2. HEAVY QUARK PRODUCTION

In this chapter a brief overview of the theoretical and experimental knowledge of heavy quark production is given. In particular the production of open beauty and $J/\psi$ in hadronic collisions at fixed target energies is discussed.

2.1 Introduction

Heavy quarks are top ($t$), beauty ($b$) and charm ($c$) quarks. Due to their large mass, the energy scales involved in their production are large. In contrast to the lighter quarks, the calculations of their production cross sections in perturbative quantum chromodynamics (pQCD) are well behaved and stable. After introducing some basic kinematic relations in section 2.2, we discuss in sections 2.3 onwards, the theoretical framework of heavy quark production.

In this thesis we measure the production cross section of beauty quarks at HERA. The presence of $b$ quarks is indicated through the decay products of the $b$-carrying hadrons. In particular we are interested in the inclusive decay of $B$ mesons into $J/\psi$ (a bound $c\bar{c}$ state). At HERA most of the $J/\psi$ are not the decay product of a $b$-carrying hadron, but are instead produced directly in the proton nucleon interaction. We measure the beauty quark cross section relative to the known cross section of these direct $J/\psi$. Therefore, it is also important to discuss the production of direct $J/\psi$. This is done in section 2.6.

At HERA the heavy quarks are produced by colliding protons on a fixed target of different nuclear materials (pN collisions). Heavy quark production is a hard scattering process and therefore we only consider the collisions between a proton and a nucleon. However, the nuclear material plays a role in the production cross section of hadrons. These effects are discussed in section 2.7.
2. Heavy quark production

2.2 Kinematic observables

The nucleons involved in pN collisions are composite objects. They consist of three valence quarks and an indefinite number of sea quarks, anti-quarks and gluons. The hard interaction of two nucleons in which a heavy quark pair is produced can be described as a collision between two of the elementary constituents (the quarks, anti-quarks and gluons, called partons) of the nucleons.

Figure 2.1 shows a schematic picture of a collision between two nucleons with four-momenta $P_1$ and $P_2$, creating a final state in which a heavy quark pair (QQ) is produced. The centre of mass (c.m.s.) energy of the system is given by:

$$\sqrt{s} = \sqrt{(P_1 + P_2)^2}.$$  \hfill (2.1)

We define $x_1$ and $x_2$ as the fraction of the momenta of the nucleons that is carried by the partons. The four momenta of the partons can be written as:

$$p_1 = x_1 \cdot P_1,$$
$$p_2 = x_2 \cdot P_2.$$  \hfill (2.2)

The kinematics of the produced particles are described in terms of the momentum coordinates parallel ($p_z$) and perpendicular ($p_x$, $p_y$) to $P_1 - P_2$. Commonly, the transverse momentum $p_T$ and the variable $x_F$ (Feynman-x, introduced by Feynman in [4]) are used:

$$p_T \equiv \sqrt{p_x^2 + p_y^2},$$  \hfill (2.3)

$$x_F \equiv \frac{p_z}{p_{z,\text{max}}} \approx \frac{2p_z}{\sqrt{s}},$$  \hfill (2.4)

where $p_z$ is the longitudinal momentum in the centre of mass frame, and $p_{z,\text{max}}$ the maximum in this frame. In fact, $x_F$ is related to the parton momentum fractions:

$$x_F \approx x_1 - x_2.$$  \hfill (2.5)

2.3 Heavy quark production cross section

2.3.1 Renormalisation

Figure 2.2 shows two examples of QCD Feynman diagrams. The amplitude of the lowest order diagrams (e.g. figure 2.2a) can be calculated. But when calculating
the amplitude of loop diagrams, such as in figure 2.2b, the integrals involved in the calculations become infinite. To solve the problem of these ultraviolet divergences, the theory has to be renormalised. In the renormalisation procedure, the infinities are absorbed by redefining the bare coupling constant and quark masses. The effective coupling constant depends on the momentum transfer ($q^2$):

\[ \alpha_s(q^2) = \frac{\alpha_s(\mu_R^2)}{1 + \left(\alpha_s(\mu_R^2)/12\pi\right) \cdot \beta_0 \cdot \ln \left(q^2/\mu_R^2\right)}, \]

in which $\mu_R$ is the renormalisation scale. We can define a constant $\Lambda_{QCD}$ such that:

\[ \alpha_s(q^2) = \frac{12\pi}{\beta_0 \cdot \ln \left(q^2/\Lambda_{QCD}^2\right)}. \]

The mass of the produced quarks introduces a large energy scale ($q^2$) involved in the scattering. If this scale is much larger than $\Lambda_{QCD}$, $\alpha_s$ will be much smaller than

---

**Figure 2.1.** Schematic picture of two colliding nucleons with momentum $P_1$ and $P_2$. Two partons inside the nucleons, with momentum fraction $x_1$ and $x_2$, interact to create the final state with a heavy quark pair $Q\bar{Q}$. 
one, so that perturbative calculations in $\alpha_s$ can be used. In pQCD only Feynman diagrams up to a certain order in $\alpha_s$ are calculated; the rest appear as higher order corrections. For decreasing $q^2$, $\alpha_s$ increases and at some point perturbation series no longer converge.

The fact that at large $q^2$ the coupling between quarks becomes small is a property referred to as asymptotic freedom. It allows at small distances (high $q^2$) the treatment of the quarks as nearly free particles.

### 2.3.2 Factorisation

The parton interactions can be described by pQCD, since the energy scale involved is of the order of the mass of the heavy quark pair produced. The initial state of the nucleons, however, involves low energy (soft) processes and cannot be calculated perturbatively. The factorisation theorem allows the separation of the soft and hard processes involved in pN collisions. This means that the total production cross section for a final state $Y$ can be written as the convolution of the partonic cross section for two partons of types $i, j$ ($\hat{a}^{1Y}_{ij}$) and the parton densities ($f_i(x)$). The parton densities can be interpreted as the probability of finding a parton of type $i$, with momentum fraction $x$ inside the nucleon. A key result of the factorisation theorem is that the parton density functions are universal, i.e. are independent of the hard scattering process involved.

At leading order the amplitudes of the Feynman diagrams describing heavy quark production can be calculated. If however higher order diagrams are included, three kinds of divergences appear, called ultraviolet, infrared and collinear.

The ultraviolet divergences, which appear when calculating the amplitude of loop diagrams have already been discussed for the general case in section 2.3.1. The problem of ultraviolet divergences is solved by renormalisation of the Lagrangian, introducing the renormalisation scale $\mu_R$. 

Figure 2.2. Examples of QCD Feynman diagrams: a) Lowest order quark-anti-quark interaction. b) Example of a loop diagram.
Infrared divergences appear when very low momentum gluons are radiated or exchanged as virtual gluons. When calculating both contributions these divergences cancel out.

The collinear divergences arise from gluon emission collinear with the parton. These gluons do not necessarily need to have low momenta. The problem of collinear divergence can be solved by a second renormalisation procedure, in which the infinities are absorbed inside the parton distributions functions. In this renormalisation a second scale, the factorisation scale $\mu_F$, is introduced.

Both $\mu_R$ and $\mu_F$ are arbitrary parameters, i.e. physical quantities do not depend on them. For simplicity usually equal values are chosen. They should be of the order of the scale of the hard interaction. In heavy quark production this is set by the heavy quark mass:

$$\mu_R = \mu_F \approx m_q.$$  

(2.8)

Although the physical cross section does not depend on the scale, in finite order calculations some scale dependencies will be present; the higher the order of the calculations, the weaker the dependence on the value of the scale. To estimate the uncertainty associated with the order of the perturbative expansion the scale is usually varied.

The factorised production cross section $\sigma$ is:

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2_{\mu_F}) f_j(x_2, \mu^2_{\mu_F}) \hat{\sigma}_{ij}(p_1, p_2, \mu^2_{\mu_R}, \mu^2_{\mu_F}),$$  

(2.9)

where $\hat{\sigma}_{ij}$ represents the hard scale partonic cross section, calculated as a power series expansion in $\alpha_s(\mu^2_{\mu_R})$. In the following sections we elaborate on the different factors in the cross section equation.

### 2.3.3 Initial state

As mentioned before, the parton distribution functions are universal. They cannot be calculated in QCD, but their dependence on $\mu_F$ is governed by the Altarelli-Parisi (DGLAP) [5] equations. Therefore, once the distributions are known for a given scale, they can be calculated for any scale. Since they are universal, they can be determined by global fits to a selected set of data, e.g. from deep inelastic scattering experiments.

Several groups, among which CTEQ (Coordinated theoretical-experimental project on QCD) [6] and MRS (Martin, Roberts, Stirling)[7], provide the parton
distributions, updated when new data or theoretical developments become available. Figure 2.3 shows MRST parton densities for $u, \bar{u}$ quarks and gluons in the proton at the scale $\mu_F^2 = 25 \text{ GeV}^2$.

2.3.4 Hard scattering

Figure 2.4 shows the dominating leading order (LO) Feynman diagrams for heavy quark production. Both quark-antiquark annihilation and gluon fusion contribute to the production cross section, whereas the contribution from quark gluon scattering at HERA – B energies is only of the order of a few percent.

The partonic cross sections for heavy quark hadroproduction have been calculated at next-to-leading order (NLO)[8, 9, 10, 11]. Examples of Feynman diagrams contributing at NLO are shown in figure 2.5. The corrections include the effects of gluon radiation as well as virtual gluons. A full next-to-next-to leading order (NNLO) calculation of heavy quark production does not yet exist; attempts at calculations of higher order corrections to beauty production near threshold are discussed in section 2.5.
2.4 Hadronisation, kinematics of the final state

So far, we have described the production of heavy quarks, where we treated the quarks as free particles. However, we only observe bound states of quarks (hadrons). We still need to describe the process in which the produced quarks form the bound state.

In the case of open heavy quark production, the hadronisation process has no effect on the total cross section. This is not the case for quarkonium production (such as $J/\psi$ production), in which the heavy quark pair produced must form a bound state. $J/\psi$ production is discussed in section 2.6. Here we discuss the effect of hadronisation on the kinematics of the final state in open heavy quark production.

**Figure 2.4.** Leading order Feynman diagrams for $Q\bar{Q}$ hadroproduction. Both quark-antiquark annihilation (a) and gluon fusion (b,c,d) contribute to the production cross section.

**Figure 2.5.** Examples of NLO Feynman diagrams for heavy quark hadroproduction.
Hadronisation cannot be understood solely from perturbation theory. Since the energy scales involved are low, phenomenological models must be used to parameterise the hadronisation.

In the hadronisation process, the produced quarks lose part of their energy through fragmentation. The probability of creating a hadron \((H)\) with a fraction of the energy \(z\) of the parton \((p)\) is given by the fragmentation function \(f^H_p(z)\). For beauty quarks, the fragmentation functions are \textit{hard}, i.e. their maximum is close to one. Possible shapes for \(f^H_p(z)\) for heavy quarks are the Kartvelishvili\([12]\):

\[
z^\alpha \cdot (1 - z),
\]

or the Peterson\([13]\) function:

\[
\frac{1}{z} \cdot (1 - \frac{1}{z} - \frac{\epsilon}{1 - z})^{-2}.
\]

Fig. 2.6 shows these distributions for \(b\) quark fragmentation as well as the experimental data of the ALEPH collaboration\([14]\).

\textbf{Figure 2.6.} Aleph results on \(b\) quark fragmentation. Superimposed are the best fitting Peterson and Kartvelishvili shapes.
Theoretical and experimental status of beauty production

Figure 2.7. Scale dependence of $b\bar{b}$ production cross section for LO, NLO and LO + NLL calculations, taken from [15], for c.m.s. energy close to that of HERA-B. Even for NLO calculations the cross section shows a large scale dependence. After NLL corrections the theoretical predictions for the production cross section become less dependent on the choice of the scale $\mu$.

The scale dependence of the $b\bar{b}$ cross section at a c.m.s. energy (39.2 GeV) close to that of HERA-B, calculated using the MRS parton distribution functions, is shown in figure 2.7, for LO and NLO calculations[15]. Even after NLO corrections the scale dependence still causes a large theoretical uncertainty. This means that the contribution from higher order corrections to the total cross section might be quite large. We discuss here attempts to estimate the higher order corrections.

The diagrams which contribute most to the NLO cross section calculations are those in which a soft gluon is emitted. The reason for this is that production of $b\bar{b}$ pairs at HERA-B happens close to the kinematic threshold, where the logarithmic corrections which arise from soft gluon emission are large[16, 17].
Production near threshold is suppressed due to gluon radiation. But, since part of the soft gluon effects are already taken into account in the pdfs, the effective corrections for soft gluon emission to the partonic cross section are positive. It is assumed that in higher order calculations the same kind of diagrams dominate.

In Laplace transformed space, the large logarithms generated by soft gluon emission appear in the hard partonic cross section as follows:

\[
\hat{\sigma}(N) = \sigma_0 \cdot \exp \left[ g_1(\alpha_s \log N) \cdot \log N + g_2(\alpha_s \log N) + \mathcal{O}(\alpha_s^{n+1} \log N^n) \right],
\]

(2.12)

where \(N\) is the Laplace conjugate variable to \(m_{th}/m_Q\), and \(m_{th}\) is a mass parameter related to the typical distance to the threshold of the production process:

\[
m_{th} = \frac{x_1 x_2 s - 4 m_Q^2}{2 m_Q}.
\]

(2.13)

The functions \(g_{1,2}\) are expansions in \(\alpha_s \log N\). Leading logarithmic (LL) resum-mations of the logarithmic terms involve only the term \(g_1\), whereas in next to
leading logarithmic (NLL) resummations the term $g_2$ is also included. To obtain numerical values for $\hat{\sigma}$, one must perform an inverse Laplace transform.

Two different approaches have been used to resum the large logarithms arising from soft gluon emission. In the first approach[18], the full NLO calculations are taken and the corrections for the NNLL-NNLO gluon emission are added. The other approach[15] calculates the NLL corrections from soft gluons to any order and adds them to the NLO cross section.

The calculated cross section of the second approach is shown in figure 2.7 as a function of the scale $\mu$. The reduction in scale dependence due to the NLL calculations can be observed. Figure 2.8 shows the predicted $b\bar{b}$ cross section for fixed target production of ref. [18], for two different kinematic schemes.

Even after the soft gluon corrections, the theoretical predictions still contain large uncertainties. A major uncertainty arises from the unknown mass of the $b$ quark, which is assumed to be between 4.5 and 5 GeV. Near threshold, this 10% uncertainty leads to a large uncertainty in the cross section.

The current theoretical predictions for $\sigma(b\bar{b})$ at the c.m.s energy of HERA $- B$ are:

$$\sigma(b\bar{b}) = 25^{+20}_{-13} \text{ nb/nucleon}^1,$$

$$\sigma(b\bar{b}) = 30 \pm 13 \text{ nb/nucleon}[18]^2.$$

The only two existing measurements with incident protons are from two Fermilab experiments: E771[19] and E789[20]. These experiments used an 800 GeV proton beam on a fixed target of Gold and Silicon, respectively. The values for the $b\bar{b}$ cross section they obtained are given in table 2.1. Their results differ by almost a factor of eight. Although the statistical uncertainties of the measurements are large, this is still a difference of more than 2 $\sigma$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Year</th>
<th>Target</th>
<th>Proton Momentum</th>
<th>$\sigma(b\bar{b})$ nb/nucleon</th>
</tr>
</thead>
<tbody>
<tr>
<td>E789</td>
<td>1995</td>
<td>Au</td>
<td>800 GeV/c</td>
<td>$5.7 \pm 1.5 \pm 1.3$</td>
</tr>
<tr>
<td>E771</td>
<td>1999</td>
<td>Si</td>
<td>800 GeV/c</td>
<td>$43^{+27}_{-12} \pm 7$</td>
</tr>
</tbody>
</table>

Table 2.1. Present experimental situation on the $b\bar{b}$ cross section.

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$^1$ Result from [15] but corrected for the updated MRST pdf in [7]

$^2$ Obtained with the CTEQ5 pdf [6].
2.6 \( J/\psi \) production

The \( J/\psi \) production mechanism differs from open heavy quark production, since the \( c\bar{c} \) pair created must form a bound state. The bound state of a quark and its antiquark is called quarkonium. For the bound state of a \( c\bar{c} \) pair, the name charmonium is used.

The production of charmonium is not well understood. There are several models for it. The colour singlet model [21, 22] (CSM) assumes the \( c\bar{c} \) pair is created in a colour neutral state with the same quantum numbers as the charmonium state. As a result, not all the diagrams which contribute to \( c\bar{c} \) production can contribute to the charmonium state. For example, for \( J/\psi \) production, which has charge conjugation \( C = -1 \), the lowest order diagrams must contain a radiated gluon, and therefore have an order \( \mathcal{O}(\alpha_s^3) \) (figure 2.9).

The leading order predictions of the CSM do not correspond with the data and it is now generally believed that colour octet states contribute to the production of charmonium.

![Figure 2.9. Leading order contribution for \( J/\psi \) production.](image)

The colour evaporation model [23] (CEM) and the colour octet model [24] (COM) do allow the production of a coloured \( c\bar{c} \) pair, which can become colour neutral later again through the emission of soft gluons. The predictive power of CEM is restricted; it cannot predict absolute cross sections. The COM is a non-relativistic QCD (NRQCD) approach to charmonium production. It includes both the octet and singlet production of charmonium states. It successfully describes the high-\( p_T \) behaviour of \( J/\psi \) production, where the CSM fails. However, since there are many free parameters in the theory, there are few other predictions that can be tested. A problem for the COM are the polarisation measurements from CDF, which seem to be conflicting with the predictions.

The \( J/\psi \) cross section is, however, well established experimentally. Figure 2.10 shows the results of the inclusive \( J/\psi \) cross sections at different c.m.s. energies. The data have been fit to a functional form \( a e^{-b\sqrt{\tau}} \), with \( \tau = M_{J/\psi}^2/s[25] \). The
two measurements at c.m.s. energy closest to the HERA – B case are provided by the E789[25] and E771[26] experiments. Both experiments made use of an 800 GeV proton beam on a fixed target (Gold for E789 and Silicon for E771). The results for the $J/\psi$ cross section are:

\[
\sigma(pN \rightarrow J/\psi) = (442 \pm 2_{\text{stat}} \pm 88_{\text{sys}}) \text{ nb/nucleon} \quad \text{(E789)}
\]
\[
\sigma(pN \rightarrow J/\psi) = (375 \pm 4_{\text{stat}} \pm 30_{\text{sys}}) \text{ nb/nucleon} \quad \text{(E771)}
\]

The $x_F$ and $p_T$ dependence of $J/\psi$ production has been measured at HERA – B [27]. A measurement of the total $J/\psi$ production cross section at HERA – B can be found in [28].
2.7 Nuclear dependence

So far we have discussed the production of heavy quarks in proton proton collisions. At HERA-B, protons collide with targets of different materials. Therefore, we have to take into account the nuclear environment. In a simplified model, one would expect that for hard interactions, such as heavy quark production, the probability of an interaction simply scales according to the number of nucleons in the nucleus (or with the volume of the nucleus). One usually factorises the nuclear dependence of the cross section:

\[ \sigma_{pA} = \sigma_{pp} \cdot A^\alpha, \]  

(2.14)

with \( A \) the atomic number of the nucleus and \( \alpha \) the nuclear dependence. In the simplified model we would expect \( \alpha = 1 \).

For \( J/\psi \) production, \( \alpha \) has been measured to be a little smaller than one[29]. There are several explanations for this nuclear suppression; most of them involve final state effects[30], i.e. effects during or after the formation of the bound charmonium states. The value of \( \alpha \) depends on the kinematic properties of the \( J/\psi \). Figure 2.11 shows the dependence of \( \alpha \) on \( x_F \) and \( p_T \) as measured by the FNAL experiment E866 [29]. HERA-B measures \( J/\psi \) with \( x_F \) between -0.25 and 0.15. The mean \( p_T \) at HERA-B is about 1.2 GeV/c. In this kinematic region, the measured value for \( \alpha_s \) is 0.955 ± 0.005. A recent measurement of nuclear effects in \( J/\psi \) production at HERA-B can be found in [31].

For open heavy quark production, the final state effects on the production cross section can be ignored and only initial state effects, like shadowing [32], should be taken into account. These effects are expected to be small and therefore, in open heavy quark production, no suppression is expected. Measurements of D meson production [33] have shown a dependence of the cross section on \( A \) consistent with \( \alpha = 1 \).
Figure 2.11. $p_T$ (a) and $x_F$ (b) dependence of suppression factor $\alpha$ as measured by E866[29]. In the $x_F$ spectrum the HERA-B values as obtained in [31] are also shown.
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