Chapter 4

Interaction of high-velocity Pulsars with old Supernova Remnants

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abstract

Hydrodynamical simulations are presented of a pulsar wind associated with a supersonically moving pulsar. The pulsar moves through the interstellar medium or, in the more interesting case, through the supernova remnant created at its birth event. In both cases there exists a bow shock around the pulsar wind nebula. Using hydrodynamical simulations we study the behaviour of the pulsar wind nebula inside a supernova remnant, and in particular the interaction with the outer shell of swept-up interstellar matter and the blast wave bounding the remnant which occurs when the pulsar breaks out of the supernova remnant.
4.1 Introduction

A supernova explosion of a massive star will result in an expanding supernova remnant (SNR). In some cases the fossil of the progenitor star is a pulsar moving at high velocity. Even though the precise physical mechanism responsible for imparting a large kick velocity to single radio pulsars at birth has not been identified, observations of the pulsar distribution with respect to the plane of the galaxy indicate that they are born with a velocity in the range $V_{\text{psr}} \sim 100 - 1000$ km/s (Harrison et al. 1992; Lyne & Lorimer 1994). A similar range of values is obtained from a sample of SNR-pulsar associations (Frail et al. 1994).

The expansion of a SNR is decelerated due to the mass-loading by the swept-up interstellar medium (ISM) or by material from a progenitor wind. As the pulsar moves with a constant velocity it will ultimately break through the SNR shell. Two observed systems are often presented as an illustration of this scenario:

*CTB80:* in this supernova remnant the pulsar PSR 1951+32 is located (in projection) just inside the outer edge of the remnant. The spectral index of the synchrotron emission in the vicinity of the pulsar system indicates that there is a plerion around the pulsar, see for example Strom (1987) and Strom & Stappers (2000).

*G5.4-1.2:* in this case the pulsar is located well outside the supernova remnant. At radio frequencies an emission bridge connects the pulsar B1757-24 and the associated pulsar wind nebula (PWN) with the supernova remnant (Frail & Kulkarni 1991). New upper limits on the proper motion of B1757-24 (Gaensler & Frail 2000), which put the transverse component of the pulsar velocity at $V_{\perp \text{psr}} \leq 600$ km/s for an assumed distance of 5 kpc, leads to a discrepancy between the characteristic pulsar age obtained from its spin period derivative ($P/2 \dot{P} \sim 16$ kyr) and the dynamical age obtained from the offset distance $R_{\text{psr}}$ from the center of G5.4-1.2 ($R_{\text{psr}}/V_{\text{psr}} \sim 39$ kyr).

Both systems are clearly brightened at radio wavelengths near the position of the pulsar, suggesting that the associated pulsar wind is rejuvenating the radio emission from the SNR shell by the injection of fresh relativistic electrons (Shull et al. 1989). In this chapter we will investigate the hydrodynamical aspects of the interaction between a pulsar wind and a SNR shell. In chapter 6 we will discuss the question of rejuvenation of the radio-emitting electrons.
Most pulsars have a lifetime \((10^6 - 10^7 \text{ yr})\) which is much larger than the age \(\leq 10^4 \text{ yr}\) of a SNR in the Sedov phase. Therefore pulsars will remain visible long after the associated SNR has dissolved into the interstellar medium and is no longer visible. In that case an isolated pulsar will move through the interstellar medium, and can form a pulsar wind nebula bow shock system. A typical example of such a system is the Guitar Nebula around PSR 2224+65 which has been detected both in X-rays (Romani et al. 1997) and in H\(\alpha\) (Cordes et al. 1993), but which has no associated SNR.

In this chapter we consider the case where the pulsar’s kick velocity is sufficiently high so that it leaves the supernova remnant while it is still in the Sedov stage. We describe three different stages in the evolution of the pulsar-SNR system: (1) the stage where the PWN/bow shock resides inside the SNR, (2) the PWN/bow shock breaking through the shell of the SNR and (3) the stage where the PWN/bow shock moves through the ISM.

### 4.2 Physics of a PWN bow shock inside a SNR

#### 4.2.1 Dynamics of the pulsar/SNR system

In rapidly rotating (young or recycled) pulsars, it is believed that a pulsar wind is driven by the spindown luminosity,

\[
L = I\dot{\Omega}\Omega, 
\]

of a pulsar with rotation period \(P = 2\pi/\Omega\) and moment of inertia \(I\). This relativistic wind is presumably generated in the pulsar magnetosphere, and accelerates electrons, positrons and possibly nuclei to ultra-relativistic speeds.

The pulsar wind blows a bubble (pulsar wind nebula: PWN) into the surrounding medium. The PWN is initially located well within the interior of the SNR created at the birth of the neutron star. During the free expansion stage of the SNR evolution the typical expansion speed of the stellar ejecta as determined by the mechanical energy \(E_{\text{snr}}\) released in the explosion and the ejecta mass \(M_{\text{ej}}\),

\[
V_{\text{ej}} \sim \sqrt{\frac{10}{3} \frac{E_{\text{snr}}}{M_{\text{ej}}}} \sim 10,000 \text{ km/s},
\]

is generally much larger than the kick velocity of the pulsar. As a result the PWN is located relatively close to the center of the SNR at this stage.
Only when the SNR expansion slows down as it enters the Sedov stage after some \( \sim 500 - 1,000 \) yr, a situation is possible where the pulsar position becomes strongly excentric with respect to the SNR.

The Sedov stage of SNR expansion lasts until internal (radiative) cooling becomes important. The SNR then enters the so-called pressure-driven snowplow (PDS) stage. The relevant transition time is calculated by Cioffi et al. (1988):

\[
t_{\text{PDS}} = 1.33 \times 10^4 E_{51}^{3/14} \zeta_m^{-4/15} n_0^{-4/7} \text{ yr.} \tag{4.1}
\]

Here \( E_{51} \) is the explosion energy \( E_{\text{snr}} \) in units of \( 10^{51} \) ergs, \( \zeta_m \) denotes the metallicity and \( n_0 \) is the number density in the ISM, assuming \( n_{\text{He}}/n_H = 0.1 \).

We will describe the physics of a pulsar wind interaction with the shell of a SNR in the Sedov stage. Since the proper motion of the pulsar is supersonic with respect to the surrounding medium the outer rim of the PWN will deform its shape which results in the formation of a bow shock. Consequently the results presented below only apply for certain range of values for the pulsar velocity \( V_{\text{psr}} \).

Equating the distance travelled by the pulsar,

\[
R_{\text{psr}} = V_{\text{psr}} t,
\]

with the radius for a SNR embedded in a homogeneous interstellar medium of density \( \rho_{\text{ism}} \) in the Sedov stage,

\[
R_{\text{snr}} \approx 1.15 \left( \frac{E_{\text{snr}}}{\rho_{\text{ism}} V_{1000}^5} \right)^{1/5} t^{2/5}, \tag{4.2}
\]

one gets the crossing time for the pulsar:

\[
t_{\text{cr}} = 1.27 \left( \frac{E_{\text{snr}}}{\rho_{\text{ism}} V_{1000}^5} \right)^{1/3} \approx 1.4 \times 10^4 E_{51}^{1/3} V_{1000}^{-5/3} n_0^{-1/3} \text{ yr.} \tag{4.3}
\]

Here \( V_{1000} \) is the velocity of the pulsar in units of 1,000 km/sec, and \( n_0 \) the number density of the ISM in units of cm\(^{-3}\). The requirement \( t_{\text{cr}} \leq t_{\text{PDS}} \) yields the minimum velocity a pulsar needs in order to break out of the SNR while it is still in the Sedov stage:

\[
V_{\text{psr}} \geq 1,030 \zeta_m^{4/25} n_0^{1/7} E_{51}^{1/14} \text{ km/s.} \tag{4.4}
\]

Although this is a rather high value, it is still in the range of values observed by Harrison et al. (1992).
4.2 Physics of a PWN bow shock inside a SNR

One can use the Rankine-Hugoniot relations to determine the pressure just behind the Sedov-Taylor blast wave bounding the SNR (assuming a gas with specific heat ratio $\gamma = 5/3$)

$$P_{sh} = \frac{3}{4} \rho_{ism} V_{snr}^2,$$  \hspace{1cm} (4.5)

where

$$V_{snr} \equiv \frac{dR_{snr}}{dt} = \frac{3}{5} \frac{R_{snr}}{t}$$

is the SNR expansion speed. Using this expression plus the expression (4.3) for the crossing time and the Sedov solution (4.2) one can derive the following equations valid at the moment of break-through. The speed of the pulsar is related to the SNR expansion speed by

$$V_{psr} = \frac{5}{2} V_{snr},$$  \hspace{1cm} (4.6)

while the material in the shell behind the SNR blast wave moves with a velocity

$$V_{sh} = \frac{3}{4} V_{snr}.$$  \hspace{1cm} (4.7)
This corresponds to a relative speed between pulsar and post-shock material equal to

\[ V_{\text{rel}} \equiv V_{\text{psr}} - V_{\text{sh}} = \frac{7}{4} V_{\text{snr}}. \]  

(4.8)

The density in the shell is \( \rho_{\text{sh}} = 4\rho_{\text{ism}} \), so the Mach number \( M_{\text{psr}} \) of the pulsar motion through the shell material satisfies

\[ M_{\text{psr}} = \frac{V_{\text{rel}}}{\sqrt{\frac{\gamma P}{4\rho_{\text{ism}}}}} = \frac{7}{\sqrt{5}} \approx 3.13. \]  

(4.9)

### 4.2.2 Pulsar Wind

A pulsar wind is believed to consist of an ultra-relativistic, cold flow with a large bulk Lorentz factor (\( \Gamma_w \geq 10^6 \)) (Kennel & Coroniti 1984). In such a wind, the energy flux \( S \) and momentum flux \( M \) are approximately given by

\[ M = \frac{S}{c} \approx \Gamma_w^2 n_w m c^2, \]  

(4.10)

where \( n_w \) is the proper density in the wind and \( m \) the mean mass per particle. The cold wind is terminated by a termination shock which thermalizes the flow, leading to a relativistically hot downstream state with sound speed \( s \sim c / \sqrt{3} \). The typical pressure behind the ultra-relativistic termination shock, located at some radius \( R_{\text{ts}} \), is (e.g. Blandford & McKee, 1976)

\[ P_{\text{ts}} = \frac{2}{3} \Gamma_w^2 n_w m c^2 \approx \frac{L}{6\pi R_{\text{ts}}^2 c}. \]  

(4.11)

The last equality in terms of the total luminosity \( L \) of the wind is approximate because of deviations of sphericity of the pulsar wind region, induced by the proper motion of the pulsar. The shocked pulsar wind material is separated from material that has gone through the bow shock by a contact discontinuity. Because of the high internal sound speed, both in the pulsar wind material and in the material that has passed through the bow shock, and because of the small size of the region between the termination shock and bow shock, the region between termination shock and bow shock can be considered to be isobaric to lowest approximation.
At the stagnation point at the head of the bow shock surrounding the PWN momentum flux conservation on the central streamline puts the pressure at
\[ P_{bs} = \rho_{sh} V_{rel}^2 + P_{sh} = 13 \rho_{ism} V_{snr}^2. \] (4.12)

After the pulsar has broken through the shell the pulsar wind is completely confined by the ram pressure of the cold ISM and the stagnation-point pressure drops to
\[ P_{bs} = \rho_{ism} V_{psr}^2 = \frac{25}{4} \rho_{ism} V_{snr}^2, \] (4.13)
a pressure reduction by roughly 50% as the pulsar leaves the SNR. The fact that the region between termination shock and bow shock is almost isobaric implies
\[ P_{bs} \approx P_{ts}, \]
This determines the stand-off distance of the pulsar wind termination shock as
\[ R_{ts} = \eta \left( \frac{L}{6\pi \rho_{ism} V_{psr}^2 c} \right)^{1/2}, \] (4.14)
where the numerical factor \( \eta \) takes the value \( \eta = \sqrt{25/52} \approx 0.693 \) when the pulsar is still just inside the SNR, and \( \eta = 1.0 \) when the pulsar moves through the ISM. This is also the typical stand-off distance of the bowshock, which is always close to the termination shock at the head of the pulsar wind nebula.

These expressions allow us to calculate the relative size of the pulsar wind to the supernova remnant at the moment of break through. From the expression (4.3) for the crossing time one has
\[ R_{snr} (t_{cr}) = V_{psr} t_{cr} = 13.6 E_{51}^{1/3} V_{1000}^{-2/3} n_0^{-1/3} \text{ pc}. \]
The termination shock radius is of order
\[ R_{ts} \approx 57.6 \eta L_{34}^{1/2} n_0^{-1/2} V_{1000}^{-1/3} \text{ AU}. \]
Here \( L_{34} = L/(10^{34} \text{ erg/s}) \). Note that the size of the SNR shell is much larger than the size of the PWN. For this reason we will neglect the curvature of the SNR blast wave and perform a hydrodynamical simulation where the pulsar moves with a Mach number of \( M = 3.13 \) through the post-shock flow of a strong plane-parallel shock, ultimately crossing this shock into the unshocked medium.
Table 1: Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulsar wind luminosity $L_0$ (erg/s)</td>
<td>$1.3 \times 10^{34}$</td>
</tr>
<tr>
<td>ISM mass density $\rho_0$ (g/cm$^3$)</td>
<td>$1.0 \times 10^{-24}$</td>
</tr>
<tr>
<td>Terminal Velocity $v_\infty$ (cm/sec)</td>
<td>$3.0 \times 10^9$</td>
</tr>
<tr>
<td>Number of grid cells (in r-direction)</td>
<td>300</td>
</tr>
<tr>
<td>Number of grid cells (in z-direction)</td>
<td>180</td>
</tr>
<tr>
<td>Grid size (pc) (in r-direction)</td>
<td>0.1</td>
</tr>
<tr>
<td>Grid size (pc) (in z-direction)</td>
<td>0.06</td>
</tr>
</tbody>
</table>

4.3 Hydrodynamics of the PWN bow shock

4.3.1 Simulation Method

We simulate a pulsar wind using the Versatile Advection Code (VAC), a non-relativistic hydrodynamics code developed by Gábor Tóth at the Astronomical Institute in Utrecht (Tóth & Odstrčil 1996). The configuration of interest is depicted in figure 1, showing both shocks which are of interest: the pulsar wind termination shock and the bow shock bounding the PWN. This system is assumed to be axially symmetric around the direction of motion of the pulsar. We use a TVD-Lax-Friedrich scheme to solve the equations of hydrodynamics in conservative form. (For an overview of different schemes see for example LeVeque 1998).

4.3.2 Starting a pulsar wind

The hydrodynamics code can only handle a non-relativistic fluid with a single equation of state. We use an ideal fluid with specific heat ratio $\gamma = 5/3$. We simulate the pulsar wind by continuously depositing thermal energy at a constant rate $L$ (the spin-down luminosity) in a small volume, together with an associated
mass injection $\dot{M}_{\text{pw}}$. The hydrodynamics code itself then develops a wind with terminal velocity $v_\infty$ before the wind is thermalized by a termination shock. The mechanical luminosity $L$ and mass deposition rate $\dot{M}_{\text{pw}}$ are chosen such that the terminal velocity of the wind as determined from these two parameters is close to the speed of light:

$$v_\infty = \sqrt{2L/\dot{M}_{\text{pw}}} \approx c.$$  

This choice will result in the correct global behaviour of the PWN. This method is similar to the method as described in Van der Swaluw et al. (2001). We employ a non-uniform grid with largest resolution near the pulsar in order to resolve the pulsar wind. The radius of the termination shock, given by Eqn. (4.14) in the relativistic case, is replaced by its non-relativistic equivalent,

$$R_{\text{ts}} \approx \eta \left( \frac{L}{2\pi \rho_{\text{ism}} V_{\text{psr}}^2 v_\infty} \right)^{1/2},$$

and will have roughly the correct value when $v_\infty \approx c$.

### 4.3.3 Steady PWN and bow shock in a uniform medium

Our calculations are performed in the pulsar rest frame. The pulsar wind nebula is allowed to evolve in a uniform medium, moving at a constant speed $V_{\text{psr}}$ at large distances from the pulsar. This medium represents the interior of the supernova remnant (shocked ISM) close to the blast wave. The velocity $V_{\text{psr}}$ is supersonic with respect to the internal sound speed of the medium so that a bow shock develops around the PWN. We let the hydrodynamics code evolve the system until the large-scale flow is steady.

In order to determine when the system is steady, we employ a recipe of Toth et al.(1998). This recipe compares all $N_{\text{var}}$ flow variables at time $t_i$ (denoted by $U_n(i|k)$ at grid point $k$) with their values at the previous time $t_{i-1}$. We then calculate the residue $\text{Res}$ defined as

$$\text{Res} \equiv \sqrt{\frac{1}{N_{\text{var}} \sum_{n=1}^{N_{\text{var}}} \sum_k [U_n(i+1|k) - U_n(i|k)]^2}{\sum_k U_n^2(i|k)}}, \quad (4.15)$$

and halt the calculation when $\text{Res}$ has a value less then a predetermined critical value.
Figure 4.2: Comparison between the numerical result for the bow shock with a low Mach Number with the equation as given by Wilkin (1996). The contour plot gives the pressure profile, whereas the solid line is the profile as was given by Wilkin.
Figure 4.3: Density profile of a PWN bow shock with the parameters as denoted in table 1. The gray-scale corresponds to the density.
Wilkin (1996) has given an analytical equation for the geometry of a wind bow shock. His solution, in terms of the distance $r$ to the wind source and polar angle $\theta$ with respect to the symmetry axis, reads:

$$
\frac{r(\theta)}{R_0} \equiv \frac{1}{\sin \theta} \sqrt{3 \left(1 - \frac{\theta}{\tan \theta}\right)}.
$$

(4.16)

Here $R_0 \approx R_{ts}$ is the stand-off distance of the bow shock on the symmetry axis ($\theta = 0$). We compare our morphology with Wilkin’s result, where we equate $R_0$ to the stand-off distance of the bow shock in the simulations. The is depicted in figure 4.3. As one can see the cone of the geometry of Wilkin's solution is much narrower. This is because his solution comes from balancing the ram pressures of the wind and the ambient medium, i.e. the limit $M_{psr} \gg 1$, while in our case the Mach number is moderate: $M_{psr} \approx 3.13$.

The figures 4.3 and 4.4 show density profiles of the PWN bow shock of a pulsar moving through a uniform medium. One clearly sees the difference between shocked pulsar wind material and the much denser shocked ISM. The synchrotron emission coming from plerionic PWN is expected to come from the shocked pulsar wind material, whereas the material swept-up by the bow shock can show up as H\(\alpha\) emission. Figure 4.5 shows the pressure distribution and
4.3 Hydrodynamics of the PWN bow shock

Figure 4.5: Pressure profile of a PWN bow shock with the parameters as denoted in table 1. The gray-scale corresponds to pressure.

Figure 4.6 shows a pressure profile along the symmetry axis.

One can see the pulsar wind region around the pulsar, located at $z = 0$, the pulsar wind termination shock at $z \sim 0.025$ pc ahead of the pulsar in the direction of motion, and at $z \sim 0.042$ pc behind the pulsar. The bow shock bounding around the PWN at located at $z \sim -0.015$ pc. The region between the pulsar wind termination shock and the bow shock is almost isobaric. As shown by Van der Swaluw et al. (2001), this is also the case for a PWN around a stationary pulsar located at the center of the SNR.

4.3.4 Interaction of the PWN with a shock

In this section we present results of the break-through of the PWN bow shock through the shell of a supernova remnant. This results are again performed in
the rest frame of the pulsar. We initialise a steady-state configuration of the PWN bow shock as described above, and use the Rankine-Hugoniot relations to implement a strong shock front moving towards the pulsar such that equations (6)-(8) hold true. This simulation has been performed with parameters as denoted in table 1. At the end of the simulation, when the strong shock is almost at the upper boundary of the grid, numerical instabilities arise. Therefore we stop the simulation after the configuration as shown in the figures 4.7-4.9, when the influence of the numerical instabilities are not influencing the solution too strongly.

As stated in section 2, the PWN bow shock is much smaller than the radius of the SNR, so we can safely approximate the SNR blast wave as a plane-parallel strong shock. Figures 4.7 and 4.8 show the system after the SNR shock has passed the head of the bow shock. In figure 4.9 one can see that the pulsar wind nebula has expanded roughly by a factor 1.5 after it leaves the SNR. This reflects the reduction in the confining (ram-)pressure calculated in Section 4.2.

During the interaction between the pulsar wind and the shell of the SNR the PWN bow shock and the SNR blastwave intersect. This interaction produces an additional pressure gradient which results in an accumulation of mass. The pressure and density enhancements can be seen at the region of intersection in the figures 4.7 and 4.8 as bright spots. When the bow shock moves through the

Figure 4.6: Pressure profile of a PWN bow shock where a cut has been made along the z-axis of figure 4.3. In this figure, the pulsar’s position corresponds to $Z = 0$. 
Figure 4.7: *Pressure profile of a PWN bow shock with the same parameters as denoted in table 1, except on a lower resolution. Here the PWN is interacting with the shell of the SNR. The gray-scale corresponds to pressure.*
Figure 4.8: Density profile of a PWN bow shock with the same parameters as denoted in table 1 at a low resolution. Here the PWN is interacting with the shell of the SNR. The gray-scale corresponds to the density.
4.4 Conclusions

We have considered the case of a pulsar wind breaking through the shell of a SNR in the Sedov-Taylor stage. We have shown that only high-velocity pulsars reach the edge of the SNR while the SNR is still in the Sedov stage of its evolution. At moment of break-through, the ratio of the pulsar velocity and SNR expansion speed is fixed at $V_{\text{psr}}/V_{\text{snr}} = 5/2$, and the Mach number associated with the pulsar motion equals $M_{\text{psr}} = 7/\sqrt{5}$. These conclusions are independent of the explosion energy $E_{\text{snr}}$ or the pulsar speed $V_{\text{psr}}$.

Our simulations show that the break-through of the PWN does not lead to a significant disruption. The reduction of stagnation pressure by about 50% leads to a moderate expansion of the PWN where its radius increases by a factor $\sim 1.5$. The latter result can also be obtained analytically.
There is good agreement between our numerical results and analytical estimates, based on pressure balance arguments, for the size of the bowshock surrounding the PWN. The only clear indication of the interaction between the PWN bow shock and the SNR (Sedov-Taylor) blast wave is a density- and pressure enhancement at the intersection of these two shocks.

In chapter 6 we will consider the effects of the energetic particles which are injected by the pulsar wind into the surroundings. There we will show that the rejuvenation mechanism as proposed by Shull et al. (1989) can not be maintained because of diffusion arguments.
Bibliography
