Chapter 3

Baryon magnetic moments

In this chapter, mainly based upon Refs. [75,81], the magnetic moments of the baryons are calculated. We have obtained reasonable agreement with experiment without introducing constituent quark masses, and with the use of a single parameter, the string tension σ, as is shown in Section 3.1. This is in contrast with the non-relativistic quark models where constituent quark masses form an essential ingredient.

Following Ref. [71], where the systematic exploitation of the field correlator method shows that meson degrees of freedom can be included in a Lagrangian together with confinement, we assume the presence of virtual pseudoscalar mesons. Higher order contributions to the baryon magnetic moment come from two-body currents like one-pion-exchange currents, and mesonic one-loop corrections which give rise to the anomalous magnetic moment of the quark. In Section 3.2 an estimate of these higher order corrections to the magnetic moment is performed. The chapter ends with some conclusions in Section 3.3

3.1 Single quark current contribution

3.1.1 Calculation in coordinate space

Since the magnetic moments as well as baryon masses are static quantities, the calculation does not involve large momentum transfers, and one can use for that purpose the baryonic bound state equation, Eq. (2.40). According to the results of Section 2.2, H_i can be represented as

$$H_i = m_i\beta^{(i)} + \mathbf{p}^{(i)}\boldsymbol{\alpha}^{(i)} + \beta^{(i)}M^{(i)}(\mathbf{r}^{(i)} - \mathbf{r}^{(0)}). \qquad (3.1)$$

Solutions of this equation are discussed in Section 2.3. In the following, we shall use only the lowest orbitals (solutions with the lowest eigenvalues) for quarks, $0(1/2)^{++}$, and therefore the orbital excitation indices are everywhere omitted. We also assume that the baryon wave function is written in the factorizable form, Eq. (2.43). To define the magnetic moment one may introduce an external electromagnetic field $\mathbf{A}$, $\mathbf{p}^{(i)} \to \mathbf{p}^{(i)} - e_q^{(i)}\mathbf{A}$, $\mathbf{A} = \frac{1}{2}(\mathbf{H}\times\mathbf{r})$, and calculate perturbatively the energy shift,

$$\Delta E = -\boldsymbol{\mu}\mathbf{H}. \tag{3.2}$$

Due to the symmetry of the problem, it suffices to consider only the perturbation of one orbital, say for the first quark,

$$H_1 \to H_1 + \Delta H_1, \qquad \Delta H_1 = -e_q^{(1)}\boldsymbol{\alpha}^{(1)}\mathbf{A}. \tag{3.3}$$

If we denote

$$\psi^{(1)} = \begin{pmatrix} \phi^{(1)} \\ \chi^{(1)} \end{pmatrix}, \tag{3.4}$$

the energy shift $\langle \Delta H_1\rangle$ becomes

$$\begin{aligned} \langle \Delta H_1\rangle &= -e_q^{(1)}\left(\phi^{(1)},\chi^{(1)}\right)^* \begin{pmatrix} 0 & \boldsymbol{\sigma}^{(1)}\mathbf{A} \\ \boldsymbol{\sigma}^{(1)}\mathbf{A} & 0 \end{pmatrix} \begin{pmatrix} \phi^{(1)} \\ \chi^{(1)} \end{pmatrix}, \\ &= -e_q^{(1)}\left(\phi^{(1)}\boldsymbol{\sigma}^{(1)}\mathbf{A}\chi^{(1)} + \chi^{(1)}\boldsymbol{\sigma}^{(1)}\mathbf{A}\phi^{(1)}\right). \end{aligned} \tag{3.5}$$

From Eq. (2.45) we find $\phi = g\Omega_{jlm_j}$ and $\chi = if\Omega_{jl'm_j}$ with $l' = 2j - l$. Taking into account that $\Omega_{jl'm_j} = -(\boldsymbol{\sigma}\mathbf{n})\Omega_{jlm_j}$, one easily obtains

$$\langle \Delta H_1\rangle = -\frac{1}{2}e_q^{(1)}\int d^3r\,(g^*f + f^*g)\,r\Omega^*_{jlm_j}\big\{(\boldsymbol{\sigma}\mathbf{n})(\mathbf{n}\mathbf{H}) - \boldsymbol{\sigma}\mathbf{H}\big\}\Omega_{jlm_j}. \tag{3.6}$$

Eq. (3.6) contains the matrix element $\int d\mathbf{n}\Omega^*_{jlm_j}n_in_k\Omega_{jlm_j}$, which simplifies when $l = 0$, so that $\langle n_in_k\rangle = \frac{1}{3}\delta_{ik}$. In this case, one obtains for the contribution of the first quark to the magnetic moment operator in spin space, taking into account relation $\langle \Delta H_1\rangle = \Delta E = -\boldsymbol{\mu}^{(1)}\mathbf{H}$,

$$\boldsymbol{\mu}^{(1)} = -\frac{1}{3}e_q^{(1)}\int \big(g^*(r)f(r) + f^*(r)g(r)\big)rd^3r\Omega^*_{jlm_j}\boldsymbol{\sigma}^{(1)}\Omega_{jlm_j}. \tag{3.7}$$

Thus, for the lowest orbital $j = \frac{1}{2}$, $l = 0$, we find for the magnetic moment operator†,

$$\mu_z \equiv 3\mu_z^{(1)} = -2e_q^{(1)}\sigma_z^{(1)}\int g^*(r)f(r)rr^2dr, \tag{3.8}$$

where the superscript 1 denotes the contribution of the first quark to the magnetic moment. Due to the symmetry of the wave function under the exchange of two quarks,

†In the special case of a local scalar potential $U(r)$, one can further express $f(r)$ through $g(r)$ using the Dirac equation for the one-quark state

$$rf(r) = \frac{1}{\varepsilon + m + U(r)}\left(\frac{d}{dr}\,(rg(r)) + \frac{\kappa}{r}rg(r)\right).$$

the contribution of the second and third quark to the magnetic moment can be included by the factor three in Eq. (3.8). The normalization condition is,

$$\int (|g|^2 + |f|^2) r^2 dr = 1. \tag{3.9}$$

Note that everywhere we put $\mathbf{r}^{(1)} - \mathbf{r}^{(0)} = \mathbf{r}$.

The magnetic moment operator Eq. (3.8) must be evaluated on the $3q$ wave function. The fully symmetric spin-isospin wave function for the nucleon $\psi^{p,n}_{sym}$ is easily constructed when the quarks are all in their ground state, which is the case considered in this chapter. For the proton with total spin up, one finds

$$\begin{aligned}\Psi^p_{sym} = N' \Big\{ &\frac{2}{3}\big[u_+(1)d_-(2) + d_-(1)u_+(2)\big]u_+(3) \\ -\frac{1}{3}\big[d_+(1)u_-(2) + u_-(1)d_+(2)\big]u_+(3) &- \frac{1}{3}\big[u_+(1)u_-(2) + u_-(1)u_+(2)\big]d_+(3) \\ -\frac{1}{3}\big[u_+(1)d_+(2) + d_+(1)u_+(2)\big]u_-(3) &+ \frac{2}{3}u_+(1)u_+(2)d_-(3)\Big\},\end{aligned} \tag{3.10}$$

where $N' = 1/\sqrt{2}$, and subscripts $(\pm)$ refer to the spin projection. The fully symmetric spin-isospin wave function for the neutron can be found by the replacement $u \leftrightarrow d$. The explicit formula is given in Appendix B. The matrix elements are easily computed

$$\left\langle \Psi^p_{sym} \left| e_q^{(1)} \sigma_z^{(1)} \right| \Psi^p_{sym} \right\rangle = \frac{1}{3} e, \tag{3.11}$$

$$\left\langle \Psi^n_{sym} \left| e_q^{(1)} \sigma_z^{(1)} \right| \Psi^n_{sym} \right\rangle = -\frac{2}{9} e, \tag{3.12}$$

where e is the charge of the proton. From Eqs. (3.11)-(3.12), one immediately gets the famous relation [5],

$$\frac{\mu^{(n)}}{\mu^{(p)}} = -\frac{2}{3}, \tag{3.13}$$

which is one of early successes of the quark model. For identical orbitals, the magnetic moment can be written as a product

$$\mu_B = 3 \left\langle \Psi_{sym} \left| e_q^{(1)} \sigma_z^{(1)} \right| \Psi_{sym} \right\rangle \lambda, \tag{3.14}$$

Substituting this equation into Eq. (3.7) and integrating by parts, one obtains

$$\mu_z^{(i)} = \frac{e_q^{(i)} \sigma_z^{(i)}}{3} \int \frac{|g(r)|^2}{(\varepsilon + m + U)^2} (3(\varepsilon + m + U) - rU'(r)) r^2 dr.$$

For $U(r) = \sigma r$ one can express $\mu_z^{(i)}$ through $g(r)$ only as,

$$\mu_z^{(i)} = \frac{e_q^{(i)} \sigma_z^{(i)}}{3} \int_0^\infty \frac{|g(r)|^2\, r^2\, (2\sigma r + 3\varepsilon)}{(\varepsilon + \sigma r)^2} dr.$$

Table 3.1: *Ground state energy ϵ_0 of the orbitals and the predicted magnetic moments of the nucleons in units of the nuclear magneton for various values of σ. Results from the calculation in coordinate space, Eq. (3.14), and momentum space, Eq. (3.20), are both given. The experimental values are also listed.*

σ	$\epsilon_0(u,d)$	$\epsilon_0(s)$	coordinate space		momentum space	
(GeV)^2	(MeV)	(MeV)	μ_{proton}	$\mu_{neutron}$	μ_{proton}	$\mu_{neutron}$
0.09	297	440	2.81	−1.87	2.78	−1.85
0.12	342	482	2.44	−1.63	2.41	−1.61
0.15	380	520	2.20	−1.46	2.16	−1.44
		experiment	2.79	−1.91	2.79	−1.91

where

$$\lambda \equiv -\frac{2}{3}\int g^*(r)f(r)r^3\, dr\ . \tag{3.15}$$

For the single-quark orbitals we have taken the ground state solution of the Dyson-Schwinger-Dirac equation with a nonlocal kernel from Eq. (2.46). It is clear that inclusion of higher orbitals will change the magnetic moment of proton and neutron, similarly to the case of tritium and ^{3}He, where the admixture of the orbital momentum $L = 2$ changes the magnetic moment by 7-8%. These components appear in the wave function due to mixing through the tensor and spin-orbit forces between quarks. However, in the case considered in this chapter, all three quarks contribute equally to the orbital momentum. Therefore these components do not contribute at this stage. Contributions of inclusion of excited orbitals are studied in Section 5.2, where Eqs. (3.14)-(3.15) are generalized when the quarks have different orbital wave functions.

Using ground state orbitals $0(1/2)^+$ for each quark we calculate the nucleon magnetic moment for various values of the string tension σ. The results are listed in Table 3.1. From the table we see that the predictions depend sensitively on the string tension σ. Increasing the value of σ leads to a larger ground state energy of the orbitals and smaller size of the magnetic moment. This is in accordance with the analysis, where the small component of the orbital is treated perturbatively. Similarly, the presence of the Coulomb interaction yields a lower ground state energy of the orbital, resulting in a larger value in magnitude of the magnetic moment. Close agreement with the experimental values of the magnetic moment is found when $\sigma = 0.09$ (GeV)^2. In this case the mass of the nucleon is predicted to be 891 MeV. It is gratifying to see, that the magnetic moments are reasonable in the regime where also the predicted mass of the nucleon is close to the experimental value.

The magnetic moments of the other baryons from the lowest baryon octet and decuplet representation of the $SU(3)$-flavor group can also be calculated. The explicit

Table 3.2: *The magnetic moment of the baryons in units of the nuclear magneton for various values of σ. Calculations in coordinate space, Eq. (3.14), and experimental results.*

B	μ_B $\sigma = 0.09$ (GeV)^2	μ_B $\sigma = 0.12$ (GeV)^2	μ_B $\sigma = 0.15$ (GeV)^2	exp
p	2.81	2.44	2.20	2.79
n	−1.87	−1.63	−1.46	−1.91
Λ	−0.66	−0.60	−0.56	−0.61
Σ^+	2.72	2.37	2.14	2.46
Σ^0	0.85	0.74	0.67	
Σ^-	−1.03	−0.89	−0.79	−1.16
Ξ^0	−1.51	−1.34	−1.23	−1.25
Ξ^-	−0.57	−0.53	−0.50	−0.65
Δ^{++}	5.62	4.89	4.39	4.52
Δ^+	2.81	2.44	2.20	
Δ^0	0.00	0.00	0.00	
Δ^-	−2.81	−2.44	−2.20	
Σ^{+*}	3.09	2.66	2.37	
Σ^{0*}	0.27	0.21	0.18	
Σ^{-*}	−2.54	−2.23	−2.02	
Ξ^{0*}	0.55	0.43	0.35	
Ξ^{-*}	−2.26	−2.02	−1.84	
Ω^-	−1.99	−1.80	−1.67	−2.02

forms of Ψ_{sym} for these baryons are given in Appendix B. Note, that due to the strange quark mass their orbitals are different from those of u and d quarks, and therefore the decomposition Eq. (3.14) has to be modified. Some useful formulas can be found in Appendix B.

The resulting values for baryon magnetic moments are given in Table 3.2, where they are compared with experimental values. Considering the case of $\sigma = 0.12$ (GeV)^2 we see, that there is a rather close agreement with the experimental values for magnetic moments, where the largest deviations are found for the nucleon and Σ^-. As discussed for the case of the nucleon improvement of the predicted mass of the composite system also leads to magnetic moments closer to the experimental values. This also applies to the case of the Δ-isobar. Hence we may hope that the inclusion of the Coulomb and hyperfine-splitting interaction will improve the predictions, effects which are calculated in Chapter 5. Moreover, pionic effects are expected to be present. As a result, significant mesonic current contributions to the magnetic moments may occur. In Section 3.2 we study the dominant corrections from the pion to the one- and two-body current.

3.1.2 Calculation in momentum space

We now turn to the calculation of the baryon magnetic moment using translationally invariant baryon wave functions. The Ansatz from Subsection 2.3.2 is used for this purpose. Since the Ansatz for the baryon wave function is most easily written down in momentum space, Eq. (2.49), the magnetic moment expression has to be rederived in momentum space.

Our starting point is the electromagnetic current matrix element:

$$M_\mu = \langle \Psi \, | J_\mu(Q) | \, \Psi \rangle \,, \tag{3.16}$$

where Ψ is the 3-quark wave function and Q is the photon momentum. For the wave function normalization Eq. (3.9) for the single particle orbitals is chosen. Due the symmetry of the baryon wave function under the permutation of any two quarks, it is sufficient to consider only the single quark current operator acting on the first quark, and multiply by a factor of three. We therefore define for the current operator:

$$J_\mu^{\gamma qq} \equiv 3 J_\mu^{\gamma qq}(1) = 3 e_q^{(1)} \gamma_\mu^{(1)} \prod_{i=2}^{3} \gamma_0^{(i)}. \tag{3.17}$$

This choice has the nice property that the zeroth component of the current at $Q = 0$ gives the correct charge of the 3-quark system:

$$M_0 = \langle \Psi \, | J_0(Q=0) | \, \Psi \rangle = \sum_{i=1}^{3} e_q^{(i)}. \tag{3.18}$$

The result for the magnetic moment, obtained in the previous section can readily be recovered from our single quark current matrix element. Following Ref. [44], the magnetic moment can be calculated by taking the curl of the space component of the current matrix element in the Breit system. In doing so, the magnetic moment can be deduced from the electromagnetic current as

$$\mu_z = \frac{e}{2M_p} G_{mag}(Q=0) = -\frac{i}{2} \left[\boldsymbol{\nabla}_Q \times \mathbf{M} \right]_z (Q=0), \tag{3.19}$$

where M_p is the proton mass, e the proton charge and G_{mag} is the Sachs magnetic form factor [57]. The matrix element Eq. (3.19) can be easily evaluated in momentum space. Introducing the Fourier transform of the wave function of the single quark orbital as was done in Eq. (2.51), we may after some algebra reduce Eq. (3.19) in momentum space to an expression similar to Eq. (3.14),

$$\mu_z = 3\mu_z^{(1)} = 3 \left\langle \psi_{sym} \left| e_q^{(1)} \sigma_z^{(1)} \right| \psi_{sym} \right\rangle \tilde{\lambda}, \tag{3.20}$$

where ψ_{sym} is the fully symmetric spin-isospin wave function of the nucleon as shown in explicitly in Appendix B. We thus find,

$$\tilde{\lambda} = \frac{-1}{2N} \iint d^3p d^3q \prod_{n=2}^{3} \left(|\tilde{g}(k_n)|^2 + |\tilde{f}(k_n)|^2 \right) \times \left(\tilde{g}(k_1) \frac{4}{3k_1} \tilde{f}(k_1) - \frac{\partial \tilde{g}(k_1)}{\partial k_1} \frac{2}{3} \tilde{f}(k_1) + \tilde{g}(k_1) \frac{2}{3} \frac{\partial \tilde{f}(k_1)}{\partial k_1} \right)_{Q^2=0}, \quad (3.21)$$

where N is the normalization factor,

$$N = \iint d^3p d^3q \prod_{n=1}^{3} \left(|\tilde{g}(k_n)|^2 + |\tilde{f}(k_n)|^2 \right), \quad (3.22)$$

and we have used

$$\left\langle \Omega_{jlm_j}(\hat{k}_1) \middle| (\hat{\mathbf{k}}_1)_i (\hat{\mathbf{k}}_1)_j \middle| \Omega_{jlm_j}(\hat{k}_1) \right\rangle = \frac{1}{3} \delta_{ij}, \quad (3.23)$$

which is valid for $l = 0$. The momenta are expressed in terms of the Jacobi coordinates,

$$\begin{aligned} \mathbf{k}_1 &= -\frac{2}{\sqrt{3}}\mathbf{q} + \frac{1}{3}\mathbf{P}, & \mathbf{k}'_1 &= -\frac{2}{\sqrt{3}}\mathbf{q}' + \frac{1}{3}\mathbf{P}', \\ \mathbf{k}_2 &= \mathbf{p} + \frac{1}{\sqrt{3}}\mathbf{q} + \frac{1}{3}\mathbf{P}, & \mathbf{k}'_2 &= \mathbf{p}' + \frac{1}{\sqrt{3}}\mathbf{q}' + \frac{1}{3}\mathbf{P}', \\ \mathbf{k}_3 &= -\mathbf{p} + \frac{1}{\sqrt{3}}\mathbf{q} + \frac{1}{3}\mathbf{P}, & \mathbf{k}'_3 &= -\mathbf{p}' + \frac{1}{\sqrt{3}}\mathbf{q}' + \frac{1}{3}\mathbf{P}'. \end{aligned} \quad (3.24)$$

Imposing the Breit system, $\mathbf{P} + \mathbf{P}' = 0$, and momentum conservation gives $\mathbf{P}' = -\mathbf{P} = \mathbf{Q}/2$, $\mathbf{p}' = \mathbf{p}$ and $\sqrt{3}(\mathbf{q} - \mathbf{q}') = \mathbf{Q}$. The magnetic moment expression (3.14) from the previous subsection is readily recovered when we replace the integration over the Jacobi momenta in Eqs. (3.21-3.22) by $\prod_{n=1}^{3} dk_n$.

Results on the nucleon magnetic moment are shown in Table 3.1. It can be seen that the values deviate only by a small amount from the result calculated in coordinate space, also shown in Table 3.1. The difference is due to the contribution of the center of mass motion to the baryon magnetic moment. This erroneous contribution is absent in the calculation in momentum space which uses translationally invariant $3q$ wave functions, but is present in the calculation in coordinate space. From the results given in Table 3.1, we conclude that the center of mass motion hardly contributes to the magnetic moment. For simplicity, most calculations in this thesis are therefore performed in coordinate space using Eq. (3.14)

3.2 Mesonic contributions

In this section we estimate in our single-orbital model, the magnitude of the pionic current corrections to the magnetic moment of the nucleon. Due to the quark coupling

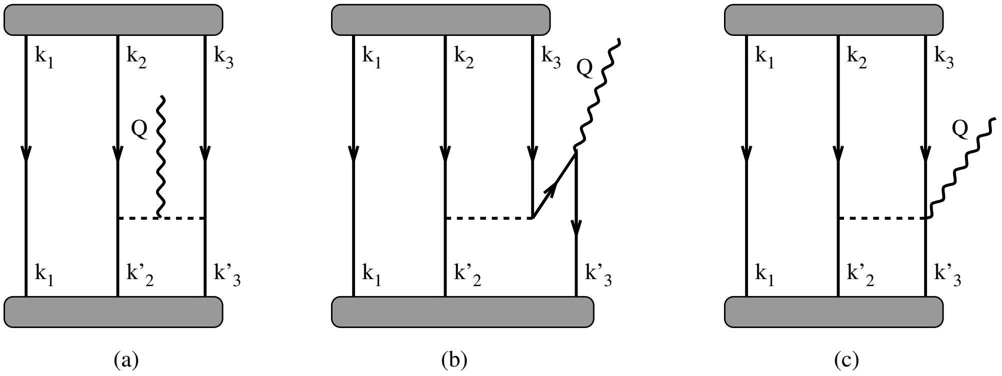

Figure 3.1: *The diagrams corresponding to the pionic contributions to the two-body current: (a) the pion-in-flight diagram, (b) the pair term, (c) the contact term. The contact term is only present when a PV coupling is assumed. The bound state of the quarks is represented by the blobs at the beginning and the end of the diagrams.*

to effective mesonic degrees of freedom, one and two-body current contributions to the magnetic moments of the baryons arise from the virtual excitations of mesons.

3.2.1 Meson exchange current

Assuming as in Refs. [27, 28, 80] that there exists an effective one meson exchange between quarks in the three-quark system this leads to meson exchange current contributions to the magnetic moment. The leading correction is due to the pion-in-flight and pair term [80]. Effects from the heavier mesons like the ρ are in general less important. These meson exchange currents were first used in few nucleon systems such as ^{3}H and ^{3}He [44], and later applied in quark models as was first done in Ref. [7].

The electromagnetic current matrix element is again taken as starting point,

$$M_\mu = \langle \Psi \,| J_\mu(Q) |\, \Psi \rangle \,, \tag{3.25}$$

where Ψ is the 3-quark wave function and Q is the photon momentum. The pionic two-body current contributions are shown in Fig. 3.1. For the coupling of the pion to the quark, we consider two possible forms: the pseudoscalar (PS) and the pseudovector (PV) coupling. The subsequent pseudoscalar coupling vertex of the pion to the quark can be written as,

$$\Gamma^a_{\pi qq}(k) = g_{\pi qq}\gamma_5 \tau^a F_{\pi qq}(k) \qquad \text{(PS)}, \tag{3.26}$$

where a monopole form factor $F_{\pi qq}(k) = \Lambda_\pi^2 / \left(\Lambda_\pi^2 - k^2\right)$ has been used. The pseudovector coupling is found by applying the replacement

$$g_{\pi qq}\gamma_5 \tau_a \rightarrow \frac{g_{\pi qq}}{2m_{\text{eff}}}\gamma_5 \not{k} \tau^a, \tag{3.27}$$

leading to

$$\Gamma^a_{\pi qq}(k) = \frac{g_{\pi qq}}{2m_{\text{eff}}}\gamma_5 \not{k}\tau^a F_{\pi qq}(k^2) \qquad \text{(PV)}. \tag{3.28}$$

The effective mass m_{eff} is a scaling mass which is inserted to make the PV coupling constant dimensionless. In this chapter, it is set equal to the constituent mass of the quark, $\epsilon_0(u,d)$, which are given in Table 3.1. The resulting pion-in-flight, pair-current and contact current operators, shown in Fig. 3.1, are given by:

$$J^{(23)}_{\gamma\pi\pi,\mu} = i\Gamma^a_{\pi qq}(k'_2 - k_2)\Delta_\pi(k_2 - k'_2)\Gamma^{ab}_{\gamma\pi\pi,\mu}(k'_2 - k_2, k_3 - k'_3)\Delta_\pi(k_3 - k'_3) \times \Gamma^b_{\pi qq}(k'_3 - k_3)\left(1 - \frac{i}{\Lambda^2_\pi}\frac{F_{\pi qq}(k_3 - k'_3)}{\Delta_\pi(k_2 - k'_2)} - \frac{i}{\Lambda^2_\pi}\frac{F_{\pi qq}(k_2 - k'_2)}{\Delta_\pi(k_3 - k'_3)}\right), \tag{3.29a}$$

$$J^{(23)}_{\gamma N\bar{N},\mu} = i\Gamma^a_{\pi qq}(k'_2 - k_2)\Delta_\pi(k_2 - k'_2)\Big(\Gamma^a_{\pi qq}(k_2 - k'_2)S_q(k_3 + Q)\Gamma_{\gamma qq,\mu} + \Gamma_{\gamma qq,\mu}S_q(k'_3 - Q)\Gamma^a_{\pi qq}(k_2 - k'_2)\Big), \tag{3.29b}$$

$$J^{(23)}_{\gamma\pi N\bar{N},\mu} = i\Gamma^a_{\pi qq}(k'_2 - k_2)\Delta_\pi(k_2 - k'_2)\Gamma^a_{\gamma\pi qq,\mu}(k_2 - k'_2). \tag{3.29c}$$

The last current is only present when a PV coupling is employed. In Eqs. (3.29), Q is the photon momentum. The propagators of the quark and pion are given by [10]

$$S_q(p) = \frac{i\,(\not{p} + M_q)}{p^2 - M_q^2 + i\epsilon}, \tag{3.30}$$

$$\Delta_\pi(p) = \frac{i}{p^2 - m_\pi^2}. \tag{3.31}$$

The photon-pion vertex is described by an effective interaction Lagrangian

$$\mathcal{L}_{\pi\pi\gamma} = -\frac{1}{2}eA_\mu\left(\vec{\pi}\times\partial^\mu\vec{\pi}\right)_z + \frac{1}{2}eA_\mu\left(\partial^\mu\vec{\pi}\times\vec{\pi}\right)_z, \tag{3.32}$$

which results in the following expression for the vertex

$$\Gamma^{ab}_{\gamma\pi\pi,\mu}(k',k) = -e\epsilon^{ab3}\left(k_\mu + k'_\mu\right). \tag{3.33}$$

For the coupling of the photon to the quark, we use the standard quark-photon vertex:

$$\Gamma_{\gamma qq,\mu} = -ie_q\gamma_\mu. \tag{3.34}$$

It can easily be verified that the charge of the quark and pion depends on the isospin as

$$e_q = \left(\tfrac{1}{6} + \tfrac{1}{2}\tau_z\right)e, \qquad e_\pi = -i\epsilon_{ab3}e, \tag{3.35}$$

where $e = |e|$ is the elementary charge.

The last two terms in the last factor in Eq. (3.29a) correspond to contact terms, which are needed to satisfy current conservation. The pair contribution consists of four terms where the photon can interact with quark 2 and 3 before and after the pion-quark interaction. In Fig. (3.1), only one term is shown, in Eq. (3.29b) the two terms where the photon couples to quark 3 are written down. The coupling to quark 2 can be easily found by interchanging $2 \leftrightarrow 3$. The quark propagator in Eq. (3.29b) has been replaced by its negative energy part,

$$\frac{i}{\not p - m} \Rightarrow \frac{i}{2\sqrt{\mathbf{p}^2 + m^2}} \frac{\mathbf{p}\boldsymbol{\gamma} - m + \sqrt{\mathbf{p}^2 + m^2}\gamma^0}{p^0 + \sqrt{\mathbf{p}^2 + m^2}} \approx \frac{i}{4m}\left(\gamma^0 - 1\right), \tag{3.36}$$

since the positive energy part has already been included in the single quark current matrix element [17]. In case of a PV coupling of the pion the minimal substitution of the electromagnetic field gives rise to the contact interaction in Eq. (3.29c),

$$\Gamma^a_{\gamma\pi qq,\mu}(k) = ie\frac{g}{2m_{\text{eff}}}\gamma_5\gamma_\mu\tau_b\epsilon_{ba3}F_{\pi qq}(k), \qquad \text{(PV)}. \tag{3.37}$$

In Eq. (3.29c) only the photon coupling to quark 3 is written down. The coupling to quark 2 can again be found by interchanging $2 \leftrightarrow 3$.

The interaction is assumed to be instantaneous. This means that the time component in the momenta of the quarks is neglected, $(k_2 - k_2')^2 \approx -(\mathbf{k}_2 - \mathbf{k}_2')^2$. From the 2-body operators $\mathbf{J}_{2b}$, Eq. (3.29) we may write down the current matrix element between the 3-quark state

$$\mathbf{M}_{2b} = 3\mathbf{M}_{2b}^{(1)} = 3\frac{1}{N}\iint d^3p d^3q \bar{\Psi}\gamma_0^{(1)}\mathbf{J}_{2b}^{(23)}\Psi, \tag{3.38}$$

where the factor 3 again originates from symmetry considerations of the baryon wave function. Taking the curl of Eq. (3.38), the magnetic moment can be determined. The resulting expressions are given in Appendix C. As a check using the obtained magnetic moment operators we have determined the exchange magnetic moment contribution to the trinucleon system. Our results agree with those obtained by Kloet and Tjon [44].

To get an estimate of the exchange current contributions in the 3-quark case we have used for the couplings and cutoff mass the values from Ref. [80]. They are taken to be $g^2_{qq\pi}/4\pi = 0.67$ and $\Lambda_\pi = 675$ MeV, respectively. The results for the magnetic moments are shown in Table 3.3. Our estimates are in strong disagreement with those obtained in Ref. [80]. The pion-in-flight contribution is substantially smaller than found in Ref. [80] using the chiral constituent model [27]. This may be partially due to the 3-quark wave function used, which has a matter radius smaller than in our case. Moreover, it contains only non-relativistic components.

The calculation using a PV pion-quark coupling yields somewhat smaller values than the calculation with a PS coupling. The two-body contribution gets larger for stronger string tensions σ, an effect which is stronger when a PS coupling is used. For small values of σ, the two-body contributions are found to be comparable, leading to an almost cancellation of the pionic current contributions.

Table 3.3: *The single quark current contribution* $\mu_N^{(1)}$ *to the magnetic moment in units of the nuclear magneton, together with the two-body corrections and the anomalous correction* $\delta\mu_N^{(1)}$ *arising from the pion one-loop diagrams. Also are shown the total combined prediction of our calculations and the experimental results.*

N	$\mu_N^{(1)}$	$\mu_N^{(\pi\pi\gamma)}$	$\mu_N^{(N\bar{N}\gamma)}$	$\mu_N^{(N\bar{N}\pi\gamma)}$	$\delta\mu_N^{(1)}$	μ_N^{tot}	exp
			$\sigma = 0.09$ (GeV)^2, PS coupling				
p	2.81	0.20	−0.16		0.17	3.02	2.79
n	−1.87	−0.20	0.16		−0.22	−2.13	−1.91
			$\sigma = 0.09$ (GeV)^2, PV coupling				
p	2.81	0.15	−0.04	−0.10	0.17	2.99	2.79
n	−1.87	−0.15	0.04	0.10	−0.22	−2.10	−1.91
			$\sigma = 0.12$ (GeV)^2, PS coupling				
p	2.44	0.19	−0.13		0.15	2.65	2.79
n	−1.63	−0.19	0.13		−0.19	−1.88	−1.91
			$\sigma = 0.12$ (GeV)^2, PV coupling				
p	2.44	0.15	−0.03	−0.09	0.15	2.62	2.79
n	−1.63	−0.15	0.03	0.09	−0.19	−1.85	−1.91
			$\sigma = 0.15$ (GeV)^2, PS coupling				
p	2.20	0.19	−0.11		0.14	2.42	2.79
n	−1.46	−0.19	0.11		−0.17	−1.71	−1.91
			$\sigma = 0.15$ (GeV)^2, PV coupling				
p	2.20	0.15	−0.03	−0.07	0.14	2.39	2.79
n	−1.46	−0.15	0.03	0.07	−0.17	−1.68	−1.91

3.2.2 Quark anomalous magnetic moment

The presence of mesonic degrees of freedom will modify the single quark current. The resulting electromagnetic current operator can in general be characterized by a large number of off-shell form factors [9, 78], which reduces to two when we assume that the initial and final quark are on their mass shell. Using this approximation, we may estimate the resulting anomalous magnetic κ term due to the mesonic contributions.

Near $Q^2 = 0$ the single quark current can be written as,

$$J_\mu^{\gamma qq} = e_q\gamma_\mu + \kappa_q \frac{ie}{2M_p}\sigma_{\mu\nu}q^\nu, \tag{3.39}$$

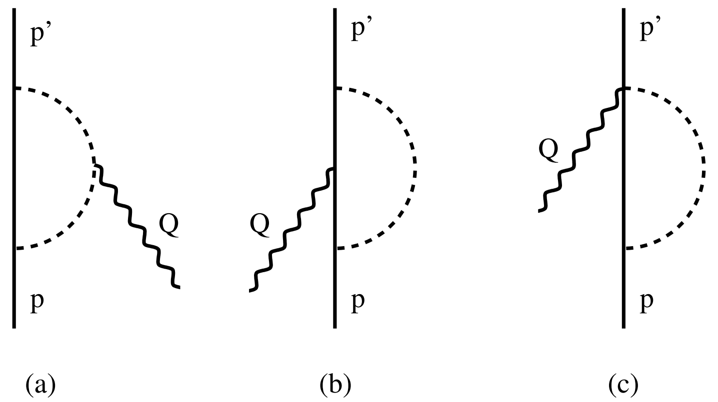

Figure 3.2: *The diagrams contributing to the anomalous magnetic moment of the single quark. Diagram (c) is only present when a PV coupling is assumed.*

where $\kappa_q = \kappa_s + \kappa_v \tau_z$ for the u, d-quark. The κ coefficients can be determined in a simple model, assuming that the loop corrections are given by only the one-loop pionic contributions to the electromagnetic vertex. Similarly, as in the two-body current case, we approximate the single quark orbital by free quark propagation with a constituent mass given by the ground state orbital energy. With the above simplifying assumptions, the calculation amounts to calculating the magnetic moment contributions of the diagrams shown in Fig. 3.2. Using the same cutoff mass regularization as for the two-body currents we can write,

$$J_\mu^{(a)} = i\int \frac{d^4k}{(2\pi)^4}\Gamma^a_{\pi qq}(k+Q)S_q(p-k)\Delta_\pi(k+Q)\Gamma^{ab}_{\gamma\pi\pi,\mu}(k,k+Q) \times \Delta_\pi(k)\Gamma^b_{\pi qq}(-k)\left(1 - \frac{i}{\Lambda_\pi^2}\frac{F_{\pi qq}\left((k+Q)^2\right)}{\Delta_\pi(k)} - \frac{i}{\Lambda_\pi^2}\frac{F_{\pi qq}\left((k)^2\right)}{\Delta_\pi(k+Q)}\right), \quad (3.40a)$$

$$J_\mu^{(b)} = i\int \frac{d^4k}{(2\pi)^4}\Gamma^a_{\pi qq}(k)S_q(p+Q-k)\Gamma_{\gamma qq,\mu}S_q(p-k)\Delta_\pi(k)\Gamma^a_{\pi qq}(-k), \quad (3.40b)$$

$$J_\mu^{(c)} = i\int \frac{d^4k}{(2\pi)^4}\Big(\Gamma^a_{\pi qq}(k)S_q(p+Q-k)\Gamma^a_{\gamma\pi qq,\mu}(-k) + \Gamma^a_{\gamma\pi qq,\mu}(k)S_q(p-k)\Gamma^a_{\gamma\pi qq,\mu}(-k)\Big)\Delta_\pi(k). \quad (3.40c)$$

The notation from the previous section is used, Eqs. (3.30) and (3.31) for the pion and quark propagators, and Eqs. (3.33) and (3.34) for the photon-pion and photon-quark vertices. Both PS and PV couplings of the pion to the quark are considered, Eqs. (3.26) and (3.28). The last current Eq. (3.40c) involves the contact interaction, Eq. (3.37), and is only present when a PV coupling is assumed. The last two terms in the last factor in Eq. (3.40a) are similar to the terms added in the pion-in-flight current, Eq (3.29a). Again they correspond to contact terms which are needed to satisfy the Ward-Takahashi identity to second order [78],

$$Q_\mu \Gamma^{\mu,(2)}_{\gamma qq} = e_q \left({S_q^{(2)}}^{-1}(p+Q) - {S_q^{(2)}}^{-1}(p) \right), \tag{3.41}$$

where $S_q^{(2)}$ represents the quark propagator dressed with one pion loop. The three-point vertex $\Gamma^{\mu,(2)}_{\gamma qq}$ is given by the sum of the currents Eqs. (3.40).

The currents can now be simplified by shifting the γ_5's through the expression and assuming that the incoming and outgoing quarks are on mass-shell. From these currents the anomalous magnetic moment has to be extracted. For this purpose the currents are written as

$$J_\mu^{(a,c)} = -2ie\tau_z \left(\gamma^\nu C_{\mu\nu}^{(a,c)} + C_\mu^{(a,c)} \right), \tag{3.42}$$

$$J_\mu^{(b)} = ie\frac{1-\tau_z}{2} \left(\gamma^\nu C_{\mu\nu}^{(b)} + C_\mu^{(b)} \right). \tag{3.43}$$

The tensors $C_{\mu\nu}^{(i)}$ and the 4-vectors $C_\mu^{(i)}$, with $i = a, b, c$, depend only on the initial and final momenta. Therefore they can be decomposed as:

$$C_{\mu\nu}^{(i)} = A_1^{(i)} K_\mu K_\nu + A_2^{(i)} K_\mu Q_\nu + A_3^{(i)} Q_\mu K_\nu + A_4^{(i)} Q_\mu Q_\nu + A_5^{(i)} g_{\mu\nu}, \tag{3.44}$$

$$C_\mu^{(i)} = B_1^{(i)} K_\mu + B_2^{(i)} Q_\mu, \tag{3.45}$$

where $A_n^{(i)}$ and $B_n^{(i)}$ are Lorentz invariants. By applying the Gordon decomposition to the current Eq. (3.39) near $Q^2 = 0$, it can be seen that the anomalous magnetic moment κ is the term proportional to $-\frac{e}{2M}K_\mu$ with $K_\mu = p_\mu + p'_\mu$. So only the first terms, $A_1^{(i)}$ and $B_1^{(i)}$, contribute to the anomalous magnetic moment. Substituting Eqs. (3.44) and (3.45) into Eqs. (3.42) and (3.43), and taking the initial and final quark on mass shell we find the anomalous magnetic moment corrections:

$$\kappa^{(a,c)} = 4iM_p\tau_z \left[2m_q A_1^{(a,c)} + B_1^{(a,c)} \right], \tag{3.46}$$

$$\kappa^{(b)} = -2iM_p \frac{1-\tau_z}{2} \left[2m_q A_1^{(b)} + B_1^{(b)} \right]. \tag{3.47}$$

Eq. (3.46) corresponds to the coupling of the photon to the pion and Eq. (3.47) to the coupling of the photon to the quark. Formally, the contact term is also represented by Eq. (3.46) in case of a PV coupling, this term however vanishes and does not contribute to the quark anomalous magnetic moment.

The Lorentz invariant expressions $A_1^{(i)}$ and $B_1^{(i)}$ can immediately be determined from the tensor $C_{\mu\nu}^{(i)}$. We obtain the expressions:

$$A_1^{(i)} = \frac{1}{3K^4} \left(4K^\mu K^\nu - K^2 g^{\mu\nu} \right) C_{\mu\nu}^{(i)}, \tag{3.48}$$

Table 3.4: *The quark anomalous magnetic moments in units of the nucleon magneton in the one-loop approximation for various values of the string tension σ. The first set is the predictions for the pion loops alone, while in the second set both pion and kaon loops are included.*

σ (GeV)^2	κ_u	κ_d	κ_s
		pion loops	
0.09	0.101	−0.160	0.0
0.12	0.092	−0.140	0.0
0.15	0.085	−0.126	0.0
		pion and kaon loops	
0.09	0.132	−0.151	−0.034
0.12	0.121	−0.133	−0.032
0.15	0.112	−0.120	−0.031

$$B_1^{(i)} = \frac{1}{K^2} K^\mu C_\mu^{(i)}. \tag{3.49}$$

Details on the calculation of the integrals and explicit expressions for $A_1^{(i)}$ and $B_1^{(i)}$ can be found in Appendix D.

The kaon one-loop diagrams can be calculated in the same manner. The starting point is the expressions Eqs. (3.40), where the mass of the pion is replaced by the mass of the kaon. Note that the interaction of the quark with the kaon changes the flavor of the quark, $(u, d) \leftrightarrow s$, which implies that the internal and external quarks have different constituent masses. As a second consequence, the effective mass $m_{\rm eff}$ in the PV coupling is in case of the kaon set equal to the average of the constituent masses of the (u, d) and s quark, $m_{\rm eff} = (m_q(u, d) + m_q(s))/2$. The isospin structure is changed as $\tau_z \to (\tau_z + 3Y)/2$ and $(1 - \tau_z)/2 \to -(\frac{2}{9} + \frac{4}{3}Y)$ respectively in Eqs. (3.46) and (3.47) where Y is the hypercharge. The coupling constant $g_{\kappa qq}$ and the cutoff Λ_κ are in first approximation the same as for the pion loop.

In Table 3.4 we show the calculated anomalous magnetic moments of the u, d and s quarks for $\Lambda = 675\ MeV$ for various choices of σ. Clearly, the results depend on the constituent quark masses. These are given in Table 3.1 for the string tensions considered.

From the calculation it is found that the results using a PV coupling can easily be related to the outcome using the PS coupling,

$$A_1^{(a,b)}(PV) = \left(\frac{m_q + M_q}{2m_{\rm eff}}\right)^2 A_1^{(a,b)}(PS), \tag{3.50a}$$

$$B_1^{(a,b)}(PV) = \left(\frac{m_q + M_q}{2m_{\text{eff}}}\right)^2 B_1^{(a,b)}(PS), \tag{3.50b}$$

where m_q is the constituent mass of the external quark and M_q is the constituent mass of the internal quark, both given by their respective ground-state orbital energies ϵ_0. In case of pion loops both internal and external quarks are u, d-quarks, $m_q = M_q$. Kaons, however, change u and d quarks into s quarks and back. As a consequence, we have $m_q \neq M_q$. Setting the effective mass m_{eff} in the PV coupling equal to the average of the involved constituent quark mass m_q and M_q, the use of either PS or PV couplings give the same value.

The analysis performed shows that the contact term does not contribute to the anomalous magnetic moment of the quark.

Using Eq. (3.19) the κ-term in Eq. (3.39) yields a nucleon magnetic moment correction

$$\delta\mu_z = 3\delta\mu_z^{(1)} = 3\left\langle \psi_{sym}\left|\kappa_q(1)\sigma_z(1)\right|\psi_{sym}\right\rangle \rho, \tag{3.51}$$

with

$$\rho = \frac{\int r^2 dr\big(|g|^2 + |f|^2/3\big)}{\int r^2 dr\big(|g|^2 + |f|^2\big)}, \tag{3.52}$$

where the identity Eq. (3.23) has been used.

In Table 3.3, the predictions for the nucleon, including also the one-pion loop contributions Eq. (3.51) and two-body currents, are shown. Our results obtained for the one-loop corrections are smaller than reported by Glozman and Riska [29]. This is due to the inclusion of the lower component in the single quark orbitals. Neglecting these, we recover the results of Ref. [29]. From Table 3.3, we see that the proton and neutron magnetic moments are in reasonable agreement with experiment for a string tension of $\sigma = 0.1$ (GeV)^2. For this value of the string tension the model predicts a nucleon mass of 940 MeV, remarkably close to the experimental value. The anomalous magnetic moment contributions are found to be of the order of 10%.

Due to the one-loop contributions, the magnetic moments of the other baryons are modified. Corrections from kaon loops have also been considered. Because of the larger kaon mass, the contributions are in general expected to be smaller in magnitude than those of the pion loops. In Table 3.4 the calculated anomalous moment of the strange quark due to the kaon one-loop corrections are given. In the calculations, a cutoff mass of $\Lambda = 675$ MeV has been used. The isoscalar and isovector anomalous magnetic moment terms are also changed by the kaon loop contributions. From Table 3.4, we see that the kaon loop contributions are indeed smaller in magnitude as compared to the pion loop ones. The complete results for the magnetic moments of the baryon octet and decuplet, including the pionic exchange currents and the pion and kaon one-loop contributions are summarized in Table 3.5 and plotted in Fig. 3.3. For the value of the string tension $\sigma = 0.1$ (GeV)^2, the overall agreement with the experimental data is reasonable. From the table we see that the anomalous magnetic moment contribution leads to an improvement of the predictions.

Table 3.5: *The magnetic moment μ_B of the baryon octet and decuplet in units of the nuclear magneton, including the anomalous contribution $\delta\mu_B$ arising from the pion and kaon one-loop diagrams and the pion exchange corrections for different string tension σ. Also are shown the experimental results*

B	$\sigma = 0.09$ (GeV)^2		$\sigma = 0.12$ (GeV)^2		$\sigma = 0.15$ (GeV)^2		exp
	$\delta\mu_B$	μ_B	$\delta\mu_B$	μ_B	$\delta\mu_B$	μ_B	
p	0.20	3.05	0.18	2.68	0.17	2.43	2.79
n	−0.22	−2.13	−0.19	−1.88	−0.17	−1.71	−1.91
Λ	−0.03	−0.69	−0.03	−0.63	−0.03	−0.58	−0.61
Σ^+	0.17	2.89	0.15	2.52	0.14	2.28	2.46
Σ^0	0.00	0.85	0.00	0.74	0.00	0.67	
Σ^-	−0.17	−1.20	−0.15	−1.04	−0.13	−0.92	−1.16
Ξ^0	−0.08	−1.59	−0.08	−1.42	−0.07	−1.30	−1.25
Ξ^-	0.00	−0.57	0.00	−0.53	0.00	−0.50	−0.65
Δ^{++}	0.35	5.97	0.32	5.21	0.30	4.69	4.52
Δ^+	0.10	2.91	0.10	2.54	0.09	2.29	
Δ^0	−0.15	−0.15	−0.13	−0.13	−0.11	−0.11	
Δ^-	−0.40	−3.21	−0.35	−2.79	−0.32	−2.52	
Σ^{+*}	0.20	3.29	0.18	2.84	0.17	2.54	
Σ^{0*}	−0.05	0.23	−0.04	0.17	−0.04	0.14	
Σ^{-*}	−0.30	−2.84	−0.27	−2.50	−0.24	−2.26	
Ξ^{0*}	0.05	0.60	0.05	0.48	0.04	0.39	
Ξ^{-*}	−0.20	−2.46	−0.18	−2.18	−0.16	−2.01	
Ω^-	−0.09	−2.08	−0.09	−1.89	−0.09	−1.76	−2.02

3.3 Conclusion

Using the formalism described in Chapter 2, a nonlinear equation for the single quark propagator S (attached to the string in a gauge invariant way) has been obtained. The equation has been solved in the Gaussian correlator approximation. The resulting three-quark wave function has been used to determine the magnetic moments of the baryons. This was done for both the octet and decuplet of the $SU(3)$-flavor group.

Comparing the predictions we find that the magnetic moments are mostly in close overall agreement with the experiment for a string tension of $\sigma = 0.1$ (GeV)^2. We find that the predicted magnetic moment of the nucleon is improved substantially once we choose a string tension that gives a reasonable nucleon mass. The same applies to the

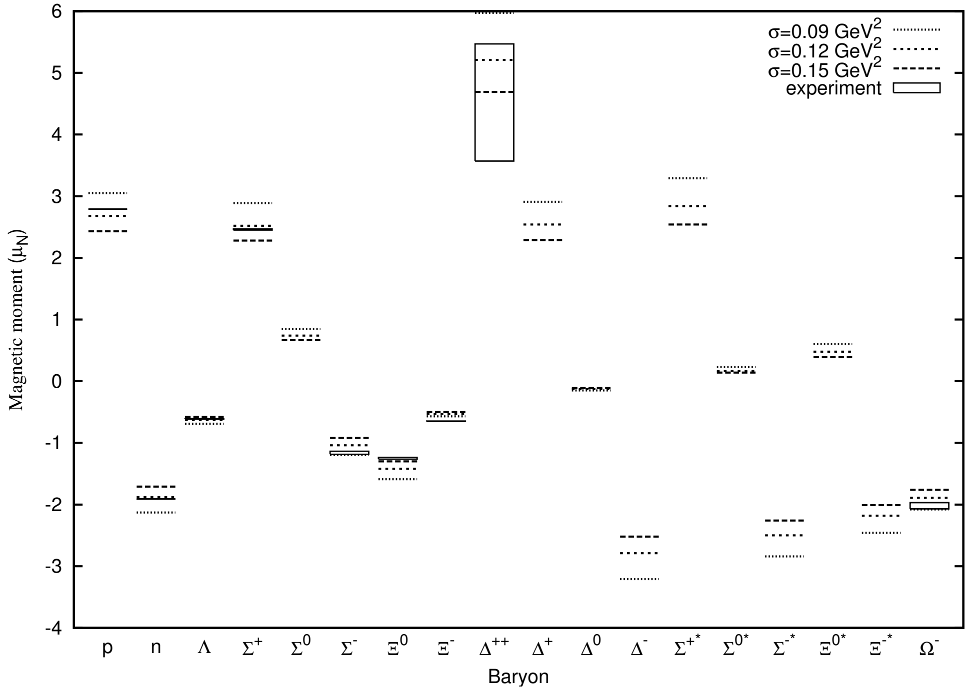

Figure 3.3: *For easy comparison the data of Table 3.5 is plotted. The boxes indicate the uncertainty in the experimental values.*

Δ-isobar. Effects due to the presence of virtual mesons are in general expected to be important. We have estimated the pionic one-loop and one pion exchange contributions to the magnetic moment. The single quark corrections from pionic loops are found to be of the order of 10%, whereas the total effect of 2-body current contributions are predicted to be small. This is in contrast to the results of Ref. [80]. This is due to the cancellation of the pion-in-flight and pair term in the present model. Because of the anomalous magnetic contributions, the predictions are somewhat better.

We have assumed that the baryon wave function can be described as a product of single quark orbitals, i.e. neglecting correlation effects. Our results for the magnetic moments of baryons in this approximation are encouraging, but are in need of including higher-order corrections. In particular, the mass spectrum obtained from our lowest-order approximation does not contain the $N - \Delta$ mass splitting. This is due to neglecting contributions like the hyperfine interaction arising from the one gluon interaction. This induces correlations in the three-quark wave function, and its magnitude may give us insight whether our basic description in this chapter in terms of single quark orbitals is a reasonable one. Moreover, it is clearly of interest to investigate how the magnetic moments are changed when effects from color Coulomb and hyperfine interaction are accounted for. This will be partially realized in Chapter 5, where we will study the influence of the excited quark orbitals on the magnetic moment of the baryon ground state.