Chapter 1

Pulsar Wind Nebulae and Supernova Remnants

1.1 Introduction

Supernovae are one of the most energetic explosive events in our Galaxy and other galaxies. These events take place when massive stars end their life. The stellar core collapses and they blow off their outer layers. The only thing which remains of the exploded star is a high density neutron star, which may manifest itself as a pulsar, or possibly a black hole. If the supernova explosion is asymmetric, the pulsar must gain a velocity to conserve total momentum. This thesis will deal with phenomena that take place after the supernova explosion. Another mechanism which can cause a supernova explosion is the disruption of a white dwarf in a binary. These explosions will however not result in a neutron star, therefore we will not consider these events in this thesis.

In the supernova explosion there is an energy release of roughly $10^{53}$ erg; 99% of this energy is radiated away in the form of neutrinos. This means that the total mechanical energy of the supernova remnant (SNR) is about $10^{51}$ erg. This mechanical energy provides the kinetic energy of the SNR expansion and the thermal energy of the SNR interior.

The expansion of the SNR can be divided in four stages (Woltjer 1972). In this chapter we will discuss all these different stages. We will also discuss the case when a pulsar wind is embedded within the interior of the SNR.
In the first stage, the *free expansion stage*, the mass of the SNR essentially consists of the ejected mass, $M_{ej}$, of the progenitor star. These ejecta expand freely, where the expansion rate is controlled by the conservation of the kinetic energy. The expansion speed exceeds the sound speed of the interstellar medium (ISM) and a shock will be formed. During the expansion, the SNR will sweep up matter from the ISM, leading to an increase of the mass of the SNR. It will not be long before the swept-up mass $M(t)$ exceeds the ejected mass $M_{ej}$. From this point on the expanding SNR is described by the Sedov solution (Sedov 1959).

In contrast to the free expansion stage, which lasts only $\sim 100\text{-}1,000$ years, the Sedov stage lasts much longer, around $\sim 10,000$ years. In this stage of the SNR expansion the ram pressure of the swept-up material is balanced by the internal pressure. The expansion rate is controlled, as before, by energy conservation: radiation losses can still be neglected.

Ultimately the total mechanical energy of the SNR decreases due to significant radiation losses. At this point the shell of swept-up interstellar gas will cool down and will be driven through the ISM by the pressure of the interior of the SNR. This stage is called the pressure-driven snowplow stage. This stage is followed by the momentum-conserving snowplow stage, when the pressure of the interior of the SNR becomes comparable with the pressure of the ISM: the SNR will start merging with the ISM.

For those SNRs which contain a pulsar, the evolution can be influenced by the presence of a pulsar wind nebula (PWN). A PWN is driven by the spindown energy from a rapidly rotating pulsar. Part of this energy provides the pressure which pushes against the gas of the SNR interior. This leads to the formation of a (relativistically hot) bubble around the pulsar.

The main subject of this thesis is the interaction between the PWN and the SNR. This chapter consists of a short overview of SNRs and PWNe from an observational point of view, which is followed by a theoretical discussion of the several evolutionary stages of a SNR and a PWN.
1.2 Observations of SNRs and PWNe

Supernova remnants can be observed from radio frequencies up to X-ray frequencies. From the morphology of a specific remnant and the spectral information at radio frequencies, one can classify it in a basic category. In the following section I briefly discuss these categories.

1.2.1 Shell type remnants

Shell type remnants are characterised by extended emission at both X-ray frequencies and radio frequencies. The morphology of these type of remnants is a roughly spherical shell. I will make a distinction between observations at radio frequencies and at X-ray frequencies.

Radio emission

Most of the SNRs are of this category. According to Green’s catalogue which contains 225 SNRs (Green 2000), there are about 193 galactic remnants of this type, although the morphology can be very distinct. At radio frequencies the emission is clearly nonthermal, with a spectrum that can be characterized by a single power law, \( S_\nu \propto \nu^{-\alpha} \), over several orders of magnitude in frequency. The power law index varies from \( \alpha = 0.6 \) for young remnants to \( \alpha = 0.5 \) for the older ones. The nonthermal emission is attributed to synchrotron radiation, due to relativistic electrons gyrating around the magnetic fields in the shells of the SNR. These relativistic electrons are probably produced by the mechanism known as diffusive shock acceleration (DSA) (see chapter 2 for a detailed discussion). The idea of DSA is that particles can cross the shock many times due to scattering in a turbulent magnetic field. Each time a particle cycles across the shock it gains energy, but at each crossing cycle there is a finite chance of escape. In this way particles can be accelerated quite efficiently, where simplified DSA predicts a resulting momentum distribution \( N(p) \propto p^{-s} \) (with \( s = (r + 2)/(r - 1) \) and \( r \) is the shock compression ratio). The power law index \( \alpha \) is related to the power law index \( s \) by \( \alpha = (s - 1)/2 \), which yields a power law index \( \alpha = 0.5 \) for a strong shock with compression ratio \( r = 4 \). This is near the observed value for the spectral distribution of older remnants, which seems to confirm the idea of DSA taking place at the blastwaves bounding SNRs.
Figure 1.1: Radio image (at 6 cm) of SNR 0102 in the LMC (Amy & Ball 1993). The observations were performed with the ATCA instrument. The contours were drawn at 0.1, 0.2, ..., 0.9 the peak brightness. The greyscaling on the other hand is displayed logarithmically. This is a typical example of a shell type remnant.
1.2 Observations of SNRs and PWNe

Figure 1.2: Same data as in figure 1. Here the greyscaling corresponds with the X-ray data taken from the Chandra archive. One can see that in this remnant the peaks in the radio and the X-ray frequencies are not at the same positions.
In the free expansion stage the material of the SNR consists mainly of expanding ejecta, bounded by a forward shock. This forward shock is accompanied by a reverse shock, due to the deceleration of the SNR by the ISM. The reverse shock is driven back into the interior of the SNR at a later stage, heating the interior so that the SNR makes the transition to the Sedov-Taylor stage.

Radio observations of SNRs clearly show the presence of the forward shock, which propagates in the ISM. Clear evidence for the presence of a reverse shock, which is driven in the ejected material of the progenitor star, has never been observed in any SNR at these frequencies. This implies that DSA is not efficient at the reverse shock. The reason for this is the much lower value of the magnetic field strength in the ejected material, which slows down the acceleration process significantly. This underlines the importance of a strong magnetic field in the medium in which a shock propagates, in order to make the shock an efficient accelerator. This may also explain the gap between very young remnants like SN 1987A with ages of \( \sim 10 \) years and young remnants like CasA with ages of \( \sim 100 \) years: in the first few hundred years, the forward shock of the remnant is propagating in the bubble of material from the progenitor stellar wind. Again the magnetic field strength seems to be too low to let the shock act as an efficient accelerator. However, when the forward shock has caught up with the front of the stellar wind bubble, it starts to propagate through the medium of the ISM. In this medium the magnetic field is sufficiently strong so that electrons can be accelerated efficiently to radio emitting frequencies.

**X-ray emission**

At X-ray frequencies most of the observed radiation from SNRs has a strong thermal component. The thermal radiation comes from the hot shocked gas inside the remnant at temperatures of \( \propto 10^6 - 10^7 \) Kelvin, from material located between the forward shock and the reverse shock. The radiation mechanism responsible for thermal emission from this hot material are severe: thermal bremsstrahlung, line emission and free-bound emission. The emitting gas can be both the ejected material from the progenitor star and the swept-up interstellar gas. For young remnants, the X-ray emission is dominated by shocked ejecta. With the XMM-Newton satellite, it is possible to investigate the distribution of the heavy elements of the ejecta in a SNR like Tycho (Decourchelle et al. 2001). In older remnants, the temperatures of the gas decreases, due to the deceleration of the blastwave, which makes these remnants harder to detect at X-ray frequencies.
Some remnants are peculiar, such as W44, which has a centrally peaked X-ray surface brightness. Cox et al (1999) suggest that thermal conduction has transported energy to the center of the remnant. There are several other remnants with a similar morphology in X-rays (W28, 3C400.2, Kes 27, MSH 11-61A, 3C391, and CTB 1)(Rho 1995).

The recent discovery of nonthermal X-ray emission from SNRs have provided evidence for particle acceleration at SNR shocks up to energies of 10-100 TeV. The X-ray emission from SN1006 (Koyama et al. 1995) and G347.3-0.5 (Koyama et al. 1997, Slane et al. 1999) is dominated by this nonthermal component. Cas A, Kepler, Tycho and RCW 86 (Allen, Gotthelf, & Petre 1999) also show a nonthermal component, but in these systems there is also a significant thermal component. Very recently another nonthermal shell-type SNR, G266.2-1.2 (Slane et al. 2001) has been added to the list of SNRs which are dominated by a nonthermal component. This is strong evidence for the scenario that cosmic rays up to the knee (around $10^{15}$ eV) of the cosmic ray spectrum, indeed originate from shocks bounding SNRs. The cosmic ray spectrum observed at Earth has a power law spectrum in energy, $N(E)dE \propto E^{-s}dE$, with $s = 2.75$, which has been modified by the transport of cosmic rays in our galaxy. The source spectrum of these particles should have an index close to the value of $s = 2.2$ (Drury et al. 1994). This is again remarkably close to the value of $s = 2.0$ expected from DSA.

### 1.2.2 Plerionic type remnants

A plerion is a filled-center remnant (Weiler & Panagia 1978) at radio and X-ray frequencies. It is thought to contains a pulsar, which converts a significant fraction of its spin-down energy into a pulsar wind. The kinetic energy of this wind, which is believed to be relativistic with a bulk Lorentz factor $\gamma_w \propto 10^4 - 10^6$, is converted into a relativistically hot plasma at a termination shock. This hot plasma is believed to be the source of the observed plerionic emission. At radio frequencies, a plerionic component can be distinguished from a shell-type component by its center-filled morphology and a flat spectrum, $(-0.3 \leq \alpha \leq 0, S_\nu \propto \nu^{-\alpha})$. In the galaxy there are 9 naked\(^1\) plerionic systems known, of which the Crab nebula is the best-known example.

A similar system is 3C58, except that in this plerion no pulsar has been detected. Although all plerions are thought to be driven by a pulsar, this does

\(^1\)Plerions without an observable shell.
not mean that the pulsar itself is always detected: so far 10 of the 32 known galactic plerions contain a detectable pulsar. In general pulsars are thought to have a proper motion with respect to the site of the supernova explosion. This can be due to the asymmetric supernova explosion of the progenitor star. Measurements of pulsar velocities show a range from 100-1000 km/sec (see e.g. Lyne & Lorimer 1994, Hartman 1997). This has two consequences: (1) the position of the pulsar is not always at the center of the plerion, and (2) the pulsar is not necessarily located at the brightest part of the plerion. It can be displaced with respect to the region where the pulsar wind deposited most of its rotational energy. Due to the large synchrotron lifetime of the electrons at radio frequencies compared with the age of the plerion, this region can still have the largest surface brightness. At X-ray frequencies, the pulsar’s position is located at the brightest spot. Here the synchrotron lifetime is shorter then the age of the plerion. This gives an indication of the pulsar’s position in the SNR, even when the pulsar itself has not been detected.

### 1.2.3 Composite remnants

Composite remnants are by far the most interesting category for this thesis. In the Galaxy there are 23 of these systems known. In this type both the plerionic component and the shell component are present. The properties of the plerionic nebula and the outer shell are the same as described above. Probably the best examples for the composite remnants are 0540 in the Small Magelanic Cloud, and MSH15-56 in the Galaxy.

A sub-class of composite remnants are those where the pulsar wind seems to interact with the SNR shell. The prototypes for this kind of system is the remnant CTB80. In this system it seems like the pulsar is about to penetrate the shell of the SNR, although this could be a projection effect. If the latter is indeed the case the pulsar has already broken out of the shell. Another system which has become more controversial recently, is G5.4-1.2. In this remnant the morphology suggests that the pulsar has already broken out of its associated SNR shell. However recent measurements by Gaensler and Frail (2000) yield a proper motion for the pulsar which is too small for the pulsar to have originated from the center of the remnant, if the age of the system is given by the characteristic age of the pulsar. This suggest that either the pulsar and the SNR are not associated or the characteristic age of the pulsar seriously underestimates the true age of the pulsar. Quite recently observations in both radio and X-rays seem to suggest that IC443 is a similar kind of system (Keohane, private communication).
Figure 1.3: Example of a SNR, where the morphology suggests that the pulsar (whose position is marked is at the head of the compact nebula G5.27–0.90) has broken out of the associated SNR shell. The data were taken with the VLA (= Very Large Array), at an observing wavelength of 90 cm; contours are at levels of 10, 25, 50, 100, 150 and 200 mJy beam$^{-1}$, and the peak intensity is 150 mJy beam$^{-1}$. The resolution of the image is $60'' \times 45''$ (figure supplied by Bryan Gaensler).
Figure 1.4: Example of a composite remnant G326.3-1.8 (MSH15-56). It was produced from observations made at 0.843 GHz with a resolution of 43'' using the Molonglo Observatory Synthesis Telescope (MOST) (Whiteoak & Green 1996). The bright region corresponds with the plerionic component. The sphere in which the bright (plerionic) component is embedded corresponds with the rest of the SNR.
The morphology in these kind of systems seems to suggest an interaction between the active pulsar wind and the old SNR shell for these interacting composites: the high-velocity pulsar re-energizes its associated shell (Shull et al. 1989).

In order for this process to occur the pulsar and the SNR shell have to be physically associated and not be due to an apparent projection effect. If the pulsar and the SNR shell are indeed associated, then the next question is whether the pulsar wind is indeed capable of reenergizing the old SNR shell. This question is addressed in the chapters 4 and 6 of this thesis.

Pulsars which catch up with the SNR shell, move supersonically at the time they cross the rim of the SNR. As a result the pulsar wind nebula is deformed, and preceded by a bow shock. Systems like CTB80, G5.4-1.2 and IC443 can be described as these kind of bow shocks.

1.3 The evolution of a supernova remnant

1.3.1 The free expansion stage (duration 100 - 1,000 years)

The supernova explosion generates a shock wave which first will propagate through the outer layers of the progenitor star. The free expansion stage starts when this shock reaches the edge of the atmosphere of the star. In the expanding debris, almost all mechanical energy of the SNR, \( E_0 \), is converted into kinetic energy. Since the mass from the ejecta is still much larger compared then the swept-up mass from the ISM, this results in an almost constant expansion velocity determined by the kinetic energy, \( E_0 \). The expansion proceeds according to

\[
R_{\text{snr}} = V_{\text{snr}} t; \quad V_{\text{snr}} \simeq \sqrt{\frac{2E_0}{M_{\text{ej}}}}.
\]

1.3.2 The Sedov stage (duration 10,000 years)

In the Sedov stage, the swept-up mass of ISM material (mass density \( \rho_0 \)) exceeds the mass \( M_{\text{ej}} \) of the ejecta. The interior of the remnant has been heated by the reverse shock, and the remnant is bounded by a strong blast wave.
One can derive the expansion law in the Sedov stage from a simple approximation. The mass of the remnant is

\[ M(t) \approx \frac{4\pi}{3} \rho_0 R_{\text{snr}}^3, \quad (1.2) \]

while the typical energy of the remnant is

\[ E_{\text{snr}} = \frac{1}{2} M u^2 + \left( \frac{4\pi}{3} R_{\text{snr}}^3 \right) \frac{P}{(\gamma - 1)}. \quad (1.3) \]

The first term in this expression is the kinetic energy of the shell of swept-up material. The typical velocity is the velocity behind the strong blast wave,

\[ u \approx \frac{2}{\gamma + 1} \left( \frac{dR_{\text{snr}}}{dt} \right), \quad (1.4) \]

which follows for the shock jump conditions for a strong shock propagating into a cold medium (Landau & Lifshitz 1959). Here \( \gamma \) is the adiabatic heat ratio of the gas, usually taken to be \( \gamma = 5/3 \).

The second term is the thermal energy of the interior, here approximated as a sphere of radius \( R_{\text{snr}} \) with constant pressure \( P \). Because of the high sound speed in this interior, there must be approximate pressure equilibrium with the shell of shocked interstellar gas so that

\[ P = \frac{2}{\gamma + 1} \rho_0 \left( \frac{dR_{\text{snr}}}{dt} \right)^2. \quad (1.5) \]

Here we use the postshock pressure in the SNR. Substituting relations (1.4) and (1.5) into (1.3), and assuming that radiative losses can be neglected so that

\[ E_{\text{snr}} = E_0, \quad (1.6) \]

one finds

\[ E_0 = C_\gamma M(t) \left( \frac{dR_{\text{snr}}}{dt} \right)^2, \quad (1.7) \]

where

\[ C_\gamma = \frac{4\gamma}{(\gamma - 1)(\gamma + 1)^2}. \]
1.3 The evolution of a supernova remnant

Figure 1.5: The profiles of the pressure (dotted line), velocity (dashed line) and the density (solid line) in a Sedov SNR. Here the radius has been normalised to the radius of the SNR, $R_{\text{snr}}$, and the physical quantities have been normalised to the postshock quantities.
In this approximation, the ratio of internal to kinetic energy of the remnant is constant and the total energy is conserved. The internal pressure decreases due to the expansion. By using the equations (1.2) and (1.7), it is possible to derive an equation for the radius of the SNR (see also Zel’dovich and Raiser 1966). This yields:

\[ R_{\text{snr}}(t) = \xi_0 \left( \frac{E_0 t^2}{\rho_0} \right)^{1/5} \]  (1.8)

\[ \bar{\xi}_0 = \left( \frac{75(\gamma - 1)(\gamma + 1)^2}{64 \pi \gamma} \right)^{1/5}. \]

This derivation is only an approximation. A full solution solves the equations of fluid flow in spherical symmetry:

\[ \frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = 0, \]  (1.9)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0, \]  (1.10)

\[ \frac{\partial}{\partial t} \left[ \rho \left( \epsilon + \frac{1}{2} u^2 \right) \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \rho u \left( \epsilon + \frac{P}{\rho} + \frac{1}{2} u^2 \right) \right] = 0. \]  (1.11)

As shown by Sedov (1959), this set of equations can be simplified by making the assumption of a similarity solution, where fluid quantities depend on the two independent variables, the radius \( r \) and time \( t \), in the dimensionless combination (similarity variable)

\[ \xi = \left( \frac{\rho_0}{E_0 t^2} \right)^{1/5} r, \]  (1.12)

with the outer blast wave at a fixed value \( \xi = \xi_0 \) so that

\[ R_{\text{snr}}(t) = \xi_0 \left( \frac{E_0 t^2}{\rho_0} \right)^{1/5}. \]  (1.13)

This is the same relation as derived above in (1.8).
Writing the radial velocity $u$, density $\rho$ and pressure $P$ as

\[
u(r,t) = \frac{r}{t} V(\xi),
\]

\[
\rho(r,t) = \rho_0 \tilde{\rho}(\xi),
\]

\[
P(r,t) = \rho_0 \left(\frac{r}{t}\right)^2 \tilde{P}(\xi),
\]

equations (1.9)-(1.11) are converted to a set of ordinary differential equations for the dimensionless variables $V(\xi)$, $\tilde{\rho}(\xi)$ and $\tilde{P}(\xi)$. Now that all quantities are known as a function of radius, it is possible to calculate the correct value of $\xi_0$. Integrating the fluid energy density over the volume and equating the total energy to the mechanical explosion energy $E_0$ one finds $\xi_0 \simeq 1.1516$ for $\gamma = 5/3$ (Sedov 1959). This yields the full solution for the expansion of the SNR in the Sedov stage:

\[
R_{\text{snr}}(t) = \xi_0 \left(\frac{E_0 t^2}{\rho_0}\right)^{1/5} \simeq 1.15 \left(\frac{E_0 t^2}{\rho_0}\right)^{1/5},
\]

\[
V_{\text{snr}}(t) = \frac{dR_{\text{snr}}(t)}{dt} = \frac{2}{5} R_{\text{snr}}(t) \frac{1}{t}.
\]

Figure 1.5 shows the profiles of the pressure, velocity and density of a Sedov solution.

### 1.3.3 The snowplow stage (duration 10,000-100,000 years)

The Sedov solution is a quite accurate approximation as long as cooling due to radiation losses can be neglected and the total energy is conserved. A transition takes place when radiation losses become important in the shell of the swept-up material. At this point, the swept-up mass collapses into a thin, dense layer, but the interior of the SNR still expands adiabatically. The interior pressure pushes the thin shell through the ISM, like a snowplow would. Due to the work done the internal energy $E_T$ is no longer constant, and is controlled by the following two equations, assuming a sphere of uniform pressure:
\[
\frac{dE_T}{dt} = -4\pi R_{\text{snr}}^2 P \left( \frac{dR_{\text{snr}}}{dt} \right) \quad (1.17)
\]

\[
\frac{4\pi}{3} R_{\text{snr}}^3 P = (\gamma - 1) E_T. \quad (1.18)
\]

By combining (1.17) and (1.18) we can express the internal energy from the SNR as a function of the radius \( R_{\text{snr}} \). If we take the quantities \( R_0 \) and \( E_{T0} \) as the values at the moment of the transition from the Sedov to the snowplow stage, and restrict ourselves to the case \( \gamma = \frac{5}{3} \), we find:

\[
E_T = E_{T0} \left( \frac{R_0}{R_{\text{snr}}} \right)^2. \quad (1.19)
\]

Because the shell of the SNR is driven outwards by the interior pressure, the equation of motion of the shell is given by:

\[
\frac{d}{dt} (M V_{\text{snr}}) = 4\pi R_{\text{snr}}^2 P. \quad (1.20)
\]

If we now substitute equation (1.2) into the equation of motion and use (1.19) to express \( P \) in terms of \( E_T \) we get the following differential equation for the radius of the SNR as a function of time:

\[
\frac{d^2}{dt^2} (R_{\text{snr}}^4) = \frac{6E_{T0} R_0^2}{\pi \rho_0 R_{\text{snr}}^3}. \quad (1.21)
\]

Inserting a power law, \( R_{\text{snr}}(t) \propto t^\alpha \), we find the following solution, using \( E_{T0} = \epsilon E_0 \):

\[
R_{\text{snr}}(t) = \left( \frac{147\epsilon E_0 R_0^2}{4\pi \rho_0} \right)^{1/7} t^{2/7}. \quad (1.22)
\]

For times much later than the transition time from the Sedov to the snowplow phase, the exact solution, obtained from integration of equation (1.21), with the appropriate boundary conditions, relaxes to this power-law form. The quantity \( \epsilon \) is the fraction of the initial mechanical energy remaining in the form of the internal energy of the interior at the moment of transition. We use \( \epsilon \approx 0.2-0.3 \) (Blinnikov et al. 1983). The other quantity to be determined is the value of
1.3 The evolution of a supernova remnant

Figure 1.6: Radius of the SNR in the Sedov stage (solid line), and in the snowplow stage (dashed line). Here $E_0 = 10^{51}$ erg, $n_0 = 1$ and $\epsilon = 0.2$. The transition from the Sedov stage to the snowplow stage takes place after $\pm 40,000$ years, and at a radius $R_{\text{snr}} \simeq 21$ parsec.

$R_0$, which marks the radius of the beginning of the snowplow stage. We use the expression from Falle (1981):

$$R_0 = 20 E_{51}^{0.295} n_0^{-0.409} \text{ pc.}$$

(1.23)

Here $E_{51} = E_0/(10^{51}\text{erg})$ and $n_0$ is the number density of atoms (in cm$^{-3}$) in the ISM. In Figure 1.6 we show the radius of the SNR in the Sedov stage and in the snowplow stage for typical parameters.

The next stage will mark the beginning of the end of the SNR where it merges with the ISM. This last stage starts when the pressure in the interior of the SNR becomes comparable with the pressure of the ISM. There will be no force acting on the shell of the SNR, so momentum is conserved, while the shock of the SNR continues to sweep up interstellar gas:

$$M(t)V_{\text{snr}}(t) = \text{constant},$$

(1.24)

from which one can deduce that
\[
\frac{dR_{\text{snr}}}{dt} \propto R_{\text{snr}}^{-3}. \tag{1.25}
\]

By assuming a power-law solution, \(R_{\text{snr}}(t) \propto t^\alpha\), one gets:
\[
R_{\text{snr}}(t) \propto t^{1/4}. \tag{1.26}
\]

This last stage is called the momentum-conserving snowplow, after which the SNR merges completely with the ISM, leaving behind a cavity with a higher temperature than the surrounding ISM. The event of merging takes place \(\sim 750,000\) years after the supernova explosion (Cioffi 1990).

### 1.4 The evolution of a PWN inside a SNR

The expansion of a PWN is driven by the high interior pressure of the nebula. This pressure is the energy of a pulsar wind which has been thermalised in a termination shock. It is commonly assumed that the mechanical luminosity of the pulsar wind equals the observed spindown luminosity of the pulsar: the loss of rotational energy of the neutron star. The PWN pushes ahead a shell of swept-up material (see figure 1.7) The energy input by the pulsar is often modelled using the rotating dipole model, where the luminosity varies with time as
\[
L = \frac{L_0}{(1 + t/\tau)^p} \quad \text{with} \quad p = 2. \tag{1.27}
\]

Here \(p \equiv (n + 1)/(n - 1)\) with \(n\) is the braking index of the pulsar, which is defined in terms of the pulsar’s angular velocity \(\Omega\), and its derivatives \(\dot{\Omega} = d\Omega/dt\) and \(\ddot{\Omega} = d^2\Omega/dt^2\) as
\[
n \equiv \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2}. \]

Throughout this thesis we will use a braking index of \(n = 3\) which yields \(p = 2\). In this way the luminosity decays as \(t^{-n}\) for values of \(t \gg \tau\). The most important approximation we make in the following discussion of a pulsar wind is by taking a constant luminosity when \(t < \tau\) and setting the luminosity equal to zero afterwards. We will derive an expression for the radius of a PWN in the free expansion stage and the Sedov stage, assuming that the pulsar is located at the center of the SNR.
1.4 The evolution of a PWN inside a SNR

Figure 1.7: The spindown energy of the pulsar pushes ahead a shell which contains the swept up mass from the ambient medium. As long as the kick velocity of the pulsar is small compared with the local sound velocity, ram pressure will not influence the system.

1.4.1 The PWN in a SNR in the free expansion stage

We assume that the initial expansion of the PWN is supersonic, and check this assumption \textit{a posteriori}. In this case, we can generalize equation (1.7), used for the description of the expansion of the SNR. We replace the energy $E_0$ by the integrated luminosity driving the pulsar wind. Assuming a constant driving luminosity $L_0$, the total amount of energy put into the PWN equals $E_{\text{pwn}} = L_0 t$.

Substituting this in (1.7), one gets:

$$R_{\text{pwn}}(t) = \tilde{C} \left( \frac{L_0 t^3}{\rho(t)} \right)^{1/5}. \quad (1.28)$$

Where $\tilde{C}$ is a constant of order of unity. The density of the ambient medium is no longer constant, but depends on time due to the expansion of the SNR. As a rough estimate we take:

$$\rho(t) = \left( \frac{3 M_{\text{ej}}}{4 \pi R_{\text{snr}}^3(t)} \right)^{1/3}. \quad (1.29)$$

Using that $R_{\text{snr}} \propto t$ in the \textit{free expansion stage}, together with (1.28) and (1.29), we get an expression for the radius of the PWN, which scales as:
\[ R_{\text{pwn}}(t) = \tilde{C} \left( \frac{L_0 t}{E_0} \right)^{1/5} R_{\text{snr}}(t) \propto t^{6/5}. \] (1.30)

The velocity of the edge of the PWN is given by,
\[ V_{\text{pwn}}(t) = \frac{6}{5} \frac{R_{\text{pwn}}(t)}{t}, \] (1.31)
and we can check whether the expansion is indeed supersonic. By calculating the Mach number \( M \): the ratio between the velocity of the PWN and the local sound velocity \( c_s \).

In order to find \( c_s \) we must estimate the pressure inside the SNR. If the expansion is adiabatic we have \( P \rho^{-\gamma} = \text{constant} \). Using \( \gamma = 5/3 \) this implies that the pressure scales as:
\[ P(t) = P_0 \left( \frac{R_e}{R_{\text{snr}}(t)} \right)^5. \] (1.32)

If we take \( R_e \) to be the radius \( R_e \) of the exploding star, and use the fact that, immediately after the explosion, most of the energy is still internal energy so that
\[ P_0 \simeq 3(\gamma - 1)E_0/4\pi R_e^3, \] (1.33)
we find:
\[ P(t) \simeq \frac{3E_0(\gamma - 1)}{4\pi R_{\text{snr}}^3(t)} \left( \frac{R_e}{R_{\text{snr}}(t)} \right)^2. \] (1.34)

The typical sound speed, \( c_s = \sqrt{\gamma P/\rho} \), now follows as:
\[ c_s = \left( \frac{\gamma(\gamma - 1)E_0}{M_{\text{ej}}} \right)^{1/2} \left( \frac{R_e}{R_{\text{snr}}(t)} \right) = \left( \frac{\gamma(\gamma - 1)}{2} \right)^{1/2} \frac{R_e}{t}. \] (1.35)

The Mach number associated with the expansion of the PWN is typically
\[ M = \frac{3}{\sqrt{5}} \frac{R_{\text{pwn}}}{R_e} \gg 1, \] (1.36)
where we take \( \gamma = 5/3 \). This leads us to the conclusion that the expansion of the PWN is indeed supersonic provided \( R_{\text{pwn}} \gg R_e \), as is the case.
1.4 The evolution of a PWN inside a SNR

1.4.2 The PWN in a Sedov-stage remnant

During the free expansion stage a reverse shock is driven back into the interior of the SNR. Once the reverse shock has reached the center of the SNR, the SNR can be approximated by the Sedov stage. This reverse shock reheats the ejecta and as a result the sound velocity increases. Therefore the expansion of the PWN will be subsonic.

Driven pulsar wind nebula

The assumption of a subsonic expansion rate implies that the internal pressure of the PWN, $P_i$, roughly equals the pressure $P(R, t)$ inside the remnant at radius $R_{pwn}$:

$$P_i(t) \approx P(R_{pwn}, t). \quad (1.37)$$

We assume that the interior sound speed inside the PWN is so large such that the pressure can be considered uniform to lowest order. The expansion law then follows from the first law of thermodynamics,

$$dE_T = dQ - P_i dV_{pwn}. \quad (1.38)$$

Here $E_T$ is the internal (thermal) energy of the PWN, and $V_{pwn}$ its volume. Approximating the PWN as a homogeneous sphere, and assuming that the pulsar supplies energy at a constant rate $L_0$ so that $dQ = L_0 dt$, one has:

$$\frac{d}{dt} \left( \frac{4\pi}{3} P_i R_{pwn}^3 \right) = L_0 - 4\pi R_{pwn}^2 P_i \left( \frac{dR_{pwn}}{dt} \right). \quad (1.38)$$

This equation has a simple power-law solution ($R_{pwn} \propto t^{\alpha}$) provided the pressure imposed on the PWN (Eqn. 1.37) varies as a power-law in time. The pressure well inside a Sedov remnant behaves in this manner: it is almost uniform for $R < 0.5R_{snr}$, and varies with time as

$$P(R, t) = \tilde{C} \left( \frac{E_0}{R_{snr}^3} \right) \propto t^{-6/5}. \quad (1.39)$$

The constant $\tilde{C} \approx 0.074$. As long as $R_{pwn} \ll R_{snr}$ relations (1.37)-(1.39) yield an expansion law that can be written as

$$R_{pwn}(t) = \overline{C} \left( \frac{L_0 t}{E_0} \right)^{1/3} R_{snr} \propto t^{11/15}. \quad (1.40)$$
The constant $C$ takes the values $C \approx 0.954$ if the PWN material is non-relativistic ($\gamma = 5/3$) and $C \approx 0.851$ if the PWN material is relativistically hot ($\gamma = 4/3$). The sound speed inside a Sedov remnant satisfies (see figure 1.8)

$$c_s \geq 0.22 \left( \frac{R_{\text{SNR}}}{t} \right),$$

where the minimum value is reached at the edge of the remnant. The Mach number $\mathcal{M} = c_s^{-1} (dR_{\text{pwn}}/dt)$ of the expansion follows from (1.40) and (1.41) as

$$\mathcal{M} \leq \mathcal{M}_{\text{max}} = 3.128 \left( \frac{L_0 t}{E_0} \right)^{1/3}.$$  

For all observed systems (see Table 1.1) one can estimate $\mathcal{M}_{\text{max}} \ll 1$ from the observed pulsar spindown luminosity, the inferred age of the system and the (assumed) mechanical explosion energy $E_0 \approx 10^{51}$ erg. One finds that the assumption of subsonic expansion is a reasonable one.

Figure 1.8: *The sound speed in the SNR interior, normalised to the sound speed at the edge of the SNR.*
1.4 The evolution of a PWN inside a SNR

Table 1.1: Properties of pulsars with a PWN, this table has been taken from Frail & Scharringhausen(1997).

<table>
<thead>
<tr>
<th>PSR</th>
<th>$E$ (ergs s$^{-1}$)</th>
<th>PWN</th>
<th>distance (kpc)</th>
<th>Age (kyr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0531+21</td>
<td>$4.5 \times 10^{38}$</td>
<td>Crab Nebula</td>
<td>2.0</td>
<td>1.3</td>
</tr>
<tr>
<td>B1509-58</td>
<td>$1.8 \times 10^{37}$</td>
<td>G 320.4-1.2</td>
<td>4.2</td>
<td>1.5</td>
</tr>
<tr>
<td>B0540-69</td>
<td>$1.5 \times 10^{38}$</td>
<td>SNR 0540-693</td>
<td>55.0</td>
<td>1.7</td>
</tr>
<tr>
<td>B0833-45</td>
<td>$6.9 \times 10^{36}$</td>
<td>Vela X</td>
<td>0.5</td>
<td>11</td>
</tr>
<tr>
<td>B1757-24</td>
<td>$2.6 \times 10^{36}$</td>
<td>G 5.27-0.90</td>
<td>4.5</td>
<td>15</td>
</tr>
<tr>
<td>B1853+01</td>
<td>$4.3 \times 10^{35}$</td>
<td>W 44</td>
<td>3.0</td>
<td>20</td>
</tr>
<tr>
<td>B1951+32</td>
<td>$3.7 \times 10^{36}$</td>
<td>CTB 80</td>
<td>2.5</td>
<td>107</td>
</tr>
</tbody>
</table>

The adiabatic stage

In our simple model, where the mechanical luminosity driving the expansion of the PWN vanishes for $t > \tau$, the expansion slows down after $t = \tau$. Assuming once again subsonic expansion into a Sedov SNR ($R_{\text{snr}} \propto t^{2/5}$), the expansion law in this stage follows from (1.38) with $L_0 = 0$, which implies the adiabatic pressure-radius relation

$$P_i R_{\text{pwn}}^{-3\gamma} = \text{constant}. \quad (1.43)$$

Combining this with the (approximate) condition of pressure equilibrium (Eqn. 1.37) one finds that the radius of the PWN scales as

$$R_{\text{pwn}} \propto R_{\text{snr}}^{1/\gamma} \propto t^{2/5}. \quad (1.44)$$

The radius of the PWN at $t = \tau$ follows from (1.40),

$$R_{\text{pwn}}(\tau) = \bar{C} \left( \frac{E_*}{E_0} \right)^{1/3} R_{\text{snr}}(\tau), \quad (1.45)$$

where $E_* \equiv L_0 \tau$ is the total amount of energy injected by the pulsar into the PWN. This gives the expansion law in the adiabatic phase:

$$R_{\text{pwn}}(t) = \bar{C} \left( \frac{E_*}{E_0} \right)^{1/3} R_{\text{snr}}(\tau) \left( \frac{t}{\tau} \right)^{2/5\gamma}. \quad (1.46)$$
Figure 1.9: The evolution of the ratio $R_{pwn}/R_{snr}$ as a function of time throughout the different evolutionary stages. We have taken the timescales such that the reverse shock hits the PWN at a time, $t = 600$ years, and the pulsar wind has a constant luminosity of order $\sim 10^{38}$ ergs/sec when $t \leq 1400$ years.

If the PWN material is non-relativistic ($\gamma = 5/3$) one has $R_{pwn} \propto t^{6/25}$, if it is relativistically hot ($\gamma = 4/3$) the nebula expands as $R_{pwn} \propto t^{3/10}$. In both cases the expansion slows down, and the ratio $R_{pwn}/R_{snr}$ decreases with time as long as $\gamma > 1$.

1.4.3 More advanced models of a PWN

The Kennel & Coroniti Model

A more detailed model of a PWN was given by Kennel & Coroniti (K&C) (1984): a steady state spherically symmetric, magnetohydrodynamical model of the Crab nebula. One can find a schematic picture of their model in figure 1.10. In their model the highly relativistic pulsar wind constitutes a positronic plasma, which was created in the magnetosphere of the pulsar. This relativistic wind is terminated by a strong MHD shock, decelerating the flow and thermalizing the wind. The magnetization parameter $\sigma$, which determines the ratio of the electromagnetic energy flux, $nB^2/4\pi$ to the particle energy flux, $n\Gamma mc^2$, where $n$ is the particle density and $\Gamma mc^2$ is the energy per particle, $\Gamma = \sqrt{1 - \beta^2}$ is the Lorentz factor and $\beta = v/c$ is the dimensionless three speed and $B$ is the
1.4 The evolution of a PWN inside a SNR

Figure 1.10: Shematic picture of the Kennel & Coroniti model. The dot is the pulsar plus magnetosphere, where the positronic plasma is created. Region II is the region which contains the pulsar wind. At $R = 0.1 \text{ pc}$ the wind is terminated by a strong MHD shock, so the flow is decelerated in region III, which is the synchrotron emitting nebula. Region IV is the SNR in which the PWN is embedded. The SNR itself is bounded by a strong shock, which propagates into the interstellar medium. The scale on the $y$-axis applies for the Crab Nebula at a distance of $D \simeq 2 \text{ kpc}$.

magnetic field strength. The associated magnetization parameter $\sigma$ equals:

$$\sigma = \frac{B^2}{4\pi n \Gamma m_c^2}.$$  \hfill (1.47)

It appears in the Rankine-Hugoniot jump conditions used to determine the upstream and downstream parameters at this MHD shock. This magnetization parameter determines the efficiency of converting the energy contained by the pulsar wind into synchrotron radiation. Beyond this shock lies the shock heated PWN which emits this synchrotron radiation. The PWN itself is embedded in the surrounding SNR. K&C assume pressure balance at the boundary of the PWN. Figure 1.10 gives a schematic picture of the four regions of the K&C model.
The density profile of a supernova remnant containing a centered pulsar wind. One can identify the pulsar wind termination shock at $r \simeq 0.5$, the PWN shock at $r \simeq 3.4$, the reverse shock at $r \simeq 3.8$ and the forward shock at $r \simeq 5.6$. The expansion of the PWN is in its supersonic stage of its evolution.

The Begelman & Li Model

The Begelman & Li model (1992) is an extension of the K&C model, both models for the Crab nebula. Instead of a spherical symmetric model $(r, \theta, \phi)$, the model of Begelman & Li is an axially symmetric model $(r, z, \theta)$, which allows to include the influence of the toroidal magnetic field on the dynamics of the PWN. This introduces a pressure difference across the interior and the exterior of the pulsar wind bubble, which depends on the angle $\theta$. This results in an elongation of the PWN as is observed for the Crab nebula. Furthermore they connect the observed elongation of the Crab nebula to the aforementioned magnetization parameter $\sigma$, which is shown to be consistent with the K&C model.

Hydrodynamical simulations of PWNe

In chapter 3 we perform hydrodynamical simulations of a centered PWN inside a SNR. An example of the resulting density profile is shown in the figure 1.11: in these simulations the pulsar wind is centered at $r = 0$. The cold freely expanding wind is thermalised by a strong shock ($r \simeq 0.5$), which decelerates the flow. The PWN itself is bounded by a strong shock propagating through
1.4 The evolution of a PWN inside a SNR

the freely expanding ejecta of the progenitor star \( r \simeq 3.4 \). The SNR is in its transition from the freely expaning stage to the Sedov-Taylor stage. In this stage of its evolution there are two shocks present: the reverse shock propagating into the unshocked ejecta \( r \simeq 3.8 \) and the forward shock of the SNR \( r \simeq 5.6 \).

1.4.4 Excentric pulsars in SNRs

In the previous Sections, I considered a PWN located centrally in an expanding SNR. Here I consider the case where the pulsar, after its birth event, has a velocity \( V_{\text{psr}} \). Observations of the proper motion of pulsars by interferometric means (Harrison et al. 1993, Bailes et al. 1990) show transverse velocities in the range of \( V_{\text{psr}} \sim 100 - 500 \) km/s, while indirect estimates based on pulsar-SNR associations yield velocities in the range \( V_{\text{psr}} \sim 100 - 2000 \) km/s (Frail et al. 1994).

The expansion velocity of a SNR in the free expansion stage is of order 10,000 km/s, much larger than the inferred pulsar velocities. This means that the pulsar will only reach the edge of the SNR (if at all) in the Sedov phase or later, in the snow plow phase.

In the Sedov phase, where \( R_{\text{snr}} \propto t^{2/5} \) the condition \( V_{\text{psr}} t = R_{\text{snr}} \) fixes the ratio of the pulsar velocity and the expansion velocity of the remnant at the moment that the pulsar reaches the outer edge:

\[
V_{\text{psr}} \frac{dR_{\text{snr}}}{dt} = \frac{5}{2} \cdot \frac{dR_{\text{snr}}}{dt}, \tag{1.48}
\]

Notice that this ratio is independent of time and other parameters such as the explosion energy \( E_0 \) and density \( \rho_0 \) of the interstellar medium. The material at the edge of the SNR moves with the post-shock velocity,

\[
u(R_{\text{rms}}) = \frac{2}{\gamma + 1} \frac{dR_{\text{snr}}}{dt} = \frac{3}{4} \frac{dR_{\text{snr}}}{dt}, \tag{1.49}
\]

where I assume \( \gamma = 5/3 \). The relative velocity between pulsar and SNR material then equals

\[
V_{\text{rel}} = V_{\text{psr}} - \nu(R_{\text{rms}}) = \frac{7}{4} \frac{dR_{\text{snr}}}{dt}. \tag{1.50}
\]
Since the sound speed at the edge of the remnant is

\[ c_s(R_{\text{snr}}) \simeq 0.56 \left( \frac{dR_{\text{snr}}}{dt} \right), \]

the pulsar moves supersonically with respect to the SNR material the moment it reaches the edge of the SNR:

\[ M_{\text{psr}} = \frac{V_{\text{rel}}}{c_s} \approx 3.182. \quad (1.51) \]

Again this quantity is independent of time and other parameters. In the next section we will discuss the physics of bow shocks moving in a medium with a constant density and a negligible pressure.
Figure 1.12: Overview of the bow shock structure of a pulsar wind nebula moving through a uniform medium, from the point of view of the pulsar.
1.4.5 Theory of bow shocks

For a pulsar moving with a constant velocity through a medium with a constant density, the morphology is depicted in figure 1.12. The pulsar wind, driven by the spindown energy, is terminated by a wind termination shock. The pulsar wind nebula itself has been deformed to a bow shock due to the supersonic motion of the pulsar. By neglecting the pressure of the ISM, and only include ram pressure, it is possible to derive an equation for the radius of the wind termination shock, \( r_s \). We will first consider a non-relativistic wind, followed by the relativistic limit.

We consider a constant mass injection, \( \dot{M} \), by the pulsar wind. The density, \( \rho_w \), in the pulsar wind region due to mass conservation is given by:

\[
\rho_w \simeq \frac{\dot{M}}{4\pi r^2 V_w}. \tag{1.52}
\]

Here the velocity of the pulsar wind is denoted as \( V_w \). When the surrounding medium of the pulsar wind region is uniform and constant, we can use ram-pressure equilibrium, \( \rho_{\text{ISM}}V_{\text{psr}}^2 = \rho_w V_w^2 \), to determine the radius of the wind termination shock:

\[
r_s = \left( \frac{\dot{M}V_w}{4\pi \rho_{\text{ISM}} V_{\text{psr}}^2} \right)^{1/2}. \tag{1.53}
\]

The above equation is a non-relativistic equation. A pulsar wind is relativistic (\( \Gamma = 10^{5-6} \)), therefore we transform the above equation to an equation which approximates the relativistic solution for the radius \( r_s \). This is done by taking the limit \( V_w \to c \), and multiply the energy and momentum by a factor \( c \) to approximate the conversion to the associated relativistic counterparts of energy and momentum. This yields:

\[
r_s = \left( \frac{\dot{E}_{\text{psr}}}{4\pi \rho_{\text{ISM}} V_{\text{psr}}^2 c} \right)^{1/2}. \tag{1.54}
\]

This equation is slightly modified, by solving the problem for a relativistic (wind) flow. In that case the radius of the wind termination shock becomes:

\[
r_s = \left( \frac{\dot{E}_{\text{psr}}}{2\pi \rho_{\text{ISM}} V_{\text{psr}}^2 c} \right)^{1/2}. \tag{1.55}
\]
A more detailed calculation of bow shocks can be found in recent work done by Wilkin (2000). In chapter 4 of this thesis, we perform hydrodynamical simulations of bow shocks in a medium with a constant density.
Bibliography


