2. Basic theory

2.1 Characteristics of complex plasmas

2.1.1 Charging of a dust particle: the floating potential

The charge and potential of a dust particle immersed in a plasma is determined by the balance between the electron and ion current towards the dust particle’s surface. Since electrons are more mobile than ions, a dust particle in a plasma will acquire a negative charge and thus a negative potential with respect to the surrounding plasma potential. The floating potential of the dust particle’s adjusts itself in such a way that, by repelling the mobile electrons and attracting positive ions, the dust particle on average collects no net charge at steady state. Dust particles in a plasma behave as a microscopic probe. The general aspects of probe theory are therefore important for dusty plasmas. To determine the charge and floating potential of a dust particle in a plasma, descriptions of the electron and ion transport towards the dust particle’s surface in the perturbed potential around the particle are required. In most approximations it is assumed that the width of the sheath region around a dust particle is smaller than electron-neutral or ion-neutral collision mean free path. This means that the charged particle transport is collisionless in the sheath. If the effect of individual Coulomb collisions is neglected in comparison to collective phenomena, the problem reduces to the solutions of the charged particle Vlasov equations:

\[
v \cdot \nabla f_e(r, v) - \frac{e}{m_e} E \cdot \nabla v f_e(r, v) = 0 \tag{2.1}
\]

and

\[
v \cdot \nabla f_i(r, v) + \frac{e}{m_i} E \cdot \nabla v f_i(r, v) = 0 \tag{2.2}
\]

where \( E = -\nabla V \), \( V \) is the electric potential, \( m_e \) and \( m_i \) are the electron and ion masses and \( e \) the elementary charge. The solution of the Poisson equation is given by:

\[
-\nabla^2 V = \nabla \cdot E = \frac{e}{\epsilon_0} (n_+ - n_e). \tag{2.3}
\]

For a plasma, the charged particle distributions \( f_e(r, v) \) and \( f_+(r, v) \) are related to electron \((n_e)\) and \((n_+)\) ion densities by

\[
n_{e,+}(r) = \int f_{e,+}(r, v) d^3v, \tag{2.4}
\]
If the loss of electrons to the dust particle does not affect the electron density and if the electron distribution function in the unperturbed plasma is a Maxwellian, a good approximate solution to the Vlasov equation for electrons in a repulsive potential (i.e. potential of a negatively charged dust particle), is a Boltzmann distribution,

$$n_e(r) = n_\infty \exp \left( \frac{e[V(r) - V_\infty]}{k_B T_e} \right),$$  \hspace{1cm} (2.5)

where $n_\infty$ is the density and $V_\infty$ is the potential of the unperturbed plasma far away from the dust particle, $k_B$ is the Boltzmann constant and $T_e$ is the electron temperature. Usually in low-temperature plasmas the electron distribution function is non-Maxwellian and the above expression may be inaccurate.

**Orbital motion limited theory for a spherical dust particle**

We assume that the electron density around a spherical dust particle is given by a Boltzmann distribution,

$$n_e(r) = n_\infty \exp \left( \frac{eV(r)}{k_B T_e} \right).$$  \hspace{1cm} (2.6)

$V_\infty$ has been taken equal to 0 in equation 2.6. The problem to be solved is the spatial distribution of ions around the dust particle and the collected ion current. The first expressions of the ion current towards the dust particle’s surface in the sheath regime were obtained by Mott-Smith and Langmuir (1926). This OML (orbit motion limited) theory assumes that for every ion energy there exists an ion impact parameter that makes the ion hit the dust particle with a grazing incidence. The trajectory of an ion can be derived as in the classical collision theory except that, due to the finite radius of the dust particle, the ion can be collected if its radial position reaches the radius of the dust particle $r=a$. In case of an ion with an energy $E_0$ and angular momentum $J_0$ at infinity, and a dust particle with radius $a$ is placed at $r=0$, the conservation of energy and angular momentum gives

$$E_0 = \frac{1}{2} m_+ \dot{r}^2 + \frac{1}{2} m_+ J_0^2 + eV(r),$$  \hspace{1cm} (2.7)

the ion impact parameter $b$ is related to $J_0$ by $J_0 = m_+ v_0 b$ ($v_0$ is the initial ion velocity). Equation 2.7 can be transformed into

$$b = r \left[ 1 - \frac{eV(r)}{E_0} - \frac{1}{2} \frac{m_+ \dot{r}^2}{E_0} \right]^{1/2}. $$  \hspace{1cm} (2.8)

The ions whose impact parameter is $b_{\text{coll}}$ with

$$b_{\text{coll}} = a \left[ 1 - \frac{eV(a)}{E_0} \right]^{1/2}.$$  \hspace{1cm} (2.9)
will hit the dust particle with zero radial velocity. Therefore all ions of energy $E_0$ whose impact parameter is less than $b_{coll}$ will hit and be collected by the dust particle. The cross-section $\sigma_{coll}$ for ion collection by the dust particle can be defined as:

$$\sigma_{coll} = \pi b_{coll}^2 = \pi a^2 \left[ 1 - \frac{eV(a)}{E_0} \right]^{1/2} \quad (2.10)$$

For monoenergetic ions, the ion current to the dust particle is then

$$I_+ = n_\infty e v_0 \sigma_{coll} = \pi a^2 n_\infty e \left( \frac{2E_0}{m_+} \right)^{1/2} \left[ 1 - \frac{eV(a)}{E_0} \right] \quad (2.11)$$

This expression gives the OML current, i.e. the maximum ion current that a spherical dust particle can collect assuming collisionless and stationary plasma conditions. In the derivation above the existence of ion trajectories reflected by an ‘absorption barrier’ due to possible secondary maxima in the effective potential energy curves are neglected.

In these conditions the positive ion density can be formulated as:

$$n_+ = \frac{1}{2} n_\infty \left[ \left[ 1 - \frac{eV(r)}{E_0} \right]^{1/2} + \left[ 1 - \frac{eV(r)}{E_0} - \frac{I_+}{I_r} \right]^{1/2} \right] \quad (2.12)$$

where $I_r$ is given by

$$I_r = \pi r^2 n_\infty e \left( \frac{2E_0}{m_+} \right) \quad (2.13)$$

Equation 2.12 shows that positive ion density increases from the plasma toward the dust particle, assuming that the potential becomes more negative when the particle is approached.

Substituting equation 2.12 and equation 2.6 in the Poisson equation gives

$$\Delta V = -\frac{e}{\epsilon_0} n_\infty \left\{ \frac{1}{2} \left[ \left[ 1 - \frac{eV(r)}{E_0} \right]^{1/2} + \left[ 1 - \frac{eV(r)}{E_0} - \frac{I_+}{I_r} \right]^{1/2} \right] \right\} \quad (2.14)$$

Equation 2.14 can be linearized for large values of $r/a$. This results in

$$(\Delta - \lambda_L^{-2}) V = 0, \quad (2.15)$$

where $\lambda_L$ is the linearized Debye length given by

$$\lambda_L = \left[ \frac{en_\infty}{\epsilon_0} \left( \frac{1}{k_B T_e} + \frac{1}{2E_0} \right) \right]^{-1/2} \quad (2.16)$$
Solving equation 2.15 gives the Debye-Hückel potential or so-called screened Coulomb potential
\[ V(r) = V(a) \frac{a}{r} \exp \left( -\frac{r-a}{\lambda_L} \right). \] (2.17)

Daugherty et al. [1] have shown that for small values of \( a/\lambda_L \) the exact solution of the Poisson equation is very close to the Debye-Hückel solution for \( eV/kT_{e,+} \ll 1 \), which is not true near the dust particle. Several radii from the dust particle the real solution is given by the Coulomb potential. This is predicted by the screened Coulomb expression.

In the discussion above we assumed that the ions are monoenergetic. For a Maxwellian ion distribution, the ion current to the dust particle has a similar form as the one derived for the monoenergetic ions:
\[ I_+ = \frac{\pi a^2 n_\infty e}{4} \left( \frac{8kT_+}{\pi m_+} \right)^{1/2} \left[ 1 - \frac{eV(a)}{k_B T_+} \right]. \] (2.18)

For the electrons we take the Boltzmann distribution for the electron density, this gives an electron current
\[ I_e = -\frac{\pi a^2 n_\infty e}{4} \left( \frac{8kT_e}{\pi m_e} \right)^{1/2} \exp \left[ \frac{eV(a)}{k_B T_e} \right]. \] (2.19)

At steady state the total current towards the dust particle becomes equal to zero. The charge \( Q_d \) for a total current of zero can be obtained from
\[ Q_d = 4\pi a^2 \sigma \] (2.20)
where \( \sigma \) is the surface charge density of the dust particle related to the electric field normal to the dust particle surface by
\[ \sigma = \epsilon_0 E_r(a) = -\epsilon_0 (\nabla V)|_{r=a}. \] (2.21)

Using the expression for the Debye-Hückel potential, the charge of a dust particle becomes
\[ Q_d = 4\pi \epsilon_0 a \left( 1 + \frac{a}{\lambda_L} \right) V(a) \] (2.22)
As mentioned previously, the OML theory assumes that all the ions with an impact parameter less than \( b_{coll} \) are collected by the dust particle. This may be not true because of the specific potential distributions around the dust particle which can reflect ions that would not be reflected in a simpler Coulomb-like potential. The OML theory therefore gives an upper limit to the ion current collected by the dust particle. It is important to know when the OML theory is valid or is
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a good approximation since the calculation of the collected ion current is much more complicated in the general case. Usually the OML theory can be used if the Debye-Hückel potential is a good approximation to the potential distribution around the dust particle. This is the case when \( a / \lambda_L \ll 1 \). Daugherty et al. [1] found that the OML theory for dust particles is a good approximation even when the particle radius is comparable to the electron Debye length.

Non-Maxwellian distribution function

In the OML theory the electron velocity distribution function is assumed to be Maxwellian. This assumption is usually made to obtain simple analytical expressions for the electron current to the dust particle. In a non-equilibrium, low temperature plasma the electron distribution function is often non-Maxwellian. Therefore it is good to estimate the errors made on the charge of a dust particle in these conditions. For a given electron energy distribution function \( f(\epsilon) \) (EEDF) the electron current can be obtained from the integral:

\[
I_e = -en_e \int_0^\infty \pi b_e^2 \sqrt{\frac{\epsilon}{2m_e}} f_e d\epsilon
\]  

(2.23)

where the cross-section \( \pi b_e^2 \) is calculated from

\[
\pi b_e^2 = \begin{cases} 
\pi a^2 \left( 1 - \frac{2eV(a)}{m_e v_e^2} \right), & \text{if } \frac{1}{2} m_e v_e^2 > -eV(a); \\
0, & \text{if } \frac{1}{2} m_e v_e^2 \leq -eV(a).
\end{cases}
\]  

(2.24)

For Maxwellian electrons, this gives the electron current of equation 2.19. Matsoukas and Russell [2] have derived an analytical expression for the electron current for a Druyvesteyn electron distribution. As expected, the number of negatives charges is smaller for a Druyvesteyn distribution than for a Maxwellian, the difference is in the order of 20 %. The explanation lies in the fact that for the same mean energy, the tail of the Druyvesteyn distribution is less populated, the balance of electron and ion current to the particle is achieved with a lower repulsive potential \( V(a) \), and the number of charges carried by the dust particle decreases.

Charging time

A dust particle injected or formed in a plasma will charge up, if stochastic effects are neglected, according to the law:

\[
\frac{dQ_d}{dt} = I_+ - I_e
\]  

(2.25)
Equation 2.25 gives the time evolution of the number of charges carried by a dust particle. The evolution of the charging time of a dust particle does not have an exponential behavior, this is due to the non-linear dependence of the currents $I_e$ and $I_+$ on the dust particle potential and charge. Bouchoule [3] has derived a characteristic charging time $\tau$ for a dust particle. Using a reduced potential $y$ defined by

$$y = \frac{eV(a)}{kT_e}$$  \hspace{1cm} (2.26)

as a new variable, the relation between charge and floating potential $Q_d = 4\pi\epsilon_0aV(a)$, and the expression for $I_e$ and $I_+$ of the OML theory results in the following differential equation

$$\frac{dy}{dt} = -A \left[ 1 + y \frac{T_e}{T_+} - \frac{v_{th,e}}{v_{th,+}} \exp(-y) \right] \quad \text{with} \quad A = \frac{e}{4\epsilon_0 a n_\infty} \frac{e}{kT_e} v_{th,+}$$  \hspace{1cm} (2.27)

At equilibrium ($t \to \infty$), $\frac{dy}{dt} \to 0$. $y$ is the solution of

$$1 + y \frac{T_e}{T_+} - \frac{v_{th,e}}{v_{th,+}} \exp(-y) = 0$$  \hspace{1cm} (2.28)

We take $y^*$ as the equilibrium of $y$. $y^*$ usually has a value between 1 and 4. Bouchoule [3] has defined the time $\tau$ characterizing the evolution of $y$ to its equilibrium values after a perturbation $(y^* + \epsilon)$ around the equilibrium value $y^*$:

$$\tau = \frac{\epsilon}{\frac{d\epsilon}{dt}}$$  \hspace{1cm} (2.29)

This gives

$$\tau = \frac{1}{A \left[ 1 + (T_e/T_+)(1 + y^*) \right]}$$  \hspace{1cm} (2.30)

Usually $T_e \gg T_+$, and the charging time simplifies to

$$\tau = 4\frac{\epsilon_0}{e} \left( \frac{\pi m_+}{8e} \right)^{1/2} \frac{(kT_+/e)^{1/2}}{a n_\infty(1 + y^*)}$$  \hspace{1cm} (2.31)

From equation 2.31 it can be seen that the charging time is inversely proportional to the plasma density and dust particle radius and proportional to the square root of the ion temperature. Punset and Boeuf [3] have shown with their Monte Carlo simulation that the charging time is in the order of 20 $\mu$s for a 100 nm radius dust particle in a $5\times10^{15}$ m$^{-3}$ argon plasma, and for $T_+/T_e=0.1$ and $T_e=3$ eV. These times are considerably longer than the RF cycle time but considerably shorter than the time needed for a dust particle to find its equilibrium position.
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2.1.2 Coulomb coupling parameter

One other important characteristic of a dusty plasma is its Coulomb coupling parameter which determines the possibility of the formation of dusty plasma crystals. If two dust grains with the same charge are separated from each other by a distance \( a \), the screened Coulomb potential energy is given by

\[
E_c = \frac{q_d^2}{a} \exp \left( -\frac{a}{\lambda_D} \right)
\]  

(2.32)

and the dust thermal energy is \( k_B T_d \). The Coulomb coupling parameter is defined as the ratio of the dust potential energy to the dust thermal energy and is represented by

\[
\Gamma_c = \frac{q_d^2}{a k_B T_d} \exp \left( -\frac{a}{\lambda_D} \right).
\]  

(2.33)

A dusty plasma is a weakly coupled system when \( \Gamma_c \ll 1 \), while it is strongly coupled when \( \Gamma_c \gg 1 \). The number of charges residing on the grain, the ratio of the inter-grain distance to the Debye screening length and the dust thermal energy play decisive roles in deciding whether a dusty plasma will be strongly coupled or weakly coupled.

2.2 Forces acting on spherical dust particles

In capacitively coupled radio frequency plasmas dust particles which have obtained charge are usually trapped by an electrostatic force present in the discharge. Usually the potential in these plasmas is higher than the surrounding walls. This results in an electric field pointing towards the reactor walls and acting as an effective trap for the negatively charged dust particles. This section will give a review of the various forces acting on dust particles in radio frequency plasmas.

2.2.1 Electric force

In this paragraph important results form Daugherty, Porteous and Graves (1993) [4] and Hamaguchi and Farouki (1994) [5] will be summarized. Daugherty et al. have shown that the electric force acting on the dust particle in the presence of an external electric field is very well approximated by the vacuum force \( Q_d E_0 \) when the dust particle radius is small with respect to the linearized Debye length. The most important result is that although the sheath around a dust particle shields it from the surrounding plasma, it does not screen the particle from an externally applied electric field. Hamaguchi and Farouki (1994) have pointed out that the Debye sheath is not 'attached' to the dust particle and represents only a local
perturbation of the background plasma. This means that the ions and electrons composing the sheath do not travel with the dust particle. The resulting force found by Daugherty et al is in the direction of the applied electric field $E_0$ and is given by

$$F_e = Q_d E_0 \left[ 1 + \frac{(a/\lambda_L)^2}{3(1 + a/\lambda_L)} \right]$$  \hspace{1cm} (2.34)

The first term of equation 2.34 is the force that would be experienced by the dust particle under the electric field $E_0$ in vacuum. The second term corresponds to the dipolar force due to polarization of the surface charge. This means that the polarized surface charge creates an electric field which in turn exerts an electric force. Since in general $a \ll \lambda_L$, the electric force can be approximated by $Q_d E_0$. Hamaguchi et al. have shown that for a uniform plasma the electric field resulting from the polarization of the dust particle also exerts a force on the plasma, increasing the ion pressure on the dust particle. They have shown, assuming a Boltzmann ion density in the sheath, that the ion pressure force exactly cancels the force due to surface charge polarization resulting in the vacuum electrostatic force

$$F_e = Q_d E_0$$  \hspace{1cm} (2.35)

Hamaguchi et al. also found another force due to the deformation of the Debye sheath induced by a spatially dependent Debye length like in the pre-sheath. Bouchoule [3] has shown that this so-called 'polarization' field is negligible under usual conditions, $(a \ll \lambda_L)$.

### 2.2.2 Ion drag force

The ion drag force is due to the momentum transfer between the positive ions and the dust particle. This force becomes important in regions where the ion flux is large. It has two components one due to the momentum transfer if the positive ion is collected (collection force) and the other due to the electrostatic Coulomb interaction between a dust particle and a positive ion deflected by the potential around it (orbit force). For monoenergetic ions the collection cross-section is given by

$$\sigma_{coll} = \pi b_{coll}^2 = \pi a^2 \left[ 1 - \frac{2eV(a)}{m_+ u_+^2} \right]$$  \hspace{1cm} (2.36)

where $u_+$ is the ion drift velocity. This results in a collection force

$$F_{col}^{ion} = \pi a^2 n_+ m_+ u_+ \left[ 1 - \frac{2eV(a)}{m_+ u_+^2} \right]$$  \hspace{1cm} (2.37)

Barnes et al. [6] (1992) have given an approximate collection force for a more general ion velocity distribution function by replacing directed velocity $u_+$ by the
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total velocity, i.e. the velocity $v_{+,tot}$ related to the ion total mean energy:

$$u_+ \rightarrow v_{+,tot} = \left( u_+^2 + v_{+,th}^2 \right)^{1/2} = \left( u_+^2 + \frac{8k_B T_+}{\pi m_+} \right)^{1/2} \quad (2.38)$$

To estimate the orbit force for a monoenergetic ion beam, the potential distribution around a dust particle has to be known. To obtain analytical expressions the potential around the dust particle is approximated by a classical Coulomb distribution (reasonable approximation for $a \ll \lambda_L$). The momentum cross-section for the orbit force can derived by solving the following integral

$$\sigma_{orb} = 4\pi \int_{b_{coll}}^{\lambda_L} \frac{pdp}{1 + (p/b_{\pi/2})^2} \quad (2.39)$$

where $p$ is the impact parameter and $b_{\pi/2}$ is the parameter whose angle is $\pi/2$:

$$b_{\pi/2} = a - \frac{eV(a)}{2E_+} \quad (2.40)$$

$E_+$ is the ion energy given by $\frac{1}{2}m_+u_+^2$. The integration is performed between the collection impact parameter $b_{coll}$ and the Debye length $\lambda_L$. The parameter $b_{coll}$ is given by

$$b_{coll} = a \left[ 1 - \frac{eV(a)}{E_+} \right] \quad (2.41)$$

The cut-off for the upper integration limit is introduced because of the divergence of the integral for an infinite upper limit. Shielding is assumed to suppress the potential for larger impact parameters. The corresponding Coulomb potential is therefore called the cut-off Coulomb potential. Solving the integral Eq. 2.39 above gives the following expression for the orbit cross-section

$$\sigma_{orb} = 2\pi b_{\pi/2}^2 \ln \left( \frac{\lambda_L^2 + b_{\pi/2}^2}{b_{coll}^2 + b_{\pi/2}^2} \right) \quad (2.42)$$

For non-monoenergetic ions, Barnes et al. used the following expression for the orbit force

$$\mathbf{F}_{ion}^{orb} = n_+ m_+ \sigma_{orb} v_{+,tot} \mathbf{u}_+ \quad (2.43)$$

the directed velocity has been replaced by the total velocity and $E_+$ replaced by the total mean energy.
2.2.3 Neutral drag force

A dust particle moving in a neutral gas experiences a drag due to momentum transfer during collisions with atoms or molecules. For laboratory plasmas, the relative velocity between the dust particles and molecules is smaller than the thermal velocity of the gas. The neutral drag force can then be approximated by the Epstein relations for the neutral drag force

\[ F_n = -\frac{4}{3} \pi a^2 m_n n_n v_{th,n} (u_d - u_n) \]  (2.44)

where \( m_n \) is the mass of the atom or molecule, \( n_n \) is the gas density, \( v_{th,n} = (8kT_{\text{gas}}/\pi m_n)^{1/2} \) is the thermal velocity of the gas, \( u_d \) is the dust particle velocity and \( u_n \) is the velocity of the neutral atom or molecule.

2.2.4 Thermophoretic force

When the neutral gas temperature is not uniform, a dust particle will experience a net resulting force due to collisions with the surrounding gas molecules. Gas molecules or atoms impinging on the hot side of the dust particles will transfer more momentum to the dust particle than gas molecules or atoms impinging for the cooler side. This gives a net resulting force which is called the thermophoretic force in the direction of the heat flux. Talbot et al. [7] have derived an analytical expression for the thermophoretic force

\[ F_{th} = -\frac{32}{15} a^2 v_{th,n} \left[ 1 + \frac{5\pi}{32} (1 - \alpha) \right] \kappa_T \nabla T_n \]  (2.45)

where \( \kappa_T \) is the translational thermal conductivity of the gas and \( T_n \) is the gas temperature. Talbot et al. [7] have shown that for gas and dust particle temperatures less then 500 K the accommodation coefficient \( \alpha \) is approximately 1.

2.2.5 Gravity

If a dusty plasma is considered on earth the gravitational force has to be taken into account. For micron sized dust particles the gravitational force is usually a dominant force. In experimental cases where the gravitational force is dominant the balance of forces acting on a dust particle is mainly made by the electric and gravitational force. The gravitational force is proportional to the particle mass, i.e. to the mass density \( \rho \) and to the dust particle volume:

\[ F_g = m_d g = \frac{4}{3} \pi r_d^3 \rho g \]  (2.46)

where \( g \) is the gravitational acceleration.
References