INTRODUCTION

String theory originated in the 1960's as a model for strong interactions. Only after it was replaced by a much more successful theory in this respect, by the name of quantum chromodynamics (QCD), was it reinvented as a theory of quantum gravity. Up to that time, twentieth century physics had borne two extremely successful theories. These are quantum mechanics, which describes physics at the very small scale, and Einstein’s theory of general relativity, a theory of gravity providing a description of physics at the largest of scales.

Quantum mechanics or, more precisely, quantum field theory provides a unified description of all but one of the forces known in nature. In the form of the Standard Model it brings together the electromagnetic force with the strong and weak nuclear forces. Moreover, this framework also includes all known elementary particles. The force missing is the gravitational force. The Standard Model is exceptionally successful at describing physics at the energy scales nowadays accessible in particle accelerators. In fact, there exists no experimental data in violation thereof. Nonetheless, the Standard Model will not be the final theory of particle interactions. Besides the absence of interaction with gravity, it exhibits a certain degree of arbitrariness, or at least inelegance, due to the large number of free parameters. This has led to many attempts at formulating a more fundamental theory.

General relativity is the theory of space, time and gravity. It has drastically changed our thinking about each of these concepts. On large scales, where gravity dominates, the theory provides an accurate description of the observed physics. It predicts such remarkable things as the expansion of the universe and the bending of light. Because gravity is such a weak force, it is very difficult to test the theory on small scales. This leaves open the possibility of new gravitational physics appearing at relatively low energy scales.

The two theoretical frameworks of quantum mechanics and general relativity have almost no overlap. There is, as mentioned above, no interaction
with gravity in the Standard Model. In laboratory particle experiments this is not so much of a problem, since gravity is too weak to play a significant role. There are, however, situations in which the gravitational interaction becomes of comparable strength to the particle interactions described by the Standard Model. These include black holes, or spacetime singularities in general, as well as the early universe. For a complete description of these situations, it is then necessary to construct a quantized theory of gravity, combining quantum mechanics with general relativity. However, general relativity is — in the language of quantum field theory — a non renormalizable theory. This means that it becomes useless when covariantly quantized because irremovable infinities appear at high energies, indicating that new physics is likely to play a role there. Hence arises the need for a full-fledged theory of quantum gravity. This new theory must replicate the predictions of quantum mechanics and general relativity in their respective regimes, but in the extreme situations where the two meet it must deviate and provide a natural connection between them. This is where string theory enters the arena.

String theory offers a radical new point of view. The fundamental constituents of the theory are one-dimensional objects rather than point particles. When being considered as a theory of the strong interactions, these so called strings arose as flux tubes between quarks. One of the problems it had as a theory of hadrons was the appearance of a massless spin-2 particle in the string spectrum. After being replaced by QCD, this massless spin-2 particle was reinterpreted as the graviton: the quantum of gravity. In this way, string theory naturally provides a theory of quantum gravity. It also encompasses the Standard Model, reproducing all known particle interactions found in nature. Moreover, it does so in a strongly unified way since all interactions are different manifestations of the same object. The mediators of the forces as well as all elementary particles are simply different vibrational modes of the fundamental string.

String theory contains both open strings, with the topology of a line, and closed strings, which are loops. While matter fields and their Standard Model interactions are described by open strings, closed strings describe the gravitational interactions. The closed string spectrum consists of an infinite tower of states, starting from the massless graviton and ever increasing in mass from there. It is the addition of these massive string modes that softens the divergences encountered at high energies in the covariant quantization of general relativity. String theory in this way provides a consistent perturbative description of the gravitational interaction.

The development of string theory as a theory of quantum gravity is characterized by two important developments known as string revolutions. The first revolution came in the 1980’s when spacetime supersymmetry was brought into the theory. Supersymmetry relates bosonic and fermionic degrees of freedom.
Until that time, only the bosonic string had been considered. This theory was troubled by the appearance of a tachyonic state in the string spectrum and by the fact that it could only be formulated consistently in twenty-six spacetime dimensions. The introduction of supersymmetry not only allowed the description to be extended to fermions, it simultaneously rid the theory of the tachyon in a natural way. These superstrings live in ten spacetime dimensions, which is, alas, not as bad as twenty-six. Supersymmetry has not yet been observed in nature. It is, however, most relevant to high energy phenomena and one can imagine that while it is broken at currently accessible energies, it gets restored at higher energies. The questions that remain are how exactly it gets broken and at which energy.

As for the dimensionality of spacetime, it is an everyday fact that we only experience three spatial and one time dimension. There are several proposals as to what happens with the six extra spatial dimensions required for a consistent formulation of superstring theory. The oldest and most generally accepted resolution is that the extra dimensions are compactified on circles with such small radii as to be undetectable in current high energy experiments. More recently, a competing idea has been put forward. It utilizes the difference in energy scales up to which the Standard Model and the gravitational interactions have been experimentally checked. The proposal states that the Standard Model fields are confined to a $3+1$-dimensional hypersurface in ten dimensional spacetime. The gravitational interaction, on the other hand, is free to probe all spacetime dimensions, as it should since it describes the dynamics of spacetime itself. These extra dimensions then only need to be curled up so much as to be consistent with high energy checks of gravity, which as mentioned are much less restrictive then those of the Standard Model. These models, called brane worlds, can naturally be embedded into string theory. The hypersurface is then represented as a D-brane, which is a topological defect to which the endpoints of open strings are confined. Gravity, being represented by closed strings, is not confined to the brane. The interaction of the Standard Model fields on the brane with gravity is described by closed strings breaking open and turning into open strings on the brane. The qualification as a topological defect derives from this process in which the topology of the string changes.

In the years ensuing the first string revolution, several superstring theories were discovered. In total there appeared to be five different theories, which were all defined perturbatively. These are the type IIA, type IIB, type I and two heterotic theories. This situation was unsatisfactory as it caused a certain ambiguity. With five different theories, which is the one that describes nature? During the 1990’s relations between these different theories were discovered. These relations are known as dualities and they showed that in fact all five theories are different manifestations of one underlying theory. This important step in the development of superstring theory is referred to as the second string
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revolutions. Lacking a fundamental coupling constant, the underlying theory is intrinsically non-perturbative. Because of this the theory, known as M-theory, is notoriously difficult to describe. The five superstring theories correspond to perturbative vacua of M-theory.

Through these and more recent developments, such as the discovery by Polchinski [100] of the relevance of D-branes in a non perturbative description of the theory, string theory has evolved into a more mature theory. It is fair to say that string theory is the leading candidate for a theory of quantum gravity. That is not to say that it is the only candidate. Certainly, based on the way string theory was unveiled as a theory of quantum gravity, there is no reason to believe that there would be no other theories waiting to be discovered. The only way of determining the full merit of string theory is then by testing it. While it has already withstood many tests, with many more awaiting, these are without exception of a theoretical nature; largely confirming only the theory’s internal – mathematical – consistency. The ultimate test of any theory that strives to describe nature, is to withstand experimentation. The extreme smallness of the corrections predicted by string theory to the theories it replaces makes this very difficult. The natural energy scale of the theory, the Planck scale at which stringy effects become relevant, is many orders of magnitude larger than those attained in modern particle accelerators. To directly test whether the fundamental building blocks of nature look like tiny rubber bands will therefore not be possible any time soon. That said, there is one laboratory where the required energies are realized: the universe itself.

This brings us to the subject of cosmology. Cosmology studies the evolution of the universe, from the putative beginning in a big bang to the vast universe we observe today and onward into the future. Although cosmology has been phenomenally successful in some respects, it fails to answer fundamental questions regarding, e.g., the nature of the initial singularity and the origin of inflation. The formulation of an underlying theory is necessary to approach these basic questions and it seems natural to consider string theory in this respect. This line of research has not proven very successful. String theory, as currently formulated, is not background independent. Each background thus requires its own consistent formulation of the theory and, so far, these have not been found for realistic cosmological backgrounds. Conversely, due to the high energies involved, some of these questions provide the best available testing ground for string theory. The rather limited interaction between the two subjects is therefore surprising. One observation that has recently sparked a change in this is that the current cosmological constant may be nonzero and positive. From a cosmological point of view, this means that the universe is entering a de Sitter phase. On the other hand, it has so far proven impossible to find a de Sitter solution to string theory. This forms an important challenge for string theory and perhaps comes closest to a comparison with nature in any test
so far, although it must be noted that the observation of a positive cosmological constant is still tentative. The issue of finding a de Sitter string solution plays a major role in the second half of this thesis.

The single most important formula around which this thesis revolves is the Bekenstein-Hawking area law

\[ S = \frac{A}{4G_N}. \]

This formula expresses the entropy hidden by an event horizon in terms of the area of that horizon in Planck units. The most amazing feature of this formula is its universality. Not only does it apply to black hole and cosmological horizons alike, but also to the black strings and branes found in string theory. From the Bekenstein-Hawking formula derives a fundamental entropy bound. The maximal entropy contained in a certain volume is given by the area of that volume in Planck units. This is known as the holographic principle, originally proposed by 't Hooft [131]. It implies a radical departure from the intuitive notion that the entropy would be proportional to the volume. Moreover, the holographic principle introduces a degree of non locality into any theory of quantum gravity. That it is expected to be a generic feature of any theory of quantum gravity is based on the fact that it derives directly from black hole formation, which limits the amount of mass a region of spacetime can support. It remains to be seen, however, whether this will appear as an effective property or as a manifest underlying principle.

As for string theory, it is not yet clear whether the holographic principle is manifest. In the perturbative picture of string theory we have today, it is only partly realized. As shown by Susskind [125], the perturbative expansion breaks down before the entropy bound of one bit per Planck area is violated. However, one would expect it to be completely explicit only in a fully non perturbative formulation of the theory. Recently, such formulations have become available in certain special cases. Most notably, Maldacena [90] has proposed the AdS/CFT correspondence, relating full, non perturbative string theory on anti-de Sitter backgrounds to conformal field theories living on the boundary of the AdS space. The number of degrees of freedom in this context manifestly obeys the holographic principle, as we will review in this thesis.

Studying the holographic principle in string theory may guide us towards answering important open questions. It has, for example, not been possible so far to find a string description of most realistic black holes and singularities of general relativity. More generally, while string theory is relatively well understood at small coupling, our knowledge of the strongly coupled regime is much more limited. The holographic principle may prove to be a valuable guide in that respect as it becomes more stringent in regions of spacetime that are highly dynamical and strongly gravitating.
The approach that will be taken in this thesis is to consider the compatibility of the holographic principle with cosmological models. This approach may help in the construction of realistic cosmological models within string theory. Since cosmology lacks a fundamental underlying theory, the solutions it offers may seem rather ad hoc. Inflation, for example, clearly offers a solution to the horizon and flatness problems, but different solutions to the same problems are certainly imaginable. How then must we choose between different cosmological models? Assuming that the holographic principle turns out to be of a fundamental nature, one would expect it to be a manifest feature of a cosmological model. This provides a possible way of determining the feasibility of specific cosmological models. Lacking a fundamental theory, one cannot look for an explicit manifestation of the holographic principle as a fundamental principle. Instead, one can consider whether a certain model obeys the entropy bounds that derive from it. These entropy bounds have culminated in a covariant entropy bound as formulated by Bousso [23,24]. This proposal has been tested to be consistent with many situations that are expected to occur in the universe, such as gravitational collapse and inflation.

This thesis is organised as follows. In Chapter 2, a general overview of the concept of holography is given. Its origin in black hole physics is reviewed as well as its most concrete realization in the form of the celebrated AdS/CFT correspondence. In between, several entropy bounds that derive from it are considered.

Chapter 3, largely based on [112], applies those entropy bounds to cosmological models. A striking relation between entropy and energy formulas on the one hand and the equations that govern the cosmological evolution of a radiation dominated FRW universe is reviewed. Subsequently, this is applied to a specific brane world model from which it derives a natural explanation.

In Chapter 4, we return to the subject of holography but now in the specific context of a spacetime with positive curvature, or de Sitter space. Although much less well established than the AdS/CFT correspondence, we review the recent proposal of an analogous dS/CFT correspondence.

Finally, Chapter 5, based on [96,111], provides a new point of view towards quantum gravity in de Sitter space. The problems that plague a formulation thereof are reconsidered within the proposed model.

For a thorough introduction to superstring theory, the reader is referred to the standard textbooks [55,101].