7. PROMPT $J/\psi$ SPECTRUM

In this section we shall determine the number of prompt $J/\psi$ in our sample. First we need a description of the shape of the invariant mass spectrum. This is given in section 7.1. We extract the number of prompt $J/\psi$ from a fit to the invariant mass spectrum in section 7.2. In the last section we give the yield of $J/\psi$ on the different wires.

7.1 The invariant mass spectrum

The number of prompt $J/\psi$ must be extracted from figure 6.4. For this we need an accurate description of the shape of both the background and signal.

7.1.1 Background shape

From the reduction in background after strong electron identification cuts (see figure 6.4), it can be concluded that the background consists mainly of hadrons leaving part of their energy in the ECAL and wrongly reconstructed tracks. Before any cuts are applied, the expected shape of the background under the $J/\psi$ signal is an exponential. This shape is altered by the selection criteria at trigger level: the $E_T$ requirement for both electron candidates and the cut on the pair invariant mass (see section 3.4). First, the mass cut results in a cut-off at 2 GeV. The mass at trigger level is calculated using the ECAL cluster energy and assuming $p = E$.

The momentum of the tracks in the reconstruction is calculated from the deflection in the x-direction of the tracks due to the magnetic field. Therefore, the sharp mass cut at trigger level is smooth in the reconstructed invariant mass spectrum. The resulting spectrum is thus a convolution of the $E/p$ resolution and the sharply cut trigger distribution. The amount of smoothing depends on the applied cut on $E/p$. The $E_T$ cut at trigger level mainly affects the lower end of the invariant mass spectrum. This effect is still visible at the cut-off mass of 2 GeV/$c^2$. The top of the distribution is shifted due to the $E_T$ cut. The influence of the $E_T$ cut changes with the requirement of bremsstrahlung, since at trigger level the lost energy was
Figure 7.1 shows a simulation of the invariant mass spectrum of combinatoric background before and after the $E_T$ cut. The shift of the maximum before and after the $E_T$ cut can be observed. In the same plot, the distributions smeared with a Gaussian are shown.

To a close approximation the shape of the background can be described by the convolution of a Gaussian distribution with an exponential, where the cut-off mass is left as a free parameter, to account for the effect of the $E_T$ cut. In figure 7.2 we show the fitted spectrum of pairs of tracks with a low electron likelihood ($|E/p - 0.99| > 0.2$ for at least one of the tracks, no bremsstrahlung).

### 7.1.2 Signal shape

The signal shape has several components. First, the intrinsic resolution of the measurement will yield a Gaussian shape centred around the mass of the $J/\psi$. This shape will be distorted by the effect of bremsstrahlung. If an electron radiates, but the bremsstrahlung energy is not recovered, the reconstructed momentum and thus the reconstructed mass of the two electrons will be lowered by the energy loss, resulting in a tail at the low end of the invariant mass spectrum. The
Figure 7.2. Invariant mass spectrum for background events (No bremsstrahlung, $|E/p - 0.99| > 0.2$ for at least 1 of the tracks). The distribution is fitted with the function described in 7.1.1 showing a good $\chi^2$. The second parameter is the slope of the exponential, $\sigma$ is the width of the Gaussian distribution and the last parameter is the turn-on mass. The values of these parameters are different from what is expected for the spectrum of the selected tracks, since they depend on the $E/p$ cut.

bremsstrahlung tail of the signal should be fitted, or, if not, a correction should be applied for the number of $J/\psi$ in the tail.

To obtain an analytical function describing the bremsstrahlung tail in the invariant mass spectrum, we start with the known distribution of energy loss through bremsstrahlung for electrons. We then combine the distributions for two independently radiating electrons to obtain a description of the distribution of the measured invariant mass.

The fraction of the remaining energy for electrons passing through material with thickness $X/X_0$ (in units of radiation length $X_0$) is given by[74]:

$$P_E(x_E) = \left(\frac{\ln \frac{1}{x_E}}{x_E/(X_0 \ln 2) - 1}\right)^{X/(X_0 \ln 2)}$$

(7.1)

$x_E = E/E_0$, where $E$ is the energy after radiation loss and $E_0$ the original energy of the electron.

The invariant mass is calculated from the measured momentum, not the energy
of the electrons. As was said before (section 6.4.1, the momentum is measured through the deflection in the magnet. Radiation downstream of the magnet has no influence on the deflection. For energy loss inside the magnet the momentum obtained is only lowered by a fraction of the energy loss (depending on the position inside the magnet where the radiation took place, see equation 6.4). We assume that this can be approximated by a lower effective value for the thickness $X/X_0$. Therefore, in the momentum distribution we replace $X = (X_0 \ln 2)$ by $X_p$, a parameter that depends on the distribution of material in the detector:

$$P_p(x_p) = \frac{(\ln \frac{1}{x_p})^{X_p-1}}{\Gamma(X_p)}. \tag{7.2}$$

The invariant mass ($M$) is given by $M = \sqrt{2p_1 \cdot p_2 \cdot (1 - \cos \theta)}$, where $p_{1,2}$ are the momenta of the electrons and $\theta$ is the opening angle between the electrons. The electrons radiate independently, therefore the ratio of measured di-lepton mass over the real mass ($M/M_0$), if only energy loss is taken into account, is the convolution of two momentum loss distributions:

$$P_m(x_m) = \int_{x_m}^{1} 2x_m \cdot \frac{x_m^2}{y_p} \cdot P_p \left( \frac{x_m^2}{y_p} \right) \cdot P_p(y_p) dy_p, \tag{7.3}$$

where $x_m = \sqrt{p_1/p_{1,0} \cdot p_2/p_{2,0}} \sim M/M_0$.

Unfortunately, this integral cannot be solved analytically. We found that to a very good approximation this distribution can be described by the function:

$$f_m(x) = N \cdot ((1 - x^{1/2} X_p) \cdot \ln \frac{1}{x})^{1/2} \cdot P_{p2}(x), \tag{7.4}$$

where

$$P_{p2}(x) = 2x \cdot \frac{(\ln \frac{1}{x^2})^{X_p-1}}{\Gamma(X_p)} \tag{7.5}$$

is distribution 7.2 for $x_p \to \sqrt{x}$; $N$ is a normalisation constant.

We simulated the distribution for $\sqrt{p_1 \cdot p_2}$ for different parameters $X_p$, shown in figure 7.3. In the same plot the function $f_m(x)$ is shown. As can be seen from the figure, $f_m(x)$ describes the distribution well, for different values of $X_p$.

The complete shape of the measured invariant mass spectrum is the convolution of distribution 7.3 with a Gaussian distribution:

$$P_M(x) = \int_{0}^{1} P_m(y) \cdot e^{-0.5 \frac{(x-y)^2}{\sigma^2}} dy \tag{7.6}$$
7.1. The invariant mass spectrum

![Correction mass](image)

**Figure 7.3.** Simulation for different values of parameter $X_p$ of the distribution of the relative mass difference for the two electron mass if only energy loss through bremsstrahlung is taken into account. Superimposed is the function $f_m(x)$ (7.4).

We determine this distribution in our fitting function by performing the convolution numerically.

This gives the distribution for $M/M_0$, whereas of course we want to use the distribution for $M$. Furthermore, for a complete description of the data, the kinematic cuts at trigger level should be applied before the convolutions are calculated. As was the case for the background, both the $E_T$ and the mass cut affect the distribution around $2 \text{ GeV}/c^2$. We approximate the effects of these cuts by adding one extra parameter, effectively narrowing the distribution between some turn-on value around $2 \text{ GeV}/c^2$ and the mass of the $J/\psi (M_{J/\psi} = M_0)$. Thus we replace $x$ in equation 7.4 by: $\frac{x-M_{J/\psi}}{p_{trig}} + 1$, with $p_{trig}$ the parameter describing the trigger effects, and let the integral in equation 7.6 run from $M_{J/\psi} - p_{trig}$ to $M_{J/\psi}$. Figure 7.4 shows the reconstructed invariant mass distribution of tracks associated with simulated $J/\psi \rightarrow e^+e^-$ decays. Also shown are the results of the fits. The parameters of the fits are given in table 7.1. When the reconstructed momenta of electrons that have been identified by bremsstrahlung in front of the magnet are corrected for radiation loss, the bremsstrahlung tail is relatively small (figure 7.4a). If this momentum correction is not performed, the bremsstrahlung tail is much more pronounced (figure 7.4b). The parameterisation is still able to reproduce the shape of the spectrum well and the major change occurs in the effective thickness parameter $X_p$, as one would expect. The tail of the spectrum is expected to be smallest
Figure 7.4. Reconstructed Monte Carlo. The upper plot shows the di-electron mass where the momentum of the electrons is corrected if a bremsstrahlung cluster was recovered, in the lower plot no correction was done. The spectra are fitted with the function 7.6 described in section 7.1.2; the $\chi^2/\text{ndf}$ showing a value close to 1 in both cases. The fit parameters can be found in table 7.1.
7.1. The invariant mass spectrum

<table>
<thead>
<tr>
<th></th>
<th>brem. correction</th>
<th>no brem. correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{J/\psi}$</td>
<td>$3.068 \pm 0.002$</td>
<td>$3.069 \pm 0.003$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$0.087 \pm 0.003$</td>
<td>$0.091 \pm 0.005$</td>
</tr>
<tr>
<td>$X_p$</td>
<td>$0.033 \pm 0.008$</td>
<td>$0.408 \pm 0.008$</td>
</tr>
<tr>
<td>$p_{\text{trig}}$</td>
<td>$1.03 \pm 0.02$</td>
<td>$1.28 \pm 0.02$</td>
</tr>
</tbody>
</table>

Table 7.1. Parameters of the fit to the MC mass spectra of figure 7.4.

if we require both electrons to be identified by bremsstrahlung emission in front of the magnet. In this case, emissions before the magnet with an energy lower than 1 GeV (the bremsstrahlung recovery threshold) are also recovered as long as they enter the same cluster as the reconstructed bremsstrahlung. This is not the case if no bremsstrahlung cluster was found. Thus, if the electron is identified by bremsstrahlung in front of the magnet there is practically no energy loss due to radiation in front of the magnet, which was not recovered, and the resulting tail is only due to energy loss inside the magnet.

Combining the background (as described in section 7.1.1) and signal distributions, the complete description of the invariant mass spectrum results in a 9-parameter fit. The result of this fit on the 12 BR and 2 BR spectra is shown in figure 7.5a,b together with the data. Also shown separately in the figures are the two components (signal and background). The parameters of the fits are given in table 7.1. The width of the signal is larger for data than for MC. This is expected since the $E/p$ distribution shows a larger width. In the data the mass is shifted to a lower value. This is probably due to misalignment and other energy loss effects that are not properly simulated.

<table>
<thead>
<tr>
<th></th>
<th>12 BR</th>
<th>2 BR</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{J/\psi}$</td>
<td>$2.985 \pm 0.006$</td>
<td>$2.973 \pm 0.006$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$0.100 \pm 0.004$</td>
<td>$0.106 \pm 0.006$</td>
</tr>
<tr>
<td>$X_p$</td>
<td>$0.016 \pm 0.008$</td>
<td>$0.0025 \pm 0.0008$</td>
</tr>
<tr>
<td>$p_{\text{trig}}$</td>
<td>$1.1 \pm 0.1$</td>
<td>$1.07 \pm 0.07$</td>
</tr>
<tr>
<td>background</td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td>$-2.03 \pm 0.08$</td>
<td>$-1.6 \pm 0.6$</td>
</tr>
<tr>
<td>$\sigma_{\text{dil}}$</td>
<td>$0.26 \pm 0.08$</td>
<td>$0.27 \pm 0.10$</td>
</tr>
<tr>
<td>cut off</td>
<td>$2.37 \pm 0.01$</td>
<td>$2.6 \pm 0.1$</td>
</tr>
</tbody>
</table>

Table 7.2. Parameters of the fit to the data mass spectra of figure 7.5.
Figure 7.5. Data, di-electron invariant mass spectrum, 12 BR (a) and 2 BR (b). Superimposed is the 9 parameter fit described in the text. The gray lines show the results on signal and background separately.
7.2 

7.2 Prompt \( J/\psi \) count

The background in the 012 BR invariant mass spectrum is too large to perform a reliable fit on the spectrum to extract the number of \( J/\psi \). Since we do not require bremsstrahlung for the detached spectrum, we need the total number of prompt \( J/\psi \) without bremsstrahlung requirement, to be able to compare the number of detached \( J/\psi \) \((n_B)\) with the number of prompt \( J/\psi \) \((n_P)\). We extract the number of prompt \( J/\psi \) in the 012 BR spectrum from the 12 BR and 2 BR spectra. For this we need to determine the efficiency of the bremsstrahlung requirement \((\varepsilon_{brem})\). It can be calculated from the data as:

\[
\varepsilon_{brem} = \frac{2}{1 + N_{12}/N_2},
\]

(7.7)

with \(N_{12}\), \(N_2\) the number of \( J/\psi \) in the 12 BR and 2 BR spectra respectively. \(N_{12}\) and \(N_2\) are given in table 7.3. The calculated value for \(\varepsilon_{brem}\) is:

\[
\varepsilon_{brem} = 0.34 \pm 0.03.
\]

(7.8)

This is in close agreement with the value found for Monte Carlo:

\[
\varepsilon_{brem}^{MC} = 0.34 \pm 0.01.
\]

(7.9)

To extract the number of \( J/\psi \) in the 012 BR spectrum \((N_{012} = N_P)\), we use:

\[
N_{012} = N_2/\varepsilon_{brem}^2 = (N_2 + N_{12})^2/4N_2.
\]

(7.10)

The value found for \(N_P\) is given in the last row of table 7.3.

| \(N_{12}\)  | 4000 ± 170 |
| \(N_2\)   | 820 ± 50   |
| \(N_P\)   | 7080 ± 550 |

Table 7.3. Number of prompt \( J/\psi \) with different BR requirements. The last row shows the extrapolated number of \( J/\psi \) using equation 7.10.

7.3 Systematic error on \( N_P \)

To determine the systematic error on \(N_P\), we use two other methods to extract \(N_P\). First, with stronger requirements on \(E/p\) we can obtain a significant signal in the 012 BR spectrum, which we are able to fit. Assuming the \(E/p\) distribution to be
Table 7.4. Number of prompt $J/\psi$ with two different $E/p$ requirements. The last column shows the extrapolated number of $J/\psi$ for a $3\sigma$ cut on $E/p$.

<table>
<thead>
<tr>
<th>$E/p$ cut units of $\sigma$</th>
<th>$N_{J/\psi}$</th>
<th>$3\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5\sigma$</td>
<td>1100 ± 65</td>
<td>7460 ± 480</td>
</tr>
<tr>
<td>$1.5\sigma$</td>
<td>5790 ± 380</td>
<td>7680 ± 570</td>
</tr>
</tbody>
</table>

Gaussian, we can calculate the number of $J/\psi$ with a $3\sigma$ cut on $E/p$. Secondly, to check the stability of the fit, we treat the bremsstrahlung tail of the signal shape as background and simply fit a Gaussian to the signal. The reconstruction efficiency for $J/\psi$ decreases this way, but can be estimated from the Monte Carlo by adding the background shape of the real data to the simulated $J/\psi$ data. The latter method was used in a previous analysis [56] to determine the number of prompt $J/\psi$.

Figure 7.6. Data, di-electron invariant mass spectrum, 012 BR. Superimposed is the 9 parameter fit described in the text. The gray lines show the results on signal and background separately.

Figure 7.6 shows the fitted 012 BR invariant mass spectrum with a strong cut on $E/p \ (|E/p - \text{mean}| < 1.5\sigma)$. With this stronger requirement on $E/p$, a clear signal is visible. We obtained the number of $J/\psi$ for different requirements on $E/p$, given in table 7.4. In the last column we give the extrapolated number for $N_p$. 
7.3. Systematic error on $N_\rho$

Figure 7.7. Monte Carlo prompt $J/\psi$ invariant mass with background spectrum added. The shape and size of the background is taken in such a way as to obtain a shape comparable with the 12 BR data spectrum. a: The tail of the signal is taken into account in the fit. b: Events in the tail are treated as background events. The filled histograms show the pure MC spectrum. The gray lines show the results on signal and background separately.
In figure 7.7 we show the simulated invariant mass spectrum where we added the shape and size of the background for the 12BR spectrum. In figure 7.7a we fitted the distribution with the parameterisation as described in section 7.1. In figure 7.7b we used the same parameterisation for the background, but for the signal shape a Gaussian was used. The resulting number of $J/\psi$ is lower since the bremsstrahlung tail is treated as background.

We applied both fits on several distributions of signal plus background to obtain an estimate of the efficiency of the fit where the tail is treated as background (i.e. the ratio of $J/\psi$ found inside the Gaussian over the number of $J/\psi$ found with the tail fitted). Within errors the same fractions were found on Monte Carlo and data. The results are summarised in table 7.5.

The results of this fit on the number of $J/\psi$ in the 12 BR and 2 BR spectra are given in table 7.6. We used the MC efficiencies obtained to extract $N_{12}$ and $N_2$, given in the second column of table 7.6. In the last row we report the value obtained for $N_P$, using equation 7.10.

The three values we obtained for $N_P$ agree within statistical uncertainties. From the differences in the values, we estimated a systematic uncertainty on $N_P$ of 8%. Our final value for the number of prompt $J/\psi$ is:

\[ N_P = 7080 \pm 550_{\text{stat}} \pm 570_{\text{sys}}. \]
7.4 statistics per wire

The number of \( J/\psi \) per wire is shown in table 7.7. We determined the relative luminosity per wire assuming \( \alpha = 0.955 \). From the statistical size of the samples on the two different target materials we can obtain the A dependence of \( \sigma_r \) in equation 4.7:

\[
A^{\alpha-1} = 0.93 \cdot A_C^{\alpha-1} + 0.07 \cdot A_{Ti}^{\alpha-1} = 0.89 \pm 0.04 \tag{7.12}
\]

From this and the value of \( \sigma(pN \rightarrow J/\psi) \) (table 4.2) it follows that:

\[
\sigma_r = 317 \pm 2_{stat} \pm 31_{sys} \text{ nb/nucleon.} \tag{7.13}
\]

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>nr. ( J/\psi )</td>
<td>5403 ± 444</td>
<td>1492 ± 213</td>
</tr>
<tr>
<td>fraction</td>
<td>78%</td>
<td>22%</td>
</tr>
<tr>
<td>relative luminosity using ( \alpha = 0.955 )</td>
<td>93%</td>
<td>7%</td>
</tr>
</tbody>
</table>

Table 7.7. Number of events per wire. The last row shows the fraction of events per wire scaled with the cross section \( \sigma(J/\psi)^A \).
7. Prompt $J/\psi$ spectrum