Chapter 6

Rejuvenating Shells of Supernova Remnants by Pulsar Winds

E. van der Swaluw, A. Achterberg and Y. A. Gallant

abstract

We reconsider the rejuvenation mechanism as proposed by Shull et al. (1989). These authors suggest that an active pulsar can rejuvenate the shell of its associated supernova remnant (SNR), at the moment when the pulsar breaks through the shell of the SNR. The morphology of the remnants G5.4-1.2 and CTB80 seem to confirm this rejuvenation mechanism. The spindown energy deposited by the pulsar, as a relativistic pulsar wind, has a magnitude, which is large enough to explain the observed radio emission structures observed in remnants where the rejuvenation mechanism seems at work.

Shull et al. (1989) did not explain the observed lengthscales of the rejuvenated parts of the SNR shell. Therefore one needs to consider the (diffusive) transport of the injected electrons by the pulsar wind. In this chapter, we investigate the physical diffusive conditions, needed to explain the observed lengthscales. We make a distinction between diffusion along the magnetic field line and perpendicular to the magnetic field line, which is parameterised by the gyrofactor $\eta$. We show that one has to assume a high value for the gyrofactor $\eta \simeq 10^{3-4}$, i.e. diffusion of the electrons along the magnetic field line is much faster than perpendicular to the magnetic field line, in order for the rejuvenation mechanism to work on the observed lengthscales. We discuss the consequences of this assumption on the overall distribution of the electrons at the shell of the SNR.
6.1 Introduction

A supernova remnant (SNR) results from the supernova explosion of a massive star. In some of these explosions the stellar core collapses to a pulsar, which gains a kick velocity at birth. Several mechanisms have been considered for giving the pulsar this kick velocity, such as an anisotropic explosion of the progenitor star, although none of them can be favoured from an observational point of view.

The expansion of the supernova remnant itself is decelerated by the surrounding interstellar medium (ISM), whereas the pulsar moves ballistically at a constant speed. This ultimately results in a break-through of the pulsar through the shell, which has been considered in Chapter 4 of this thesis. In that chapter we concluded that only high-velocity pulsars can break through the supernova remnant shell while the SNR is in the Sedov stage of its expansion. Furthermore we concluded that the interaction between the pulsar wind nebula bow shock around the pulsar and the shell of the remnant leads to a growth of the pulsar wind region, plus a slight enhancement of the density and pressure in the region between the pulsar wind bow shock and the shell of the SNR.

Shull et al. (1989) have argued that, during the interaction between the pulsar wind and the SNR shell, the radio emission from the SNR shell can be rejuvenated. They explain the radio brightening as due to the pulsar's relativistic electrons that have encountered the magnetic field lines in the shell. These electrons can propagate along the magnetic field lines and radiate at radio frequencies. Besides an apparently rejuvenated SNR shell, a diffuse plerionic component around the pulsar is observed in systems like CTB80 (Angerhofer et al. 1981) and G5.4-1.2 (Frail & Kulkarni 1991). For both cases one can see a trail of radio emission which, in the model of a pulsar catching up with the shell of the remnant, is a wake of radio electrons originating from the pulsar wind. In this respect the model seems quite consistent with observations. In this chapter we describe the propagation of the particles injected by the pulsar wind and describe the associated diffusion process. We estimate the diffusive length scales of the injected electrons in the lifetime of a SNR or their radiative lifetime. This will enable us to compare these results with actual lengthscales observed in these kind of systems. In this way, we can constrain the physical conditions needed for the rejuvenation mechanism to work. This chapter is organised as follows. Section 6.2 considers the physical configuration of the pulsar wind-SNR system. Section 6.3 discusses the lengthscales one gets for electron diffusion in the source.
Section 6.4 will apply this to three candidates where the rejuvenation process might be at work: CTB80, G5.4-1.2, and G341.2+0.9 (Frail et al. 1994). Section 6.5 will consider the results of section 6.4 in more detail by using a numerical simulation. Section 6.6 contains the conclusions.

6.2 Rejuvenation

We consider the case where a pulsar has just caught up with the front of a SNR. In this situation there are two main components. First there is the decelerating shell of the SNR. Secondly there is the pulsar wind nebula (PWN) associated with the pulsar moving at constant speed. For typical parameters, the SNR shell at this stage of its evolution is a slowly fading radio source. The reason for the increasingly weak emission is twofold: (1) due to the deceleration of the SNR shock, it is less efficient in accelerating particles to radio-emitting energies, (2) the accelerated particles from the first (more energetic) stage of the SNR have lost a significant amount of their energy due to synchrotron losses, and adiabatic losses in the remnant’s interior. The pulsar wind associated with the pulsar, on the other hand, consists of an ultra-relativistic, cold flow with a high Lorentz factor ($\Gamma_w \geq 10^6$). This flow is thermalised by a termination shock, leading to a relativistically hot bubble. Because of the low sound speed in the SNR interior with respect to the pulsar velocity, the hot bubble is deformed to a bow shock. The size of the pulsar wind nebula is very small ($\sim 0.01$ parsec) compared with the size of the SNR ($\sim 30$ parsec).

In principle there are two ways to achieve a rejuvenation of the SNR shell: (1) the bow shock bounding the pulsar wind nebula (PWN) reaccelerates electrons already present in the SNR shell to higher energies; (2) the pulsar wind itself rejuvenates the shell of the SNR by injecting fresh already highly relativistic electrons. In this chapter we will take the pulsar wind as the source for radio electrons, and do not discuss the details about the origin of these electrons. The main point of this chapter is how the freshly injected electrons diffuse in the PWN/SNR system. The basic geometry of the system is illustrated in figure 6.1.
Figure 6.1: Configuration of the system considered in our calculations: the pulsar wind is terminated by a wind shock (dashed line). The pulsar wind nebula itself is bounded by a bow shock. Closeby is the supernova remnant shock. The re-energised particles of the supernova remnant of the freshly injected particles of the pulsar wind nebula have to propagate along this SNR shock in order to be visible at radio frequencies.
6.2 Rejuvenation

6.2.1 Diffusion of radio electrons

General considerations

Relativistic electrons in astrophysical flows radiate part of their energy as synchrotron radiation due to the presence of a magnetic field, $B$ in the plasma. During this emission process a relativistic electron, with Lorentz factor $\gamma$, mass $m_e$ and charge $e$, radiates at a characteristic frequency $\nu_s$ (the frequency where the emission spectrum peaks), which equals (Rybicky & Lightman 1979):

$$\nu_s = \frac{3 \gamma^2 qB}{4 \pi m_e c},$$

$$\simeq 4.67B_{\mu G} E_{GeV}^2 \text{ MHz}. \quad (6.1)$$

The associated timescale $\tau_{\text{loss}}$, after which half of the energy of the electron has been lost due to synchrotron radiation, equals:

$$\tau_{\text{loss}} = \frac{6 \pi m_e c}{\sigma_T B^2 \gamma},$$

$$\simeq 2 \times 10^9 B_{\mu G}^{-3/2} \nu_{\text{MHz}}^{-1/2} \text{ yr}. \quad (6.2)$$

Here $\sigma_T$ is the Thomson cross section, $\sigma_T = 6.6 \times 10^{-25} \text{ cm}^2$. The propagation of relativistic particles through the flow of the plasma is a combination of advection by the large-scale flow, and diffusion with respect to this flow, mediated by the (gyro-)resonant interaction with Alfven waves (e.g. Skilling 1975a&b). For the diffusion mechanism we consider the limit of Bohm diffusion, where the mean free path $\lambda$, equals the gyroradius $r_g$ defined as $r_g = pc/qB$, where $c$ is the light speed. The Bohm diffusion coefficient equals $\kappa_B = \frac{1}{3} c \lambda = \frac{1}{3} c r_g$. Expressing $\kappa_B$ in terms of the characteristic frequency $\nu_s$ at which the synchrotron spectrum peaks, the diffusion coefficient can be written as:

$$\kappa_B(\nu) = 7.5 \times 10^{31} \nu_{\text{MHz}}^{1/2} B_{\mu G}^{-3/2} \text{ cm}^2/\text{sec}. \quad (6.3)$$

Now consider particles injected at $t = 0$. The diffusion lengthscale, $\Delta R_{\text{syn}}$, at the synchrotron loss time $\tau_{\text{loss}}$ equals for Bohm diffusion:

$$\Delta R_{\text{syn}} = 9.7 B_{\mu G}^{-3/2} \text{ parsec}. \quad (6.4)$$
Notice that the observing frequency drops out in this expression. These values will be used for reference in the more general case where the mean free path $\lambda$ satisfies $\lambda \gg r_g$.

**Radio electrons from a pulsar wind**

Consider the case where the pulsar wind is sufficiently close to the shell of a SNR. We investigate the length scales associated with the injected radio electrons in such a considered configuration. Because of the small size of the PWN, when compared with the SNR, we can approximate the source of radio electrons as a point source in the SNR interior. Normalising the timescale to the crossing time for a pulsar moving at speed $V_{psr}$ with the SNR shell, $R_{snr}/V_{psr} \propto 10^4$ years, we can write the diffusion lengthscale for radio electrons as:

$$\Delta R_{\text{radio}} \approx 2.2 \times 10^{-2} \nu_{1/4} B_{-3/4} (R_{snr}/V_{psr})^{1/2} \text{ parsec}, \quad (6.5)$$

where $R_{snr}$ is the radius of the SNR in parsec, $V_{psr}$ is the velocity of the pulsar in units of 100 km/sec, and we again assume Bohm diffusion. This lengthscale is similar to the size of the PWN itself, i.e. the radio electrons are more or less confined to the immediate surroundings of the PWN. In order to diffuse into the SNR over distances as large as the observations seem to imply, we extend our diffusion model in the following way: we make a distinction between diffusion along the magnetic field lines ($\kappa_\parallel$) and perpendicular to the magnetic field ($\kappa_\perp$).

The mean free path along the magnetic field line then equals $\lambda_\parallel = \eta r_g$, with $\eta$ the gyrofactor, which is related to the turbulence level $\delta B$ in the magnetic field by $\eta = (\delta B/B)^{-2}$. The diffusion coefficient along the magnetic field lines equals:

$$\kappa_\parallel = \eta \kappa_B \quad (6.6)$$

One usually assumes $\eta \gg 1$. Perpendicular to the magnetic field lines we follow Jokipii (1987) and assume that a particle scatters one gyroradius across field lines for every parallel scattering length, so:

$$\kappa_\perp = \eta \kappa_\parallel / (1 + \eta^2) \simeq \kappa_\parallel / \eta \quad (6.7)$$

With this description for diffusion particles diffuse faster along the magnetic field lines than compared with quasi-isotropic Bohm diffusion. Using this description together with the timescale for synchrotron losses, $\tau_{\text{loss}}$, one can write the diffusion length scale at time $t$ of the radio electrons as:
6.3 Comparison with observations

Table 1: Investigated systems

<table>
<thead>
<tr>
<th>SNR</th>
<th>pulsar</th>
<th>age (in years)</th>
<th>frequency</th>
<th>distance (in parsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTB80</td>
<td>PSR 1951+32</td>
<td>10$^9$</td>
<td>600 MHz</td>
<td>2.0</td>
</tr>
<tr>
<td>G5.4-1.2</td>
<td>PSR B1757-24</td>
<td>1.6 $\times$ 10$^4$</td>
<td>330 MHz</td>
<td>2.0</td>
</tr>
<tr>
<td>G341.2+0.9</td>
<td>PSR 1643-43</td>
<td>3.3 $\times$ 10$^4$</td>
<td>1.5 GHz</td>
<td>6.9</td>
</tr>
</tbody>
</table>

$$\Delta R_\parallel = \Delta R_{\text{syn}} \times \left(\frac{\kappa_\parallel}{\kappa_B}\right)^{1/2} \times \left(\frac{t}{\tau_{\text{loss}}}\right)^{1/2} \text{ parsec},$$  \hspace{1cm} (6.8)

with $\Delta R_{\text{syn}}$ given by equation (6.4). This gives the diffusive lengthscale along the magnetic field, where the diffusion proceeds rapidly.

6.3 Comparison with observations

In this section, we will use equation (6.8) to determine a value for $\eta$ in order to get to length scales $\Delta R$ observed in a number of remnants where rejuvenation processes are thought to occur. We make the following two assumptions, both of which lead to a lower limit for the gyrofactor $\eta$:

1) the timescale $t$ for the interaction between the pulsar wind and the shell of the remnant is taken to be the age of the remnant, instead of the interaction time which will be less;

2) the diffusion process is assumed to take place in a uniform magnetic field rather than in a curved, position-dependent magnetic field.

We consider three different SNRs, observed at radio frequencies. From the distance to the SNR, the observed frequency and the age one can derive all the parameters of interest needed to derive the length scales of the radio arms, and the minimum value for the gyrofactor $\eta$ needed to explain these radio arms using the rejuvenation mechanism. The table summarises all the relevant parameters of the considered remnants.
6.3.1 CTB80 and PSR 1951+32

Figure 6.2 shows the SNR CTB80. The radio arms are clearly visible, which started the idea of a rejuvenation mechanism. The radio arms emanate from a more compact core where the pulsar is located, together with the associated PWN. The PWN shows a wake of particles, one expects for a PWN associated with a supersonically moving pulsar. The observation frequency equals 600 MHz. At an assumed distance of $\sim 2$ kpc (Strom & Stappers, 2000), each radio arm has a lengthscale of $\sim 17$ parsec. The assumed age of the system equals $10^5$ years, derived from the characteristic age of the associated pulsar. With the above parameters one gets $\tau_{\text{loss}}(600 \text{ MHz}) = 8.2 \times 10^7 B^{-3/2}_{\mu G}$ years. Combining this with equation (6.8), we derive a minimum value for the gyrofactor needed to get the observed lengthscales from field-aligned diffusion of the radio electrons:

$$\eta \geq 2.5 \times 10^3 B^{3/2}_{\mu G}.$$  \hspace{1cm} (6.9)

6.3.2 G5.4-1.2 and PSR B1757-24

The system of G5.4-1.2 was observed recently by Gaensler and Frail (2000). They derived a new limit for the velocity of the pulsar. Their map is at a frequency of 330 MHz. The characteristic age of the pulsar equals $1.6 \times 10^4$ yr and the assumed distance to the system is $\sim 2.0$ kpc. For this distance the radio arms observed in the associated remnant have a length of $\sim 15$ parsec. From these parameters we derive a value for the loss time $\tau_{\text{loss}}(330 \text{ MHz}) = 10^8 B^{-3/2}_{\mu G}$ years. Together with equation (6.8) the derived loss time corresponds to

$$\eta \geq 1.4 \times 10^4 B^{3/2}_{\mu G}.$$  \hspace{1cm} (6.10)

6.3.3 G341.2+0.9 and PSR 1643-43

The system of G341.2+0.9 was observed by Frail et al. (1994) at 1.5 GHz. The system is at a distance of $\sim 6.9$ kpc (Taylor & Cordes 1993) and the pulsar’s characteristic age is 32.6 kyr. This implies that the radio arms observed have a size $\sim 15$ parsec. The synchrotron loss time at this frequency equals $\tau_{\text{loss}}(1.5 \text{ GHz}) = 5.2 \times 10^7 B^{-3/2}_{\mu G}$ yr. These values when substituted in equation (6.8) yield:

$$\eta \geq 3.6 \times 10^3 B^{3/2}_{\mu G}.$$  \hspace{1cm} (6.11)
Figure 6.2: Radio image of the SNR CTB80. The data are from WSRT at a frequency of 600 MHz (R. Strom & B. Stappers 2000).
Conclusions

We have shown that, in order to explain the observed radio arms in three SNRs by a rejuvenation process as suggested by Shull et al. (1989), one has to introduce a minimum value for the gyrofactor of order $\eta \geq 10^{3-4}$. This means that the level of turbulence in these remnants must be small $\delta B \simeq B/\sqrt{\eta} \simeq 0.01 - 0.03B$. In section 6.5 we investigate the consequences of such a value for the observed radio profiles in a SNR.

6.4 Numerical Simulations

In Chapter 4 of this thesis, we presented results of hydrodynamical simulations of a pulsar wind nebula bounded by a bow shock. In this section we combine the hydrodynamics code with a Monte-Carlo method, which traces the position of test particles injected at the pulsar wind termination shock.

Formally, in order to calculate the propagation and acceleration of particles in astrophysical plasmas, one has to solve a Fokker-Planck equation:

$$\frac{\partial F(Z,t)}{\partial t} = \frac{\partial}{\partial Z} \cdot \left( -\dot{Z} F(Z,t) + \frac{\partial}{\partial Z} \cdot [D F(Z,t)] \right).$$

(6.12)

Here $Z \equiv (x(t), p(t))$ describes the position of the particle in phase space, $F(Z,t)$ is the particle distribution function of the system at time $t$, $\dot{Z}$ is the advection velocity in phase space and $D$ is the diffusion tensor. It has been shown (e.g. Gardiner 1983, Saslaw 1985) that the Fokker-Planck equation corresponds to the stochastic differential equation (SDE) of the Itô form:

$$dZ = \dot{Z}(Z,t) \, dt + 2^{1/2} \sqrt{D} \, dW.$$

(6.13)

The term $dW$ entering in the stochastic part of this equation is a 2N-dimensional Wiener process, where N is the number of degrees of freedom in phase space (e.g. MacKinnon & Craig (1991), Krülls & Achterberg (1994)). The Wiener process is chosen at each time step $dt$ from a Gaussian distribution, in such a way that $\langle dW_i \rangle = 0$ and the variance satisfies (in component form) $\langle dW_i \, dW_j \rangle = \delta_{ij} \, dt$.

This correspondence between an Itô stochastic differential equation and the Fokker-Planck equation allows for a fast simulation method (Achterberg & Krülls (1992)) capable of calculating the particle acceleration and particle propagation in astrophysical flows. We do not consider particle acceleration in this chapter,
Figure 6.3: Configuration of the system of pulsar wind and bow shock. The magnetic field lines are aligned with the x-axis. Diffusion of electrons along these magnetic field lines proceeds faster than perpendicular to the magnetic field lines ($\eta \gg 1$). Except in the pulsar wind bubble where diffusion is taken isotropic ($\eta = 1$).
Figure 6.4: Synchrotron map at X-ray frequencies for a simulation with gyrofactor $\eta = 1$ (right) and gyrofactor $\eta = 10$ (left). For a gyrofactor $\eta = 10$, the emission profile become much sharper compared with emission profile with $\eta = 1$. 
as we are only interested in the distribution of particles continuously injected at
the pulsar wind termination shock. The position of the particles is changed each
time step according to:

\[ dx = U_{\parallel} \, dt + \sqrt{2D(x)} \, dt \, \xi. \tag{6.14} \]

The velocity \( U_{\parallel} \) is obtained from the hydrodynamics code. The stochastic term \( \sqrt{2D(x)} \, dt \) uses a random number \( \xi \), drawn from a Gaussian distribution
with zero mean and unit variance, in effect writing \( dW = \sqrt{dt} \, \xi. \)

The geometry of the pulsar wind-bow shock system is taken to be axially
symmetric around the y-axis (see figure 6.3). The magnetic field is taken along
the x-axis (see figure 6.3). We employ anisotropic diffusion as described in section
6.2, which means that the Monte Carlo method has to be applied in 3D.

The simulations use a value of the diffusion coefficient \( \kappa \sim 10^{28} \), assuming
Bohm diffusion. For these values, the corresponding electrons would radiate at
X-ray frequencies in a \( \sim 10 - 100 \mu G \) magnetic field. However the simulations
here are performed to give a qualitatively result, which can be used to illustrate
the effect of a gyrofactor \( \eta \neq 1.0. \)

We present results from two simulations. In the first simulation the diffusion
coefficient is quasi-isotropic \( (\eta = 1) \), in the second simulation the diffusion is
mostly along the magnetic field lines, with a value for the gyrofactor \( \eta = 10. \)
The results are shown in the figures 6.4 and 6.5. Two effects are visible from the
comparison between these two cases: 1) When the diffusion is not isotropic, the
particles can diffuse further away from their place of injection as compared with
particles which diffuse isotropically, as expected. 2) Figure 6.5 on the other hand
illustrates that dependent on the angle between the observer and the magnetic
field, the effect introduced by the gyrofactor has its maximum effect when the
PWN is observed from a direction perpendicular to the magnetic field in which
the PWN is embedded.

### 6.5 Discussion

We have reconsidered the rejuvenation mechanism as proposed by Shull et al.
(1989). This was done by deriving values for the gyrofactor, \( \eta \), needed to get
diffusive length scales which are comparable with the observed size of the radio
arms in three supernova remnants: CTB80, G5.4-1.2 and G341.2+0.9. A nu-
merical simulation was conducted to see the consequences of a high value for
this gyrofactor. The conclusions of this work are as follows:
Figure 6.5: Synchrotron map at X-ray frequencies, illustrating the effect of different angles of observation with the gyrofactor, $\eta = 10$. The figure has arm-like features for $\phi = 90^\circ$, which disappear for $\phi = 0^\circ$. 
6.5 Discussion

- The gyrofactor has to have a minimum value $\eta \geq 10^{3-4}$ in order to let rejuvenated particles diffuse over length scales, comparable with the observed size of the radio arms;

- As a consequence, this implies that the diffusion of particles perpendicular to the magnetic field proceeds very slowly. Dependent on the orientation of the magnetic field with the shock this influences the precursor of the SNR shock.

Under the circumstances we assumed, i.e. the diffusion time equals the age of the SNR and the diffusion is along a uniform planar field the derived value for the gyrofacor $\eta$ is a minimum value. The reason is twofold:

- The velocity of the pulsar has to be perpendicular to the magnetic field orientation where the pulsar crosses the shock. If this is not the case the value for the gyrofactor has to be increased.

- The angle of observation has to be such that the system is observed almost perpendicular to the pulsar’s velocity, otherwise one sees part of the rejuvenated shell in projection.

We can summarise by mentioning that there are problems with the rejuvenation scenario as an explanation for the brightening of SNR shells to occur and brighten the supernova remnant over large scales. Although energetically there might be no problem, one needs a diffusion mechanism with a gyrofactor in the range $\eta \geq 10^{3-4}$ and a special field geometry. Since there are more SNRs observed where only one shell is brightened at radio frequencies without a pulsar wind connected with the system, it becomes questionable whether the pulsar wind is indeed responsible for the brightening of the shell of a SNR like CTB80, G5.4-1.2 and G341.2+0.9. or whether environmental reasons are responsible.
Bibliography