td>
<td>1/2(1/2+)</td>
<td>-2</td>
<td>-1</td>
<td>uus</td>
<td>1315</td>
<td>-1.25</td>
</tr>
<tr>
<td>Ξ^-</td>
<td>1/2(1/2+)</td>
<td>-2</td>
<td>-1</td>
<td>dss</td>
<td>1321</td>
<td>-0.65</td>
</tr>
<tr>
<td>Δ++</td>
<td>5/2(3/2+)</td>
<td>0</td>
<td>1</td>
<td>uuu</td>
<td>1232</td>
<td>4.52</td>
</tr>
<tr>
<td>Δ^+</td>
<td>3/2(3/2+)</td>
<td>0</td>
<td>1</td>
<td>uud</td>
<td>1232</td>
<td></td>
</tr>
<tr>
<td>Δ^0</td>
<td>3/2(3/2+)</td>
<td>0</td>
<td>1</td>
<td>udd</td>
<td>1232</td>
<td></td>
</tr>
<tr>
<td>Δ^-</td>
<td>3/2(3/2+)</td>
<td>0</td>
<td>1</td>
<td>ddd</td>
<td>1232</td>
<td></td>
</tr>
<tr>
<td>Σ^0*</td>
<td>1(3/2+)</td>
<td>-1</td>
<td>0</td>
<td>uus</td>
<td>1383</td>
<td></td>
</tr>
<tr>
<td>Σ^-*</td>
<td>1(3/2+)</td>
<td>-1</td>
<td>0</td>
<td>uds</td>
<td>1384</td>
<td></td>
</tr>
<tr>
<td>Ξ^0*</td>
<td>1/2(3/2+)</td>
<td>-2</td>
<td>-1</td>
<td>uss</td>
<td>1532</td>
<td></td>
</tr>
<tr>
<td>Ξ^-*</td>
<td>1/2(3/2+)</td>
<td>-2</td>
<td>-1</td>
<td>dss</td>
<td>1535</td>
<td></td>
</tr>
<tr>
<td>Ω^-</td>
<td>0(3/2+)</td>
<td>-3</td>
<td>-2</td>
<td>sss</td>
<td>1672</td>
<td>-2.02</td>
</tr>
</tbody>
</table>

build in an ‘eightfold way’ the baryon multiplet which is given in Table 1.1 and shown in Figure 1.1. Quarks appear to be fundamental, without any internal structure, and have mass, fractional charge and spin 1/2. Some of the quark properties are listed in Table 1.2. Later on the discovery of even more exotic hadrons needed the introduction of heavier quarks which were called ‘charm’ (c), ‘bottom’ (b) and ‘top’ (t).

The quarks are not just convenient mathematical constructs providing an appealing explanation of the symmetry of the hadrons, but actually are real building blocks. In the SLAC-MIT experiment analogous to Rutherford scattering on the atom, the internal structure of the nucleon (proton or neutron) was sampled by high energy electron beams [43]. From the results it had been concluded that the nucleon actually
consists of three point-like objects having spin 1/2 and fractional electric charges.

Two main problems were associated with the quark model. First, the quarks seemed to occur only in combination with other quarks or anti-quarks, never alone. This property is called (quark) confinement. Second, no two identical particles with half-integer spin can exist in the same state, as this would violate the Pauli exclusion principle. However, from Table 1.1 it can be seen that the quark content of, for example, the $\Delta^{++}$ consists of three identical particles, $uuu$, in the same state. Such an arrangement is clearly prohibited by the exclusion principle.

To overcome these problems the quantum number ‘color’ was introduced, which is just a name for one of the properties of quarks and has nothing to do with our experience with color in everyday live. The three colors red (R), green (G) and blue (B) are ascribed to the quarks. The behavior of these colored particles is described by a relativistic quantum field theory called quantum chromodynamics (QCD). The symmetry $SU(3)$-color group forms the heart of QCD. At low energies QCD requires that quarks occur in colorless combinations. This confinement of quarks is non-trivially contained in the low-energy regime of QCD. Mesons are build up as color anti-color particles, $q\bar{q}$. Baryons are made of one red, one blue and one green quark yielding an object without color, which is a color singlet of the $SU(3)$-color group. Note that due to this extra quantum number color, we can have objects like $\Delta^{++}$ which consist of three identical quarks in different color states, $u_Ru_Gu_B$. 

Figure 1.1: The isospin versus strangeness charts for the baryon octet (spin-1/2) and the decuplet (spin-3/2).
Table 1.2: Some properties and quantum numbers of the up, down and strange quark [32]. The symbols J, I, P, S and Y represent spin, isospin, parity, strangeness and hypercharge, respectively.

<table>
<thead>
<tr>
<th>q</th>
<th>I(J^P)</th>
<th>charge e_q</th>
<th>S</th>
<th>Y</th>
<th>current mass (MeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>$\frac{1}{2}(\frac{1}{2}^+)$</td>
<td>$2e/3$</td>
<td>0</td>
<td>$1/3$</td>
<td>1.5-4.5</td>
</tr>
<tr>
<td>d</td>
<td>$\frac{1}{2}(\frac{1}{2}^+)$</td>
<td>$-e/3$</td>
<td>0</td>
<td>$1/3$</td>
<td>5-8.5</td>
</tr>
<tr>
<td>s</td>
<td>0($\frac{1}{2}^+$)</td>
<td>$-e/3$</td>
<td>−1</td>
<td>$-2/3$</td>
<td>80-155</td>
</tr>
</tbody>
</table>

Local gauge symmetry of the $SU(3)$-color group requires the exchange of massless particles between the quarks which are called gluons. These gluons take care of the exchange of color among the quarks. As both the quarks and the gluons carry color, apart from the quark-gluon couplings, there also exists gluon-gluon couplings. This property reflects the non-Abelian character of the $SU(3)$-color group.

From Table 1.2 it can be seen that rather low current quark masses have been determined for the $u, d$-quarks, typically a few MeV/c^2, while the corresponding baryons have masses of the order of 1 GeV/c^2, see Table 1.1. QCD appears to be such that the quarks acquire a mass through their interaction with the gluon field. In this way quite heavy compound objects, such as the baryon, can be made out of very light quarks. This property of QCD is called spontaneous chiral symmetry breaking (CSB), and reflects the loss of the (almost) massless symmetry of the quarks.

The coupling constant $\alpha_s$ in QCD is a running parameter, being large for low energies and small for high energies. In the high-energy regime of QCD one can therefore use perturbation theory. Detailed tests of perturbative QCD can be performed at high-energy colliders [76, 86]. Since leptons and photons are color neutral, they are not subject to the strong force, and therefore they form an ideal probe to examine the short-distance structure of hadrons. This examination can be performed by, for example, scattering of virtual photons, emitted by high-energy electron beams, from hadrons, thus revealing the quark and gluon distributions. As another example, electron-positron annihilation, $e^+e^- \rightarrow q\bar{q}$, can be used to examine the materialization of the quarks into hadrons, which yields detailed information on the short-distance coupling in QCD. At this high-energy scale, predictions based on perturbative QCD are in good agreement with experiment. However, at low energies, needed for the description of bound states of the quarks, perturbation theory breaks down due to the large coupling constant $\alpha_s$ in this region.

To deal with the characteristics of QCD at low energies, the picture of the elementary quarks of QCD as being dressed by clouds of virtual gluons and quarks is widely used. In this way, extended quarks are formed which are called constituent quarks. In constituent quark models, they are considered as the basic degrees of freedom, which have the same quantum numbers as the elementary quarks but have much larger
masses. Non-relativistic constituent quark models are very successful in describing the level structure of the masses of mesons and baryons. The electromagnetic properties are also well reproduced. For the confining interaction, a phenomenological potential is used, usually a harmonic oscillator potential or a QCD-inspired linear potential with one-gluon or one-pion exchanges, and rather heavy constituent quarks, see for example Refs. [27–29, 31, 37–39]. Relativistic effects can be included by using a relativistic kinetic term in the Hamiltonian, as was done in, for example, Refs. [14–16]. Mesons are made of constituent quark anti-quark pairs, baryons of three constituent quarks. One expects good behavior of these models in heavy hadrons where heavy quark flavors are involved. Surprisingly, however, the models also do well in light quark systems.

This theoretical description can, however, not be considered completely satisfactory. As QCD is generally accepted as the theory of strong interactions, $\bar{q}q$ and $3q$ dynamics should be derived from QCD in a fully covariant way. This starting point is highly non-trivial due to the large coupling constant of QCD at low energies, and due to the non-Abelian character of QCD.

These difficulties can be treated numerically by simulating QCD on a lattice giving an ab initio way of calculating low energy aspects of QCD. Results on the static quark potential, the hadron spectrum, the hadron structure, and the running coupling constant can thus be obtained. Some reviews can be found in Refs. [53,55].

The mentioned characteristics of QCD can also be dealt with by using analytic non-perturbative methods. Such methods should reproduce in a covariant way the two basic phenomena that govern the QCD dynamics of $\bar{q}q$ and $3q$ systems at low energies: confinement and chiral symmetry breaking (CSB). Confinement for static quarks is usually reflected in the area law of the Wilson loop [85], or equivalently through field correlators in the Field Correlator Method (FCM) [22,23,62].

The formalism of the FCM and the Feynman–Schwinger path integral representation is well suited for the treatment of relativistic quarks when spin-dependent mass corrections are neglected [62,72]. The method yields confinement as the area law also for light quarks in a covariant way. As a result, Regge trajectories have been found in Ref. [25] with the correct string slope $(2\pi\sigma)^{-1}$. The main difficulty which was always present in the method, was the perturbative treatment of spin-dependent interactions (which led to incorrect results, e.g. a collapse of the $q\bar{q}$-system for the pion), and absence of spontaneous CSB effects in general [24].

Recently, a new type of formalism was suggested to treat simultaneously confinement and CSB in a gauge invariant way, and a nonlinear equation was derived for a light quark in the field of a heavy anti-quark [66,67]. This equation, which is derived directly from the QCD Lagrangian, was found to produce linear confinement and CSB for the light quark. The explicit form of the nonlocal effective quark mass operator $M(x,y)$ was defined obeying both these properties. As the mass operator $M(x,y)$ can be considered as a string-like object connecting the light quark to the heavy anti-quark, the concerning model can be called: QCD-string model.

The eigenvalues and eigenfunctions of the nonlocal and nonlinear equations have been determined, and a nonzero condensate was computed in Ref. [73], confirming that CSB is really present in the equations. In an additional study [74], it was demonstrated
that magnetic-field correlators do not contribute to the large-distance confinement, however strongly modify the confinement for lowest levels, and heavy-light masses corrected in this way are favorably compared in Ref. [74] to experiment and results of other calculations.

Since the method of Refs. [66,67] is quite general and allows to treat also multi-quark systems, it can be applied to the $q\bar{q}$ and $3q$ systems, to find dynamical equations for them, which contain confinement and CSB [68]. To make these equations tractable, the large $N_c$-limit was systematically exploited, and the calculation was mostly confined to the simplest field correlators – the so-called Gaussian approximation; it was in particular shown in Ref. [66] that the sum over all correlators does not change the qualitative results, but in this case the resulting kernel of the equations becomes much more complicated. Moreover, it was shown in Ref. [70] that lattice data strongly support the dominance of the Gaussian (bilocal) correlator, estimating the correction due to higher correlators to 1-2%.

In this thesis, we study the baryons in the QCD-string model using the field correlator method described above as developed by Simonov and others [66–68, 73, 74]. Using this model, we have calculated the mass spectrum and magnetic moments of the lowest baryon octuplet and decuplet representations of the $SU(3)$-flavor group.

The thesis is organized as follows. In Chapter 2 the general effective quark Lagrangian from the standard QCD Lagrangian is obtained by integrating out gluonic degrees of freedom, and the nonlinear equation for the single quark propagator $S$ (attached to the string in a gauge invariant way) is derived, following the procedure in Ref. [68]. It is shown that the baryon Green’s function can be expressed in the lowest order of our approximation scheme (neglecting gluon and pion exchange) in terms of three independent quark Green’s functions, resulting in a Hamiltonian as a sum of three single-quark terms. The subsequent Dirac-like equation is solved self-consistently, and the baryon wave function is obtained. In this first approximation, the baryon wave function is given by the product of three independent single-particle wave functions. However, a translationally symmetric Ansatz for the baryon wave function is also considered as a second, more involved, approximation.

Using these baryon wave functions, we calculate the magnetic moments in Chapter 3 for both the baryon octet and decuplet. Higher-order corrections to these results are calculated from the presence of virtual pseudoscalar mesons. Effects from one-body currents have been estimated by performing mesonic one-loop calculations which contribute to the anomalous magnetic moment of the quark. The contributions from two-body currents due to the exchange of pions between the quarks are predicted to be small. For the interaction of the pseudoscalar mesons with the quarks either the pseudoscalar or the pseudovector coupling is used.

The virtual pseudoscalar mesons in Chapter 3 and the following chapters are introduced as effective representations of correlated quark anti-quark pairs. This replacement is supported by Ref. [71], where Simonov has shown that an effective chiral Lagrangian, containing both confinement and pion-quark couplings, can be obtained by systematically exploiting the field correlator method. From this derivation, we adopt the view that quarks effectively interact with the virtual pseudoscalar mesons.
A schematic picture of this statement is shown in Figure 1.2. As a consequence, quark-gluon and quark-meson interactions are employed together in this thesis. These perturbative interactions are considered in addition to the confining QCD-string, and thus the model is extended by spin and flavor dependencies, which are absent in the QCD-string.

Chapter 4 is devoted to the calculation of the mass spectrum. By introducing one gluon and pseudoscalar meson exchanges on top of the confining interaction induced by the QCD-string, refinements on the baryon mass spectrum are calculated. Results on the baryon multiplet are given of which mainly the mass splitting between the nucleon and the $\Delta$ is considered. We show that for realistic values of the coupling constants, the gluon exchange cannot account for the large mass splitting between $J = 1/2$ and $J = 3/2$ baryons. The one-pion exchange appears to be large and crucial to reproduce the baryon mass spectrum.

The lowest baryon wave functions in general consist of the ground state three-particle wave functions and (small) mixings of excited three-particle states. The mixing of these higher states into the baryon ground state modifies the predictions for the baryon properties. We consider the higher three-particle states as being (partially) built up of excited single particle orbitals. In Chapter 5, we study these effects by forming excited baryon states out of the orbital and radial excitations of the single-quark wave functions and using these as a basis to diagonalize the Hamiltonian. The results of these higher states on the baryon masses and magnetic moments are calculated for the lowest $SU(3)$-flavor octet and decuplet representation. Both pseudoscalar and pseudovector couplings are studied for the interaction of the pseudoscalar mesons with the quarks.

In Chapter 6 we conclude with a general discussion of the results of the present work. In the appendices, some technical details of the calculations are presented. Throughout the thesis, the Bjorken and Drell convention is used for the Dirac $\gamma$-matrices [10], and the natural units $\hbar = c = 1$ are adopted.
Chapter 2

QCD string model

As QCD is generally accepted as the theory of strong interactions, \( \bar{q}q \) and 3\( q \) dynamics should be derived from QCD in a fully covariant way. This is a formidable task due to the large gluon-quark coupling constant at low energies and the non-Abelian character of QCD. This chapter describes how baryon wave functions can be obtained from the QCD partition function in a number of steps using the formalism developed by Simonov and others \([23, 66–69, 75]\). In Section 2.1, an effective quark Lagrangian is obtained from the QCD action using the formalism of field correlators. From the effective Lagrangian, Green’s functions have been written down in Section 2.2 leading to nonlinear and nonlocal Dirac-like equations for the quarks. In Section 2.3, we briefly discuss the baryon wave functions satisfying these equations. In the last section, a little excursion is made to include perturbative corrections into the equations.

2.1 Effective quark Lagrangian

In recent years, the formalism of field correlators was set up to derive equations from QCD which deal with the two main features of quark-systems: confinement and chiral symmetry breaking (CSB). Using the field correlator method (FCM), one can average over background gluonic fields and obtain an effective quark Lagrangian. In this way a nonlinear equation has been derived for a light quark in the field of a heavy anti-quark \([66, 67]\). In this section it is shown that developed method can be extended to treat the light quark systems \( \bar{q}q \) and 3\( q \). The procedure mainly follows Refs. \([66, 67]\), but now special attention is paid to the dependence on the contour in the definition of the contour gauge, and introducing the operation of averaging over the contour manifold.

The QCD partition function for quarks and gluons can be written as

\[
Z = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\psi^\dagger \exp \left[ L_0 + L_1 + L_{\text{int}} \right],
\]  

(2.1)
where we are using the Euclidean metric and define

\[ L_0 = -\frac{1}{4} \int (F_{\mu\nu}^a)^2 d^4 x, \]  

\[ L_1 = -i \int \psi^f \dagger (x)(\partial + m_f)\psi^f (x) d^4 x, \]  

\[ L_{\text{int}} = \int \psi^f \dagger (x)g\gamma^\mu A_\mu (x)\psi^f (x) d^4 x. \]  

Here, \( m_f \) is the (current) mass of the quark field \( \psi^f \), with flavor \( f \), color \( k \) and bispinor index \( \alpha \). We have combined the gluon field \( A_\mu^a \) with the generators \( \lambda^a_{kl} \) of the SU(3)-color group and used the notation

\[ (A_\mu^a(x))_{kl} = A_\mu^a(x)\lambda^a_{kl}, \]  

with \( a = 1 \ldots 8 \).

To express \( A_\mu^a(x) \) through \( F_{\mu\nu}^a \), one can use the so called generalized Fock–Schwinger gauge\(^1\) which was first introduced in Ref. [40] and refined in Ref. [60]. For any point \( x \), a smooth contour \( C(x) \) is introduced which connects the point \( x \) with \( x_0 \), which can also be at infinity. The contours are parameterized by \( z_\mu(s, x) \) with the condition \( z_\mu(0, x) = x_0^\mu \) and \( z_\mu(1, x) = x^\mu, s \in [0, 1] \). A special class of contours is considered which satisfy the requirement that for any \( s \) and \( s' \) there exists an \( s'' \) such that

\[ z_\mu(s, x) = z_\mu(s'', z(s', x)). \]  

Geometrically this means that the contour \( C(y) \) for a point \( y \) lying somewhere on the contour \( C(x) \), coincides with the corresponding part of \( C(x) \). The gauge condition is defined as

\[ A_\mu^a(x)t^\mu(x) = 0, \]  

where

\[ t_\mu(x) = \frac{\partial z_\mu(s, x)}{\partial s} \bigg|_{s=1}. \]  

Using this gauge, one can express the gluon field \( A_\mu^a(x) \) in terms of the field \( F_{\mu\nu}^a \):

\[ A_\mu^a(x) = \int_0^1 \frac{d}{ds} \left( A_\beta^a(z) \frac{\partial z^\beta(s, x)}{\partial x^\mu} \right) ds, \]

\[ = \int_0^1 \left( \partial_\lambda A_\beta^a(z) \frac{\partial z^\lambda(s, x)}{\partial s} \frac{\partial z^\beta(s, x)}{\partial x^\mu} + A_\beta^a(z) \frac{\partial}{\partial s} \frac{\partial z^\beta(s, x)}{\partial x^\mu} \right) ds, \]

\(^1\)This gauge is a generalization of a radial gauge, \( (x-x_0)_\mu A_\mu^a = 0 \). It is known that the so-called ‘ghost’ fields decouple when a radial gauge is used [40, 47]. In case the straight lines \( (x-x_0) \) in the radial gauge are replaced by arbitrary contours \( C(x) \), we expect that the ghost fields influence the properties of the resulting QCD-string when it is treated dynamically. However, in this thesis the QCD-string is considered statically and thus the ghost fields have been neglected.
\[\int_0^1 \left( \partial_\lambda A_\beta^a(z) \frac{\partial z^\lambda(s, x)}{\partial s} \frac{\partial z^\beta(s, x)}{\partial x^\mu} - \frac{\partial A_\beta^a(z)}{\partial s} \frac{\partial z^\beta(s, x)}{\partial x^\mu} \right) ds,\]

\[= \int_0^1 F_\lambda^{a\beta}(z) \frac{\partial z^\beta(s, x)}{\partial x^\mu} \frac{\partial z^\lambda(s, x)}{\partial s} ds. \tag{2.9}\]

More details on this gauge can be found in Ref. [60].

Now one can integrate out the gluonic field \( A_\mu^a(x) \), and introduce an arbitrary integration over the set of contours \( C(x) \) with weight \( D_\kappa(C) \). Since \( Z \) is gauge invariant it does not depend on the contour \( C(x) \). One obtains

\[Z = \int D_\kappa(C) D\psi D\bar{\psi} \exp \left[ L_1 + L_{\text{eff}} \right], \tag{2.10}\]

where the effective quark Lagrangian \( L_{\text{eff}} \) is defined as

\[\exp(L_{\text{eff}}) = \int DA \exp \left[ \int \left( \bar{\psi} \gamma^\mu A_\mu(x) \psi f(x) - \frac{1}{4} (F_{\mu\nu}(x))^2 \right) \right] d^4x \]

\[= \left\langle \exp \int \bar{\psi} f^\dagger(x) g_\gamma^\mu A_\mu(x) \psi f(x) d^4x \right\rangle_A. \tag{2.11}\]

When the quark fields are treated statically the right-hand side of Eq. (2.11) reduces to the Wilson loop [85]. The integration over \( DA \) in Eq. (2.11) is the gluon vacuum averaging process, and is denoted by \( \langle \ldots \rangle_A \). To study the averaging of the gluonic field configurations, we adopt the correlator method, based on the series expansion of the exponential. Using the cluster expansion [22, 23, 62], \( L_{\text{eff}} \) can be written as an infinite sum containing averages \( \langle (A^\mu)^k \rangle_A \),

\[\left\langle \exp \int \psi_k^\dagger(x) g_\gamma^\mu A_\mu^k(x) \psi f(x) d^4x \right\rangle_A = \]

\[\exp \left[ \int \psi_k^\dagger(x) g_\gamma^\mu \left\langle A_\mu^k(x) \right\rangle_A \psi f(x) d^4x \right. + \frac{1}{2} \int \psi_k^\dagger(x) g_\gamma^\mu \psi f(x) \left\langle A_\mu^k(x) A_\nu^{k'}(y) \right\rangle_A \psi f(y) g_\gamma^\nu \psi f(y) d^4xd^4y + \ldots \right]. \tag{2.12}\]

Due to the gauge and Lorentz invariance of the vacuum the first term \( \langle (A^\mu)^2 \rangle \) vanishes. At this point the Gaussian approximation can be exploited. This means that from the higher-order correlators, only the bilocal gluon correlator \( \langle (A^\mu)^2 \rangle \) is kept. All other correlators of higher orders which are indicated by \( \ldots \) are neglected. Numerical accuracy of this approximation was discussed and tested in Refs. [61, 70]. One expects that for static quarks, corrections to the Gaussian approximation amount to less than 2-3%.

Using Eq. (2.9), the bilocal correlator can be expressed as

\[\left\langle A_\mu^k(x) A_\nu^{k'}(y) \right\rangle_A = \int_{C(x)} \frac{\partial u_\omega}{\partial x^\mu} du_\varepsilon \int_{C(y)} \frac{\partial v_\omega'}{\partial y^\nu} dv_{\varepsilon'} \left\langle F_{\varepsilon\omega}(u) F_{\varepsilon'\omega'}(v) \right\rangle_A \lambda_\mu^a \lambda_\nu^b, \tag{2.13}\]

QCD string model
where $\lambda_{kl}^a$ are the generators of the $SU(3)$-color group. Since the vacuum is color neutral, the bilocal correlator has to be proportional to a colorless object [21,66,71],

$$
\left\langle F_{\varepsilon \omega}^a(u) F_{\varepsilon' \omega'}^{a'}(v) \right\rangle_A = \frac{\delta_{ab}}{N_c^2 - 1} \left\langle F_{\varepsilon \omega}^c(u) F_{\varepsilon' \omega'}^{c'}(v) \right\rangle_A 
= \frac{1}{2} \frac{\delta_{ab}}{N_c^2 - 1} \text{tr} \left\langle F_{\varepsilon \omega}^a(u) F_{\varepsilon' \omega'}^{a'}(v) \right\rangle_A ,
$$

(2.14)

where $N_c$ is the number of colors. The explicit form of $\left\langle F_{\varepsilon \omega}^a(x) F_{\varepsilon' \omega'}^{a'}(y) \right\rangle_A$ is specified later. The product of $SU(3)$ generators in Eq. (2.13) can be rewritten using

$$
\lambda_{kl}^a \lambda_{k'l'}^{a'} = 2 \left( \delta_{kl} \delta_{k'l'} - \frac{1}{N_c} \delta_{kl} \delta_{k'l'} \right) .
$$

(2.15)

The resulting effective Lagrangian is quartic in $\psi$:

$$
L_{(4)}^{\text{eff}} = \frac{1}{2} \int d^4 x \, d^4 y \left( \psi_{\alpha_1}^f \psi_{\beta_1}^f \psi_{\gamma_2}^g \psi_{\delta_2}^g \right) J_{\alpha_1 \beta_1 ; \gamma_2 \delta_2}(x,y) + O(\psi^6) ,
$$

(2.16)

where

$$
J_{\alpha \beta, \gamma \delta}(x,y) = (\gamma_{\mu})_{\alpha \beta} (\gamma_{\nu})_{\gamma \delta} J_{\mu \nu}(x,y) .
$$

(2.17)

Using Eqs. (2.13) and (2.14), $J_{\mu \nu}$ can be expressed as

$$
J_{\mu \nu}(x,y) = g^2 \int_{C(x)} \frac{du_\omega^2}{dx_\mu} \int_{C(y)} \frac{d\nu_{\omega'}}{dy_\nu} \frac{1}{N_c^2 - 1} \text{tr} \left\langle F_{\varepsilon \omega}^a(u) F_{\varepsilon' \omega'}^{a'}(v) \right\rangle_A .
$$

(2.18)

The effective Lagrangian $L_{(4)}^{\text{eff}}$, Eq. (2.16), is written in the contour gauge [60]. It can be identically rewritten in a gauge–invariant form by the use of parallel transporters,

$$
\Phi(x,y) = P \exp \left[ ig \int_x^y A_\mu(z) dz^\mu \right] ,
$$

(2.19)

where the operator $P$ takes care of the path ordering. These parallel transporters perform a gauge transformation which turn the field $A_\mu$ defined in the modified Fock–Schwinger gauge into some arbitrary gauge

$$
A'_\mu(x) = \Phi^\dagger(x,x_0) A_\mu(x) \Phi(x,x_0) + i g \Phi^\dagger(x,x_0) \partial_\mu \Phi(x,x_0) .
$$

(2.20)

The parallel transporters are identically equal to unity in the gauge Eq. (2.7). If $\psi(x)$ and $\psi(y)$ in Eq. (2.16) are multiplied by $\Phi(x,x_0)$ and $\Phi(y,x_0)$, respectively, and $F(u)$ in Eq. (2.18) is replaced by $\Phi(x,u) F(u) \Phi(u,x_0)$ and similarly for $F(v)$, the effective Lagrangian $L_{\text{eff}}$ becomes gauge invariant. However, if only the quartic term is kept
QCD string model

(Eq. (2.16)), and all terms of higher degree are neglected, it becomes in general contour dependent.

A similar problem occurs in the cluster expansion of the Wilson loop, when one keeps only lowest-order correlators, leading to the (incorrect) surface dependence of the result. The situation is the same as with a sum of QCD perturbation series, which depends on the normalization scale $\mu$ for any finite number of terms in the expansion. This unphysical dependence is usually treated by fixing $\mu$ at some physically reasonable value $\mu_0$ (of the order of the inverse size of the system).

The integration over contours $D_κ(C)$ in Eq. (2.10) resolves this difficulty in a similar way. Namely, the partition function $Z$ formally does not depend on contours (since it is integrated over a set of contours) but depends on the weight $D_κ(C)$. We choose this weight in such a way, that the contours generate the string of minimal length between the quarks. Thus the physical choice of the contour corresponds to the minimization of the meson (baryon) mass over the class of strings, in the same way as the choice of $\mu = \mu_0$ corresponds to the minimization of the higher perturbative terms that are neglected. As a practical outcome, we shall keep the integral $D_κ(C)$ till the end and finally use it to minimize the string between the quarks.

Up to this point only one approximation has been made: neglecting all field correlators except the Gaussian one. Recent lattice calculations (see Refs. [3,19,64]) estimate the accuracy of this approximation at the level of few percent. Now one must make another approximation. It was shown in Ref. [66], that the colorless product in Eq. (2.16)

$$\psi^f_{l\beta}(x)\psi^g_{k\gamma} (y) = \text{tr} \left[ \psi^f_{\beta}(x)\Phi(x,x_0)\Phi(x_0,y)\psi^g_{\gamma} (y) \right]$$

can be replaced by the quark Green’s function. Thus we assume

$$\psi^f_{l\beta}(x)\psi^g_{k\gamma} (y) \rightarrow \delta_{fg}\delta_{lk}S_{\beta\gamma}(x,y). \quad (2.21)$$

After replacing different combinations of $\psi^f_{l\beta}(x)$ and $\psi^g_{k\gamma} (y)$ in Eq. (2.16) by the quark Green’s function, $L_{\text{eff}}^{(4)}$ takes the form

$$L_{\text{eff}}^{(4)} = -i \int d^4x d^4y \, \psi^f_{k\alpha}(x)M_{\alpha\delta}^f(x,y)\psi^f_{k\delta}(y), \quad (2.22)$$

where the quark mass operator is

$$M_{\alpha\delta}^f(x,y) = - \left( N_c - \frac{1}{N_c} \right) J_{\mu\nu}(x,y)(\gamma^\mu S^f(x,y)\gamma^\nu)_{\alpha\delta}. \quad (2.23)$$

From Eq. (2.22), it is evident that $S^f$ satisfies

$$(-i \, \partial_x - im_f)S^f(x,y) = \int M^f(x,z)d^4z S^f(z,y) = \delta^{(4)}(x-y). \quad (2.24)$$

Note that assuming the large $N_c$-limit from the start by neglecting $1/N_c$-terms in Eqs. (2.13) and (2.14), leads to exactly the same result, Eq. (2.24). Eqs. (2.23) and (2.24) were first derived in the large $N_c$-limit in Ref. [66]. From Eqs. (2.10) and (2.22),
one should expect that the $q\bar{q}$ and $3q$ dynamics are expressed through the quark mass operator Eq. (2.23), which should contain both confinement and CSB. Indeed, the analysis performed in Refs. [66, 73, 74] for a light quark in the field of a heavy antiquark reveals that confinement is present in the long-distance form of $M(x, y)$, when both distances $|x|, |y|$ of the light quark from the heavy anti-quark (placed at the origin) are large.

We shall now make several simplifying assumptions, to clarify the structure of $M(x, y)$. First of all we take the class of contours $C$ going from any point $x = (x, x_4)$ to the chosen point $x_0 = (r^{(0)}, x_4)$ and then to $(r^{(0)}, -\infty)$ along the $x_4$-axis. For this class, the corresponding gauge was studied in Ref. [4]. Secondly, we take the dominant part of $J_{\mu\nu}$ in Eq. (2.23), namely $J_{44}$, which is proportional to the correlator of color-electric fields. This yields a linear confining interaction, while the other components $J_{ik}, J_{i4}, J_{4i}, i = 1, 2, 3$ have been neglected, containing magnetic fields and yielding momentum-dependent corrections. The contribution of the magnetic correlators is studied in Ref. [74] where a more detailed analysis is performed.

The correlator $\langle F_{\epsilon\omega}(u)F_{\epsilon'\omega'}(v)\rangle$ in Eq. (2.18) can be expressed through the scalar correlator $D(x)$, defined as [22, 23],

$$\frac{g^2}{N_c} \text{tr} \left( F_{\alpha\beta}(u)F_{\gamma\delta}(v) \right)_A = D(u - v)(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}) + O(D_1). \tag{2.25}$$

The correlator $D_1$ is a full derivative, which only contains short distance behavior. Therefore it does not contribute to confinement, and is neglected. The expression can again be made gauge independent by inserting appropriate parallel transporters $\Phi(x, y)$, as explained above. The coefficient $\sigma$ in the area law of the Wilson loop, $\langle W(C) \rangle = \exp(-\sigma \text{area})$ is related to the correlator $D(x)$ as [23, 67],

$$\sigma = \frac{1}{2} \int \int d^2 u D(u), \tag{2.26}$$

where the integral is two-dimensional in, for example, the $(u_1, u_4)$-plane. The parameter $\sigma$ represents the string tension. To simplify the calculations, a Gaussian form of $D(x)$ is assumed. It can be written as [73],

$$D(u) = D(0) \exp \left( -\frac{u^2}{4T_g^2} \right), \quad D(0) = \frac{\sigma}{2\pi T_g^2}, \tag{2.27}$$

where $T_g$ is the correlation time, characterizing the time scale of correlations in the fluctuations of the gluon background field. It has been studied in lattice gauge simulations [19] and found to be of the order of $\frac{1}{4} fm$. Following Ref. [73], we have adopted a value $T_g = 0.24 fm$.

Following Refs. [73, 74] the quark mass operator $M_f$ can generally be written as,

$$M_{C_{x_4}}^f(x, y) = M_f^{(0)} I + M_f^{(i)} \tilde{\sigma}_i + M_f^{(4)} \gamma_4 + M_f^{(i)} \gamma_i, \tag{2.28}$$

with

$$\tilde{\sigma}_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}. \tag{2.29}$$
It was shown in Refs. [67, 74] that $M_{f,0}^f$ is the only part of $M^f$ which is increasing. It behaves linearly at large values of $|x|, |y|$ and in the most simple case of a Gaussian form of $D(x)$, Eq. (2.27), it can be written as

$$M_{f,0}^f(x, y) = \frac{1}{2T_g \sqrt{\pi}} \left| \frac{x + y}{\sigma} \right| \exp \left( -\frac{(x_4 - y_4)^2}{4T_g^2} \right) \tilde{\delta}^{(3)}(x - y). \quad (2.30)$$

In the space dimension we assume for $\tilde{\delta}$ in Eq. (2.30) a smeared $\delta$-function, which can be represented as [73, 74]

$$\tilde{\delta}^{(3)}(x - y) \approx \exp \left( -\frac{|x - y|^2}{b^2} \right) \left( \frac{1}{b \sqrt{\pi}} \right)^3, \quad b \sim 2T_g. \quad (2.31)$$

Again $T_g$ is the gluon correlation length. Note that the mass operator $M_{f,0}^f$ is of a Lorentz-scalar type. In Chapters 4 and 5, we also allow for a constant term in the confining interaction, corresponding to the next-to-leading order corrections to the area law. For asymptotically large $|x|$, it has been shown that the kernel Eq. (2.30) leads for a fixed $r^{(0)}$ to a linear confining interaction $\sigma|\mathbf{r}^{(i)} - \mathbf{r}^{(0)}|$ [73, 74]. We are now in the position to derive the $3q$ Green’s function, which will be done in the next section.

### 2.2 Equations for the baryon Green’s function

Equations for both the $3q$ and the $q\bar{q}$ systems can similarly be written as was shown in Ref. [68]. In this section, only the derivation of the $3q$ Green’s function is given.

We assume the large $N_c$-limit in the sense, that $1/N_c$ corrections from $q\bar{q}$ pairs to the quark Green’s function and the effective mass can be neglected. We now write down the explicit expressions for $N_c = 3$. The initial and final field operators are

$$\Psi_{in}(x, y, z) = e_{k_1k_2k_3} \Gamma^{\alpha\beta\gamma} \psi_{k_1\alpha}(x, C(x)) \psi_{k_2\beta}(y, C(y)) \psi_{k_3\gamma}(z, C(z)) \quad (2.32)$$

with the notations: $k_i$ are color indices, $\alpha, \beta, \gamma$ are Lorentz bispinor indices. The tensor $e_{k_1k_2k_3}$ is a totally antisymmetric tensor which is added to make the field operator $\Psi_{in}$ color neutral. The transported quark operators are

$$\psi_{k\alpha}(x, C(x)) = P \exp \left[ ig \int_x^{\bar{x}} dz^\mu A_\mu(z) \right]_{kl} \psi_{l\alpha}(\bar{x}),$$

$$= (\Phi_C(x, \bar{x}) \psi_{\alpha}(\bar{x}))_k. \quad (2.33)$$

The contour $C(x)$ from $x$ to $\bar{x}$ in $\Phi_C$ can be arbitrary, but it is convenient to choose it in the same class of contours that is used in $D_K(C)$ and in the generalized Fock–Schwinger gauge [60]. Thus we set $\bar{x}$ equal to $x_0$. $\Gamma^{\alpha\beta\gamma}$ is the Lorentz spinor tensor securing proper baryon quantum numbers. One can also choose other operators, but it does not influence the resulting equations. In Eq. (2.32) we have omitted flavor indices in $\Gamma$ and $\psi(x, C)$, to be easily reinstated in final expressions.
Using the effective Lagrangian Eq. (2.22), we obtain for the 3q Green’s function,

\[ G^{(3q)}(x,y,z|\mathbf{x}',\mathbf{y}',\mathbf{z}') = \frac{1}{N} \int D\kappa(C) D\psi D\psi^\dagger S_{\text{fin}}(\mathbf{x}',\mathbf{y}',\mathbf{z}') \Psi_{\text{fin}}(x,y,z) \exp(L_1 + L_{\text{eff}}). \]  

Integrating out the quark degrees of freedom and neglecting the determinant at large \( N_c \) in the sense that virtual quark anti-quark pairs are suppressed, one has

\[ G^{(3q)} = \int D\kappa(C)(e\Gamma)(e'\Gamma') \{ S(x,x')S(y,y')S(z,z') + \text{perm.} \}, \] 

where for simplicity color and bispinor indices are suppressed together with parallel transporters in initial and final states. One can also define unprojected (without \( \Gamma, \Gamma' \)) 3q Green’s function \( G^{(3q)}_{\text{un}} \) with three initial and three final bispinor indices instead of projected by \( \Gamma, \Gamma' \) quantum numbers of the baryon.

The set of contours \( C(x) \) in Eq. (2.35) should be chosen to yield a stationary point of the action Eq. (2.10). For the particular case of small correlation time \( T_g \), i.e. taking \( x_4 = y_4 = z_4 \), we may assume that this can be achieved by a single choice of contours passing through the point \( x_0 = (r^{(0)}, x_4) \) (see Fig. 2.1). This can readily be generalized to the non-instantaneous case.

One can write for \( G^{(3q)}_{\text{un}} \):

\[ (-i\hat{\partial}_x - im_1 - i\hat{M}_1)(-i\hat{\partial}_y - im_2 - i\hat{M}_2)(-i\hat{\partial}_z - im_3 - i\hat{M}_3)G^{(3q)}_{\text{un}} = \delta^{(4)}(x - x')\delta^{(4)}(y - y')\delta^{(4)}(z - z') \]

with e.g. \( \hat{M}_1 G \equiv \int M(x,u)G(u,x')d^4u \). One can simplify the form Eq. (2.36) for \( G^{(3q)} \) taking into account that \( M(x,x') \) actually does not depend on \( (x_4 + x'_4)/2 \). Hence the
interaction kernel of $G^{(3q)}$ does not depend on relative energies, as in Ref. [58]. Similarly to Refs. [58,59], one can introduce the Fourier transform of $G^{(3q)}$ in time components, and take into account energy conservation $E = E_1 + E_2 + E_3$. One obtains

$$G^{(3q)}(E, E_2, E_3) \simeq \int D\kappa(C)(e\Gamma)(e'\Gamma') \frac{1}{(E - E_2 - E_3 - H_1)(E_2 - H_2)(E_3 - H_3)},$$

where we have used the notation

$$H_i = m_i\beta(i) + p(i)\alpha(i) + \beta(i)M(r^{(i)} - r^{(0)}).$$

(2.38)

Moreover, we have taken in $M(x, x')$ the limit of small $T_g$. As in Ref. [58], one can now integrate over $E_2, E_3$. Thus Eq. (2.37) reduces to

$$G^{(3q)}(E, r_1, r_1') \simeq \int D\kappa(C)(e\Gamma)(e'\Gamma') \frac{1}{(E - H_1 - H_2 - H_3)}. \hspace{1cm} (2.39)$$

From Eq. (2.39) one obtains the equation for the $3q$ wave function similar to that of the $q\bar{q}$ system,

$$(H_1 + H_2 + H_3 - E)\psi(r_1, r_2, r_3) = 0. \hspace{1cm} (2.40)$$

Solutions of Eq. (2.40) are discussed in the next section.

### 2.3 Solutions of the baryonic bound state equation

In this section properties and solutions of the nonlinear and nonlocal Dirac-like equation,

$$\sum_{i=1}^{3} \left[ m_i\beta^{(i)} + p^{(i)}\alpha^{(i)} + \beta^{(i)}M (r^{(i)} - r^{(0)}) \right] \Psi = E\Psi, \hspace{1cm} (2.41)$$

as has been obtained in the previous section, are discussed. In the non-relativistic approximation $m_i \gg \sqrt{\sigma}$, and considering large $r^{(i)}$, Eq. (2.41) reduces to

$$\sum_{i=1}^{3} \left[ \frac{(p^{(i)})^2}{2m_i} + \sigma |r^{(i)} - r^{(0)}| \right] \Psi = \varepsilon\Psi, \hspace{0.5cm} \varepsilon = E - \sum m_i. \hspace{1cm} (2.42)$$

For a (arbitrary) given point $r^{(0)}$ the solution of Eq. (2.41) can be factorized. Hence when we treat $r^{(0)}$ as a constant parameter, the three-quark wave function is simply expressed in terms of single-quark orbitals. However, in general the point $r^{(0)}$ should be found by minimizing the interaction between the three quarks, yielding for $r^{(0)}$ the so-called Torricelli point. As a result, we obtain for the effective string not an additive two-body confining interaction, but a single string Y-junction, which is of a genuine three-body nature. Some formulas on the position of the Torricelli point can be found in Appendix A.
We do not consider in Eq. (2.40) \( r^{(0)} \) to be expressed through three-quark positions, as required by the Torricelli point, but in first approximation take it as a constant parameter. This allows us to have the three-quark solutions in factorized form, leaving the calculation of the dynamical correlations induced by nonfactorizability to a further study.

Due to the treatment of \( r_0 \) as a constant instead of adopting the Torricelli point \( r_T \), the distance \( \sum_{i=1}^{3} |r_i - r_0| \) is about \( \sim 1.5 \) times bigger than the minimal distance \( \sum_{i=1}^{3} |r_i - r_T| \). This means that the string tension \( \sigma \) has to be chosen \( \sim 1.5 \) times smaller to yield about the same energy. Recent lattice simulations on static quarks give a value of about \( \sigma = 0.15 \) (GeV)\(^2\) for the three-quark Y-shaped interaction which is close to the value for quark anti-quark interactions [77]. This result suggests that string tensions as low as \( \sigma = 0.10 \) (GeV)\(^2\) can be used in our calculations.

### 2.3.1 Factorizable solution

In the approximation where \( r^{(0)} \) is considered as a constant, the baryon solution of Eq. (2.41) can be represented as the product of three single particle solutions:

\[
\Psi_{JM} = \Gamma^{\alpha\beta\gamma}_{JM}(f_1 f_2 f_3)e_{k_1 k_2 k_3}^{f_1 f_2 f_3}(r^{(1)} - r^{(0)}) \psi_{k_1 \alpha}^{f_1}(r^{(2)} - r^{(0)}) \psi_{k_2 \beta}^{f_2}(r^{(3)} - r^{(0)}),
\]

where \( k_i \) and \( f_i \) are the color and flavor indices respectively. The orbital and radial excitations are indicated by \( \alpha, \beta \) and \( \gamma \). Because of the Pauli principle, the baryon wave function has to be a totally antisymmetric function of the three quark coordinates. As the baryon is a color white object (antisymmetric in color), the wave function Eq. (2.43) has to be totally symmetric in flavor, orbital and radial excitations of the single particle wave functions \( \psi_{k_i \alpha_i}^{f_i} \). The functions \( e_{k_1 k_2 k_3} \) and \( \Gamma^{\alpha\beta\gamma}_{JM} \) take care of this (anti)-symmetrization. Each single-particle solution satisfies the Dirac-like equation,

\[
H_i \psi_{k_i \alpha_i}^{f_i} = \varepsilon_n^{(i)} \psi_{k_i \alpha_i}^{f_i}, \quad E = \sum_{i=1}^{3} \varepsilon_n^{(i)},
\]

where \( H_i \) is given by Eq. (2.38). From now on the color index will be omitted. The only remnant of color is the requirement that \( \Psi_{JM} \) be symmetric in all coordinates besides color.

The orbital wave function can be decomposed in partial waves in the standard way

\[
\psi_{\alpha}^{f}(\rho) = \frac{1}{\rho} \begin{pmatrix} G_n(\rho) \Omega_{jm} & iF_n(\rho) \Omega_{jm} \\ iF_n(\rho) \Omega_{jm} & G_n(\rho) \Omega_{jm} \end{pmatrix}, \quad \rho = r - r^{(0)},
\]

with \( l' = 2j - l \). The angular part of the orbital wave function is contained in \( \Omega_{jm} \).

The partial wave decomposition yields for Eq. (2.44),

\[
\begin{aligned}
\frac{dF_n}{dr} - \frac{\kappa}{r} F_n + (\varepsilon_n - m) G_n - M_{11} G_n - i M_{12} F_n &= 0, \\
\frac{dG_n}{dr} + \frac{\kappa}{r} G_n - (\varepsilon_n + m) F_n - M_{22} F_n + i M_{12} G_n &= 0,
\end{aligned}
\]
Table 2.1: Ground state energy $\epsilon_0$ in units of MeV of the single particle orbitals (spo) for various values of $\sigma$ in units of (GeV)$^2$ for both the $u, d$-quark and the $s$-quark. For the current quark masses the values $m_u = m_d = 5$ MeV and $m_s = 200$ MeV are adopted.

<table>
<thead>
<tr>
<th>spo</th>
<th>$\sigma = 0.06$</th>
<th>$\sigma = 0.09$</th>
<th>$\sigma = 0.12$</th>
<th>$\sigma = 0.15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0(1/2)^{++}$</td>
<td>243</td>
<td>297</td>
<td>342</td>
<td>380</td>
</tr>
<tr>
<td>$1(1/2)^{++}$</td>
<td>435</td>
<td>535</td>
<td>617</td>
<td>688</td>
</tr>
<tr>
<td>$2(1/2)^{--}$</td>
<td>375</td>
<td>460</td>
<td>531</td>
<td>592</td>
</tr>
<tr>
<td>$0(3/2)^{++}$</td>
<td>339</td>
<td>415</td>
<td>478</td>
<td>533</td>
</tr>
<tr>
<td>$1(3/2)^{++}$</td>
<td>501</td>
<td>616</td>
<td>711</td>
<td>793</td>
</tr>
<tr>
<td>$0(5/2)^{++}$</td>
<td>419</td>
<td>514</td>
<td>593</td>
<td>661</td>
</tr>
</tbody>
</table>

with $\kappa = \pm (j + \frac{1}{2})$ when $j = l \pm \frac{1}{2}$ and $n$ indicates the radial excitation. Eqs. (2.46) have to be solved self consistently, leading to confining solutions. Details can be found in Ref. [73]. In Table 2.1 some values of $\epsilon_n$ are shown. The solutions are listed as

$$n(j)^{\text{sign}(\kappa)\text{sign}(\epsilon_n)}.$$  

From these single-quark orbitals the baryon spectrum $M_B = E$ and the corresponding baryon wave functions, Eq. (2.43), can immediately be constructed.

### 2.3.2 Translationally symmetric Ansatz

The baryon wave function, Eq. (2.43), has the undesirable property of not being translationally invariant. As a consequence, the center of mass motion can give unwanted extra contributions to the properties of the baryons which are calculated in later chapters.

To remove the center of mass motion, one can average out the position of the center of mass $\mathbf{R} = (\mathbf{r}^{(1)} + \mathbf{r}^{(2)} + \mathbf{r}^{(3)})/3$ by integrating over $\mathbf{R}$ as,

$$\hat{\Psi}_{JM} = \int \Psi_{JM} d^3R = \Gamma_{JM}^{\alpha\beta\gamma}(f_1f_2f_3)\epsilon_{k_1k_2k_3} \int \psi_{k_1\alpha}^{f_1}(\mathbf{r}^{(1)} - \mathbf{r}^{(0)})\psi_{k_2\beta}^{f_2}(\mathbf{r}^{(2)} - \mathbf{r}^{(0)})\psi_{k_3\gamma}^{f_3}(\mathbf{r}^{(3)} - \mathbf{r}^{(0)})d^3R,$$

(2.48)
where the translationally invariant baryon wave function is indicated by $\hat{\Psi}_{JM}$. By applying Fourier transforms this equation can easily be rewritten in momentum space:

$$\tilde{\Psi}_{JM} = \Gamma_{JM}^{\alpha\beta\gamma} (f_1 f_2 f_3) e_{k_1 k_2 k_3} \psi_{k_1 \alpha} (k_1) \psi_{k_2 \beta} (k_2) \psi_{k_3 \gamma} (k_3),$$  \hspace{1cm} (2.49)

with the boundary condition $P = k_1 + k_2 + k_3 = 0$. This explicitly removes the total movement of the 3q-system.

The single-particle orbital Eq. (2.45) can be rewritten in momentum space by using the expansion of a plane wave in terms of spherical Bessel functions $j_l$ and spherical Harmonics $Y_{lm}$ [42]:

$$e^{ikr} = 4\pi \sum_{l=0}^{\infty} i^l j_l (kr) \sum_{m=-l}^{l} Y_{lm}(\theta_r, \phi_r) Y_{lm}^{*}(\theta_k, \phi_k).$$  \hspace{1cm} (2.50)

One obtains

$$\tilde{\psi}_f^{\alpha} (k) = \int \psi_f^{\alpha} (\rho) e^{ik\rho} d^3\rho$$

$$= \left( \begin{array}{c} \tilde{g}(k) \Omega_{jl}^{lm} \\ \tilde{f}(k) \Omega_{jl}^{l'm} \end{array} \right) = 4\pi \int \left( \begin{array}{c} (-i)^l j_l (k\rho) g(\rho) \Omega_{jl}^{lm} \\ i(-i)^l j_l (k\rho) f(\rho) \Omega_{jl}^{l'm} \end{array} \right) \rho^2 d\rho,$$  \hspace{1cm} (2.51)

which is substituted into Eq. (2.49). In this way, a translationally symmetric baryon wave function is obtained and a baryon spectrum $M_B = E$.

### 2.4 Perturbative corrections to factorized solutions

The effective Lagrangian and the effective mass operator $M(x, y)$, Eqs. (2.16) and (2.23), do not take into account the perturbative interactions between the quarks in the baryon. In order to include them in the model, we separate the gluonic field $A_\mu$ into a background $B_\mu$ and perturbative parts, $A_\mu = B_\mu + a_\mu$ and use the background field method (see Refs. [1, 20, 33–35]) to integrate in the partition function independently over both parts of $A_\mu$ as was done in Refs. [63, 65, 69].

We shall use the following representation of gauge transformations

$$B_\mu \rightarrow U^\dagger (B_\mu + \frac{i}{g} \partial_\mu) U, \quad a_\mu \rightarrow U^\dagger a_\mu U,$$  \hspace{1cm} (2.52)

and keep for $a_\mu$ the background gauge condition,

$$D_\mu (B) a_\mu = 0, \quad D_\mu (B) = \partial_\mu - ig B_\mu.$$  \hspace{1cm} (2.53)

As a result of the perturbative gluon exchange between different quarks in the baryon there will appear an additional vertex in the effective Lagrangian [73],

$$\Delta L = g^2 \int \psi^\dagger (x) \gamma^f \psi (x) \int \psi^\dagger (y) \gamma^2 \psi (y) \langle a_\mu (x) a_\nu (y) \rangle dxdy.$$  \hspace{1cm} (2.54)
Figure 2.2: A schematic view of the gluon propagating inside the world sheet of the string.

Using Eq. (2.54) and, for simplicity reasons, considering only the color Coulomb interaction and assuming the simplest form of gluon propagator leads to,

\[
\int \langle a_\mu(x) a_\nu(y) \rangle d(x_4 - y_4) = \frac{\delta_{\mu\nu} C_2}{4\pi^2} \int \frac{d(x_4 - y_4)}{(\bar{x} - \bar{y})^2 + (x_4 - y_4)^2} = \frac{\delta_{\mu\nu} C_2}{4\pi|x - y|}. \tag{2.55}
\]

where as a first approximation the influence of the background field has been neglected. When the background is taken into account, one arrives at the picture of the gluon \(a_\mu\) propagating inside the film – the world sheet of the string, created by the background between three-quark world lines and the string junction, as shown in Fig. 2.2. Depending on the choice of \(r^{(0)}\), we will in general get an effective interaction of a two-body or three-body nature. Due to the presence of the QCD background, the strength of the resulting Coulomb interaction is expected to be different from the perturbative OGE contribution and as a result different from the interaction used for example in the Breit equation [12, 13].

Due to its attractive nature the color Coulomb contribution leads to smaller baryon masses and giving rise to composite systems with smaller radii. As a result the magnetic moments become smaller. In the next chapter the influence of these exchanges are neglected. A more involved analysis, where apart from the color Coulomb interaction also the hyperfine interaction and pion exchanges are included, is presented in the Chapter 4.
Chapter 3

Baryon magnetic moments

In this chapter, mainly based upon Refs. [75,81], the magnetic moments of the baryons are calculated. We have obtained reasonable agreement with experiment without introducing constituent quark masses, and with the use of a single parameter, the string tension $\sigma$, as is shown in Section 3.1. This is in contrast with the non-relativistic quark models where constituent quark masses form an essential ingredient.

Following Ref. [71], where the systematic exploitation of the field correlator method shows that meson degrees of freedom can be included in a Lagrangian together with confinement, we assume the presence of virtual pseudoscalar mesons. Higher order contributions to the baryon magnetic moment come from two-body currents like one-pion-exchange currents, and mesonic one-loop corrections which give rise to the anomalous magnetic moment of the quark. In Section 3.2 an estimate of these higher order corrections to the magnetic moment is performed. The chapter ends with some conclusions in Section 3.3.

3.1 Single quark current contribution

3.1.1 Calculation in coordinate space

Since the magnetic moments as well as baryon masses are static quantities, the calculation does not involve large momentum transfers, and one can use for that purpose the baryonic bound state equation, Eq. (2.40). According to the results of Section 2.2, $H_i$ can be represented as

$$H_i = m_i \beta^{(i)} + p^{(i)} \alpha^{(i)} + \beta^{(i)} M^{(i)} (r^{(i)} - r^{(0)}).$$

Solutions of this equation are discussed in Section 2.3. In the following, we shall use only the lowest orbitals (solutions with the lowest eigenvalues) for quarks, $0(1/2)^{++}$, and therefore the orbital excitation indices are everywhere omitted. We also assume that the baryon wave function is written in the factorizable form, Eq. (2.43). To
define the magnetic moment one may introduce an external electromagnetic field $A$, $p^{(i)} \rightarrow p^{(i)} - e_q^{(i)} A$, $A = \frac{1}{2} (H \times r)$, and calculate perturbatively the energy shift,

$$\Delta E = -\mu H.$$ (3.2)

Due to the symmetry of the problem, it suffices to consider only the perturbation of one orbital, say for the first quark,

$$H_1 \rightarrow H_1 + \Delta H_1, \quad \Delta H_1 = -e_q^{(1)} \alpha^{(1)} A.$$ (3.3)

If we denote

$$\psi^{(1)} = \left( \begin{array}{c} \phi^{(1)} \\ \chi^{(1)} \end{array} \right),$$ (3.4)

the energy shift $\langle \Delta H_1 \rangle$ becomes

$$\langle \Delta H_1 \rangle = -e_q^{(1)} \left( \phi^{(1)} \chi^{(1)} \right)^* \left( \begin{array}{cc} 0 & \sigma^{(1)} A \\ \sigma^{(1)} A & 0 \end{array} \right) \left( \begin{array}{c} \phi^{(1)} \\ \chi^{(1)} \end{array} \right),$$

$$= -e_q^{(1)} \left( \phi^{(1)} \sigma^{(1)} A \chi^{(1)} + \chi^{(1)} \sigma^{(1)} A \phi^{(1)} \right).$$ (3.5)

From Eq. (2.45) we find $\phi = g \Omega_{jlm_j}$ and $\chi = i f \Omega_{jl'm_j}$ with $l' = 2j - l$. Taking into account that $\Omega_{jl'm_j} = - (\sigma n) \Omega_{jlm_j}$, one easily obtains

$$\langle \Delta H_1 \rangle = -\frac{1}{2} e_q^{(1)} \int d^3 r (g^* f + f^* g) r \Omega^*_{jlm_j} \{ (\sigma n) (nH) - \sigma H \} \Omega_{jlm_j}. \quad \text{(3.6)}$$

Eq. (3.6) contains the matrix element $\int d^3 n \Omega^*_{jlm_j} n_i n_k \Omega_{jlm_j}$, which simplifies when $l = 0$, so that $\langle n_i n_k \rangle = \frac{1}{3} \delta_{ik}$. In this case, one obtains for the contribution of the first quark to the magnetic moment operator in spin space, taking into account relation $\langle \Delta H_1 \rangle = \Delta E = -\mu^{(1)} H$,

$$\mu^{(1)} = -\frac{1}{3} e_q^{(1)} \int (g^* (r) f (r) + f^* (r) g (r)) r d^2 r \Omega^*_{jlm_j} \sigma^{(1)} \Omega_{jlm_j}. \quad \text{(3.7)}$$

Thus, for the lowest orbital $j = \frac{1}{2}$, $l = 0$, we find for the magnetic moment operator$^1$,

$$\mu_z \equiv 3 \mu_z^{(1)} = -2 e_q^{(1)} \sigma_z^{(1)} \int g^* (r) f (r) r dr,$$ (3.8)

where the superscript 1 denotes the contribution of the first quark to the magnetic moment. Due to the symmetry of the wave function under the exchange of two quarks,

$^1$In the special case of a local scalar potential $U(r)$, one can further express $f(r)$ through $g(r)$ using the Dirac equation for the one-quark state

$$rf(r) = \frac{1}{\epsilon + m + U(r)} \left( \frac{d}{dr} (rg(r)) + \frac{\kappa}{r} rg(r) \right).$$
the contribution of the second and third quark to the magnetic moment can be included by the factor three in Eq. (3.8). The normalization condition is,

\[
\int (|g|^2 + |f|^2) r^2 dr = 1. \quad (3.9)
\]

Note that everywhere we put \( r^{(1)} - r^{(0)} = r \).

The magnetic moment operator Eq. (3.8) must be evaluated on the 3q wave function. The fully symmetric spin-isospin wave function for the nucleon \( \psi^{p,n}_{sym} \) is easily constructed when the quarks are all in their ground state, which is the case considered in this chapter. For the proton with total spin up, one finds

\[
\Psi_{sym}^p = N' \left\{ \frac{2}{3} \left[ u^+(1) d^-(2) + u^-(1) d^+(2) \right] u^+(3) - \frac{1}{3} \left[ u^+(1) u^-(2) + u^-(1) u^+(2) \right] d^+(3) - \frac{1}{3} \left[ u^+(1) d^+(2) + d^+(1) u^+(2) \right] u^-(3) + \frac{2}{3} u^+(1) u^+(2) d^-(3) \right\}, \quad (3.10)
\]

where \( N' = 1/\sqrt{2} \), and subscripts \((\pm)\) refer to the spin projection. The fully symmetric spin-isospin wave function for the neutron can be found by the replacement \( u \leftrightarrow d \). The explicit formula is given in Appendix B. The matrix elements are easily computed

\[
\left\langle \Psi_{sym}^p \left| e_q^{(1)} \sigma_z^{(1)} \right| \Psi_{sym}^p \right\rangle = \frac{1}{3} e, \quad (3.11)
\]

\[
\left\langle \Psi_{sym}^n \left| e_q^{(1)} \sigma_z^{(1)} \right| \Psi_{sym}^n \right\rangle = \frac{2}{9} e, \quad (3.12)
\]

where \( e \) is the charge of the proton. From Eqs. (3.11)-(3.12), one immediately gets the famous relation \[5\],

\[
\frac{\mu(n)}{\mu(p)} = - \frac{2}{3}, \quad (3.13)
\]

which is one of early successes of the quark model. For identical orbitals, the magnetic moment can be written as a product

\[
\mu_B = 3 \left\langle \Psi_{sym} \left| e_q^{(1)} \sigma_z^{(1)} \right| \Psi_{sym} \right\rangle \lambda, \quad (3.14)
\]

Substituting this equation into Eq. (3.7) and integrating by parts, one obtains

\[
\mu_z^{(i)} = \frac{e_q^{(1)} \sigma_z^{(1)}}{3} \int \frac{|g(r)|^2}{(\varepsilon + m + U)^2} (3(\varepsilon + m + U) - r U'(r)) r^2 dr.
\]

For \( U(r) = \sigma r \) one can express \( \mu_z^{(i)} \) through \( g(r) \) only as,

\[
\mu_z^{(i)} = \frac{e_q^{(1)} \sigma_z^{(1)}}{3} \int_0^\infty \frac{|g(r)|^2 r^2 (2\sigma r + 3\varepsilon)}{(\varepsilon + \sigma r)^2} dr.
\]
Table 3.1: Ground state energy $\epsilon_0$ of the orbitals and the predicted magnetic moments of the nucleons in units of the nuclear magneton for various values of $\sigma$. Results from the calculation in coordinate space, Eq. (3.14), and momentum space, Eq. (3.20), are both given. The experimental values are also listed.

<table>
<thead>
<tr>
<th>$\sigma$ (GeV)$^2$</th>
<th>$\epsilon_0(u,d)$ (MeV)</th>
<th>$\epsilon_0(s)$ (MeV)</th>
<th>$\mu_{\text{proton}}$</th>
<th>$\mu_{\text{neutron}}$</th>
<th>$\mu_{\text{proton}}$</th>
<th>$\mu_{\text{neutron}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>297</td>
<td>440</td>
<td>2.81</td>
<td>-1.87</td>
<td>2.78</td>
<td>-1.85</td>
</tr>
<tr>
<td>0.12</td>
<td>342</td>
<td>482</td>
<td>2.44</td>
<td>-1.63</td>
<td>2.41</td>
<td>-1.61</td>
</tr>
<tr>
<td>0.15</td>
<td>380</td>
<td>520</td>
<td>2.20</td>
<td>-1.46</td>
<td>2.16</td>
<td>-1.44</td>
</tr>
<tr>
<td>experiment</td>
<td></td>
<td></td>
<td>2.79</td>
<td>-1.91</td>
<td>2.79</td>
<td>-1.91</td>
</tr>
</tbody>
</table>

where

$$\lambda \equiv -\frac{2}{3} \int g^*(r)f(r)r^3 \, dr .$$

(3.15)

For the single-quark orbitals we have taken the ground state solution of the Dyson-Schwinger-Dirac equation with a nonlocal kernel from Eq. (2.46). It is clear that inclusion of higher orbitals will change the magnetic moment of proton and neutron, similarly to the case of tritium and $^3$He, where the admixture of the orbital momentum $L = 2$ changes the magnetic moment by 7-8%. These components appear in the wave function due to mixing through the tensor and spin-orbit forces between quarks. However, in the case considered in this chapter, all three quarks contribute equally to the orbital momentum. Therefore these components do not contribute at this stage. Contributions of inclusion of excited orbitals are studied in Section 5.2, where Eqs. (3.14)-(3.15) are generalized when the quarks have different orbital wave functions.

Using ground state orbitals $0(1/2)^+$ for each quark we calculate the nucleon magnetic moment for various values of the string tension $\sigma$. The results are listed in Table 3.1. From the table we see that the predictions depend sensitively on the string tension $\sigma$. Increasing the value of $\sigma$ leads to a larger ground state energy of the orbitals and smaller size of the magnetic moment. This is in accordance with the analysis, where the small component of the orbital is treated perturbatively. Similarly, the presence of the Coulomb interaction yields a lower ground state energy of the orbital, resulting in a larger value in magnitude of the magnetic moment. Close agreement with the experimental values of the magnetic moment is found when $\sigma = 0.09$ (GeV)$^2$. In this case the mass of the nucleon is predicted to be 891 MeV. It is gratifying to see, that the magnetic moments are reasonable in the regime where also the predicted mass of the nucleon is close to the experimental value.

The magnetic moments of the other baryons from the lowest baryon octet and decuplet representation of the $SU(3)$-flavor group can also be calculated. The explicit
Table 3.2: The magnetic moment of the baryons in units of the nuclear magneton for various values of $\sigma$. Calculations in coordinate space, Eq. (3.14), and experimental results.

<table>
<thead>
<tr>
<th>B</th>
<th>$\mu_B$ for $\sigma = 0.09$ (GeV)$^2$</th>
<th>$\mu_B$ for $\sigma = 0.12$ (GeV)$^2$</th>
<th>$\mu_B$ for $\sigma = 0.15$ (GeV)$^2$</th>
<th>exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>2.81</td>
<td>2.44</td>
<td>2.20</td>
<td>2.79</td>
</tr>
<tr>
<td>$n$</td>
<td>-1.87</td>
<td>-1.63</td>
<td>-1.46</td>
<td>-1.91</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>-0.66</td>
<td>-0.60</td>
<td>-0.56</td>
<td>-0.61</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>2.72</td>
<td>2.37</td>
<td>2.14</td>
<td>2.46</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>0.85</td>
<td>0.74</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>-1.03</td>
<td>-0.89</td>
<td>-0.79</td>
<td>-1.16</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>-1.51</td>
<td>-1.34</td>
<td>-1.23</td>
<td>-1.25</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>-0.57</td>
<td>-0.53</td>
<td>-0.50</td>
<td>-0.65</td>
</tr>
<tr>
<td>$\Delta^{++}$</td>
<td>5.62</td>
<td>4.89</td>
<td>4.39</td>
<td>4.52</td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>2.81</td>
<td>2.44</td>
<td>2.20</td>
<td></td>
</tr>
<tr>
<td>$\Delta^0$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>$\Delta^-$</td>
<td>-2.81</td>
<td>-2.44</td>
<td>-2.20</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^{++}$</td>
<td>3.09</td>
<td>2.66</td>
<td>2.37</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^{0*}$</td>
<td>0.27</td>
<td>0.21</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^{-*}$</td>
<td>-2.54</td>
<td>-2.23</td>
<td>-2.02</td>
<td></td>
</tr>
<tr>
<td>$\Xi^{0*}$</td>
<td>0.55</td>
<td>0.43</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>$\Xi^{-*}$</td>
<td>-2.26</td>
<td>-2.02</td>
<td>-1.84</td>
<td></td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>-1.99</td>
<td>-1.80</td>
<td>-1.67</td>
<td>-2.02</td>
</tr>
</tbody>
</table>

forms of $\Psi_{sym}$ for these baryons are given in Appendix B. Note, that due to the strange quark mass their orbitals are different from those of $u$ and $d$ quarks, and therefore the decomposition Eq. (3.14) has to be modified. Some useful formulas can be found in Appendix B.

The resulting values for baryon magnetic moments are given in Table 3.2, where they are compared with experimental values. Considering the case of $\sigma = 0.12$ (GeV)$^2$ we see, that there is a rather close agreement with the experimental values for magnetic moments, where the largest deviations are found for the nucleon and $\Sigma^-$. As discussed for the case of the nucleon improvement of the predicted mass of the composite system also leads to magnetic moments closer to the experimental values. This also applies to the case of the $\Delta$-isobar. Hence we may hope that the inclusion of the Coulomb and hyperfine-splitting interaction will improve the predictions, effects which are calculated
in Chapter 5. Moreover, pionic effects are expected to be present. As a result, significant mesonic current contributions to the magnetic moments may occur. In Section 3.2 we study the dominant corrections from the pion to the one- and two-body current.

3.1.2 Calculation in momentum space

We now turn to the calculation of the baryon magnetic moment using translationally invariant baryon wave functions. The Ansatz from Subsection 2.3.2 is used for this purpose. Since the Ansatz for the baryon wave function is most easily written down in momentum space, Eq. (2.49), the magnetic moment expression has to be rederived in momentum space.

Our starting point is the electromagnetic current matrix element:

$$M_\mu = \langle \Psi | J_\mu(Q) | \Psi \rangle,$$

(3.16)

where $\Psi$ is the 3-quark wave function and $Q$ is the photon momentum. For the wave function normalization Eq. (3.9) for the single particle orbitals is chosen. Due the symmetry of the baryon wave function under the permutation of any two quarks, it is sufficient to consider only the single quark current operator acting on the first quark, and multiply by a factor of three. We therefore define for the current operator:

$$J^{\gamma qq}_\mu = 3 J^{\gamma qq}_\mu(1) = 3 e^{(1)}_q \gamma^{(1)}_\mu \prod_{i=2}^{3} \gamma^{(i)}_0.$$

(3.17)

This choice has the nice property that the zeroth component of the current at $Q = 0$ gives the correct charge of the 3-quark system:

$$M_0 = \langle \Psi | J_0(Q = 0) | \Psi \rangle = \sum_{i=1}^{3} e^{(i)}_q.$$

(3.18)

The result for the magnetic moment, obtained in the previous section can readily be recovered from our single quark current matrix element. Following Ref. [44], the magnetic moment can be calculated by taking the curl of the space component of the current matrix element in the Breit system. In doing so, the magnetic moment can be deduced from the electromagnetic current as

$$\mu_z = \frac{e}{2M_p} G_{mag}(Q = 0) = -\frac{i}{2} [\nabla_{Q} \times \mathbf{M}]_{z}(Q = 0),$$

(3.19)

where $M_p$ is the proton mass, $e$ the proton charge and $G_{mag}$ is the Sachs magnetic form factor [57]. The matrix element Eq. (3.19) can be easily evaluated in momentum space. Introducing the Fourier transform of the wave function of the single quark orbital as was done in Eq. (2.51), we may after some algebra reduce Eq. (3.19) in momentum space to an expression similar to Eq. (3.14),

$$\mu_z = 3 \mu_z^{(1)} = 3 \left\langle \psi_{sym} \left| e^{(1)}_q \sigma^{(1)}_z \psi_{sym} \right| \lambda \right\rangle,$$

(3.20)
where $\psi_{sym}$ is the fully symmetric spin-isospin wave function of the nucleon as shown explicitly in Appendix B. We thus find,

$$
\tilde{\lambda} = \frac{-1}{2N} \int \int d^3p d^3q \prod_{n=2}^{3} \left( |\tilde{g}(k_n)|^2 + |\tilde{f}(k_n)|^2 \right) \times \left( \frac{4}{3k_1} \tilde{f}(k_1) - \frac{\partial \tilde{g}(k_1)}{\partial k_1} \frac{2}{3} \tilde{f}(k_1) + \tilde{g}(k_1) \frac{2}{3} \frac{\partial \tilde{f}(k_1)}{\partial k_1} \right),
$$

where $N$ is the normalization factor,

$$
N = \int \int d^3p d^3q \prod_{n=1}^{3} \left( |\tilde{g}(k_n)|^2 + |\tilde{f}(k_n)|^2 \right),
$$

and we have used

$$
\langle \Omega_{jlm_j}(\hat{k}_1) \mid (\hat{k}_1)_i \mid \Omega_{jlm_j}(\hat{k}_1) \rangle = \frac{1}{3} \delta_{ij},
$$

which is valid for $l = 0$. The momenta are expressed in terms of the Jacobi coordinates,

$$
k_1 = -\frac{2}{\sqrt{3}} q + \frac{1}{3} P, \quad k'_1 = -\frac{2}{\sqrt{3}} q' + \frac{1}{3} P',
$$

$$
k_2 = p + \frac{1}{\sqrt{3}} q + \frac{1}{3} P, \quad k'_2 = p' + \frac{1}{\sqrt{3}} q' + \frac{1}{3} P',
$$

$$
k_3 = -p + \frac{1}{\sqrt{3}} q + \frac{1}{3} P, \quad k'_3 = -p' + \frac{1}{\sqrt{3}} q' + \frac{1}{3} P'.
$$

Imposing the Breit system, $P + P' = 0$, and momentum conservation gives $P' = -P = Q/2$, $p' = p$ and $\sqrt{3} (q - q') = Q$. The magnetic moment expression (3.14) from the previous subsection is readily recovered when we replace the integration over the Jacobi momenta in Eqs. (3.21-3.22) by $\prod_{n=1}^{3} dk_n$.

Results on the nucleon magnetic moment are shown in Table 3.1. It can be seen that the values deviate only by a small amount from the result calculated in coordinate space, also shown in Table 3.1. The difference is due to the contribution of the center of mass motion to the baryon magnetic moment. This erroneous contribution is absent in the calculation in momentum space which uses translationally invariant $3q$ wave functions, but is present in the calculation in coordinate space. From the results given in Table 3.1, we conclude that the center of mass motion hardly contributes to the magnetic moment. For simplicity, most calculations in this thesis are therefore performed in coordinate space using Eq. (3.14)

### 3.2 Mesonic contributions

In this section we estimate in our single-orbital model, the magnitude of the pionic current corrections to the magnetic moment of the nucleon. Due to the quark coupling
to effective mesonic degrees of freedom, one and two-body current contributions to the magnetic moments of the baryons arise from the virtual excitations of mesons.

3.2.1 Meson exchange current

Assuming as in Refs. [27, 28, 80] that there exists an effective one meson exchange between quarks in the three-quark system this leads to meson exchange current contributions to the magnetic moment. The leading correction is due to the pion-in-flight and pair term [80]. Effects from the heavier mesons like the $\rho$ are in general less important. These meson exchange currents were first used in few nucleon systems such as $^3\text{H}$ and $^3\text{He}$ [44], and later applied in quark models as was first done in Ref. [7].

The electromagnetic current matrix element is again taken as starting point,

$$ M_\mu = \langle \Psi | J_\mu (Q) | \Psi \rangle, \quad (3.25) $$

where $\Psi$ is the 3-quark wave function and $Q$ is the photon momentum. The pionic two-body current contributions are shown in Fig. 3.1. For the coupling of the pion to the quark, we consider two possible forms: the pseudoscalar (PS) and the pseudovector (PV) coupling. The subsequent pseudoscalar coupling vertex of the pion to the quark can be written as,

$$ \Gamma^{a}_{\pi qq}(k) = g_{\pi qq} \gamma_5 \tau^a F_{\pi qq}(k) \quad (\text{PS}), \quad (3.26) $$

where a monopole form factor $F_{\pi qq}(k) = \Lambda_\pi^2 / (\Lambda_\pi^2 - k^2)$ has been used. The pseudovector coupling is found by applying the replacement

$$ g_{\pi qq} \gamma_5 \tau_a \rightarrow \frac{g_{\pi qq}}{2m_{\text{eff}}} \gamma_5 \frac{k \tau_a}{}, \quad (3.27) $$
leading to

$$\Gamma^a_{\pi qq}(k) = \frac{g_{\pi qq}}{2m_{\text{eff}}} k^a F_{\pi qq}(k^2) \quad \text{(PV)}. \quad (3.28)$$

The effective mass $m_{\text{eff}}$ is a scaling mass which is inserted to make the PV coupling constant dimensionless. In this chapter, it is set equal to the constituent mass of the quark, $\epsilon_0(u,d)$, which are given in Table 3.1. The resulting pion-in-flight, pair-current and contact current operators, shown in Fig. 3.1, are given by:

$$J^{(23)}_{\gamma \pi \pi, \mu} = i\Gamma^a_{\pi qq}(k_3' - k_3) \Delta_{\pi}(k_3' - k_3) \left( 1 - i \frac{F_{\pi qq}(k_3' - k_3)}{\Lambda_{\pi}^2} \frac{\Delta_{\pi}(k_2 - k_2')}{\Delta_{\pi}(k_3 - k_3')} \right), \quad (3.29a)$$

$$J^{(23)}_{\gamma NN, \mu} = i\Gamma^a_{\pi qq}(k_4' - k_4) \Delta_{\pi}(k_4' - k_4) \left( \Gamma^a_{\pi qq}(k_2 - k_2') S_q(k_3 + Q) \Gamma^{a,\text{PV}}_{\gamma \pi qq, \mu} \right), \quad (3.29b)$$

$$J^{(23)}_{\gamma \pi N N, \mu} = i\Gamma^a_{\pi qq}(k_4' - k_4) \Delta_{\pi}(k_4' - k_4) \Gamma^a_{\gamma \pi qq, \mu}(k_2 - k_2'). \quad (3.29c)$$

The last current is only present when a PV coupling is employed. In Eqs. (3.29), $Q$ is the photon momentum. The propagators of the quark and pion are given by [10]

$$S_q(p) = \frac{i(p + M_q)}{p^2 - M_q^2 + i\epsilon}, \quad (3.30)$$

$$\Delta_{\pi}(p) = \frac{i}{p^2 - m_{\pi}^2}. \quad (3.31)$$

The photon-pion vertex is described by an effective interaction Lagrangian

$$\mathcal{L}_{\pi \pi \gamma} = -\frac{1}{2} e A_\mu (\vec{\pi} \times \partial^\mu \vec{\pi})_z + \frac{1}{2} e A_\mu (\partial^\mu \vec{\pi} \times \vec{\pi})_z, \quad (3.32)$$

which results in the following expression for the vertex

$$\Gamma^{ab}_{\gamma \pi \pi, \mu}(k', k) = -e e_{ab3} (k_\mu + k'_\mu). \quad (3.33)$$

For the coupling of the photon to the quark, we use the standard quark-photon vertex:

$$\Gamma_{\gamma qq, \mu} = -ie_q \gamma_\mu. \quad (3.34)$$

It can easily be verified that the charge of the quark and pion depends on the isospin as

$$e_q = \left( \frac{1}{6} + \frac{1}{2} \tau_z \right) e, \quad e_\pi = -ie_{ab3} e, \quad (3.35)$$

where $e = |e|$ is the elementary charge.

The last two terms in the last factor in Eq. (3.29a) correspond to contact terms, which are needed to satisfy current conservation. The pair contribution consists of
four terms where the photon can interact with quark 2 and 3 before and after the pion-quark interaction. In Fig. (3.1), only one term is shown, in Eq. (3.29b) the two terms where the photon couples to quark 3 are written down. The coupling to quark 2 can be easily found by interchanging $2 \leftrightarrow 3$. The quark propagator in Eq. (3.29b) has been replaced by its negative energy part,

$$i \not{p} - m \Rightarrow \frac{i}{2\sqrt{\not{p}^2 + m^2}} \not{p} \gamma - m + \sqrt{\not{p}^2 + m^2} \gamma^0 \approx \frac{i}{4m} (\gamma^0 - 1), \quad (3.36)$$

since the positive energy part has already been included in the single quark current matrix element [17]. In case of a PV coupling of the pion the minimal substitution of the electromagnetic field gives rise to the contact interaction in Eq. (3.29c),

$$\Gamma^\alpha_{\pi qq, \mu}(k) = ie g_{\text{eff}} \gamma_5 \gamma_\mu \tau_b \epsilon_{ba} F_{\pi qq} (k), \quad (PV). \quad (3.37)$$

In Eq. (3.29c) only the photon coupling to quark 3 is written down. The coupling to quark 2 can again be found by interchanging $2 \leftrightarrow 3$.

The interaction is assumed to be instantaneous. This means that the time component in the momenta of the quarks is neglected, $(k_2 - k'_2)^2 \approx -(k_2 - k'_2)^2$. From the 2-body operators $J_{2b}$, Eq. (3.29) we may write down the current matrix element between the 3-quark state

$$M_{2b} = 3M_{2b}^{(1)} = 3 \frac{1}{N} \int d^3p d^3q \bar{\Psi}(1) (23) J_{2b}^{(23)} \Psi, \quad (3.38)$$

where the factor 3 again originates from symmetry considerations of the baryon wave function. Taking the curl of Eq. (3.38), the magnetic moment can be determined. The resulting expressions are given in Appendix C. As a check using the obtained magnetic moment operators we have determined the exchange magnetic moment contribution to the trinucleon system. Our results agree with those obtained by Kloet and Tjon [44].

To get an estimate of the exchange current contributions in the 3-quark case we have used for the couplings and cutoff mass the values from Ref. [80]. They are taken to be $g_{qq\pi}^2/4\pi = 0.67$ and $\Lambda_\pi = 675$ MeV, respectively. The results for the magnetic moments are shown in Table 3.3. Our estimates are in strong disagreement with those obtained in Ref. [80]. The pion-in-flight contribution is substantially smaller than found in Ref. [80] using the chiral constituent model [27]. This may be partially due to the 3-quark wave function used, which has a matter radius smaller than in our case. Moreover, it contains only non-relativistic components.

The calculation using a PV pion-quark coupling yields somewhat smaller values than the calculation with a PS coupling. The two-body contribution gets larger for stronger string tensions $\sigma$, an effect which is stronger when a PS coupling is used. For small values of $\sigma$, the two-body contributions are found to be comparable, leading to an almost cancellation of the pionic current contributions.
Table 3.3: The single quark current contribution $\mu_{N}^{(1)}$ to the magnetic moment in units of the nuclear magneton, together with the two-body corrections and the anomalous correction $\delta\mu_{N}^{(1)}$ arising from the pion one-loop diagrams. Also are shown the total combined prediction of our calculations and the experimental results.

<table>
<thead>
<tr>
<th>N</th>
<th>$\mu_{N}^{(1)}$</th>
<th>$\mu_{N}^{(\pi\gamma)}$</th>
<th>$\mu_{N}^{(NN\gamma)}$</th>
<th>$\mu_{N}^{(NN\pi\gamma)}$</th>
<th>$\delta\mu_{N}^{(1)}$</th>
<th>$\mu_{N}^{tot}$</th>
<th>exp</th>
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<tr>
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<td>0.17</td>
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</table>

$\sigma = 0.09$ (GeV)$^2$, PS coupling

<table>
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<th>$\mu_{N}^{(NN\gamma)}$</th>
<th>$\mu_{N}^{(NN\pi\gamma)}$</th>
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$\sigma = 0.12$ (GeV)$^2$, PS coupling

<table>
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$\sigma = 0.12$ (GeV)$^2$, PV coupling

<table>
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<tr>
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<tr>
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$\sigma = 0.15$ (GeV)$^2$, PS coupling

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$\sigma = 0.15$ (GeV)$^2$, PV coupling

<table>
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<th>$\mu_{N}^{(\pi\gamma)}$</th>
<th>$\mu_{N}^{(NN\gamma)}$</th>
<th>$\mu_{N}^{(NN\pi\gamma)}$</th>
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<td>2.39</td>
<td>2.79</td>
</tr>
<tr>
<td>n</td>
<td>-1.46</td>
<td>-0.15</td>
<td>0.03</td>
<td>0.07</td>
<td>-0.17</td>
<td>-1.68</td>
<td>-1.91</td>
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### 3.2.2 Quark anomalous magnetic moment

The presence of mesonic degrees of freedom will modify the single quark current. The resulting electromagnetic current operator can in general be characterized by a large number of off-shell form factors [9, 78], which reduces to two when we assume that the initial and final quark are on their mass shell. Using this approximation, we may estimate the resulting anomalous magnetic $\kappa$ term due to the mesonic contributions.

Near $Q^2 = 0$ the single quark current can be written as,

$$J_{\mu}^{\gamma qq} = e_q \gamma_{\mu} + \kappa_q \frac{ie}{2M_p} \sigma_{\mu\nu} q^{\nu},$$

(3.39)
where \( \kappa_q = \kappa_s + \kappa_v \tau_z \) for the \( u,d \)-quark. The \( \kappa \) coefficients can be determined in a simple model, assuming that the loop corrections are given by only the one-loop pionic contributions to the electromagnetic vertex. Similarly, as in the two-body current case, we approximate the single quark orbital by free quark propagation with a constituent mass given by the ground state orbital energy. With the above simplifying assumptions, the calculation amounts to calculating the magnetic moment contributions of the diagrams shown in Fig. 3.2. Using the same cutoff mass regularization as for the two-body currents we can write,

\[
J^{(a)}_{\mu} = i \int \frac{d^4k}{(2\pi)^4} \Gamma_{\pi qq}^{a}(k + Q) S_q(p - k) \Delta_\pi(k + Q) \Gamma_{\gamma\pi\pi,\mu}^{ab}(k, k + Q) \\
\times \Delta_\pi(k) \Gamma_{\pi qq}^{b}(-k) \left( 1 - i \frac{F_{\pi qq}((k + Q)^2)}{\Lambda_\pi^2} \frac{1}{\Delta_\pi(k)} - i \frac{F_{\pi qq}((k)^2)}{\Lambda_\pi^2} \frac{1}{\Delta_\pi(k + Q)} \right),
\]

(3.40a)

\[
J^{(b)}_{\mu} = i \int \frac{d^4k}{(2\pi)^4} \Gamma_{\pi qq}^{a}(k) S_q(p + Q - k) \Gamma_{\gamma qq,\mu} \Delta_\pi(k) \Gamma_{\pi qq}^{a}(-k),
\]

(3.40b)

\[
J^{(c)}_{\mu} = i \int \frac{d^4k}{(2\pi)^4} \left( \Gamma_{\pi qq}^{a}(k) S_q(p + Q - k) \Gamma_{\gamma\pi qq,\mu}^{a}(-k) \right)
\]

\[
+ \Gamma_{\gamma\pi qq,\mu}^{a}(k) S_q(p - k) \Gamma_{\gamma\pi qq,\mu}^{a}(-k) \Delta_\pi(k).
\]

(3.40c)

The notation from the previous section is used, Eqs. (3.30) and (3.31) for the pion and quark propagators, and Eqs. (3.33) and (3.34) for the photon-pion and photon-quark vertices. Both PS and PV couplings of the pion to the quark are considered,

Figure 3.2: The diagrams contributing to the anomalous magnetic moment of the single quark. Diagram (c) is only present when a PV coupling is assumed.
Eqs. (3.26) and (3.28). The last current Eq. (3.40c) involves the contact interaction, Eq. (3.37), and is only present when a PV coupling is assumed. The last two terms in the last factor in Eq. (3.40a) are similar to the terms added in the pion-in-flight current, Eq (3.29a). Again they correspond to contact terms which are needed to satisfy the Ward-Takahashi identity to second order [78],

\[ Q_\mu \Gamma^{\mu, (2)}_{\gamma qq} = -e_q \left( S^{(2)}_{q}^{-1} (p + Q) - S^{(2)}_{q}^{-1} (p) \right), \]  

where \( S^{(2)}_{q} \) represents the quark propagator dressed with one pion loop. The three-point vertex \( \Gamma^{\mu, (2)}_{\gamma qq} \) is given by the sum of the currents Eqs. (3.40).

The currents can now be simplified by shifting the \( \gamma_5 \)'s through the expression and assuming that the incoming and outgoing quarks are on mass-shell. From these currents the anomalous magnetic moment has to be extracted. For this purpose the currents are written as

\[ J^{(a,c)}_{\mu} = -2ie \tau_z \left( \gamma^\nu C^{(a,c)}_{\mu \nu} + C^{(a,c)}_{\mu} \right), \]  
\( (3.42) \)

\[ J^{(b)}_{\mu} = ie \frac{1 - \tau_z}{2} \left( \gamma^\nu C^{(b)}_{\mu \nu} + C^{(b)}_{\mu} \right). \]  
\( (3.43) \)

The tensors \( C^{(i)}_{\mu \nu} \) and the 4-vectors \( C^{(i)}_\mu \), with \( i = a, b, c \), depend only on the initial and final momenta. Therefore they can be decomposed as:

\[ C^{(i)}_{\mu \nu} = A_{1}^{(i)} K_\mu K_\nu + A_{2}^{(i)} K_\mu Q_\nu + A_{3}^{(i)} Q_\mu K_\nu + A_{4}^{(i)} Q_\mu Q_\nu + A_{5}^{(i)} g_{\mu \nu}, \]  
\( (3.44) \)

\[ C^{(i)}_{\mu} = B_{1}^{(i)} K_\mu + B_{2}^{(i)} Q_\mu, \]  
\( (3.45) \)

where \( A_{n}^{(i)} \) and \( B_{n}^{(i)} \) are Lorentz invariants. By applying the Gordon decomposition to the current Eq. (3.39) near \( Q^2 = 0 \), it can be seen that the anomalous magnetic moment \( \kappa \) is the term proportional to \( \frac{-e}{2M} K_\mu \) with \( K_\mu = p_\mu + p'_\mu \). So only the first terms, \( A_{1}^{(i)} \) and \( B_{1}^{(i)} \), contribute to the anomalous magnetic moment. Substituting Eqs. (3.44) and (3.45) into Eqs. (3.42) and (3.43), and taking the initial and final quark on mass shell we find the anomalous magnetic moment corrections:

\[ \kappa^{(a,c)} = 4iM_p \tau_z \left[ 2m_q A_{1}^{(a,c)} + B_{1}^{(a,c)} \right], \]  
\( (3.46) \)

\[ \kappa^{(b)} = -2iM_p \frac{1 - \tau_z}{2} \left[ 2m_q A_{1}^{(b)} + B_{1}^{(b)} \right]. \]  
\( (3.47) \)

Eq. (3.46) corresponds to the coupling of the photon to the pion and Eq. (3.47) to the coupling of the photon to the quark. Formally, the contact term is also represented by Eq. (3.46) in case of a PV coupling, this term however vanishes and does not contribute to the quark anomalous magnetic moment.

The Lorentz invariant expressions \( A_{1}^{(i)} \) and \( B_{1}^{(i)} \) can immediately be determined from the tensor \( C^{(i)}_{\mu \nu} \). We obtain the expressions:

\[ A_{1}^{(i)} = \frac{1}{3K^4} (4K^\nu K^\nu - K^2 g^{\mu \nu}) C^{(i)}_{\mu \nu}, \]  
\( (3.48) \)
Table 3.4: The quark anomalous magnetic moments in units of the nucleon magneton in the one-loop approximation for various values of the string tension $\sigma$. The first set is the predictions for the pion loops alone, while in the second set both pion and kaon loops are included.

<table>
<thead>
<tr>
<th>$\sigma$ (GeV)$^2$</th>
<th>$\kappa_u$</th>
<th>$\kappa_d$</th>
<th>$\kappa_s$</th>
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</thead>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.0</td>
</tr>
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<td>0.092</td>
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<td>0.0</td>
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<tr>
<td>0.15</td>
<td>0.085</td>
<td>-0.126</td>
<td>0.0</td>
</tr>
<tr>
<td>pion and kaon loops</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.09</td>
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<td>0.12</td>
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<td>-0.133</td>
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<tr>
<td>0.15</td>
<td>0.112</td>
<td>-0.120</td>
<td>-0.031</td>
</tr>
</tbody>
</table>

$$B_1^{(i)} = \frac{1}{K^2} K^\mu C_\mu^{(i)}.$$  \hfill \(3.49\)

Details on the calculation of the integrals and explicit expressions for $A_1^{(i)}$ and $B_1^{(i)}$ can be found in Appendix D.

The kaon one-loop diagrams can be calculated in the same manner. The starting point is the expressions Eqs. (3.40), where the mass of the pion is replaced by the mass of the kaon. Note that the interaction of the quark with the kaon changes the flavor of the quark, $(u,d) \leftrightarrow s$, which implies that the internal and external quarks have different constituent masses. As a second consequence, the effective mass $m_{\text{eff}}$ in the PV coupling is in case of the kaon set equal to the average of the constituent masses of the $(u,d)$ and $s$ quark, $m_{\text{eff}} = (m_q(u,d) + m_q(s))/2$. The isospin structure is changed as $\tau_z \rightarrow (\tau_z + 3Y)/2$ and $(1 - \tau_z)/2 \rightarrow -(2/3 + 4/3 Y)$ respectively in Eqs. (3.46) and (3.47) where $Y$ is the hypercharge. The coupling constant $g_{\kappa qq}$ and the cutoff $\Lambda_\kappa$ are in first approximation the same as for the pion loop.

In Table 3.4 we show the calculated anomalous magnetic moments of the $u$, $d$ and $s$ quarks for $\Lambda = 675 \text{ MeV}$ for various choices of $\sigma$. Clearly, the results depend on the constituent quark masses. These are given in Table 3.1 for the string tensions considered.

From the calculation it is found that the results using a PV coupling can easily be related to the outcome using the PS coupling,

$$A_1^{(a,b)}(PV) = \left(\frac{m_q + M_q}{2 m_{\text{eff}}}\right)^2 A_1^{(a,b)}(PS),$$ \hfill \(3.50a\)
Baryon magnetic moments

\[ B_1^{(a,b)}(PV) = \left( \frac{m_q + M_q}{2m_{\text{eff}}} \right)^2 B_1^{(a,b)}(PS), \]  

(3.50b)

where \( m_q \) is the constituent mass of the external quark and \( M_q \) is the constituent mass of the internal quark, both given by their respective ground-state orbital energies \( \epsilon_0 \). In case of pion loops both internal and external quarks are \( u, d \)-quarks, \( m_q = M_q \). Kaons, however, change \( u \) and \( d \) quarks into \( s \) quarks and back. As a consequence, we have \( m_q \neq M_q \). Setting the effective mass \( m_{\text{eff}} \) in the PV coupling equal to the average of the involved constituent quark mass \( m_q \) and \( M_q \), the use of either PS or PV couplings give the same value.

The analysis performed shows that the contact term does not contribute to the anomalous magnetic moment of the quark.

Using Eq. (3.19) the \( \kappa \)-term in Eq. (3.39) yields a nucleon magnetic moment correction

\[ \delta \mu_z = 3 \delta \mu_z^{(1)} = 3 \left( \langle \psi_{\text{sym}} | \kappa_q(1) \sigma_z(1) | \psi_{\text{sym}} \rangle \right) \rho, \]  

(3.51)

with

\[ \rho = \frac{\int r^2 dr (|g|^2 + |f|^2 / 3)}{\int r^2 dr (|g|^2 + |f|^2)}, \]  

(3.52)

where the identity Eq. (3.23) has been used.

In Table 3.3, the predictions for the nucleon, including also the one-pion loop contributions Eq. (3.51) and two-body currents, are shown. Our results obtained for the one-loop corrections are smaller than reported by Glozman and Riska [29]. This is due to the inclusion of the lower component in the single quark orbitals. Neglecting these, we recover the results of Ref. [29]. From Table 3.3, we see that the proton and neutron magnetic moments are in reasonable agreement with experiment for a string tension of \( \sigma = 0.1 \) (GeV)\(^2\). For this value of the string tension the model predicts a nucleon mass of 940 MeV, remarkably close to the experimental value. The anomalous magnetic moment contributions are found to be of the order of 10%.

Due to the one-loop contributions, the magnetic moments of the other baryons are modified. Corrections from kaon loops have also been considered. Because of the larger kaon mass, the contributions are in general expected to be smaller in magnitude than those of the pion loops. In Table 3.4 the calculated anomalous moment of the strange quark due to the kaon one-loop corrections are given. In the calculations, a cutoff mass of \( \Lambda = 675 \) MeV has been used. The isoscalar and isovector anomalous magnetic moment terms are also changed by the kaon loop contributions. From Table 3.4, we see that the kaon loop contributions are indeed smaller in magnitude as compared to the pion loop ones. The complete results for the magnetic moments of the baryon octet and decuplet, including the pionic exchange currents and the pion and kaon one-loop contributions are summarized in Table 3.5 and plotted in Fig. 3.3. For the value of the string tension \( \sigma = 0.1 \) (GeV)\(^2\), the overall agreement with the experimental data is reasonable. From the table we see that the anomalous magnetic moment contribution leads to an improvement of the predictions.
choose a string tension that gives a reasonable nucleon mass. The same applies to the
overall agreement with the experiment for a string tension of $\sigma = 0.1$ (GeV)$^2$. We find
that the predicted magnetic moment of the nucleon is improved substantially once we
choose a string tension that gives a reasonable nucleon mass. The same applies to the

### 3.3 Conclusion

Using the formalism described in Chapter 2, a nonlinear equation for the single quark
propagator $S$ (attached to the string in a gauge invariant way) has been obtained.
The equation has been solved in the Gaussian correlator approximation. The resulting
three-quark wave function has been used to determine the magnetic moments of the
baryons. This was done for both the octet and decuplet of the $SU(3)$-flavor group.

Comparing the predictions we find that the magnetic moments are mostly in close
overall agreement with the experiment for a string tension of $\sigma = 0.1$ (GeV)$^2$. We find
that the predicted magnetic moment of the nucleon is improved substantially once we
choose a string tension that gives a reasonable nucleon mass. The same applies to the

<table>
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</table>
Figure 3.3: For easy comparison the data of Table 3.5 is plotted. The boxes indicate the uncertainty in the experimental values.

Δ-isobar. Effects due to the presence of virtual mesons are in general expected to be important. We have estimated the pionic one-loop and one pion exchange contributions to the magnetic moment. The single quark corrections from pionic loops are found to be of the order of 10%, whereas the total effect of 2-body current contributions are predicted to be small. This is in contrast to the results of Ref. [80]. This is due to the cancellation of the pion-in-flight and pair term in the present model. Because of the anomalous magnetic contributions, the predictions are somewhat better.

We have assumed that the baryon wave function can be described as a product of single quark orbitals, i.e. neglecting correlation effects. Our results for the magnetic moments of baryons in this approximation are encouraging, but are in need of including higher-order corrections. In particular, the mass spectrum obtained from our lowest-order approximation does not contain the $N - \Delta$ mass splitting. This is due to neglecting contributions like the hyperfine interaction arising from the one gluon interaction. This induces correlations in the three-quark wave function, and its magnitude may give us insight whether our basic description in this chapter in terms of single quark orbitals is a reasonable one. Moreover, it is clearly of interest to investigate how
the magnetic moments are changed when effects from color Coulomb and hyperfine interaction are accounted for. This will be partially realized in Chapter 5, where we will study the influence of the excited quark orbitals on the magnetic moment of the baryon ground state.
Chapter 4

Baryon Spectrum

Until now the picture of quarks moving in a confining sea of gluons is used. This leads to confinement and chiral symmetry breaking, but it lacks spin-dependent interactions. Typical spin-dependent structures in the baryon spectrum such as the hyperfine splitting between the nucleon and the $\Delta$ are therefore not present in this simplified model. In the present chapter, we refine the baryon spectrum by introducing perturbative pseudoscalar meson and gluon exchanges on top of the non-perturbative confining QCD-string. This approach is supported by Ref. [71] where a quark-meson Lagrangian is obtained from the QCD Lagrangian by systematically exploiting the field correlator expansion. This Lagrangian contains both confinement through the QCD-string and quarks interacting with meson fields.

In the first section, the formalism on exchange potentials is defined. In Section 4.2, the coupling constants are defined which are used in Section 4.3 for the calculation of the baryon mass spectrum. In the last section the same calculation is performed in momentum space using translationally invariant baryon wave functions. The computation yields results that are comparable to those obtained before. This chapter is based upon Ref. [82].

4.1 Interaction potential

In this section, perturbative particle exchanges are introduced to calculate the mass spectrum of the lowest baryon octet and decuplet representations of the $SU(3)$-flavor group. The exchanges induce spin-dependent interactions needed to reproduce the mass splittings in the baryon spectrum. First we split the gluonic field in a non-perturbative background part, leading to the confining interaction as described in Chapter 2, and a perturbative part. Focusing now on the perturbative gluon field, we introduce a one-gluon-exchange interaction. The Coulomb part of the interaction is expected to lower the baryon masses and the color magnetic part is expected to give rise to a splitting between the $J = 1/2$ and $J = 3/2$ states. Beside the one-gluon interaction, we also
Figure 4.1: A schematic view of the exchange of a pion (a) and a gluon (b) between two quarks in a baryon.

introduce a perturbative pseudoscalar meson-exchange which can be considered as an effective interaction representing the exchange of correlated quark anti-quark pairs. The interactions are schematically shown in Figure 4.1.

The baryonic bound state equation, Eq. (2.40), is equivalent to the Bethe-Salpeter equation with an instantaneous interaction. Of all possible three particle wave functions of the type (2.43), it couples only \((+++)\) and \((---)\), where \(\pm\) indicates the sign of \(\epsilon_n\) as in Eq. (2.47). All other wave functions decouple. Within such an equal time approximation, we now consider the effects of the perturbative exchange. Following Refs. [49, 51, 52], where the instantaneous three-particle Bethe-Salpeter equation is considered, we assume that the perturbative gluon and meson exchanges only take place between these purely positive and negative energy components. As discussed in Ref. [49], we write for the interaction potential in the Hamiltonian

\[
H^{(23)}_{\text{int}}(P, P'_{\rho}, P'_{\lambda}) = \left( \Lambda^+(p_1) \Lambda^+(p_2) \Lambda^+(p_3) + \Lambda^-(p_1) \Lambda^-(p_2) \Lambda^-(p_3) \right)
\times \int \int \frac{d^3 p'_{\rho}}{(2\pi)^3} \frac{d^3 p'_{\lambda}}{(2\pi)^3} \gamma^{0}_{(2)} \gamma^{0}_{(3)} V^{(23)}_{\text{ex}}(p_{\rho}, p_{\lambda}; p'_{\rho}, p'_{\lambda}) \psi(P, p'_{\rho}, p'_{\lambda}),
\]

where the energy projection operators are defined by

\[
\Lambda^\pm(p) = \frac{\omega \pm H(p)}{2\omega}, \quad \omega = |\epsilon_n|.
\]

The interactions between the other quark pairs, \(H^{(12)}_{\text{int}}\) and \(H^{(13)}_{\text{int}}\), can similarly be written down and be included. The Dirac-like equation Eq. (2.40) becomes

\[
H \psi(r_1, r_2, r_3) = \left( H_1 + H_2 + H_3 + H^{(12)}_{\text{int}} + H^{(13)}_{\text{int}} + H^{(23)}_{\text{int}} \right) \psi(r_1, r_2, r_3)
= (E + \Delta E) \psi(r_1, r_2, r_3).
\]
The exchange interactions are treated perturbatively:

$$\Delta E = 3 \langle \Psi_{JM} | H_{\text{int}}^{(23)} | \Psi_{JM} \rangle,$$  \hspace{1cm} (4.4)

where $\Psi_{JM}$ represents the baryon wave function, Eq. (2.43). Because of symmetry considerations, the interactions between the other quark pairs can simply be included by a factor of three in Eq. (4.4).

The exchange potential $V_{\text{ex}}^{(23)}$ in Eq. (4.1) for non-strange baryons can be separated into two contributions, the one-gluon exchange (OGE) and the one-pion exchange (OPE). They are explicitly written as

$$V_{\text{oge}}^{(23)}(k) = -\frac{4\pi}{3} \alpha_s \frac{\gamma_\mu(2)\gamma_\mu(3)}{k^2},$$  \hspace{1cm} (4.5a)

$$V_{\text{ope}}^{(23)}(k) = 4\pi g_{qq\pi}^2 \tau(2) \cdot \tau(3) \gamma_5(2)\gamma_5(3) \frac{1}{k^2 + m_\pi^2} \left( \frac{\Lambda^2}{k^2 + \Lambda^2} \right)^2,$$  \hspace{1cm} (4.5b)

where a pseudoscalar (PS) coupling for the pion is assumed. The factor $2/3$ in the OGE originates from the color content of the baryons and the term $\tau(2) \cdot \tau(3)$ in the OPE takes care of the isospin.

Using the baryon wave function Eq. (2.43), the energy shift can be calculated after some modifications of the exchange potential. First, the exchange potentials are rewritten into coordinate space by performing Fourier transformations

$$V_{\text{oge}}^{(23)}(r_2 - r_3) = -\frac{2}{3} \alpha_s \frac{\gamma_\mu(2)\gamma_\mu(3)}{|r_2 - r_3|},$$  \hspace{1cm} (4.6a)

$$V_{\text{ope}}^{(23)}(r_2 - r_3) = g_{qq\pi}^2 \tau(2) \cdot \tau(3) \gamma_5(2)\gamma_5(3) \frac{1}{k^2 + m_\pi^2} \left( \frac{\Lambda^2}{k^2 + \Lambda^2} \right)^2 \left( -e^{-m_\pi|r_2 - r_3|} + e^{-\Lambda|r_2 - r_3|} + \frac{\Lambda^2 - m_\pi^2 e^{-\Lambda|r_2 - r_3|}}{2} \right).$$  \hspace{1cm} (4.6b)

Second, expanding the potentials in terms of spherical harmonics $Y_{lm}$, one obtains

$$V(r_2, r_3) = \sum_{l=0}^{\infty} V_l(r_2, r_3) \frac{2l + 1}{4\pi} P_l(x)$$

$$= \sum_{l=0}^{\infty} V_l(r_2, r_3) \sum_{m=-l}^{l} Y_{lm}^*(\Omega_2)Y_{lm}(\Omega_3),$$  \hspace{1cm} (4.7)

where $P_l$ is a Legendre polynomial and $x = \cos(\gamma)$ the angle between the vectors $r_2$ and $r_3$. The function $V_l(r_2, r_3)$ can be found by using the orthonormality condition of the Legendre polynomials,

$$V_l(r_2, r_3) = \frac{4\pi}{2} \int_{-1}^{1} V(r_2, r_3) P_l(x) \, dx.$$  \hspace{1cm} (4.8)
In the special case of the Coulomb potential 
\[ V(r_2, r_3) = \frac{1}{r_2 - r_3}, \]
the integral can be done analytically and the expansion reduces to,

\[ V_l(r_2, r_3) = \frac{4\pi}{2l + 1} \frac{r_l^l}{r_{l+1}^{l+1}}, \quad (4.9) \]

with \( r_<(r_>) \) the smaller (larger) of \( r_2 \) and \( r_3 \). The advantage of this expansion is that it is easy to evaluate analytically the integrals over the angles appearing in the calculation of the matrix elements in Eq. (4.4).

The matrix element Eq. (4.4) can now be written as

\[
\left\langle \Psi_{JM} \left| H_{\text{int}}^{(23)} \right| \Psi_{JM} \right\rangle = \frac{1}{N} \int d^3r_1 \int d^3r_2 \int d^3r_3 \Psi_{JM}^\dagger(r_1, r_2, r_3) \times \left[ \Lambda^+(1)\Lambda^+(2)\Lambda^+(3) + \Lambda^-(1)\Lambda^-(2)\Lambda^-(3) \right] \times \gamma_0(2)\gamma_0(3)V(r_2, r_3)\Psi_{JM}(r_1, r_2, r_3), \quad (4.10)
\]

with the normalization

\[
N = \int d^3r_1 \int d^3r_2 \int d^3r_3 \Psi_{JM}^\dagger(r_1, r_2, r_3)\Psi_{JM}(r_1, r_2, r_3), \quad (4.11)
\]

and the projection matrices \( \Lambda^\pm \) are defined in Eq. (4.2). The baryon wave function \( \Psi_{JM} \) is a product of three single quark orbitals, Eq. (2.43), which can each be decomposed in partial waves as shown in Eq. (2.45). The integral over the first quark factorizes and drops out. The remaining part contains two angular integrals over three spherical harmonics each. The product of three spherical harmonics can be evaluated analytically as (see for example Ref. [56]):

\[
\int d\Omega Y^{*}_{l'm'}Y_{LM}Y_{lm} = \sqrt{\frac{(2l + 1)(2L + 1)}{4\pi (2l'+1)}}C(lLl'; mMm')C(lLl'; 000). \quad (4.12)
\]

The remaining radial integrals over \( r_2 \) and \( r_3 \) are done numerically.

When a pseudovector (PV) pion-quark coupling is assumed, some modifications have to be made. In Eq. (4.5b) the pseudovector coupling is obtained by the replacement

\[
g_{\pi qq}\gamma_5\tau_a \rightarrow \frac{g_{\pi qq}}{2m_{\text{eff}}}\gamma_5 \kappa \tau^a, \quad (4.13)
\]

where \( m_{\text{eff}} \) is a scaling mass of the order of the constituent quark mass, to be discussed later. By Fourier transforming, the momenta in Eq. (4.13) become derivatives in coordinate space. In this way, the matrices \( \kappa \) can be evaluated when they act on the quark wave functions resulting in some derivatives on the radial wave function and a modified angular dependence. As the derivatives act on the wave functions, the actual potential \( V_{\text{ope}}^{(23)}(r_2 - r_3) \) can still be expanded in terms of spherical harmonics in exactly the same way as is done for PS coupling. The result is

\[
\left\langle \Psi_{JM} \left| H_{\text{int}}^{(23)} \right| \Psi_{JM} \right\rangle = \frac{1}{N} \int d^3r_1 \int d^3r_2 \int d^3r_3 \Psi_{JM}^\dagger(r_1, r_2, r_3) \times \left( \Lambda^+(1)\Lambda^+(2)\Lambda^+(3) + \Lambda^-(1)\Lambda^-(2)\Lambda^-(3) \right) \gamma_0(2)\gamma_0(3) \]

The arrows point in the direction in which the derivatives act. Again the integral over $r_1$ factorizes and drops out. The derivative on the wave function has to be calculated, which can be done by using

$$
\sigma \nabla \tilde{g}(r) \Omega_{j'l'} = - (\sigma \tilde{r}) \Omega_{jlm} \quad \text{and} \quad \kappa = \pm (j + \frac{1}{2}) \text{ as } j = l \pm \frac{1}{2}.
$$

The integral over the angles can again be evaluated using Eq. (4.12), while the integral over the radial wave functions, also containing derivatives on the radial wave functions, is done numerically.

The model can easily be extended to include the strange baryons. Apart from pion exchange, we also have to add kaon and eta exchanges. These exchanges can be included by changing the $SU(2)$ isospin matrices $\tau_2 \cdot \tau_3$ in Eq. (4.5b) into $SU(3)$-flavor matrices $\sum_{a=1}^{8} \lambda_a^2 \lambda_a^3$ and substituting the correct coupling constants, meson and cutoff masses.

### 4.2 Coupling constants

For the coupling of the gluon to the quark, a running coupling constant is used which depends on the distance. Following Ref. [2] the coupling is parameterized as

$$
\alpha^{(2)}(q) = \alpha^{(1)}(q) \left( 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln t_B}{t_B} \right), \quad \alpha^{(1)}(q) = \frac{4\pi}{\beta_0 t_B},
$$

with

$$
t_B = \ln \frac{q^2 + m_B^2}{\Lambda_V^2},
$$

and

$$
\beta_0 = 11 - \frac{2}{3} n_f, \quad \beta_1 = 102 - \frac{38}{3} n_f.
$$

Since the calculation is performed in coordinate space, the coupling constant has to be Fourier transformed which can be done by using

$$
\tilde{\alpha}(r) = \frac{2}{\pi} \int_0^{\infty} dq q \sin(qr) \alpha(q) = \frac{2}{\pi} \int_0^{\infty} dx \frac{\sin x}{x} \alpha(x/r).
$$

The constants are fixed at $m_B = 1.0 \, GeV$, $\Lambda_V = 385 \, MeV$ and $n_f = 3$ as discussed in Ref. [2].
Table 4.1: The parameters involved in the interaction of the pseudoscalar meson with the quark for various values of the string tension $\sigma$, which is given in units of $(\text{GeV})^2$. We assume for axial coupling constant, $g_A^q = 1$. The decay constants $f_m$ and all (cutoff) masses are in units of MeV, the coupling constant $g_{mqq}$ is dimensionless.

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<th>$M_m$</th>
<th>$f_m$</th>
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<td>$g_{mqq}^2/4\pi$</td>
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Following Ref. [27], we exploit the Goldberger-Treiman relation [30] for the pion-quark vertex to find the coupling constant of the pseudoscalar meson $m$ to the quark

$$g_{mqq} = m_{\text{eff}} \frac{g_A^q}{f_m},$$

(4.20)

where $m_{\text{eff}}$ is the scaling mass from the pseudovector coupling Eq. (4.13), $g_A^q$ is the quark axial coupling constant and $f_m$ is the decay constant. From Refs. [32, 45], we take the decay constants $f_\pi = 93$ MeV, $f_\kappa = 113$ MeV and $f_\eta = 112$ MeV.

The axial coupling constant of the quark is not well known. In the static quark model, it can be related to the nucleon axial coupling constant as, $g_A^q = \frac{2}{3} g_A^N$, which would give $g_A^q = 0.75$ for $g_A^N = 1.26$. In the large $N_c$-limit, however, the coupling would be $g_A^q = 1$ as was derived in Ref. [83] and confirmed by Simonov using the field correlator method in Ref. [71]. If $1/N_c$ corrections are taken into account the coupling decreases to $g_A^q = 0.87$ [84]. For consistency reasons, we try to keep close to the field correlator method, and choose $g_A^q = 1$. The resulting values for $g_{mqq}$ are determined from the Goldberger-Treiman relation and shown in Table 4.1. The parameterization of the cutoff mass is the same as in Ref. [27]:

$$\Lambda_m = \Lambda_0 + \kappa M_m$$

(4.21)

with the parameters $\Lambda_0 = 565$ MeV, $\kappa = 0.81$. The results are listed in Table 4.1.

The $m_{\text{eff}}$, defined in Eq. (4.13), can be looked at as the effective constituent quark mass. In case of the pion coupling $m_{\text{eff}}$ is the effective constituent mass of the $u,d$-quark, in case of the kaon and eta coupling $m_{\text{eff}}$ is a mixture of $u$, $d$- and $s$-quark masses. The mass is chosen such that the one-pion and one-kaon exchanges using PS coupling give the same value as using the PV coupling in only positive energy channels. Therefore, we require

$$V_{\text{ome}}^{(23)}(PS) = V_{\text{ome}}^{(23)}(PV) \left( \frac{1 + \gamma_0}{2} \right)^{(2)} \left( \frac{1 + \gamma_0}{2} \right)^{(3)}$$

(4.22)

\[ m = \pi, \kappa. \]
Table 4.2: The mass is calculated for the baryon multiplet for the string tension $\sigma = 0.06 \text{ (GeV)}^2$. $C_0$ is adjusted such as to yield the correct value for the nucleon. The contributions of the exchange potentials are also shown. All numbers are in units of MeV.

<table>
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<th>V_{\pi}</th>
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The effective mass $m_{\text{eff}}$ for the eta is set equal to the $m_{\text{eff}}$ for the K-meson. The parameters are summarized in Table 4.1. It can be seen that the coupling constants $g_{mqq}$ are almost equal, as would be the case in a chiral symmetric world. In case of $\sigma = 0.09 \text{ (GeV)}^2$, we find values which are somewhat smaller than $g_{mqq}/4\pi = 0.67$ as was used by Glozman et al. [27,28], in case of $\sigma = 0.12 \text{ (GeV)}^2$ somewhat larger.

### 4.3 Results and discussion

Using these coupling constants, the perturbative exchanges are calculated, where in first approximation the baryon wave function Eq. (2.43) is used. Results are given in Tables 4.2, 4.3 and 4.4 for string tensions of $\sigma = 0.06 \text{ (GeV)}^2$, $\sigma = 0.09 \text{ (GeV)}^2$ and $\sigma = 0.12 \text{ (GeV)}^2$ respectively, and plotted in Figure 4.2.

As can be seen from Tables 4.2, 4.3 and 4.4, an extra parameter $C_0$ has been introduced which is of a Lorentz vector nature. This parameter is added to the confining
The mass is calculated for the baryon multiplet for the string tension $\sigma = 0.09 \ (GeV)^2$. $C_0$ is adjusted such as to yield the correct value for the nucleon. The contributions of the exchange potentials are also shown. All numbers are in units of MeV.

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| PV  | p,n               | -159           | -11        | -222       | 0       | 3     | 939             | 939  |
|     | $\Lambda$        | -167           | -11        | -133       | -39     | 3     | 1123            | 1116 |
|     | $\Sigma$         | -167           | -9         | -15        | -65     | -16   | 1199            | 1193 |
|     | $\Xi$            | -177           | -10        | 0          | -65     | -27   | 1335            | 1318 |
|     | $\Delta$         | -159           | 11         | -44        | 0       | -3    | 1132            | 1232 |
|     | $\Sigma^*$       | -167           | 10         | -15        | -26     | 6     | 1279            | 1385 |
|     | $\Xi^*$          | -177           | 9          | 0          | -26     | -5    | 1415            | 1533 |
|     | $\Omega$         | -187           | 8          | 0          | 0       | -37   | 1539            | 1672 |

potential:

$$M(r^{(i)} - r^{(0)}) \rightarrow M(r^{(i)} - r^{(0)}) + \beta^{(i)} C_0.$$  

(4.23)

In the derivation of the confining potential in Refs. [68,73], the large distance behavior of the interaction was examined. The actual dependence at short distances is only poorly known. There can be contributions from the Lorentz-vector or -scalar type, even spin–spin interactions are possible. Our results seem to suggest that a constant $C_0$ with a value of about $170 - 190$ MeV has to be added in case of the PS coupling.

When the PV coupling is used, we find smaller values of the meson exchanges as compared to the PS coupling. Considering only positive energy components, both couplings give the same results by definition, Eq. (4.22). The inclusion of negative energy components decreases the effect of meson exchanges such that a smaller value of $C_0$, in the range of $C_0 \approx 140 - 150$ MeV, is needed in case of a PV coupling.

The hyperfine splitting and the OPE show some symmetry. The origin of the symmetry can be understood by performing a non-relativistic reduction on the exchanges.
Table 4.4: The mass is calculated for the baryon multiplet for the string tension $\sigma = 0.12 \text{ (GeV)}^2$. $C_0$ is adjusted such as to yield the correct value for the nucleon. The contributions of the exchange potentials are also shown. All numbers are in units of MeV.

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Eqs. (4.5). The resulting Breit interaction yields the $\sigma_2 \cdot \sigma_3$ structure which causes the well known $-3:1$ splitting between the spin singlet and triplet state. From Appendix B, it can be seen that the $\Delta$ solely consists of spin triplet ($j_{23} = 1$) states, while the nucleon is build up from a sum of spin singlet ($j_{23} = 0$) and spin triplet ($j_{23} = 1$) states. This results exactly in the $-1:1$ hyperfine splitting observed for the nucleon and the $\Delta$ in Tables 4.2, 4.3 and 4.4. The splitting $-5: -1$ in the OPE can similarly be understood when it is realized that in the non-relativistic reduction of Eq. (4.6b) the spin-isospin structure looks like $\mathbf{\sigma}_2 \cdot \mathbf{\sigma}_3 \cdot \mathbf{\tau}_2 \cdot \mathbf{\tau}_3$.

When the string tension $\sigma$ is increased the single particle orbitals tend to become more compact. Due to the $1/r$ behavior of the exchange potentials this results in larger values in the perturbative calculation Eq. (4.4).

The results for the baryon octet ($J = 1/2$) are quite close to the experimental masses in case of the PS coupling for $\sigma = 0.06 \text{ (GeV)}^2$ and in case of the PV coupling for $\sigma = 0.09 \text{ (GeV)}^2$. From Table 4.3, it can be seen that the PS coupling does somewhat better for the baryon decuplet ($J = 3/2$) where the PV calculation misses
Figure 4.2: For easy comparison the data of Tables 4.2, 4.3 and 4.4 is plotted.

about 100 MeV. For $\sigma = 0.12 \text{ (GeV)}^2$ in Table 4.4 the PV coupling leads to a reasonable overall agreement while the PS calculation produces values which are too large.

4.4 Translationally invariant calculation

The calculation of the exchange potentials can also be performed translationally symmetrically by using a translationally symmetric Ansatz for the baryon wave functions, Eq. (2.49). As this Ansatz, Eq. (2.49), is most easily formulated in momentum space the calculation is also performed in momentum space. This means that the exchange potentials given in Eqs. (4.5) are used instead of the formulation in coordinate space. Because the total momentum vanishes, $\mathbf{P} = \mathbf{P}' = 0$ is imposed on the coordinates of the single-quark orbitals $\mathbf{k}_i$ and $\mathbf{k}'_i$. These coordinates differ from the integration variables $\mathbf{p}$, $\mathbf{p}'$ and $\mathbf{q}$, and the integration over the angles can therefore not be done analytically. An expansion of the exchange potentials in terms of spherical harmonics is useless in this case.

The exchange interaction Eq. (4.4) can now be written as

$$\left\langle \tilde{\Psi}^{JM}_{J'M'} \left| H_{\text{int}}^{(23)} \right| \tilde{\Psi}^{JM}_J \right\rangle = \frac{1}{N} \int d^3q \int d^3p \int d^3p' \tilde{\Psi}^{JM}_{J'}(\mathbf{p}', \mathbf{q})$$
× \left[ \Lambda^+(1)\Lambda^+(2)\Lambda^+(3) + \Lambda^-(1)\Lambda^-(2)\Lambda^-(3) \right] \gamma_0(2)\gamma_0(3)V(p - p')\tilde{\Psi}_{JM}(p, q), \quad (4.24)

with the normalization

\[ N = \int d^3 q \int d^3 p \tilde{\Psi}_{JM}^\dagger(p, q)\tilde{\Psi}_{JM}(p, q). \quad (4.25) \]

The projection matrices \( \Lambda^\pm \) are defined in Eq. (4.2). In contrast to the calculation in coordinate space in Section 4.1, no parts of the integral factorize and in principle all the integrations have to be done numerically resulting in a nine-dimensional integral. However, if for example the baryon wave functions \( \tilde{\Psi}_{JM} \) and \( \tilde{\Psi}_{JM}' \) equal the wave function of the ground state, the integral can be simplified somewhat.

In the calculation in momentum space a PV coupling meson-quark coupling can be introduced with relative ease. The involved matrices \( \Omega \) can be evaluated by using the identity \( \Omega_{jl'm_j} = (\sigma \hat{k})\Omega_{jlm_j} \) with \( l' = 2j - l \).

As an example of the results obtained using the translationally symmetric Ansatz, the integrals are calculated for a string tension of \( \sigma = 0.09 \) (GeV)\(^2\) and the parameters given in Table 4.1. Only non-strange baryons are considered. The resulting values are given in Table 4.5.

The modifications when using the translationally symmetric Ansatz Eq. (2.49) can be understood by considering a non-relativistic calculation of the exchange potentials using Gaussian wave functions for the single-particle ground state, \( 0(\frac{1}{2})^++ \). Let the baryon wave function be composed of single-particle orbitals as,

\[ \Psi(p, q, P) = \prod_{i=1}^{3} \phi_i(k_i), \quad \phi_i(k_i) = A \exp(-\alpha k_i^2) \left( \frac{1}{\sigma . k_i / 2m} \right) \chi_s, \quad (4.26) \]

where \( \chi_s \) is the spin function and using the Jacobi coordinates Eqs. (3.24). The coefficients for symmetrization and coupling to the correct total (iso)spin are omitted for simplification. In the non-relativistic limit, we keep only the terms lowest in order of \( m \) and we find for the Coulomb part of the one-gluon exchange

\[ \Delta E \sim \frac{1}{N} \left\langle \Psi \left| \frac{1}{(p - p')^2} \right| \Psi \right\rangle = \frac{1}{N} \int d^3 p d^3 p' d^3 q d^3 P \frac{1}{(p - p')^2} \exp \left( -2\alpha \left( p^2 + p'^2 + 2q^2 + P^2 / 3 \right) \right), \quad (4.27) \]

with the normalization

\[ N = \langle \Psi | \Psi \rangle = \int d^3 p d^3 q d^3 P \exp \left( -2\alpha \left( 2p^2 + 2q^2 + P^2 / 3 \right) \right). \quad (4.28) \]

Eq. (4.27) is independent of the total momentum \( P \) since the integral over \( P \) clearly factorizes. This explains why the modifications in Table (4.5) are small for the Coulomb interaction. The deviations are caused by the use of the single-orbital solutions of the
Table 4.5: The mass and the contributions of the exchange potentials are calculated in momentum space for the nucleon and the $\Delta$ using a string tension with the value of $\sigma = 0.09 \text{ (GeV)}^2$, and the translationally symmetric Ansatz. The $C_0$ is adjusted such as to yield the correct value for the nucleon. For comparison reasons the corresponding values calculated in coordinate space from the previous section are also shown. All numbers are in units of MeV.

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</table>

nonlocal equation (2.46), instead of the simple Ansatz Eq. (4.26). For the hyperfine part of the one-gluon exchange we find

$$\Delta E \sim \frac{1}{N} \left\langle \Psi \left| \frac{\alpha_2 \alpha_3}{(p - p')^2} \right| \Psi \right\rangle$$

$$= \frac{1}{N} \int d^3p d^3p' d^3q d^3P \left[ \sigma^{(2)} . k_2 . \sigma^{(3)} . k_3 + \sigma^{(2)} . k' . \sigma^{(2)} . \sigma^{(3)} . \sigma^{(3)} . k_3 \right.$$

$$+ \sigma^{(3)} . k'_3 . \sigma^{(2)} . \sigma^{(2)} . k_2 + \sigma^{(2)} . \sigma^{(3)} . \sigma^{(3)} . k_2 . \sigma^{(3)} . k_3 \left. \right] \frac{1}{4m^2 (p - p')^2}$$

$$\times \exp \left( -2\alpha \left( p^2 + p'^2 + 2q^2 + P^2/3 \right) \right),$$

$$= \frac{1}{N} \int d^3p d^3p' d^3q d^3P \frac{P^2 - (p - p')^2 + \frac{1}{2} \sigma^{(2)} . \sigma^{(3)} (p - p')^2}{4m^2 (p - p')^2}$$

$$\times \exp \left( -2\alpha \left( p^2 + p'^2 + 2q^2 + P^2/3 \right) \right),$$

(4.29)

which explains the different offset for $V_{hf}$ as is shown in Table 4.5. The one-pion exchange can be analyzed in the same manner.

Since the numerical calculation of the exchange potentials using the translationally symmetric Ansatz.
symmetric Ansatz for the baryon wave function is much more involved than the calculation in coordinate space, and the results are comparable, we mostly confine ourselves using factorizable baryon wave functions in coordinate space in first approximation.
Chapter 5

Multichannel calculation

In this chapter, we consider the influence of excited single-quark orbitals to the baryon ground state. In the first section, the changes to the baryon mass spectrum are estimated. In Section 5.2 the consequences for the baryon magnetic moment are shown. Conclusions are drawn in Section 5.3. This chapter is based upon Ref. [82].

5.1 Mass spectrum

The baryon wave functions used so far in the calculations performed, contain only the ground state of the single-quark orbital. That is, the baryon wave function Eq. (2.43) can schematically be written as,

$$\Psi_{JM} = \prod_{k=1}^{3} \left( 0 \left( \frac{1}{2} \right)^{++} \right)_k,$$

(5.1)

where the notation from Eq. (2.47) has been used. Coefficients needed for symmetrization and coupling to the proper angular momentum are suppressed for simplicity. Quarks, however, can also be in excited states which means that generally baryon wave functions which are (partly) build up from excited single-particle solutions also contribute to the baryon. Thus contributions like

$$\Psi_{JM} = \left( 1 \left( \frac{1}{2} \right)^{++} \right)_1 \prod_{k=2}^{3} \left( 0 \left( \frac{1}{2} \right)^{++} \right)_k,$$

(5.2a)

$$\Psi_{JM} = \left( 0 \left( \frac{1}{2} \right)^{++} \right)_1 \left( 0 \left( \frac{1}{2} \right)^{--} \right)_2 \left( 1 \left( \frac{1}{2} \right)^{++} \right)_3,$$

(5.2b)

$$\Psi_{JM} = \left( 0 \left( \frac{3}{2} \right)^{++} \right)_1 \left( 0 \left( \frac{1}{2} \right)^{++} \right)_2 \left( 0 \left( \frac{1}{2} \right)^{--} \right)_3,$$

(5.2c)

can mix into the baryon and change the energy. Similarly, as was done in Refs. [37,38] for non-relativistic quark models and in Ref. [14] for a relativistic model, we take wave functions as Eqs. (5.2) as a basis for diagonalizing the Hamiltonian Eq. (4.3).
As the color content of the baryon takes care of the anti-symmetrization, the resulting three-particle wave function has to be totally symmetric with respect to particle interchanges when the color is disregarded. Spin, isospin, orbital and radial excitations are taken into account in the symmetrization procedure.

The 3-quark state can be characterized in the following way. Let us consider the representation, where quark 1 plays a special role. Starting from the single-quark orbitals we may couple quark 2 and 3 to a $j_{23}$ and $i_{23}$ state. These states can in general be made symmetric by adding the permutation $2 \leftrightarrow 3$ to these states, which results in

\[
\sum \text{C.G.} \left( |2\rangle |3\rangle + |3\rangle |2\rangle \right) = \sum C(i_2, i_3, i_{23}; m_{i_2}, m_{i_3}, m_{i_{23}})C(j_2, j_3, j_{23}; m_{j_2}, m_{j_3}, m_{j_{23}})
\]

\[
|n_{2j_2l_2m_{j_2}}|n_{3j_3l_3m_{j_3}}\rangle \otimes |i_2m_{i_2}\rangle |i_3m_{i_3}\rangle
\]

\[
+ |n_{3j_3l_3m_{j_3}}|n_{2j_2l_2m_{j_2}}\rangle \otimes |i_2m_{i_2}\rangle |i_3m_{i_3}\rangle
\] (5.3)

where for the moment the notation

\[
|n_{kjl_lm_{l_k}}\rangle \otimes |i_{km_{m_k}}\rangle,
\] (5.4)

is adopted to represent the single-orbital solution. The coefficients $C$ are the Clebsch-Gordon coefficients in the Rose notation [56]. An appropriate choice for the quantum numbers $j_{23}$ and $i_{23}$ has to be made such that the wave function does not vanish. To form the 3-quark state with total quantum numbers $J$ and $I$, the single-quark orbital of quark 1 is added. Taking the proper linear combinations of the Faddeev components formed from the first term by the interchanges $1 \leftrightarrow 2$ and $1 \leftrightarrow 3$, it is assured that the whole wave function is totally symmetric under the interchange of any two particles. The result is:

\[
\Psi_{J,I}(j_{23}, i_{23}) = \sum \text{C.G.} \left\{ |1\rangle (|2\rangle |3\rangle + |3\rangle |2\rangle ) + (1 \leftrightarrow 2) + (1 \leftrightarrow 3) \right\}
\]

\[
= \sum C(i_1, i_{23}, J; m_{i_1}, m_{i_{23}}, M_J)C(j_1, j_{23}, J; m_{j_1}, m_{j_{23}}, M_J)
\]

\[
\times C(i_2, i_3, i_{23}; m_{i_2}, m_{i_3}, m_{i_{23}})C(j_2, j_3, j_{23}; m_{j_2}, m_{j_3}, m_{j_{23}})
\]

\[
x \left\{ |n_{1j_1l_1m_{j_1}}\rangle \otimes |i_{1m_{i_1}}\rangle \left( |n_{2j_2l_2m_{j_2}}\rangle |n_{3j_3l_3m_{j_3}}\rangle \otimes |i_{2m_{i_2}}\rangle |i_{3m_{i_3}}\rangle +
\right.
\]

\[
\left. |n_{3j_3l_3m_{j_3}}\rangle |n_{2j_2l_2m_{j_2}}\rangle \otimes |i_{3m_{i_3}}\rangle |i_{2m_{i_2}}\rangle \right) + (1 \leftrightarrow 2) + (1 \leftrightarrow 3) \right\}. \quad \text{(5.5)}
\]

An allowed choice of $j_{23}$ and $i_{23}$ does not always lead to unique three-particle wave functions. For example, if one takes three identical single particle orbitals with $j_1 = j_2 = j_3 = 1/2$ and $i_1 = i_2 = i_3 = 1/2$, the wave function $\psi_{1/2,1/2}(0,0)$ equals $\psi_{1/2,1/2}(1,1)$.
Table 5.1: The masses in units of MeV of the nucleon and the Δ are calculated for a string tension of $\sigma = 0.09 \ (GeV)^2$ and the parameters from Table 4.1, only one-pion and one-gluon exchanges are taken into account. By adjusting $C_0$, the nucleon is set to its experimental value. The importance of admixture of excited single-particle orbitals can now be seen by comparing the predictions for the Δ-mass $m_\Delta(1)$ with only $0(\frac{1}{2})^{++}$ single-particle orbitals taken into account and $m_\Delta(2)$ where the single-particle orbitals listed in the first column are also taken into account.

<table>
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<th>PS coupling</th>
<th>PV coupling</th>
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</tr>
<tr>
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</tr>
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<td>1212</td>
</tr>
<tr>
<td>$1(\frac{3}{2})^{++}$</td>
<td>1213</td>
<td>1215</td>
</tr>
<tr>
<td>$0(\frac{5}{2})^{++}$</td>
<td>1213</td>
<td>1219</td>
</tr>
<tr>
<td>$0(\frac{7}{2})^{++}$</td>
<td>1213</td>
<td>1214</td>
</tr>
</tbody>
</table>

Independence of these basis functions is tested by calculating the determinant of the matrix $\langle \Psi_\alpha | \Psi_\beta \rangle$ where $\Psi_\alpha$, $\Psi_\beta$ are wave functions such as Eq. (5.5). A simple example of this procedure is shown in Appendix B, where the totally symmetric nucleon- and Δ-wave functions are constructed.

The wave functions $\psi_\alpha$ are taken as a basis to diagonalize the full Hamiltonian, including one-gluon and one-pion exchange. We therefore consider

$$\langle \psi_\alpha | V_{ex} | \psi_\beta \rangle + \langle \psi_\alpha | [H_0 - E] | \psi_\beta \rangle = 0,$$

with

$$H_0 |\psi_\beta\rangle = E_\beta |\psi_\beta\rangle, \quad V_{\alpha \beta} = \langle \psi_\alpha | V_{ex} | \psi_\beta \rangle, \quad A_{\alpha \beta} = \langle \psi_\alpha | \psi_\beta \rangle,$$

and $E_\beta = \epsilon^{(1)} + \epsilon^{(2)} + \epsilon^{(3)}$. From Eq. (5.6) we obtain

$$[A_{\alpha \gamma}^{-1} V_{\gamma \beta} + E_\beta \delta_{\alpha \beta}] = H_{\alpha \beta},$$

from which the eigenvalues have to be found.

In Table 5.1 the results of the multichannel calculations are shown for non-strange quarks. It can easily be seen that admixture of the single-quark orbitals $1(\frac{1}{2})^{++}$, $0(\frac{3}{2})^{++}$ and to a smaller extent $0(\frac{1}{2})^{-+}$ give the largest change of the spectrum in case...
Table 5.2: The result for the mass in units of MeV and the magnetic moment in units of the nuclear magneton of the multichannel calculation where the single-particle orbitals (spp’s) \( 0\left(\frac{1}{2}\right)^{++} \), \( 0\left(\frac{3}{2}\right)^{++} \), \( 0\left(\frac{5}{2}\right)^{++} \) and either \( 1\left(\frac{1}{2}\right)^{++} \) (PS) or \( 0\left(\frac{3}{2}\right)^{++} \) (PV) are used as input. The string tension is fixed at \( \sigma = 0.06 \text{ (GeV)}^2 \), the \( C_0 \) is adjusted to yield the correct nucleon mass, in case of the PS coupling \( C_0(1) = 174 \text{ MeV}, C_0(4) = 204 \text{ MeV} \) and in case of the PV coupling \( C_0(1) = 157 \text{ MeV}, C_0(4) = 182 \text{ MeV} \).

<table>
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<td>( \Omega^- )</td>
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of a PS coupling. These orbitals have been used in a multichannel calculation for both the u, d-quark and the s-quark. That is, all possible combinations of \( 0\left(\frac{1}{2}\right)^{++} \) \((u,d)\), \( 1\left(\frac{1}{2}\right)^{++} \) \((u,d)\), \( 0\left(\frac{3}{2}\right)^{++} \) \((u,d)\), \( 0\left(\frac{1}{2}\right)^{--} \) \((u,d)\), \( 0\left(\frac{1}{2}\right)^{++} \) \((s)\), \( 1\left(\frac{1}{2}\right)^{++} \) \((s)\), \( 0\left(\frac{3}{2}\right)^{++} \) \((s)\) and \( 0\left(\frac{5}{2}\right)^{--} \) \((s)\) which couple to some specific total angular momentum \( J \) and total isospin \( I \) are taken into account. When a PV coupling is exploited, the single-particle orbital \( 0\left(\frac{5}{2}\right)^{++} \) is taken instead of \( 1\left(\frac{1}{2}\right)^{++} \) since it was found to give a larger contribution in this case. The results are shown in Tables 5.2, 5.3 and 5.4 for string tensions of
Table 5.3: The result for the mass in units of MeV and the magnetic moment in units of the nuclear magneton of the multichannel calculation where the single particle orbitals (spo’s) $0(\frac{1}{2})^{++}$, $0(\frac{1}{2})^{-+}$, $0(\frac{3}{2})^{++}$ and either $1(\frac{1}{2})^{++}$ (PS) or $0(\frac{3}{2})^{++}$ (PV) are used as input. The string tension is fixed at $\sigma = 0.09$ (GeV)$^2$, the $C_0$ is adjusted to yield the correct nucleon mass, in case of the PS coupling $C_0(1) = 175$ MeV, $C_0(4) = 221$ MeV and in case of the PV coupling $C_0(1) = 146$ MeV, $C_0(4) = 190$ MeV.

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<tr>
<td>$\Omega^-$</td>
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<td>-2.07</td>
<td>1743</td>
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$\sigma = 0.06$ (GeV)$^2$, $\sigma = 0.09$ (GeV)$^2$ and $\sigma = 0.12$ (GeV)$^2$ respectively, and plotted in Figure 5.1.

The inclusion of the excited quark orbitals, changes the spectrum significantly. All ground state masses decrease. An effect which is stronger for the baryon octet, which causes the nucleon-$\Delta$ splitting to increase by about 100 MeV. In case of $\sigma = 0.09$ (GeV)$^2$ the situation for the baryon decuplet thus improves considerably, leading to a rather close prediction for the PV calculation as can be seen in Table 5.3. The baryon octet however is quite well reproduced in Table 5.2 for a string tension of
As a second consequence of the lower masses, larger $C_0$ values have to be used. The mass splittings inside the baryon octet and decuplet also get larger. In most cases, this behavior makes the predictions within the baryon octet somewhat poorer, being already too large by a small amount in the calculation presented in the previous chapter.

Table 5.4: The result for the mass in units of MeV and the magnetic moment in units of the nuclear magneton of the multichannel calculation where the single particle orbitals (spo’s) $0(\frac{1}{2})^{++}$, $0(\frac{1}{2})^{-1+}$, $0(\frac{3}{2})^{-++}$ and either $1(\frac{1}{2})^{++}$ (PS) or $0(\frac{3}{2})^{++}$ (PV) are used as input. The string tension is fixed at $\sigma = 0.12$ (GeV)$^2$, the $C_0$ is adjusted to yield the correct nucleon mass, in case of the PS coupling $C_0(1) = 184$ MeV, $C_0(4) = 242$ MeV and in case of the PV coupling $C_0(1) = 143$ MeV, $C_0(4) = 204$ MeV.

<table>
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<th>PV coupling</th>
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</tr>
</thead>
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<td></td>
<td>1 spo</td>
<td>4 spo’s</td>
<td>1 spo</td>
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<tr>
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<td>$m_N$</td>
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<td>$\Omega^-$</td>
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</tbody>
</table>

$\sigma = 0.06$ (GeV)$^2$ and a PV coupling. The PS calculation with the string tension of $\sigma = 0.09$ (GeV)$^2$ yields values somewhat too large, while the results in Table 5.4 are much too high for the decuplet.
Figure 5.1: For easy comparison the data of Tables 5.2, 5.3 and 5.4 is plotted.

These results for the mass spectrum seem to point in the direction of a small string tension of about \( \sigma = 0.08 \text{ (GeV)}^2 \) and a slight preference for a PV coupling when the overall agreement is considered.

### 5.2 Magnetic moments

Now the influence of the perturbative exchanges on the mass spectrum has been calculated and the mixing of excited quark orbitals into the baryon ground state has been estimated, the question arises what might be the consequences for the baryon magnetic moments. To address this issue, the expressions obtained for the baryon magnetic moments in Chapter 3 must be generalized to arbitrary quark orbitals which are used in a multichannel calculation. For the coupling of the meson to the quark, we consider two possible forms, the pseudoscalar (PS) and pseudovector (PV) coupling as was also done in the calculation of the meson exchanges in the previous chapter.

Following the same procedure as in Section 3.1 to calculate the major contribution to the baryon magnetic moment, we introduce an external electromagnetic field \( \mathbf{A} \) into the Dirac-like equation, Eq. (2.44), by minimal substitution, \( \mathbf{p}_i \rightarrow \mathbf{p}_i - e_q(i) \mathbf{A} \), \( \mathbf{A} = \frac{1}{2}(\mathbf{H} \times \mathbf{r}) \). We obtain

\[
\langle \Delta H_1 \rangle = -e_q^{(1)} \left( \phi^{(1)} \sigma^{(1)} A \chi^{(1)} + \chi^{(1)} \sigma^{(1)} A \phi^{(1)} \right),
\]

(3.5)
which is the same expression as we found in Chapter 3. Using \( A = \frac{1}{2} (H \times r) \) and \( \Delta E = -\mu H \), we find for the magnetic moment operator,

\[
\mu_z^{(1)} = -e_q^{(1)} \frac{1}{2} \int d^3 r_1 \left\{ \phi^*(r_1) \left( \sigma^{(1)} \times r_1 \right)_z \chi(r_1) + \chi^*(r_1) \left( \sigma^{(1)} \times r_1 \right)_z \phi(r_1) \right\}. \tag{5.9}
\]

The magnetic moment operator Eq. (5.9) can be evaluated by rewriting it in terms of spherical harmonics

\[
\frac{1}{2} (\sigma \times r) = -\frac{1}{2i} \sqrt{\frac{2\pi}{3}} \left( \sigma_+ Y_{1-1} + \sigma_- Y_{11} \right) r, \tag{5.10}
\]

after which the angular part can easily be calculated analytically using Eq. (4.12). We are then left with a radial integral over \( r_1 \) that must be evaluated numerically (the integrals over \( r_2 \) and \( r_3 \) factorize and drop out),

\[
\mu_z = 3\mu_z^{(1)} = -3i \frac{1}{N} \int d^3 r_1 \int d^3 r_2 \int d^3 r_3
\]

\[
\times \Psi_{JM}^\dagger (r_1, r_2, r_3) e^{(1)}_q \sqrt{\frac{2\pi}{3}} \left( \sigma_+^{(1)} Y_{1-1} + \sigma_-^{(1)} Y_{11} \right) r_1 \Psi_{JM}(r_1, r_2, r_3), \tag{5.11}
\]

with the normalization Eq. (4.11) and symmetrized baryon wave functions Eq. (2.43). Because of symmetry considerations we can calculate the contribution of the first quark only and take the second and third quark into account by multiplying by a factor of three.

In the calculation of the baryon mass spectrum, we introduced pseudoscalar meson exchange as an effective interaction representing the exchange of a correlated quark anti-quark pair. We now study one-loop effects of the mesonic fluctuations which give rise to modifications of the single-quark current, in particular, to an anomalous magnetic moment of the quark. Near \( Q^2 = 0 \) the current can be written as:

\[
J_{\mu}^{q\bar{q}} = e_q \gamma_\mu + \kappa_q \frac{ie}{2M_p} \sigma_{\mu\nu} Q^\nu, \tag{3.39}
\]

where \( \kappa_q = \kappa_s + \kappa_t \tau_z \) is the anomalous magnetic moment for the \( u, d \)-quark. From Eq. (3.19), the magnetic moment contribution is found to be

\[
\delta \mu_z^{(1)} = -\int d^3 r_1 \left\{ \phi^*(r_1) \kappa_q^{(1)} \sigma_z^{(1)} \phi(r_1) - \chi^*(r_1) \kappa_q^{(1)} \sigma_z^{(1)} \chi(r_1) \right\}, \tag{5.12}
\]

which results in

\[
\delta \mu_z = 3\delta \mu_z^{(1)}
\]

\[
= 3 \frac{1}{N} \int d^3 r_1 \int d^3 r_2 \int d^3 r_3 \Psi_{JM}^\dagger (r_1, r_2, r_3) \kappa_q^{(1)} \sigma_z^{(1)} \gamma_0 \Psi_{JM}(r_1, r_2, r_3). \tag{5.13}
\]
Table 5.5: The anomalous magnetic moment of the quark in units of the nucleon magneton for different values of the string tension $\sigma$. The parameters are taken from Table 4.1. The first set is the predictions for the pion loops alone, while in the second set both pion and kaon loops are included, and the third set shows the results where pion, kaon and eta loops are taken into account.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 0.06$ (GeV)$^2$</th>
<th>$\sigma = 0.09$ (GeV)$^2$</th>
<th>$\sigma = 0.12$ (GeV)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PS</td>
<td>PV</td>
<td>PS</td>
</tr>
<tr>
<td>$\kappa_u$</td>
<td>0.065</td>
<td>0.091</td>
<td>0.089</td>
</tr>
<tr>
<td>$\kappa_d$</td>
<td>$-0.110$</td>
<td>$-0.153$</td>
<td>$-0.141$</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 0.06$ (GeV)$^2$</th>
<th>$\sigma = 0.09$ (GeV)$^2$</th>
<th>$\sigma = 0.12$ (GeV)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PS</td>
<td>PV</td>
<td>PS</td>
</tr>
<tr>
<td>$\kappa_u$</td>
<td>0.097</td>
<td>0.136</td>
<td>0.129</td>
</tr>
<tr>
<td>$\kappa_d$</td>
<td>$-0.100$</td>
<td>$-0.138$</td>
<td>$-0.129$</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>$-0.028$</td>
<td>$-0.040$</td>
<td>$-0.039$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\sigma = 0.06$ (GeV)$^2$</th>
<th>$\sigma = 0.09$ (GeV)$^2$</th>
<th>$\sigma = 0.12$ (GeV)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PS</td>
<td>PV</td>
<td>PS</td>
</tr>
<tr>
<td>$\kappa_u$</td>
<td>0.092</td>
<td>0.131</td>
<td>0.123</td>
</tr>
<tr>
<td>$\kappa_d$</td>
<td>$-0.097$</td>
<td>$-0.136$</td>
<td>$-0.126$</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>$-0.021$</td>
<td>$-0.025$</td>
<td>$-0.031$</td>
</tr>
</tbody>
</table>

Repeating the procedure in Subsection 3.2.2, the anomalous magnetic moments $\kappa$ are calculated considering one-loop mesonic contributions. As the pseudoscalar meson octet (pion, kaon and eta) was used in the calculation of the baryon mass spectrum, the eta is now also added to the one-loop mesonic contributions for consistency reasons. In case of the eta-loop, the mass of the pion is replaced by the mass of the eta in expressions Eqs. (3.40) and appropriate changes of the constituent masses of the internal and external quarks are made. Note that the eta-meson does not change the quark-flavor. The eta loop only contributes to the diagram where the photon couples to the quark since the eta is a charge-neutral meson. Therefore the isospin structure in Eqs. (3.46) and (3.47) changes into $\tau_z \rightarrow 0$ and $(1 - \tau_z)/2 \rightarrow -\frac{1}{6} + \frac{1}{6} \tau_x + \frac{1}{2} Y$ respectively where $Y$ is the hypercharge. The coupling constants $g_{mqq}$ and the cutoff masses $\Lambda_m$ are taken as discussed in Section 4.2 and as given in Table 4.1. The results for the anomalous magnetic moment of the quarks due to the different mesonic one-loop contributions are given in Table 5.5.

From the table, it can be seen that different values are obtained when PS or PV couplings are employed. The PS and PV results can be related by a factor $(m_q + M_q)^2 / 4m^2_{\text{eff}}$ as was shown in Eq. (3.50). The masses $m_q$ and $M_q$ represent the constituent mass of
the external and internal quark, respectively. These masses are approximated by the
ground state orbital energy $\epsilon_0$ given in Table 2.1. The effective mass $m_{\text{eff}}$ comes from
the definition of the PV coupling, Eq. (4.13). If the effective mass $m_{\text{eff}}$ is taken to be
the same as the constituent quark mass $m_q$, both couplings give the same value as is
the case in Section 3.2. However, in this chapter we adopt the same values for the pa-
rameters as were used for the calculation of the baryon mass spectrum in Section 4.2.
From Table 4.1, it can be seen that the effective mass in this case differs from the
ground state orbital energy $\epsilon_0$ shown in Table 2.1 resulting in somewhat larger values
when the PV coupling is employed.

Pion loops clearly provide the largest contribution to the anomalous magnetic mo-
ment as the pion mass is small. Although the masses of the kaon and eta are of
the same order, kaon-loops give larger contributions due to the flavor dependence.

As the contribution from pion exchange currents are predicted to be small [75], we
leave them out in a first approximation.

The results for the baryon magnetic moments are shown in Tables 5.2, 5.3 and
5.4 for string tensions of $\sigma = 0.06 \ (\text{GeV})^2$, $\sigma = 0.09 \ (\text{GeV})^2$ and $\sigma = 0.12 \ (\text{GeV})^2$,
Figure 5.3: For easy comparison the magnetic moment data of Tables 5.2, 5.3 and 5.4 is plotted, where a PV coupling is employed for the meson-quark coupling.

respectively, and plotted in Figures 5.2 and 5.3. The inclusion of the excited quark orbitals decreases the baryon magnetic moment. This behavior results in values that are too low in the case of $\sigma = 0.09$ (GeV)$^2$ and $\sigma = 0.12$ (GeV)$^2$ while $\sigma = 0.06$ (GeV)$^2$ yields values which are too large. The best overall agreement is obtained for a string tension of $\sigma = 0.08$ (GeV)$^2$. When a PV coupling is exploited the anomalous magnetic moment contributions are larger which causes an increase of the resulting total magnetic moments of the baryons. Although the results obtained by using either PS or PV couplings are rather similar, the PS case seems to produce results slightly closer to experiment.

5.3 Conclusions

The method described in this chapter should be looked at as a second approximation to calculate both the magnetic moments and the baryon mass spectrum in the QCD-string model. The first approximation is described in Chapter 3 where no correlations between the quarks were taken into account. This means that the baryon wave function
was described as a product of single quark orbitals. In the present chapter we improve on this by considering one-gluon and meson exchanges and taking excited single-quark orbitals into account. However, effects from neglecting the actual position of $r_0$ in the Torricelli point and instead choosing a fixed value for the parameter $r_0$ are not considered and left for further study.

From the results presented in this chapter, it appears to be possible to obtain a reasonable agreement of the baryon magnetic moments in a region where the predicted masses are close to experiment. Although there is a small preference for a PS coupling when the magnetic moments are considered, the mass spectrum puts in more weight in favor of a PV coupling. So the best overall agreement is obtained when a PV coupling is assumed and if we choose a string tension of $\sigma = 0.08$ (GeV)$^2$. 
Chapter 6
Discussion and conclusions

In this thesis we have explored the baryon wave function obtained from the QCD-Lagrangian by the field correlator method. Using this method, the gluonic degrees of freedom has been integrated out and an effective Lagrangian was obtained. In the lowest order of the approximation scheme, neglecting gluon and meson exchanges, the baryon Green’s function is given by the product of three independent single-quark Green’s functions. As a result, the Hamiltonian has been written as a sum of three quark terms. In this approximation, the resulting baryon wave function is given by a product of three independent single-quark solutions, each satisfying the Dirac-like equation with a nonlocal kernel. This kernel is the nonlocal quark mass operator $M(x, y)$, represented by the QCD string and containing both confinement and CSB.

We have used the baryon wave function obtained in this way, to calculate the magnetic moments of the lowest baryon octet and decuplet representation of the $SU(3)$-flavor group in Chapter 3. Assuming the presence of virtual pseudoscalar mesons as effective representations of correlated quark anti-quark pairs, corrections to the magnetic moments from mesonic fluctuations have been estimated. Both mesonic one-loop contributions to the anomalous magnetic moment of the quark and two-body pion exchange have been calculated. For the interaction of the pseudoscalar meson with the quark both the pseudoscalar and the pseudovector coupling have been investigated. The two-body correction appears to be quite small while the anomalous magnetic moments of the quarks contribute about 10%. Reasonable agreement with experimental baryon magnetic moments was obtained for a string tension of $\sigma = 0.10$ (GeV)$^2$ while the baryon masses were in the correct region without need for introduction of constituent quark masses.

Since the confining nonlocal operator $M(x, y)$ does not involve any spin-dependent interactions, the first stage of the model does not account for the mass splitting between the baryon octet and decuplet. We have subsequently improved this omission by considering perturbative one-gluon and pseudoscalar meson exchanges in Chapter 4. A constant term $C_0$ is added to the confining interaction which corresponds to next-to-leading order corrections to the area law and is adjusted such that it yields the
correct nucleon mass. The pseudoscalar meson exchange potential using pseudovector couplings produces smaller values for the mass splittings than the use of the corresponding pseudoscalar coupling. The best overall agreement is obtained for a string tension $\sigma = 0.12 \text{ (GeV)}^2$ and using a pseudovector coupling.

The effects of excited single-quark orbitals on the baryon ground state were considered in Chapter 5 where the masses and magnetic moments have been calculated for the baryon multiplet. The calculation is performed by using all possible combinations of four single-quark orbitals for both the $u,d$-quark and the $s$-quark to build different three-particle states. The resulting states are used as a basis to diagonalize the Hamiltonian. The mixing of the higher states in general lowers the magnetic moments, and increases the mass splittings both inside and between the baryon octet and decuplet. As a result, the best overall agreement is found when the string tension is lowered to $\sigma = 0.08 \text{ (GeV)}^2$ and a pseudovector coupling of the pseudoscalar meson to the quark is used.

In the approach considered, the central point $r_0$, in which the QCD-strings meet, is fixed, thereby explicitly breaking translational symmetry. This unappealing feature should in principle be treated by adopting the Torricelli point for $r_0$. It is, however, not clear to what extent $r_0$ should be treated as a dynamical variable in the $3q$ Green’s function or as a parameter in the Hamiltonian. The former approach yields highly nontrivial three-particle equations of genuine Y-string nature. As a practical solution, we have written down an Ansatz involving the replacement of $r_0$ by the center of mass point in the baryon wave function consisting of the product of three single-quark solutions. By integrating over the center of mass coordinate, the center of mass motion is averaged out. This treatment is equivalent to the formulation of the theory in momentum space and projecting out the center of mass momentum. In this way, one is able to make the theory manifestly translationally invariant.

This Ansatz is used in the calculations of the magnetic moments and masses of the baryons in Chapters 3 and 4. The modifications are rather minor for the magnetic moments, while the masses remain in the same energy regime. We can have some confidence in our first approximation of keeping $r_0$ fixed. However, due to the large overlap of some of the constructed higher baryon states with the baryon ground state, the multichannel calculation shows large off-diagonal elements resulting in unphysically large mass-shifts. The approach clearly fails to work in this respect.

Another possible solution to deal with the center of mass motion can be found in the nuclear shell-models used to describe atomic nuclei. These models suffer from the same problem of translational symmetry since the coordinates of the individual nucleons are taken with respect to some arbitrary origin. Several techniques have been developed to overcome this difficulty afterwards. However, except for the case of harmonic oscillator wave functions, they do not lead to consistent results and may suffer from nonorthogonality of the excited states [6, 48, 54]. This kind of approach has not been pursued in this thesis and is left for further study.

Connected to the discussion of the treatment of $r_0$ is the value of the string tension $\sigma$. Lattice simulations yield results of about $\sigma = 0.15 \text{ (GeV)}^2$ for the static three-quark interaction of a Y-shape nature [77]. Considering mesons, a value of roughly
\( \sigma = 0.18 \text{ (GeV)}^2 \) is found by studying Regge-trajectories \([11, 50]\). The value resulting from the multichannel calculation of \( \sigma = 0.08 \text{ (GeV)}^2 \) is somewhat too small, also when the larger string length due to fixing the central point \( r_0 \) is taken into account. We cannot make more definite statements before a more complete treatment of \( r_0 \) has been developed.

Several remarks concerning further developments and possible improvements of the model can be made. In the derivation of the effective confining interaction, several approximations have been made, keeping only the dominant terms. It would, however, be interesting to consider corrections of higher order gluon correlators to this potential or take color-magnetic terms of the mass operator \( M(x, y) \) into account. A more ambitious plan is the investigation of the characteristics of the confining potential at short distances and possibly determine the value of the parameter \( C_0 \) from first principles.

In the calculation of the electromagnetic form factors, the influence of Lorentz boost and recoil effects have silently been neglected. In a more accurate calculation where the center of mass motion is properly treated, these effects should be taken into account. This can be envisaged as for example in Ref. \([36]\), where the boost operator has been shifted to the electromagnetic operator. Moreover, in such a calculation the analysis can be extended to arbitrary photon momentum, which can be compared to experiment. From these structure functions, the charge radius of the baryon can also be found, which is a quantity that has been accurately measured.

Many years after QCD was proposed as the theory of strong interaction between the quarks, the treatment of its low energy regime is still an interesting field with unanswered questions and uncalculated quantities. A lot of work has already been done using phenomenological non-relativistic potentials, in some cases with relativistic kinetic operators, relativistic Bethe-Salpeter type approaches or numerical simulations of QCD on the lattice. The strong appeal of the field correlator method considered here, is that an effective quark mass operator \( M(x, y) \), which shows confinement and CSB, can be derived from the QCD-Lagrangian operator in a gauge invariant way. These features, among others, support the QCD-string model as a promising model. Further study and input are, however, needed to increase our knowledge on this subject.
Appendix A

Torricelli point

In the formalism described in Chapter 2, a point \( r_0 \) has to be found such that the string which is formed between this point and the quarks is of minimal length. In this Appendix, explicit formulas for \( r_0 = r_T \) are given for which \( \sum_{i=1}^{3} |r_i - r_T| \) is minimal. Originally this problem was posed by the mathematician Fermat and solved by Torricelli which is the reason for the name ‘Fermat point’ or ‘Torricelli point’ for \( r_T \).

A geometric construction of the Torricelli point can be found in many textbooks, for example in the book by Coxeter [18]. Consider three points \( A, B \) and \( C \) in three dimensional space. The problem is to find a point \( T \) such that the sum of the distances \( |A - T| + |B - T| + |C - T| \) is minimal. If the three points are on one line the point \( T \) is simply given by the point which is between the two end points.

Let us assume the points are not on a line such that a triangle is formed between these points. Assume for the moment also that each angle of the triangle \( \triangle ABC \) is less than 120°. Select an arbitrary point \( P \) inside the triangle and rotate \( \triangle APB \) over 60° into \( \triangle AP'C' \) as is shown in the left part of Figure A.1. By construction \( \triangle ABC' \) is equilateral and we find for the distances \( |A - P| = |A - P'| = |P - P'| \) and \( |B - P| = |C' - P'| \). Thus the total distance is given by \( |A - P| + |B - P| + |C - P| = |C' - P'| + |P - P'| + |C - P| \), which is in general a broken line with angles at \( P' \) and \( P \). This line from \( C \) to \( C' \) via \( P \) and \( P' \) is minimal when it is straight, that is, when \( P \) and \( P' \) are on the line \( CC' \). This is the case when \( \angle APC = 180° - \angle APP' = 120° \) and \( \angle APB = \angle AP'C' = 180° - \angle APP' = 120° \), which results in the Torricelli point \( T = P \).

This procedure can be repeated for the other sides. Hence, after composing equilateral triangles \( \triangle A'BC \), \( \triangle AB'C \) and \( \triangle ABC' \) the Torricelli point can be found by the intersection of the lines \( AA', BB' \) and \( CC' \) as is shown in the right part of Figure A.1. Because the angles \( \angle BTC, \angle CTA \) and \( \angle ATB \) are all equal (120°) the Torricelli point is also known as the first isogonic center.

The Torricelli point can be expressed in terms of the coordinates of the points \( A, B \) and \( C \). First the accompanying points \( A', B' \) and \( C' \) have to be found. The point
Figure A.1: *The geometric construction to find the Torricelli point.*

$A'$ can be expressed as:

$$A' = \frac{B + C}{2} + E_{A'} \frac{|B - C|}{2} \sqrt{3}.$$  \hfill (A.1)

The unit vector $E_{A'}$ is perpendicular to $B - C$, perpendicular to the surface of the triangle $(B - A) \times (C - A)$ and pointing outward. Thus we find for $E_{A'}$:

$$E_{A'} = \frac{((B - A) \times (C - A)) \times (B - C)}{|((B - A) \times (C - A)) \times (B - C)|}.$$  \hfill (A.2)

For the points $B'$ and $C'$ similar equations can be found. To summarize:

$$A' = \frac{B + C}{2} + \frac{\sqrt{3}}{2} \frac{((B - A) \times (C - A)) \times (B - C)}{|(B - A) \times (C - A) \times (B - C)|} |B - C|.$$  \hfill (A.3)

$$B' = \frac{C + A}{2} + \frac{\sqrt{3}}{2} \frac{((C - B) \times (A - B)) \times (C - A)}{|(C - B) \times (A - B) \times (C - A)|} |C - A|.$$  \hfill (A.4)

$$C' = \frac{A + B}{2} + \frac{\sqrt{3}}{2} \frac{((A - C) \times (B - C)) \times (A - B)}{|(A - C) \times (B - C) \times (A - B)|} |A - B|.$$  \hfill (A.5)

Now the intersection between the lines $AA'$ and $BB'$ has to be found. The lines can
be parameterized as
\[ L_1(\alpha) = A + \alpha (A' - A), \]  
\[ L_2(\beta) = B + \beta (B' - B). \]  
(A.6)  

(A.7)

The equation \( L_1 = L_2 \) results in
\[ (A - B) + \alpha (A' - A) - \beta (B' - B) = 0. \]  
(A.8)

This equation can be multiplied by \((A' - A)\) and by \((B' - B)\) and thus two equations are obtained:
\[ (A' - A) \cdot (A - B) + \alpha |A' - A|^2 - \beta (A' - A) \cdot (B' - B) = 0, \]  
(A.9)

\[ (B' - B) \cdot (A - B) + \alpha (B' - B) \cdot (A' - A) - \beta |B' - B|^2 = 0, \]  
(A.10)

which can be used to find the parameter \( \alpha \):
\[ \alpha = \alpha_T = \frac{s_2 \cdot s_3 - s_1}{1 - s_3^2} \cdot \frac{1}{|A' - A|}, \]  
(A.11)

with
\[ s_1 = \frac{(A' - A) \cdot (A - B)}{|B' - B|}, \]  
(A.12)

\[ s_2 = \frac{(B' - B) \cdot (A - B)}{|B' - B|}, \]  
(A.13)

\[ s_3 = \frac{(A' - A) \cdot (B' - B)}{|A' - A| \cdot |B' - B|}. \]  
(A.14)

The Torricelli point is now given by:
\[ T = A + \alpha_T (A' - A). \]  
(A.15)

If one of the angles of the triangle \( \triangle ABC \) equals 120°, say \( \angle BCA \), then the Torricelli point coincides with \( C \). However if one of the angles is even bigger than 120°, say \( \angle BCA \), a geometric solution as in Figure A.1 cannot be found. The point \( T \) with the smallest possible sum of distances to \( A, B \) and \( C \) is then given by \( T = C \).

The minimal length \( L_T = |A - T| + |B - T| + |C - T| \) can be related to the length of the sides of the triangle, \( L_\Delta = |A - B| + |B - C| + |C - A| \). From the geometric construction of the Torricelli point it is clear that \( L_T \) is given by \( |A - A'| \) when all angles of \( \triangle ABC \) are less or equal to 120°. The fraction \( L_T/L_\Delta \) is minimal for an equilateral triangle \( ABC \) in which case \( L_T = L_\Delta/\sqrt{3} \). The maximum fraction is found when all points \( A, B \) and \( C \) are on one line, which results in \( L_T = L_\Delta/2 \). So we conclude that there is a rather constant relation between \( L_T \) and \( L_\Delta \),
\[ \frac{1}{2} \leq \frac{L_T}{L_\Delta} \leq \frac{1}{\sqrt{3}}. \]  
(A.16)
Appendix B

Magnetic moment of the multiplet

In this appendix the explicit formulas of the baryon wave function are constructed for the lowest SU(3)-flavor representation of the baryon multiplet revealing the (iso)spin structure. Matrix elements for the single quark current contribution to the magnetic moment of the baryon are given for the baryon multiplet.

Assuming that all quarks are in their ground state, the spin-isospin wave function has to be symmetric under the exchange of any two quarks. The color which takes care of the total anti-symmetrization is disregarded. In this way a baryon wave function can be constructed which satisfies the Pauli exclusion principle.

From three spin-1/2 particles, different combinations can be formed which are denoted as [8, 14, 37, 38],

\[ \chi_{3/2}^S = \left| \frac{3}{2} \frac{3}{2} \right\rangle = \uparrow \uparrow \uparrow, \]
\[ \chi_{1/2}^S = \left| \frac{3}{2} \frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} (\uparrow \downarrow \downarrow + \uparrow \uparrow \downarrow + \downarrow \uparrow \uparrow), \]
\[ \chi_{1/2}^\rho = \left| \frac{1}{2} \frac{1}{2} \right\rangle_\rho = \sqrt{\frac{1}{2}} (\downarrow \uparrow \uparrow - \uparrow \downarrow \uparrow), \]
\[ \chi_{1/2}^\lambda = \left| \frac{1}{2} \frac{1}{2} \right\rangle_\lambda = \sqrt{\frac{1}{6}} (\uparrow \downarrow \uparrow + \downarrow \uparrow \uparrow - 2 \uparrow \downarrow \downarrow). \]

Negative spin functions can be obtained by flipping all spins. It can be seen that Eqs. (B.1a), (B.1b) and (B.1d) are found by coupling a \( j_{12} = 1 \) state with \( j_3 = \frac{1}{2} \), whereas Eq. (B.1c) is formed from a \( j_{12} = 0 \) state and a \( j_3 = \frac{1}{2} \) state. Isospin functions can similarly be written down resulting in \( \eta_{m_i}^S \), \( \eta_{m_i}^\rho \), and \( \eta_{m_i}^\lambda \). Eqs. (B.1) contain all possible combinations and are orthonormal. The states \( S \) are totally symmetric while \( \rho \) and \( \lambda \) are mixed-symmetric states. When the interchange of particle \( i \) and \( j \) is denoted
by $P_{ij}$ they behave as

\[ P_{12} \chi^S = \chi^S, \quad P_{13} \chi^S = \chi^S, \]  
(\text{B.2})

\[ P_{12} \chi^\rho = -\chi^\rho, \quad P_{13} \chi^\rho = -\frac{1}{2} \sqrt{3} \chi^\lambda + \frac{1}{2} \chi^\rho, \]  
(\text{B.3})

\[ P_{12} \chi^\lambda = \chi^\lambda, \quad P_{13} \chi^\lambda = -\frac{1}{2} \sqrt{3} \chi^\rho - \frac{1}{2} \chi^\lambda. \]  
(\text{B.4})

The states $\rho$ and $\lambda$ are clearly not symmetric under the permutation of any two quarks. However, some specific combination of $\chi^{\rho, \lambda}$ and $\eta^{\rho, \lambda}$ is symmetric. Actually, from the states Eqs. (B.1) only two totally symmetric states can be formed:

\[ \Delta(m_s, m_i) = \chi^S_{m_s} \cdot \eta^S_{m_i}, \quad J = \frac{3}{2}, \quad I = \frac{3}{2}, \]  
(\text{B.5})

\[ N(m_s, m_i) = \frac{1}{\sqrt{2}} (\chi^\rho_{m_s} \eta^\rho_{m_i} + \chi^\lambda_{m_s} \eta^\lambda_{m_i}), \quad J = \frac{1}{2}, \quad I = \frac{1}{2}. \]  
(\text{B.6})

Eq. (B.5) and Eq. (B.6) represent the $\Delta$ and the nucleon, respectively. The formalism can be extended to the total baryon octet and decuplet by including the $s$-quark in writing a complete orthonormal set.

In this way the baryon flavor octet with total spin $1/2$ up can be determined:

\[ \Psi^p_{\text{sym}} = \frac{\sqrt{2}}{6} \left\{ 2d_- u_+ u_- - u_- d_+ u_+ - d_+ u_- u_+ + 2u_+ d_- u_+ ight\}, \]  
(B.7)

\[ \Psi^n_{\text{sym}} = \frac{\sqrt{2}}{6} \left\{ 2u_- d_+ d_- - d_- u_+ d_+ - u_+ d_- u_+ - d_+ u_- u_+ + 2u_+ u_- d_- \right\}, \]  
(B.8)

\[ \Psi^{\Sigma^+}_{\text{sym}} = \frac{\sqrt{2}}{6} \left\{ 2s_- u_+ u_- - u_- s_+ u_+ - s_+ u_- u_+ + 2u_+ u_- s_- \right\}, \]  
(B.9)

\[ \Psi^{\Sigma^0}_{\text{sym}} = \frac{-1}{6} \left\{ u_+ d_- s_+ + d_+ u_- s_+ + s_+ d_- u_+ + s_+ u_- d_+ - 2u_+ s_- d_- \right\}, \]  
(B.10)

\[ \Psi^{\Sigma^-}_{\text{sym}} = \frac{\sqrt{2}}{6} \left\{ 2s_- d_+ d_- - d_- s_+ d_+ - s_+ d_- d_+ + 2d_+ s_- d_- \right\}, \]  
(B.11)

\[ \Psi^\lambda_{\text{sym}} = \frac{\sqrt{3}}{6} \left\{ u_- d_+ s_+ - d_- u_+ s_+ + u_- s_+ d_+ - d_- s_+ u_+ - u_+ d_- s_+ + d_+ u_- s_+ + s_+ d_- u_+ - s_+ u_- d_+ - u_+ s_- d_- + d_+ s_- u_+ \right\}. \]  
(B.12)
\[ \Psi_{\text{sym}}^{\pm} = \frac{\sqrt{3}}{6} \left\{ 2u_- s_+ + s_- u_+ s_+ - u_+ s_- s_+ + 2s_+ u_- s_+ \\ - s_+ s_- u_+ - s_- s_+ u_+ - s_+ u_+ s_- - u_+ s_+ s_- + 2s_+ s_- u_- \right\}, \]

\[ \Psi_{\text{sym}}^{-} = \frac{\sqrt{3}}{6} \left\{ 2d_- s_+ + s_- d_+ s_+ - d_+ s_- s_+ + 2s_+ d_- s_+ \\ - s_+ s_- d_+ - s_- s_+ d_+ - s_+ d_+ s_- - d_+ s_+ s_- + 2s_+ s_- d_- \right\}, \]

where the subscripts (±) refer to the spin projection. For the flavor decuplet with total spin 3/2 up we have

\[ \Psi_{\text{sym}}^{\Delta^{++}} = u_+ u_+ u_+, \quad (B.15) \]
\[ \Psi_{\text{sym}}^{\Delta^+} = \frac{1}{\sqrt{3}} \left\{ u_+ u_+ d_+ + u_+ u_+ + d_+ u_+ u_+ \right\}, \quad (B.16) \]
\[ \Psi_{\text{sym}}^{\Delta^0} = \frac{1}{\sqrt{3}} \left\{ d_+ d_+ u_+ + d_+ u_+ d_+ + u_+ d_+ d_+ \right\}, \quad (B.17) \]
\[ \Psi_{\text{sym}}^{\Delta^-} = d_+ d_+ d_+, \quad (B.18) \]
\[ \Psi_{\text{sym}}^{\Sigma^+} = \frac{1}{\sqrt{3}} \left\{ u_+ u_+ s_+ + u_+ s_+ u_+ + s_+ u_+ u_+ \right\}, \quad (B.19) \]
\[ \Psi_{\text{sym}}^{\Sigma^0} = \frac{1}{\sqrt{6}} \left\{ u_+ d_+ s_+ + d_+ u_+ s_+ + u_+ s_+ d_+ + s_+ u_+ d_+ + d_+ s_+ u_+ + s_+ d_+ u_+ \right\}, \quad (B.20) \]
\[ \Psi_{\text{sym}}^{\Sigma^-} = \frac{1}{\sqrt{3}} \left\{ d_+ d_+ s_+ + d_+ s_+ d_+ + s_+ d_+ d_+ \right\}, \quad (B.21) \]
\[ \Psi_{\text{sym}}^{\Xi^0} = \frac{1}{\sqrt{3}} \left\{ s_+ s_+ u_+ + s_+ u_+ s_+ + u_+ s_+ s_+ \right\}, \quad (B.22) \]
\[ \Psi_{\text{sym}}^{\Xi^-} = \frac{1}{\sqrt{3}} \left\{ s_+ s_+ d_+ + s_+ d_+ s_+ + d_+ s_+ s_+ \right\}, \quad (B.23) \]
\[ \Psi_{\text{sym}}^{\Omega^-} = s_+ s_+ s_+. \quad (B.24) \]

These fully symmetrical wave functions Eqs. (B.7)-(B.24) can be written symbolically as

\[ \Psi_{JM,\text{sym}}^{N} = \Gamma_{JM}^{\alpha\beta\gamma}(f_1 f_2 f_3) \psi_{\alpha}^{f_1} \psi_{\beta}^{f_2} \psi_{\gamma}^{f_3}. \quad (B.25) \]

Since the s-quark orbital is heavier than the orbitals of the u- and d-quarks, Eq. (3.14) has to be separated in contributions from the u, d-quark and from the s-quark. Using the symmetrical wave function Eq. (B.25) this is realized by writing,

\[ \mu_z = 3\mu_z^{(1)} = \]
\[ 3 \sum_{f_1 f_2 f_3} \left( \Gamma_{JM}^{\alpha\beta\gamma}(f_1 f_2 f_3) \psi_{\alpha}^{f_1} \psi_{\beta}^{f_2} \psi_{\gamma}^{f_3} \right) e_q(1) \sigma_z(1) \left| \Gamma_{JM}^{\alpha\beta\gamma}(f_1 f_2 f_3) \psi_{\alpha}^{f_1} \psi_{\beta}^{f_2} \psi_{\gamma}^{f_3} \right| \lambda_{f_1}. \quad (B.26) \]
with,

\[ \lambda_{f_i} = -\frac{2}{3} \int g_{f_i}^*(r) f_{f_i}(r) r^3 dr. \]  \hfill (B.27)

The flavor index \( f_i \) can take the values \( u, d \) or \( s \). Note that \( \lambda_u = \lambda_d \) as the same orbital is taken for the \( u \)- and \( d \)-quark. Evaluating Eq. (B.26) for the different baryon wave functions Eqs. (B.7-B.24) results in the expressions in Table B.1.

Similarly, Eq. (3.51), which is the expression for the nucleon magnetic moment correction due to the anomalous magnetic moment of the quark, can be generalized to the baryon multiplet. We obtain

\[ \delta \mu_z = 3 \delta \mu_z^{(1)} = 
3 \sum_{f_1f_2f_3} \left\langle \Gamma_{JM}^{\alpha\beta\gamma} (f_1f_2f_3) \psi_1^{f_1} \psi_2^{f_2} \psi_3^{f_3} \right| \kappa_q(1) \sigma_z(1) \left| \Gamma_{JM}^{\alpha\beta\gamma} (f_1f_2f_3) \psi_1^{f_1} \psi_2^{f_2} \psi_3^{f_3} \right\rangle \rho_{f_1}, \]  \hfill (B.28)

with,

\[ \rho_{f_i} = \int r^2 dr \left( |g_{f_i}|^2 + |f_{f_i}|^2 / 3 \right). \]  \hfill (B.29)

Once again we have \( \rho_u = \rho_d \) since the same orbital is used for the \( u \) and \( d \) quark. The results are shown in Table B.1.
Table B.1: The matrix elements of the electromagnetic current for the baryons. The corrections $\delta\mu_z^{(1)}$ due to the anomalous magnetic moment of the quark are also listed.

<table>
<thead>
<tr>
<th>N</th>
<th>$\mu_z^{(1)}$</th>
<th>$\delta\mu_z^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$\frac{1}{3} \lambda_u$</td>
<td>$\frac{1}{3} (4\kappa_u - \kappa_d) \rho_u$</td>
</tr>
<tr>
<td>$n$</td>
<td>$-\frac{2}{9} \lambda_u$</td>
<td>$-\frac{1}{3} (\kappa_u - 4\kappa_d) \rho_u$</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$-\frac{1}{9} \lambda_s$</td>
<td>$\kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$\frac{8}{27} \lambda_u + \frac{1}{27} \lambda_s$</td>
<td>$\frac{4}{3} \kappa_u \rho_u - \frac{1}{3} \kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>$\frac{2}{27} \lambda_u + \frac{1}{27} \lambda_s$</td>
<td>$\frac{1}{3} (2\kappa_u + 2\kappa_d) \rho_u - \frac{1}{3} \kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>$-\frac{4}{27} \lambda_u + \frac{1}{27} \lambda_s$</td>
<td>$\frac{4}{3} \kappa_d \rho_u - \frac{1}{3} \kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$-\frac{4}{27} \lambda_s - \frac{2}{27} \lambda_u$</td>
<td>$-\frac{1}{3} \kappa_u \rho_u + \frac{4}{3} \kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>$-\frac{4}{27} \lambda_s + \frac{1}{27} \lambda_u$</td>
<td>$-\frac{1}{3} \kappa_d \rho_u + \frac{4}{3} \kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Delta^{++}$</td>
<td>$\frac{2}{3} \lambda_u$</td>
<td>$3\kappa_u \rho_u$</td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>$\frac{1}{3} \lambda_u$</td>
<td>$(2\kappa_u + \kappa_d) \rho_u$</td>
</tr>
<tr>
<td>$\Delta^0$</td>
<td>$0$</td>
<td>$(\kappa_u + 2\kappa_d) \rho_u$</td>
</tr>
<tr>
<td>$\Delta^-$</td>
<td>$-\frac{1}{9} \lambda_u$</td>
<td>$3\kappa_d \rho_u$</td>
</tr>
<tr>
<td>$\Sigma^{++}$</td>
<td>$\frac{4}{9} \lambda_u - \frac{1}{9} \lambda_s$</td>
<td>$2\kappa_u \rho_u + \kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Sigma^{0*}$</td>
<td>$\frac{1}{9} \lambda_u - \frac{1}{9} \lambda_s$</td>
<td>$(\kappa_u + \kappa_d) \rho_u + \kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Sigma^{-*}$</td>
<td>$-\frac{2}{9} \lambda_u - \frac{2}{9} \lambda_s$</td>
<td>$2\kappa_d \rho_u + \kappa_s \rho_s$</td>
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<tr>
<td>$\Xi^{0*}$</td>
<td>$-\frac{2}{9} \lambda_s + \frac{2}{9} \lambda_u$</td>
<td>$\kappa_u \rho_u + 2\kappa_s \rho_s$</td>
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<tr>
<td>$\Xi^{-*}$</td>
<td>$-\frac{2}{9} \lambda_s - \frac{1}{9} \lambda_u$</td>
<td>$\kappa_d \rho_u + 2\kappa_s \rho_s$</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>$-\frac{1}{3} \lambda_s$</td>
<td>$3\kappa_s \rho_s$</td>
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</tbody>
</table>
Appendix C

Pionic two-body contribution to the magnetic moment

In this appendix the pion-in-flight and pair contributions to the magnetic moment of the nucleons are given for both pseudoscalar (PS) and pseudovector (PV) coupling. In case of the PV coupling, a contact term appears, which contribution has also been determined. A non-relativistic reduction is performed for both couplings in the last section.

Following Ref. [44] the contributions mentioned above are calculated by taking the curl of the pionic two-body currents Eq. (3.38). The 3-quark state $\Psi$ is given by the product of three single quark orbitals Eq. (2.43). Because of symmetry considerations it suffices to calculate the magnetic moment contribution of pion exchange between, say, the second and the third quark, and multiply the result by a factor of 3 to include the contribution of the other possible permutations of quark pairs.

The momenta of the quarks are expressed in terms of Jacobi coordinates as defined in Eqs. (3.24). From imposing the Breit system and momentum conservation, we get $P' = -P = Q/2$ and $2\sqrt{3}(q' - q) = Q$. After taking the curl, the resulting expressions can be reduced by making use of symmetry among the integration parameters valid for $Q = 0$:

\[
\begin{align*}
\left\{ \begin{array}{l} p \to -p \\ p' \to -p' \end{array} \right\} &= \left\{ \begin{array}{l} k_2 \leftrightarrow k_3 \\ k'_2 \leftrightarrow k'_3 \end{array} \right\}, \\
\text{and}

p \leftrightarrow p' &= \left\{ \begin{array}{l} k_2 \leftrightarrow k'_2 \\ k_3 \leftrightarrow k'_3 \end{array} \right\}.
\end{align*}
\]

(C.1)
C.1 Pseudoscalar coupling

Considering the pion-in-flight contribution first, Eq. (3.29a), taking the curl gives rather long expressions which can be divided into two parts:

\[
\delta \mu_z^{\text{proton}} = -\delta \mu_z^{\text{neutron}} = 3 \left( \delta \mu_z^A + 3 \delta \mu_z^B \right). \tag{C.3}
\]

The first part gives the larger contribution and can be written as,

\[
\delta \mu_z^A = \lim_{Q^2 \to 0} \frac{eg_{\pi qq}^2}{3(2\pi)^3 N} \int d^3q d^3p d^3p' \frac{1}{(\Delta^2 + m_\pi^2)^2} \left( |\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2 \right)
\times \left\{ \tilde{g}(k_2') \tilde{f}(k_2) \tilde{g}(k_3') \tilde{f}(k_3) \frac{2}{k_2 k_3} \left( p \Delta - p_z \Delta_z \right) 
+ \tilde{g}(k_2') \tilde{f}(k_2) \tilde{f}(k_3') \tilde{g}(k_3) \frac{1}{k_2 k_3'} \left( (p - p' + 2\sqrt{\frac{1}{3} q_z}) \Delta 
- (p_z - p_z' + 2\sqrt{\frac{1}{3} q_z}) \Delta_z \right) \right\} \left( 1 + 2 \frac{\Delta^2 + m_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right) \left( \frac{\Lambda_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right)^2. \tag{C.4}
\]

The second part comes from the curl applied on the wave functions and gives

\[
\delta \mu_z^B = \lim_{Q^2 \to 0} \frac{2eg_{\pi qq}^2}{3(2\pi)^3 N} \int d^3q d^3p d^3p' \frac{1}{(\Delta^2 + m_\pi^2)^2} \left( |\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2 \right)
\times \left\{ \frac{\partial \tilde{g}(k_2')}{\partial k_2} \tilde{f}(k_2) \left[ \tilde{g}(k_3') \tilde{f}(k_3) \left( \hat{k}_2 \times \hat{k}_3 \right)_z - \tilde{f}(k_3') \tilde{g}(k_3) \left( \hat{k}_2 \times \hat{k}_3' \right)_z \right] 
- \left( \frac{\partial \tilde{f}(k_2')}{\partial k_2'} - \frac{1}{k_2'} \tilde{f}(k_2') \right) \tilde{g}(k_2) \times \left[ \tilde{g}(k_3') \tilde{f}(k_3) \left( \hat{k}_2 \times \hat{k}_3 \right)_z - \tilde{f}(k_3') \tilde{g}(k_3) \left( \hat{k}_2' \times \hat{k}_3' \right)_z \right] \right\} \times \frac{1}{3} \left( \hat{k}_2' \times \Delta \right)_z \left( 1 + 2 \frac{\Delta^2 + m_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right) \left( \frac{\Lambda_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right)^2. \tag{C.5}
\]

The normalization factor $N$ is the same as used before in the single quark current contribution (Eq. (3.22)). In writing these expressions, we have made use of the spin-isospin operator sandwiched between the fully symmetric wave functions in spin-isospin and orbital space of the 3 quarks,

\[
\left\langle \psi_{\text{sym}}^p \left| \boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)} \right| \sigma_i^{(1)} \sigma_j^{(2)} \right\rangle_{\text{sym}} \psi_{\text{sym}}^p = 
- \left\langle \psi_{\text{sym}}^n \left| \boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)} \right| \sigma_i^{(1)} \sigma_j^{(2)} \right\rangle_{\text{sym}} = -\frac{2}{3} \epsilon_{ij3}. \tag{C.6}
\]
It should be noted that the spin-isospin factor (C.6) is identical to that found in the trinucleon case. For all the other baryon wave functions given in Appendix B, the matrix elements of the two-body electromagnetic operators considered here vanish, because of the isospin structure of the operator involved. Hence the two-body currents considered contribute only to the magnetic moment of the proton and neutron.

The second part $\delta \mu_B^z$ is a relativistic effect which enlarges the values by about 10% and which vanishes in the static limit as is shown in section C.3.

In the same way the pair term can be analyzed. We find:

$$\delta \mu_z^{\text{proton}} = -\delta \mu_z^{\text{neutron}} = 3 \left( \delta \mu_z^C + 3 \delta \mu_z^D \right),$$

where

$$\delta \mu_z^C = \lim_{Q^2 \to 0} \frac{e g_{\piqq}^2}{m_q (2\pi)^3 N} \int d^3 q d^3 p d^3 p' \frac{1}{\Delta^2 + m_q^2} \frac{1}{3} \left( |\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2 \right)$$

$$\times \tilde{g}(k'_2) \tilde{f}(k_2) \tilde{g}(k'_3) \tilde{g}(k_3) \frac{1}{k_2}$$

$$\times \left\{ 1 - (\Delta k_2 - \Delta_z k_2 z) \left( \frac{1}{\Delta^2 + m_q^2} + \frac{2}{\Delta^2 + \Lambda_\pi^2} \right) \right\} \left( \frac{\Lambda_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right)^2,$$

(C.7)

and

$$\delta \mu_z^D = \lim_{Q^2 \to 0} \frac{e g_{\piqq}^2}{6m_q (2\pi)^3 N} \int d^3 q d^3 p d^3 p' \frac{1}{\Delta^2 + m_q^2} \frac{1}{3} \left( |\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2 \right)$$

$$\times \left\{ - \frac{\partial \tilde{g}(k'_2)}{\partial k'_2} \tilde{g}(k_2) \left[ \tilde{g}(k'_3) \tilde{f}(k_3) \left( \hat{k}_2 \hat{k}_3 - (\hat{k}_2')_z (\hat{k}_3)_z \right) \right. \right.$$  

$$+ \tilde{f}(k'_3) \tilde{g}(k_3) \left( \hat{k}_2' \hat{k}_3 - (\hat{k}_2')_z (\hat{k}_3')_z \right) \right. \left.$$  

$$+ \tilde{g}(k'_3) \tilde{g}(k_3) \left. \left[ - \frac{\partial \tilde{g}(k'_2)}{\partial k'_2} \tilde{f}(k_2) \left( \hat{k}_2 \hat{k}_3 - (\hat{k}_2')_z (\hat{k}_3)_z \right) \right. \right.$$  

$$+ \left. \left( \frac{\partial \tilde{f}(k'_2)}{\partial k'_2} - \frac{1}{k'_2} \tilde{f}(k'_2) \right) \tilde{g}(k_2) \left( \hat{k}_2 \hat{k}_3 - (\hat{k}_2')_z (\hat{k}_3')_z \right) \right] \right\} \left( \frac{\Lambda_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right)^2. \quad \text{(C.8)}$$

C.2 Pseudovector coupling

In the same way as for the pseudoscalar case considered in the previous section, the expressions are divided into two parts,

$$\delta \mu_z^{\text{proton}} = -\delta \mu_z^{\text{neutron}} = 3 \left( \delta \mu_z^A + 3 \delta \mu_z^B \right). \quad \text{(C.10)}$$
Considering the pion-in-flight current first, the first part gives the larger contribution and can be written as:

\[
\delta \mu_z^A = \lim_{Q^2 \to 0} \frac{2e^2 g_{\pi qq}^2}{3(2\pi)^3 N m_{\pi}^2} \int d^3qd^3pd^3p' \frac{1}{(\Delta^2 + m_{\pi}^2)^2} \left((|\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2)\right)
\]

\[
\times \left\{ \tilde{g}(k_2')\tilde{g}(k_2)\tilde{g}(k_3')\tilde{g}(k_3) \frac{2}{3} \Delta^2 + \tilde{g}(k_2')\tilde{f}(k_2')\tilde{f}(k_3) \frac{1}{k_2'k_3} \times \left[ \frac{2}{3} k_2'k_3 \Delta^2 - \frac{2}{3} k_2'k_3 \Delta + \frac{4}{3} k_2' \Delta k_3 - \frac{2}{3} k_3' \Delta^2 \right] + \tilde{f}(k_2')\tilde{f}(k_2)\tilde{f}(k_3) \frac{1}{k_2'k_3} \left[ \frac{1}{3} k_2' k_3 \Delta^2 \right] \left( \frac{\Lambda_{\pi}^2}{\Delta^2 + \Lambda_{\pi}^2} \right)^2 \right\}. \tag{C.11}
\]

The second part comes from the curl applied to the wave functions and gives

\[
\delta \mu_z^B = \lim_{Q^2 \to 0} \frac{2e^2 g_{\pi qq}^2}{3(2\pi)^3 N m_{\pi}^2} \int d^3qd^3pd^3p' \frac{1}{(\Delta^2 + m_{\pi}^2)^2} \left((|\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2)\right)
\]

\[
\times \left\{ \frac{\partial \tilde{g}(k_2')}{\partial k_2'} \tilde{g}(k_2)\tilde{f}(k_3')\tilde{f}(k_3) \frac{1}{k_2'k_3} \times \left[ \frac{2}{9} \left[ k_2' k_3 (k_2' k_3 \Delta^2 - k_2' \Delta k_3 \Delta) - k_3 (k_2' k_3 \Delta^2 - k_2' \Delta k_3 \Delta) \right] \right] + \tilde{g}(k_2')\tilde{g}(k_2) \left( \frac{1}{k_2'} \tilde{f}(k_3') \right) \tilde{f}(k_3) \frac{1}{k_3} \left[ \frac{1}{2} k_3' \left( k_3' k_3 \Delta^2 - k_3' \Delta k_3 \Delta \right) - k_3 \left( k_3' \Delta^2 - k_3' \Delta k_3 \Delta \right) \right] \times \frac{2}{9} \left[ k_2' k_3' \left( k_2' k_3 \Delta^2 - k_2' \Delta k_3 \Delta \right) - k_2' \left( k_2' \Delta^2 - k_2' \Delta k_2 \Delta \right) \right] \right\} \times \left( 1 + 2 \frac{\Delta^2 + m_{\pi}^2}{\Delta^2 + \Lambda_{\pi}^2} \right) \left( \frac{\Lambda_{\pi}^2}{\Delta^2 + \Lambda_{\pi}^2} \right)^2 \tag{C.12}
\]
The second part comes from the curl applied to the wave functions and yields

\[
\delta \mu_z^{\text{proton}} = -\delta \mu_z^{\text{neutron}} = 3 \left( \delta \mu_z^C + 3 \delta \mu_z^D \right),
\]

where the first term gives the contribution:

\[
\begin{align*}
\delta \mu_z^C &= \lim_{Q^2 \to 0} \frac{e g_{\pi qq}^2}{3(2\pi)^3 N_\text{etf}} \frac{1}{m_q} \int d^3q d^3p' \frac{1}{\Delta^2 + m_r^2} \left( \frac{\Lambda_r^2}{\Delta^2 + \Lambda_r^2} \right)^2 \\
&\times \left( |\bar{g}(k_1)|^2 + |\bar{f}(k_1)|^2 \right) \left\{ \left( \frac{1}{\Delta^2 + m_r^2} + \frac{2}{\Delta^2 + \Lambda_r^2} \right) \right. \\
&\left. \times \left[ \bar{g}(k_2')\bar{g}(k_2)\bar{g}(k_3')\tilde{f}(k_3) \left( \frac{2\Delta^2 k_3\Delta}{k_3} + \frac{\tilde{f}(k_2')\tilde{f}(k_2)\bar{g}(k_3')\tilde{f}(k_3)}{k_2'k_2'k_3} \right) \right] \right. \\
&\left. + \bar{g}(k_2')\tilde{g}(k_2)\tilde{g}(k_3')\tilde{f}(k_3) \left( \frac{-2k_3\Delta}{k_3} + \frac{\tilde{f}(k_2')\tilde{f}(k_2)\tilde{g}(k_3')\tilde{f}(k_3)}{k_2'k_2'k_3} \right) \right\}.
\end{align*}
\]

The second part comes from the curl applied to the wave functions and yields

\[
\begin{align*}
\delta \mu_z^D &= \lim_{Q^2 \to 0} \frac{e g_{\pi qq}^2}{9(2\pi)^3 N_\text{etf}} \frac{1}{2m_q} \int d^3q d^3p' \frac{1}{\Delta^2 + m_r^2} \left( \frac{\Lambda_r^2}{\Delta^2 + \Lambda_r^2} \right)^2 \\
&\times \left( |\bar{g}(k_1)|^2 + |\bar{f}(k_1)|^2 \right) \left\{ \left( \frac{1}{\Delta^2 + m_r^2} + \frac{2}{\Delta^2 + \Lambda_r^2} \right) \right. \\
&\left. \times \left[ \frac{\partial \bar{g}(k_2')}{\partial k_2'} \bar{g}(k_2)\bar{g}(k_3')\tilde{f}(k_3) \left( \frac{1}{k_2'k_3} \right) \frac{3k_2'\Delta k_3\Delta - \Delta^2 k_2'k_3}{9} \right] \\
&\left. + \frac{\partial \bar{g}(k_3')}{\partial k_3'} \bar{g}(k_2)\bar{g}(k_3')\tilde{f}(k_3) \left( \frac{1}{k_2'k_3} \right) \frac{-3k_2'\Delta k_3\Delta + \Delta^2 k_2'k_3}{9} \right\} \\
&\left. + \left( \frac{\partial \tilde{f}(k_2')}{\partial k_2'} - \frac{1}{k_2'} \tilde{f}(k_2') \right) \tilde{f}(k_2)\bar{g}(k_3')\tilde{f}(k_3) \left( \frac{1}{k_2'k_2'k_3} \right) \\
&\left. \times \frac{1}{9} k_2' \left[ -2k_2'k_2k_3\Delta + k_2'k_3k_2\Delta - k_2k_3k_2'\Delta + 2k_2^2 k_3\Delta \right] \right] \\
&\left. + \left( \frac{\partial \tilde{f}(k_2')}{\partial k_2'} - \frac{1}{k_2'} \tilde{f}(k_2') \right) \tilde{f}(k_2)\bar{g}(k_3')\tilde{f}(k_3) \left( \frac{1}{k_2'k_2'k_3} \right) \\
&\left. \times \frac{1}{9} k_2' \left[ 2k_2'k_2k_3\Delta - k_2'k_3k_2\Delta + k_2k_3'k_2'\Delta - 2k_2^2 k_3\Delta \right] \right] \\
&\left. + \bar{g}(k_2')\frac{\partial \bar{g}(k_3')}{\partial k_3'} \tilde{f}(k_3) \left( \frac{1}{k_2'k_3} \right) \frac{3k_3'\Delta k_3\Delta - \Delta^2 k_3'k_3}{9} \right\}.
\end{align*}
\]
\[ + \tilde{g}(k'_2)\tilde{f}(k'_2) \left( \frac{\partial \tilde{f}(k'_3)}{\partial k'_3} - \frac{1}{k'_3} \tilde{f}(k'_3) \right) \tilde{f}(k'_3) \frac{1}{k'_3 k'_3} \frac{1}{9} \left( -3k'_3 \Delta k'_3 \Delta + \Delta^2 k'_3^2 \right) \]

\[ + \tilde{f}(k'_2)\tilde{f}(k'_2) \left( \frac{\partial \tilde{f}(k'_3)}{\partial k'_3} - \frac{1}{k'_3} \tilde{f}(k'_3) \right) \tilde{g}(k'_3) \frac{1}{k'_2 k'_2 k'_3} \]

\[ \times \frac{1}{9} \left[ k'_2^2 \left( -2k'_2 k'_3 k_3 \Delta + k'_3 k_3 k_2 \Delta - k_2 k'_3 k_3 \Delta \right) \right. \]

\[ \left. + k'_2^2 \left( 2k'_2 k'_3 k_3 \Delta - k_3 k'_3 k_2 \Delta + k_3 k'_2 k_3 \Delta \right) \right] \]

\[ + \tilde{f}(k'_2)\tilde{f}(k'_2) \left( \frac{\partial \tilde{f}(k'_3)}{\partial k'_3} - \frac{1}{k'_3} \tilde{f}(k'_3) \right) \tilde{g}(k'_3) \frac{1}{k'_2 k'_2 k'_3} \]

\[ \times \frac{1}{9} \left[ k'_2^2 \left( 2k'_2 k'_3 k_3 \Delta - k'_3^2 k_2 \Delta + k_2 k'_3 k_3 \Delta \right) \right. \]

\[ \left. + k'_2^2 \left( -2k'_2 k'_3 k_3 \Delta + k'_3^2 k_2 \Delta - k'_2 k'_3 k'_3 \Delta \right) \right] \} \right( C.15 \)

As a PV coupling is assumed, there is also a contact term which contributes. Its contribution is divided into two parts,

\[ \delta \mu^\text{proton}_z = -\delta \mu^\text{neutron}_z = 3 \left( \delta \mu^E_z + 3 \delta \mu^F_z \right), \]

of which the first part gives the larger contribution and can be written as,

\[ \delta \mu^E_z = \lim_{Q^2 \to 0} \frac{eg^2_{\pi\pi}}{(2\pi)^3 N^4 m^2_{\text{eff}}} \int d^3q d^3p d^3p' \frac{1}{\Delta^2 + m^2_\pi} \left( \frac{\Lambda^2_\pi}{\Delta^2 + \Lambda^2_\pi} \right)^2 \]

\[ \left( |\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2 \right) \left\{ \left( \frac{1}{\Delta^2 + m^2_\pi} + \frac{2}{\Delta^2 + \Lambda^2_\pi} \right) \left[ \tilde{g}(k'_2)\tilde{g}(k'_2)\tilde{g}(k'_3)\tilde{g}(k'_3) \frac{4}{9} \Delta^2 \right. \right. \]

\[ \left. + \tilde{g}(k'_2)\tilde{g}(k'_2)\tilde{f}(k'_3)\tilde{f}(k'_3) \frac{-4\Delta k_3}{9k'_3 k'_3} \left( \Delta k'_3 - 2k'_3 k'_3 \right) \right] \]

\[ + \tilde{f}(k'_2)\tilde{f}(k'_2)\tilde{f}(k'_3)\tilde{f}(k'_3) \frac{4k'_2^2}{9k'_2 k'_2 k'_3} \left( \Delta k'_3 k'_3 k'_3 + \Delta k'_3 k'_3 k'_3 \right) \]

\[ + \tilde{f}(k'_2)\tilde{f}(k'_2)\tilde{f}(k'_3)\tilde{f}(k'_3) \frac{2}{9} \left( \frac{3\Delta k'_3 + 2k'_3 k'_3}{k'_3 k'_3} \right) \]

\[ \left. + \tilde{f}(k'_2)\tilde{f}(k'_2)\tilde{f}(k'_3)\tilde{f}(k'_3) \frac{2}{9} \left( \frac{k'_3 k'_2 k'_3 + k'_2 k'_3 k'_3 - k'_2 k'_3 k'_3 + 2k'_2 k'_3 k'_3}{k'_2 k'_2 k'_3 k'_3} \right) \right\} \right( C.17 \)

The second part comes from the curl applied to the wave functions

\[ \delta \mu^F_z = \lim_{Q^2 \to 0} \frac{eg^2_{\pi\pi}}{3(2\pi)^3 N^4 m^2_{\text{eff}}} \int d^3q d^3p d^3p' \frac{1}{\Delta^2 + m^2_\pi} \left( \frac{\Lambda^2_\pi}{\Delta^2 + \Lambda^2_\pi} \right)^2 \]
In the non-relativistic limit, the lower component of the wave function can be expressed in terms of the upper component

\[
\tilde{f}(k) = -\frac{|k|}{2m_q} \tilde{g}(k),
\]  

(C.19)

where \( m_q \) is the constituent mass of the quark. As a result, the pionic two-body current contributions can be simplified considerably. When only the leading term \( 1/m_q^2 \) is kept we obtain for the pion-in-flight contribution

\[
\delta \mu^A = \frac{e g_{\pi qq}^2}{6m_q^2(2\pi)^3 N} \int d^3qd^3p d^3p' \left( |\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2 \right) \tilde{g}(k_2') \tilde{g}(k_2) \tilde{g}(k_3') \tilde{g}(k_3) \times \frac{\Delta^2 - \Delta_z^2}{(\Delta^2 + m_q^2)^2} \left( 1 + \frac{2\Delta^2 + m_q^2}{\Delta^2 + \Lambda_\pi^2} \right) \left( \frac{\Lambda_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right)^2.
\]  

(C.20)

In case of a PS coupling we get for the pair term

\[
\delta \mu^C = \frac{e g_{\pi qq}^2}{6m_q^2(2\pi)^3 N} \int d^3qd^3p d^3p' \left( |\tilde{g}(k_1)|^2 + |\tilde{f}(k_1)|^2 \right) \tilde{g}(k_2') \tilde{g}(k_2) \tilde{g}(k_3') \tilde{g}(k_3) \times \left\{ \left( \frac{\Delta^2 - \Delta_z^2}{\Delta^2 + m_q^2} + \frac{2\Delta^2 + m_q^2}{\Delta^2 + \Lambda_\pi^2} \right) - 1 \right\} \frac{1}{\Delta^2 + m_q^2} \left( \frac{\Lambda_\pi^2}{\Delta^2 + \Lambda_\pi^2} \right)^2.
\]  

(C.21)

In case of a PV coupling the pair term has no term proportional to \( 1/m_q^2 \) in the non-relativistic limit. The contact term however reduces to Eq. (C.21). All terms
which come from derivatives on the wave functions, $\delta \mu^B_z$, $\delta \mu^D_z$ and $\delta \mu^F_z$, vanish. These expressions agree with the results of Refs. [17,44].
Appendix D

Anomalous magnetic moment contributions from meson loops

In this appendix explicit formulas for the contribution of one-loop diagrams to the anomalous magnetic moment of the quarks are given. Only diagrams involving pseudoscalar mesons are considered. For the coupling of the pseudoscalar meson to the quark, a pseudoscalar coupling has been used in the first section, Eq. (3.26), and a pseudovector coupling in the second section, Eq. (3.28). In the last section some useful formulas used in the calculation of the integrals are given.

D.1 Pseudoscalar coupling

Our starting point is the electromagnetic currents, corresponding to the one-loop diagrams shown in Fig. 3.2, and assuming a $\gamma_5$ coupling of the pion to the quark. This results in

$$J^{(a)}_\mu = -2i g^{2}_{\piqq} e \tau_z$$

$$\times \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_5 (p - k + M_q) \gamma_5 (2k_\mu + Q_\mu)}{[(p - k)^2 - M_q^2 + i\epsilon] [k^2 - m_\pi^2 + i\epsilon] [(k + Q)^2 - m_\pi^2 + i\epsilon]}$$

$$\times \frac{\Lambda^2_\pi}{k^2 - \Lambda^2_\pi} \frac{\Lambda^2_\pi}{(k + Q)^2 - \Lambda^2_\pi} \left(1 + \frac{k^2 - m_\pi^2}{(k + Q)^2 - \Lambda^2_\pi} + \frac{(k + Q)^2 - m_\pi^2}{k^2 - \Lambda^2_\pi}\right)$$

$$\equiv -2ie\tau_z \left(\gamma^\nu C^{(a)}_{\mu\nu} + C^{(a)}_{\mu}\right),$$

(D.1)
\[ J^{(b)}_{\mu} = -ig^2_{\pi qq} \frac{1 - \tau_z}{2} \]

\[
\times \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_5 (p^\prime - k + M_q) \gamma_\mu (p^\prime - k + M_q) \gamma_5}{[(p^\prime - k)^2 - M_q^2 + i\epsilon][(p - k)^2 - M_q^2 + i\epsilon][k^2 - m_q^2 + i\epsilon]} \times \left( \frac{\Lambda^2_\pi}{k^2 - \Lambda^2_\pi} \right)^2 \]

\[ \equiv ie \frac{1 - \tau_z}{2} \left( \gamma^\nu C^{(b)}_{\mu\nu} + C^{(b)}_{\mu} \right). \tag{D.2} \]

Writing these equations we have used the explicit evaluation of the \(\gamma\)-matrix algebra and of the approximation that the initial and final quark are on their mass shell. To be able to discuss more general diagrams, we allow the masses of the external quark \(m_q\) and the intermediate quark \(M_q\) to be different. In case of the pionic fluctuations of the \(u\) and \(d\) quarks, the equations can be reduced using \(M_q = m_q\).

Since we have assumed a finite form factor at the \(\pi qq\) vertex, as in the two-body current case, the two additional terms are needed in the last factor of Eq. (D.1) to satisfy the Ward-Takahashi identity, Eq. (3.41).

From these currents, the anomalous magnetic moment has to be extracted. As was discussed in Subsection 3.2.2, this can be done by calculating the Lorentz-invariant terms \(A^{(a)}_1\) and \(B^{(a)}_1\) which are found as described in Eqs. (3.48) and (3.49). For the diagram where the photon couples to the meson, the expressions for \(A^{(a)}_1\) and \(B^{(a)}_1\) are

\[ A^{(a)}_1 = \frac{g^2_{\pi qq}}{6m_q^4} \int \frac{4(p \cdot k)^2 - p^2 k^2}{k^2 - 2pk + m_q^2 - M_q^2 + i\epsilon} \left[ k^2 - m_q^2 + i\epsilon \right] \left( \frac{\Lambda^2_\pi}{k^2 - \Lambda^2_\pi} \right)^2 \left( 1 + \frac{k^2 - m^2_\pi}{k^2 - \Lambda^2_\pi} \right) \]

\[ \times \left( \frac{\Lambda^2_\pi}{k^2 - \Lambda^2_\pi} \right)^2 \left( 1 + \frac{k^2 - m^2_\pi}{k^2 - \Lambda^2_\pi} \right) \]

\[ = \frac{-i}{32\pi^2} g^2_{\pi qq} \int_0^1 d\alpha \alpha (1 - \alpha)^2 \]

\[ \times \left[ \frac{1}{F_{m_\pi}} - \frac{1}{F_{\Lambda_\pi}} \right] \left( \frac{\Lambda^2_\pi}{\Lambda^2_\pi - m^2_\pi} \right)^2 \alpha \left( \frac{\Lambda^2_\pi}{F_{\Lambda_\pi}^2} \right) \left( \frac{\Lambda^2_\pi}{\Lambda^2_\pi - m^2_\pi} \right), \tag{D.3} \]

and

\[ B^{(a)}_1 = \frac{g^2_{\pi qq}}{m_q^2} \int \frac{(M_q - m_q) p \cdot k}{k^2 - 2pk + m_q^2 - M_q^2 + i\epsilon} \left[ k^2 - m^2_\pi + i\epsilon \right] \left( \frac{\Lambda^2_\pi}{k^2 - \Lambda^2_\pi} \right)^2 \left( 1 + \frac{k^2 - m^2_\pi}{k^2 - \Lambda^2_\pi} \right) \]

\[ \times \left( \frac{\Lambda^2_\pi}{k^2 - \Lambda^2_\pi} \right)^2 \left( 1 + \frac{k^2 - m^2_\pi}{k^2 - \Lambda^2_\pi} \right) \]
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\[= -\frac{i}{16\pi^2}g_{\pi qq}^2 (M_q - m_q) \int_0^1 d\alpha \alpha (1 - \alpha) \times \left[ \left( \frac{1}{F_{m\pi}} - \frac{1}{F_{\Lambda\pi}} \right) \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right)^2 - \alpha \frac{\Lambda_{\pi}^2}{F_{m\pi}^2} \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right], \quad (D.4)\]

The expressions for \( A_1^{(b)} \) and \( B_1^{(b)} \), which are related to the process where the photon couples to the intermediate quark, are

\[A_1^{(b)} = \frac{g_{\pi qq}^2}{6m_q^4} \int \frac{4 (p \cdot k)^2 - p^2k^2}{[k^2 - 2pk + m_q^2 - M_q^2 + \imath\epsilon]^2 [k^2 - m_{\pi}^2 + \imath\epsilon]} \frac{\left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - k^2} \right)^2 d^4k}{(2\pi)^4} \]
\[= \frac{-i}{32\pi^2}g_{\pi qq}^2 \int_0^1 d\alpha (1 - \alpha)^3 \times \left[ \left( \frac{1}{F_{m\pi}} - \frac{1}{F_{\Lambda\pi}} \right) \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right)^2 - \alpha \frac{\Lambda_{\pi}^2}{F_{m\pi}^2} \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right], \quad (D.5)\]

and

\[B_1^{(b)} = \frac{g_{\pi qq}^2}{m_q^2} \int \frac{(M_q - m_q) p \cdot k}{[k^2 - 2pk + m_q^2 - M_q^2 + \imath\epsilon]^2 [k^2 - m_{\pi}^2 + \imath\epsilon]} \frac{\left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - k^2} \right)^2 d^4k}{(2\pi)^4} \]
\[= \frac{-i}{16\pi^2}g_{\pi qq}^2 (M_q - m_q) \int_0^1 d\alpha (1 - \alpha)^2 \times \left[ \left( \frac{1}{F_{m\pi}} - \frac{1}{F_{\Lambda\pi}} \right) \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right)^2 - \alpha \frac{\Lambda_{\pi}^2}{F_{m\pi}^2} \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right]. \quad (D.6)\]

In these formulas, \( F_m \) is defined by

\[F_m = (1 - \alpha)^2 m_q^2 + (1 - \alpha) (M_q^2 - m_q^2) + \alpha m_q^2. \quad (D.7)\]

**D.2 Pseudovector coupling**

In case of pseudovector coupling, the PS vertex has to be changed into the PV vertex by applying Eq. (3.27). The currents can be reduced to

\[J_{\mu}^{(a)} = -2i \frac{g_{\pi qq}^2 e \tau_z}{4m_q^2} \int \frac{d^4k}{(2\pi)^4} \left\{ \frac{2k_\mu k \left( k^2 - 2p \cdot k + 2m_q (m_q + M_q) \right)}{[k^2 - 2pk + m_q^2 - M_q^2 + \imath\epsilon] [k^2 - m_{\pi}^2 + \imath\epsilon]^2} \right. \]
\[+ \left. \frac{2k_\mu \left( k^2 - 2p \cdot k \right) (M_q + m_q)}{[k^2 - 2pk + m_q^2 - M_q^2 + \imath\epsilon] [k^2 - m_{\pi}^2 + \imath\epsilon]^2} \right\} \]
The expressions for $A_1^{(a)}$ and $B_1^{(a)}$ are

$$A_1^{(a)} = \frac{g^2_{\pi q}}{24m_q^6} \int \frac{4 \left( p \cdot k \right)^2 - p^2 k^2}{(2\pi)^4} \frac{k^2 - 2p \cdot k + 2m_q (m_q + M_q)}{[k^2 - 2pk + m_q^2 - M_q^2 + ie]^2} \left( \frac{\Lambda^2_\pi}{\Lambda^2_\pi - k^2} \right)^2 \left( 1 + 2 \frac{k^2 - m^2_\pi}{k^2 - \Lambda^2_\pi} \right) d^4k$$

$$= -i \frac{g^2_{\pi q}}{32\pi^2 4m_q^2} (M_q + m_q)^2 \int_0^1 d\alpha (1 - \alpha)^2 \left[ \frac{1}{F_{m_\pi}} - \frac{1}{F_{A_\pi}} \right] \left( \frac{\Lambda^2_\pi}{\Lambda^2_\pi - m^2_\pi} \right)^2 - \alpha \frac{\Lambda^2_{\pi}}{F^2_{\Lambda_\pi}} \left( \frac{\Lambda^2_\pi}{\Lambda^2_\pi - m^2_\pi} \right)^2 ,$$

(D.11)

and

$$B_1^{(a)} = \frac{g^2_{\pi q}}{4m_q^2} \int \frac{(M_q + m_q) \left( k^2 - 2p \cdot k \right) p \cdot k}{[k^2 - 2pk + m_q^2 - M_q^2 + ie]^2} \left( \frac{\Lambda^2_\pi}{\Lambda^2_\pi - k^2} \right)^2 \left( 1 + 2 \frac{k^2 - m^2_\pi}{k^2 - \Lambda^2_\pi} \right) d^4k$$

The expressions for $A_1^{(a)}$ and $B_1^{(a)}$ are
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\[
\frac{-i}{16\pi^2} \frac{g_{\pi qq}^2}{4m_q^2} (M_q - m_q) (m_q + M_q)^2 \int_0^1 d\alpha (1 - \alpha) \times \left[ \left( \frac{1}{F_{m_q}} - \frac{1}{F_{\Lambda_q}} \right) \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right)^2 - \frac{\Lambda_{\pi}^2}{F_{\Lambda_q}^2} \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right]
\]  

(D.12)

The expressions for \( A^{(b)}_1 \) and \( B^{(b)}_1 \) are

\[
A^{(b)}_1 = \frac{g_{\pi qq}^2}{24m_q^6} \int \frac{(4(p \cdot k)^2 - p^2 k^2)}{[k^2 - 2pk + m_q^2 - M_q^2 + i\epsilon]^2 [k^2 - m_{\pi}^2 + i\epsilon]^2} \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - k^2} \right)^2 \frac{d^4k}{(2\pi)^4}
\]

\[
= \frac{-i}{32\pi^2} \frac{g_{\pi qq}^2}{4m_q^2} (m_q + M_q)^2 \int_0^1 d\alpha (1 - \alpha)^3 \times \left[ \left( \frac{1}{F_{m_q}} - \frac{1}{F_{\Lambda_q}} \right) \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right)^2 - \frac{\Lambda_{\pi}^2}{F_{\Lambda_q}^2} \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right],
\]

(D.13)

and

\[
B^{(b)}_1 = \frac{g_{\pi qq}^2}{4m_q^6} \int \frac{(M_q + m_q) p \cdot k}{[k^2 - 2pk + m_q^2 - M_q^2 + i\epsilon]^2 [k^2 - m_{\pi}^2 + i\epsilon]^2} \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - k^2} \right)^2 \frac{d^4k}{(2\pi)^4}
\]

\[
= \frac{-i}{16\pi^2} \frac{g_{\pi qq}^2}{4m_q^2} (M_q - m_q) (m_q + M_q)^2 \int_0^1 d\alpha (1 - \alpha)^2 \times \left[ \left( \frac{1}{F_{m_q}} - \frac{1}{F_{\Lambda_q}} \right) \left( \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right)^2 - \frac{\Lambda_{\pi}^2}{F_{\Lambda_q}^2} \frac{\Lambda_{\pi}^2}{\Lambda_{\pi}^2 - m_{\pi}^2} \right].
\]

(D.14)

The current resulting from the contact term is proportional to \( \gamma_{\mu} \) since \( k \) in Eq. (D.10) gets replaced through the loop integral by \( \not{p} \) and \( \not{p} \) respectively. They reduce to \( m_q \) when they act on the external quark. The contact term does therefore not contribute to the anomalous magnetic moment, \( A^{(c)}_1 = 0 \) and \( B^{(c)}_1 = 0 \).

In the formulas above \( F_m \) is defined in Eq. (D.7). Although the expressions for the currents become much more involved using a PV coupling, the final expression can simply be written in terms of the result from the previous calculation as shown in Eqs. (3.50).

### D.3 Useful formulas

In the calculation of one-loop integrals frequent use has been made of the Feynman parameterization

\[
\frac{1}{ab} = \int_0^1 d\alpha \frac{1}{[\alpha a + (1 - \alpha)b]^2}
\]

(D.15)
This formula can be generalized to

\[
\frac{1}{a_1a_2\ldots a_n} = (n - 1)! \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \ldots \int_0^{1-\alpha_1-\alpha_2-\ldots-\alpha_{n-2}} d\alpha_{n-1} \times \frac{1}{[\alpha_1a_1 + \alpha_2a_2 + \ldots + (1 - \alpha_1 - \alpha_2 - \ldots - \alpha_{n-1})a_n]^n}, \quad (D.16)
\]

which can be proven by induction. All loop integrals in the text can be reduced to one of the following forms [79]

\[
\int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{[(k-l)^2 - F_m]^2} - \frac{1}{[(k-l)^2 - F_{\Lambda_\pi}]^2} \right) \{1, k^\mu\} = -i \frac{1}{16\pi^2} \ln \left( \frac{F_m}{F_{\Lambda_\pi}} \right) \{1, l^\mu\}, \quad (D.17)
\]

\[
\int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{[(k-l)^2 - F_m]^3} - \frac{1}{[(k-l)^2 - F_{\Lambda_\pi}]^3} \right) k^\mu k^\nu = -i \frac{1}{32\pi^2} \left( -\frac{1}{2} \ln \left( \frac{F_m}{F_{\Lambda_\pi}} \right) g^{\mu\nu} + \left( \frac{1}{F_m} - \frac{1}{F_{\Lambda_\pi}} \right) l^\mu l^\nu \right), \quad (D.18)
\]

If the previous formulas do not apply, one can use \((n \geq 3)\)

\[
\int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k-l)^2 - F + i\eta]^n} \{1, k^\mu, k^\nu k^\nu\} = \frac{i(-1)^n}{(n-1)(n-2)} \frac{1}{16\pi^2} \left\{ \frac{1}{F_{n-2}}, \frac{l^\mu}{F_{n-2}}, \frac{l^\mu l^\nu}{F_{n-2}} - \frac{g^{\mu\nu}}{2(n-3)F_{n-3}} \right\}, \quad (D.19)
\]
Bibliography


Al sinds mensenheugenis zijn mensen geïnteresseerd in de wereld om hen heen en zijn zij op zoek naar de meest elementaire bouwstenen waaruit deze is opgebouwd. Deze speurtocht heeft in het begin van de twintigste eeuw geleid tot een consistent model, dat schematisch is afgebeeld in de eerste drie plaatjes in figuur 7.1.

Alle materie om ons heen is opgebouwd uit moleculen die bestaan uit een of meerdere atomen. In figuur 7.1 is een watermolecuul afgebeeld, dat bestaat uit een zuurstofatoom (O) en twee waterstofatomen (H). Als er vervolgens op een van de atomen wordt ingezoomd, dan blijkt dat deze atomen een interne structuur hebben, die bestaat uit een positief geladen kern in het midden omringd door negatief geladen elektronen. Deze elektronen worden gebonden aan de kern door de elektrische aantrekkingskracht. De kern bestaat uit een aantal protonen en neutronen. Het aantal protonen in de kern is bepalend voor het type atoom, zo bevat waterstof één, zuurstof acht en ijzer zesentwintig protonen.

Een atoom is ongeveer $10^{-10}$ m groot (een tienmiljardste meter). Op deze kleine lengte schalen gelden de normale natuurkundige wetten, zoals we die kennen uit het alledaagse leven, niet meer. Hier moet de kwantummechanica worden toegepast, een theorie die aan het begin van de twintigste eeuw is ontwikkeld. Deze theorie beschrijft het gedrag van kleine deeltjes, zoals atomen en moleculen. Omdat de elektronen erg licht zijn, kunnen zij gemakkelijk hoge snelheden bereiken die de snelheid van het licht kunnen benaderen. Vaak moet hierdoor tevens de theorie van de hoge snelheden, de speciale relativiteitstheorie, worden toegepast. Dit is de beroemde theorie die door Einstein is geformuleerd. De combinatie van beide theorieën, kwantummechanica en speciale relativiteitstheorie, heeft onder andere geleid tot kwantumelektrodynamica (QED). Deze zogenaamde kwantumveldentheorie is erg succesvol door de verbluffend nauwkeurige wijze waarop de elektromagnetische interacties van bijvoorbeeld elektronen kunnen worden beschreven.

Zoals al genoemd bestaat de kern van een atoom uit protonen (p) en neutronen (n), ook wel nucleonen genaamd, wat schematisch is weergegeven in figuur 7.1. Protonen...
Figuur 7.1: In deze figuur staat de algemene opbouw van de materie schematisch weergegeven. Linksboven is een water molecuul (H₂O) getekend, rechtsboven een atoom dat bestaat uit een kern die omgeven wordt door elektronen. Rechtsonder is de structuur van een kern weergegeven, bestaande uit protonen (p) en neutronen (n). Elk van deze nucleonen bestaat vervolgens weer uit drie gebonden quarks, linksonder getekend. Deze quarks worden gebonden door hun interactie met de gluonen. Deze interactie wordt schematisch weergegeven met een snaarachtige structuur.

Protonen dragen een positieve elektrische lading, terwijl neutronen elektrisch neutraal zijn. Door de gelijke elektrische lading worden de protonen uit elkaar gedreven. Er is echter nog een andere kracht die de kern bij elkaar houdt. Deze kracht werkt tussen de nucleonen en wordt de ‘sterke kracht’ genoemd.

De zoektocht naar meer elementaire bouwstenen is hier niet gestopt. Er zijn talloze experimenten uitgevoerd naar de aard van protonen en neutronen, onder andere door protonen en neutronen te beschieten met elektronen of met elkaar. Hieruit bleek dat nucleonen geen elementaire puntdeeltjes zijn, maar ook een interne structuur hebben. Bovendien werd duidelijk dat er tal van zwaardere deeltjes bestaan die vergelijkbaar zijn met nucleonen, maar instabiel zijn, daardoor kort leven en snel uiteenvallen in nucleonen. Deze deeltjes worden onder andere aangegeven met Griekse letters als Δ, Σ, Ξ en Ω en worden tezamen met de nucleonen baryonen genoemd.
Er zijn veel eigenschappen van deze baryonen bekend. Door de kleine afmetingen van deze baryonen moet de kwantummechanica worden toegepast en kan er geen klassiek, alledaags beeld van worden gevormd. Indien men zich baryonen echter toch zou willen voorstellen, zouden ze moeten worden beschouwd als kleine magneetjes die in de ruimte tollen en met elkaar kunnen reageren. Aan de baryonen wordt de eigenschap spin toegekend, een beschrijving van het tollen van het baryon. De sterkte van de magneet wordt gegeven door de grootte van het magnetische moment van het baryon. Daarnaast heeft het baryon een massa en specifieke voorkeuren voor de reactie met overige deeltjes. Hiervoor zijn de eigenschappen isospin en strangeness ingevoerd die hier verder niet besproken worden.

Uit verder onderzoek bleek er veel symmetrie te bestaan in deze diverse eigenschappen van deze baryonen, gedeeltelijk weergegeven in tabel 1.1 en figuur 1.1. Door middel van deze symmetrie konden de reacties van de baryonen met elkaar, en het verval naar andere deeltjes, in hoge mate voorspeld worden. Als verklaring voor deze symmetrie werd het quarkmodel geïntroduceerd.

In dit model wordt elk baryon beschouwd als een object dat is samengesteld uit drie quarks. Er bestaan diverse soorten quarks. De normale materie waaruit de wereld is opgebouwd bestaat uit up en down quarks. Zo is het proton opgebouwd uit twee up-quarks en één down-quark en het neutron uit één up-quark en twee down-quarks. Daarnaast bestaan er strange, charm, bottom en top quarks. Deze quarks spelen een rol in de zwaardere baryonen die veelal worden aangegeven met de genoemde Griekse letters als Σ, Ξ en Ω. Enkele eigenschappen van de lichtste quarks, up, down en strange, worden opgesomd in tabel 1.2. Naast de baryonen, die zijn opgebouwd uit drie quarks, zijn er ook objecten mogelijk die bestaan uit een gebonden quark-antiquark toestand. Deze objecten worden mesonen genoemd.

Aan de quarks is een eigenschap verbonden die kleur wordt genoemd. Quarks kunnen voorkomen in een rode, blauwe of groene kleur, de antiquarks in de respectievelijke complementaire kleuren. Deze eigenschap heeft overigens niets te maken met onze dagelijkse ervaring met kleur, maar is uitsluitend een verzonnen naam om het gedrag te verklaren.

In de moderne fysica van de genoemde deeltjes wordt een kracht tussen twee deeltjes vaak beschreven door de uitwisseling van een derde deeltje. Zo worden in QED (de kwantumveldentheorie van elektromagnetische interacties) de krachten tussen geladen deeltjes beschreven door het uitwisselen van fotonen. De interactie tussen gekleurde objecten, de quarks, wordt beschreven door de uitwisseling van zogenaamde gluonen. Deze gluonen zorgen voor de uitwisseling van kleur tussen de quarks en binden de quarks daarmee binnen een baryon of een meson. De kwantumveldentheorie die dit mechanisme beschrijft wordt kwantumchromodynamica (QCD) genoemd.

De uitwisseling van kleuren door gluonen werkt op een dusdanige wijze, dat alleen kleur-neutrale objecten voorkomen in de natuur. Dat wil zeggen dat baryonen zijn opgebouwd uit een rood, een blauw en een groen quark, $q_{rood}q_{blauw}q_{groen}$, en mesonen uit bijvoorbeeld een rood quark en een antirood antiquark, $q_{rood}{\bar q}_{antirood}$. Door de voortdurende interactie met de gluonen wisselen de quarks overigens frequent van kleur. Doordat alleen kleur-neutrale (witte) objecten zijn toegestaan, kunnen quarks niet
alleen voorkomen, maar zijn ze altijd gebonden binnen baryonen of mesonen. Dit opsluitmechanisme wordt confinement genoemd. Het verbod op gekleurde objecten, en het daarmee samenhangende optreden van confinement, is op een gecompliceerde wijze gevatt in het lage energiegebied van QCD.

Doordat gluonen zelf kleur dragen kunnen ze ook aan elkaar koppelen, dit in tegenstelling tot de situatie in QED waarbij de fotonen geen lading bevatten, en dus niet met elkaar wisselwerken. De situatie kan worden voorgesteld als een drietal quarks die zich in een zee van gluonen bevinden, schematisch weergegeven in het linker gedeelte van figuur 7.2.

In QED is er sprake van een kleine koppelingsconstante, waardoor de geladen deeltjes relatief zwak aan elkaar gekoppeld worden. Deze interactie wordt daarom vaak al goed beschreven door de uitwisseling één enkel foton. De bijdrage van twee fotonen is door de lage waarde van de koppelingsconstante zoveel kleiner dat deze bijdrage slechts een kleine correctie vormt. Echter, in QCD zijn de gekleurde deeltjes sterk gebonden (confinement) en is er in het lage energiegebied sprake van een grote koppelingsconstante. Het is in dit geval niet langer geoorloofd enkele gluon uitwisselingen te beschouwen, maar de hele gluonachtergrond moet in rekening worden gebracht. Een impressie van een mogelijke gluonachtergrond bijdrage is schematisch weergegeven in het linker gedeelte van figuur 7.2. Het is ondoenlijk elke mogelijke configuratie van deze talloze gluonuitwisselingen apart in rekening te brengen, terwijl QCD dit wel vereist. Dit is een moeilijkheid die het uitrekenen van eigenschappen van baryonen vanuit QCD lastig maakt.

Bovendien bevat QCD erg kleine massa’s (stroommassa of current quark mass) voor de up en down quark als ingrediënten. Door de interactie van deze quarks met de zee van gluonen op de achtergrond krijgen ze een effectieve massa, de bouwsteenmassa (constituent quark mass). Dit mechanisme wordt chirale symmetriebreking genoemd. Hierdoor kunnen zware objecten, zoals nucleonen (massa ≈ 1000 MeV/c²), worden gevormd uit drie lichte quarks (massa ≈ 5 MeV/c²). De zee van gluonen gedraagt zich als het ware als een stroop die het de quarks bemoeilijkt zich te bewegen, waardoor ze
zich gedragen als zware deeltjes.

Om de quarkinteracties toch te kunnen beschrijven zijn fenomenologische modellen opgesteld, waarbij een effectieve interactie wordt aangenomen. Voor de interactie tussen de quarks wordt vaak een harmonische of lineaire potentiaal verondersteld. Voor de quarks wordt aangenomen dat zij een vrij grote massa bezitten (bouwsteenmassa) en dat zij vervolgens niet-relativistisch of gedeeltelijk relativistisch behandeld mogen worden. Voor baryonen die bestaan uit de zware quarks (charm, bottom en top) mag binnen deze benadering nog steeds een goede beschrijving van hun eigenschappen worden verwacht. Vanwege de tamelijk hoge massa bewegen zij met tamelijk lage snelheden en is een relativistische beschrijving niet nodig. Echter, deze niet-relativistische, fenomenologische modellen blijken ook goed te werken voor de lichte quarks (up, down en strange).

Ondanks hun goede beschrijving van de experimentele waarden zijn de zojuist beschreven niet-relativistische fenomenologische modellen toch niet geheel bevredigend. In deze modellen wordt een bepaalde interactie aangenomen, terwijl men liever QCD als uitgangspunt zou willen nemen en vanuit QCD direct een effectieve interactie destilleren. Men zou over een methode willen beschikken die de gluonachtergrond sommeert en als resultaat op een relativistische wijze confinement en chirale symmetriebreking genereert.

Hiervoor zijn diverse zogenaamde niet-perturbatieve methoden ontwikkeld. De methode die in dit proefschrift beschouwd wordt, vervangt de QCD achtergrond effectief door een QCD-snaar, zoals weergegeven in figuur 7.2. Deze methode is tamelijk recent door met name Prof. Simonov ontwikkeld en maakt een niet-perturbatieve behandeling van de gluonachtergrond mogelijk. De QCD-snaar kan worden verkregen vanuit de exacte theorie, QCD, met behulp van een aantal aannames. Deze aannames kunnen worden gestaafd aan diverse numerieke berekeningen en computersimulaties van QCD op een rooster. De beschrijving is relativistisch en bevat zowel confinement als chirale symmetriebreking. Als invoer worden lichte quarks, met de stroommassa, gebruikt en er is slechts één parameter, de snaarspanning $\sigma$, waarmee wat geschoven wordt.

Het werk dat gepresenteerd wordt in dit proefschrift beschouwt de baryonen binnen het QCD-snaar formalisme. In dit model kan uit de interactie van de quarks via de QCD-snaar het gedrag van de quarks worden afgeleid. Deze informatie is gevat in de banen die de quarks volgen binnen het baryon, ook wel golffunctie genoemd. Drie quarks vormen een baryon; het product van drie golffuncties van de quarks vormt de golffunctie van het baryon. Deze golffunctie bevat de eigenschappen van het baryon en kan worden gebruikt om deze eigenschappen uit te rekenen. In dit proefschrift worden de massa’s en de magnetische momenten van de baryonen uitgerekend binnen het QCD-snaar model. Er wordt gekeken in welke mate de massa’s en de magnetische momenten correct beschreven kunnen worden in deze niet-perturbatieve methode. De uitgerokende resultaten binnen het model worden daartoe vergeleken met de experimentele waarden.

Naast de interactie via de QCD-snaar kunnen quarks ook nog direct met elkaar reageren door het uitwisselen van een perturbatief gluon. Ook kunnen quarks met elkaar van plaats verwisselen, wat effectief beschreven kan worden door een uitwisseling van
een perturbatief pseudoscalair meson. Deze interacties geven aanleiding tot correctie-
termen, verbeteringen op het resultaat voor de massa’s en magnetische momenten van
de baryonen zoals die met enkel de QCD-snaar worden verkregen. Dergelijke interacties
worden beschouwd en uitgerekend in dit proefschrift, de figuren 3.1, 3.2 en 4.1 geven
een beeld van dergelijke interacties.

Zoals hierboven genoemd is, wordt de golffunctie van het baryon opgebouwd als een
product van drie golffuncties van de quarks. In eerste benadering wordt verondersteld
dat elke quark in zijn laagste toestand zit. De quarks kunnen zich echter ook in aange-
slagen toestanden bevinden en vervolgens bijdragen aan de golffunctie van het baryon.
Hierdoor kunnen de eigenschappen van het baryon als de massa en het magnetische
moment in het model wijzigen. Ook dit soort correcties worden in dit proefschrift
uitgerekend binnen het model.

Het proefschrift is als volgt opgebouwd. Na het inleidende eerste hoofdstuk wordt
het formalisme wat betreft de QCD-snaar uiteengezet in het tweede hoofdstuk. De
QCD-snaar kan als effectieve interactie worden verkregen uit de QCD-Lagrangiaan op
ijkinvariante wijze, door gebruik te maken van de veldcorrelatormethode. Van alle
veldcorrelators wordt alleen de twee-punts correlator meegenomen. Deze zogenaamde
Gaussische benadering is binnen een paar procent nauwkeurig, zoals numerieke simu-
laties van QCD op een rooster hebben laten zien. De QCD-snaar geeft effectief lineaire
confinement, en chirale symmetrie breking kan worden aangetoond. Relativistische,
Dirac-achtige vergelijkingen kunnen worden opgeschreven voor elk van de quarks. Deze
vergelijkingen kunnen worden opgelost en daarmee kan de golffunctie voor het baryon
in laagste orde worden opgeschreven in termen van de 1-deeltjes golffuncties van de
quarks.

De aldus verkregen baryongolffunctie kan worden gebruikt voor de berekening van
de magnetische momenten van de baryonen. Dit wordt uitgevoerd in hoofdstuk 3.
Daarnaast worden correcties op de magnetische momenten door meson-uitwisselingen
tussen de quarks en meson-lussen uitgerekend. Voor een waarde van \( \sigma = 0.10 \text{ (GeV)}^2 \)
voor de snaarspanning wordt een goede overeenkomst gevonden met de experimentele
waarden van de magnetische momenten van de baryonen terwijl de gevonden massa’s
zich tevens in het correcte energiegebied bevinden.

In hoofdstuk 4 wordt de massa van het baryon nader beschouwd. De door de
QCD-snaar gegenereerde interactie bevat geen spin-afhankelijkheid. Het gevolg is dat
er geen massaverschil gevonden wordt tussen bijvoorbeeld het nucleon en de \( \Delta \), wat in
tegenspraak is met experimentele gegevens. Door het model uit te breiden met pertur-
batieve gluon- en mesonuitwisselingen tussen de quarks, naast de al aanwezige interac-
tie gegenereerd door de QCD-snaar, worden spin-afhankelijke termen geïntroduceerd.
Hierdoor worden de massa’s van diverse typen baryonen uit elkaar gedreven en wordt
het resultaat meer in overeenstemming met de experimentele gegevens gebracht.

Tot nu toe is de baryon golffunctie beschouwd als het product van drie 1-deeltjes
golffuncties die elk in hun grondtoestand zitten. Er kunnen echter ook bijdragen op-
treden waarbij een of meer van de quarks in een aangeslagen 1-deeltjes toestand zitten.
Deze bijdrage wordt beschouwd in hoofdstuk 5 waar een zogenaamde meer-kanalen
berekening wordt uitgevoerd. De uitkomsten worden vergeleken met experimentele
waarden voor de massa’s en de magnetische momenten. In het algemeen kan worden opgemerkt dat de inmenging van deze hogere toestanden de magnetische momenten en de massa’s verlaagd. Hierdoor moet tevens een lagere waarde voor de snaarspanning gekozen worden. Een waarde van $\sigma = 0.08 \text{(GeV)}^2$ lijkt het best resultaat te geven.

Uiteindelijk worden in het laatste hoofdstuk de resultaten algemeen besproken en de conclusies getrokken. Hierin wordt met name de rol van het centrale punt waar de QCD-snaar bijeenkomt besproken. In principe moet hiervoor het Torricelli punt van de drie quarks gekozen worden. Dit is het punt binnen een driehoek waarbij de som van de afstanden tot de hoekpunten minimaal is. Appendix A bevat een kleine toegift in de vorm van een uitwerking van de berekening van het Torricelli punt. Om de complexiteit van de berekeningen te beperken is er in eerste instantie voor gekozen om dit punt vast te pinnen, waardoor de translatie symmetrie echter wel gebroken wordt.

In de overige appendices worden de technische details van de diverse berekeningen in dit proefschrift expliciet uitgewerkt.

Vele jaren na de ontwikkeling van QCD als de theorie van de sterke interacties is het lage energiegebied daarvan nog altijd een interessant studieonderwerp. Het QCD-snaar model is een veelbelovend model voor de beschrijving van de interactie van de quarks met de gluon-achtergrond. Het beschrijft op ikkinvariante wijze confinement en chirale symmetriebreking. De charme van het model wordt vooral gegeven door de relativistische, ikkinvariante afleiding uit de QCD-Lagrangiaan. Met name de kennis van het gedrag van de QCD-snaar op korte afstanden en een correcte behandeling van het centrale punt van de QCD-snaar ontbreken nog. Nadere en diepere beschouwingen zijn noodzakelijk om de kennis op deze punten te vergroten.
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Curriculum Vitae

Hans Weda was born on October 5, 1976 in Heerenveen, the Netherlands. During his secondary school years at the Bornego College in Heerenveen, he managed to get into the last national round of both the Dutch Biology Olympiad and the Dutch Physics Olympiad in 1995. He took part in the Biology Olympiad only, and finished at the 11th place.

After the Dutch VWO examination in 1995, Hans started to study physics at the University of Groningen, the Netherlands, and passed his propaedeutic examination cum laude in 1996. He continued his studies in theoretical physics during the next three years. In 1998 he was allowed to participate in the International Summer Student Program at GSI, Darmstadt, Germany, and performed a study on the pion beam-line in front of the HADES-experiment. After his return, he wrote his Master thesis under the supervision of Prof. Dr. R. G. E. Timmermans and Dr. O. Scholten at the KVI in Groningen, which was titled *Spin-3/2 particles and consistent πNΔ- and γNΔ-couplings*. The Kamerlingh Onnes price 1999 has been awarded to this thesis.

In 1999 Hans obtained his Master degree cum laude, and moved to the University of Utrecht, the Netherlands, where he was given the opportunity to take his PhD in the field of theoretical intermediate energy physics under the supervision of Prof. Dr. J. A. Tjon. During the following four years, he has traveled to several places in the Netherlands, and abroad in Portugal, Czech Republic, Switzerland, Russia, USA and Slovenia for summer schools, conferences and scientific visits. He has presented his work at a number of these meetings, and in a couple of articles. Part of the results of the research carried out during this period are presented in this thesis. Furthermore, he has been a teaching assistant during these years on the problem session of an advanced undergraduate course on Maxwell theory for two years, and the Student Seminar on the renormalization group for one year.

In his sometimes rather limited spare time, he has been visiting several museums, cycling long distances, reading books, running, training regularly at the swimming club ‘Het Zinkstuk’, performing numerous tasks in the organization of this club, and winning the first price in the autoped time trial on the Afsluitdijk.