3 A COUPLED MODEL OF SLOPE HYDROLOGY AND STABILITY

3.1 Introduction

As defined in the preceding chapters, this study aims to quantify the role of rainfall-induced landslides within the hierarchy of hillslope erosion processes in a sub-humid Mediterranean environment. This hierarchy can be envisaged as a cascade-in which the downslope movement of material—or the conversion of potential into kinetic energy—is controlled by related processes of surface wash, soil production and mass movement (Van Asch, 1980).

Rainfall-induced landslides are triggered by a relative increase in pore pressure (Chapter 1). The triggering pore pressures develop often over a lithological contact, such as the transition between the soil and the underlying bedrock. Because of their limited depth, they can be triggered by rainfall that is accumulated over short periods. Thus, their occurrence can be extensive and capable of mobilising considerable volumes of material. Moreover, the destruction of the vegetation cover exposes barren material, which is the onset for surface wash processes.

Mediterrenean environments, subject to erosion processes, are extremely sensitive to changing environmental conditions (Mulligan, 1998). This can be extended to include those environments where landsliding interacts with surface wash processes. The impact of environmental changes on landslide activity comprises changes in its frequency and magnitude. Under hypothetical conditions, when the changes cannot be evaluated a posteriori, the assessment requires that the causal relation between the triggering rainfall events and landslide activity be understood. This relation is complex as the net precipitation and the antecedent moisture conditions control the triggering of landslides (Chapter 1). As a result, the evaluation of the sensitivity of an area to landsliding in the light of presumed changes in land use and climate becomes rather difficult. The use of a model may assist in the analysis whether it is directed to test a predefined hypothesis or to predict the changes in landslide activity under a presumed scenario.

The primary trait of such a model must be its capability to relate generated pore pressures to a threshold of instability. This threshold can be defined for a single landslide or for distinct type. It is either an observed pore pressure level or derived from a slope stability assessment, in which it is defined as a critical pore pressure or as a measure of overall stability (e.g. the Safety Factor; see Section 3.2.3). Irrespective of the type of threshold used, it can be compared through the model with the generated ground water. This coupling of hydrology and stability by the model makes it possible to express the sensitivity to landsliding in terms of frequency and order of magnitude. The effects of changing environmental conditions can be incorporated in the model by applying it to different scenarios.

In the following sections, a coupled hillslope model is described of which the aim is to simulate the pore pressure conditions adequately over time in order assess the activity of rainfall-induced landslides. The intended application to hypothetical scenarios postulates
that the model is physically based. Moreover, changes in the sensitivity to landsliding will only emerge on a regional scale whilst local differences in topography and land use must be considered. These requirements are the easiest accommodated by a distributed model. Adaptations of the model parameterisation that arise from environmental changes are readily accomplished when the model is embedded in a GIS environment. Consequently, the coupled hillslope model is dynamic, distributed and physically based and will be applied on a regional scale. It includes a hydrological model component named STARWARS, an acronym for STorage And Redistribution of Water on Agricultural and Revegetated Slopes. The stability analysis, PROBSTAB, is a limiting equilibrium model based on the Mohr-Coulomb failure criterion by which the probability of failure is assessed. The demands for the implementation of such a model are numerous and some general aspects that underlie the development of the model are considered first.

3.2 Development of the coupled model of slope hydrology and stability

3.2.1 Scale aspects and model considerations

Awareness of the possible impact of changing environmental conditions like land use and climate on the controlling pore pressures and the absence of long term observational records has led to an increase in the use of physically based hydrological models to evaluate the consequences for future landslide activity. These model approaches range from simple 1-D tank models (e.g. Buma & Dehn, 1998), over GIS embedded static and quasi-static 2- to 3-D approaches (e.g. Van Asch et al., 1993; Montgomery & Dietrich, 1994; Miller & Sias, 1998) to complex numerical solutions of 2- and 3-D saturated and unsaturated flow (e.g. Brooks et al., 1995; Hattendorf et al., 1999). All model categories include different levels of spatial and temporal detail.

Physically based models are favoured since they are capable of predicting alterations in the hydrological behaviour by means of the constituent equations for the incorporated processes (Grayson et al., 1992). The applicability of these models for future scenarios is, however, limited. Practical limitations are the related problems of spatial and temporal resolution, numerical stability and computation time. A further limitation is the large dataset that the more complex models require. Even if all model parameters can be acquired, it remains doubtful whether the changes in model output are discernible against the ensuing uncertainty (Nandakumar & Mein, 1997).

The uncertainty in parameter values derives from the natural variability and the discrepancy model-, process- and sample scale. Because of the use of the constituent physical relations, the model scale is inseparably bound to the scale at which the material properties have been sampled and at which the formulae have been defined. It coincides usually with a point scale and the support of the retrieved data is seldom larger than volumes of 1 dm$^3$ or 1 m$^3$ for soil properties that are taken constant over time. For soil properties, sampled over time by automated equipment, this support may even be smaller. This support may differ from the relevant process scale, often referred to as the representative elementary area or volume (REA, REV), that determines the observed behaviour (Bear, 1972; Wood, 1995).
Partly because of the spatial resolution of the data, partly because of the natural variability of the incorporated processes, the temporal scale of most models lies in the range from seconds to days. In contrast, the scale of interest is usually defined by larger natural or administrative entities (e.g. catchments and provinces) and by periods covering many years. The tendency of reducing the model resolution to cover these larger scales of interest leads to uncertainty in the estimation of parameter values (Heuvelink & Pebesma, 1999).

Further questions on the validity of these models are raised by the common practice of optimising the model performance under present conditions. This means that the tacit assumption is made that the importance and relations of the relevant processes in the modelled system will not change. Many physically based models of rainfall-induced landslides link the triggering groundwater level or matric suction in a deterministic manner to a static stability model or an observed stability threshold. This implies that the prediction of the groundwater level or matric suction as a measure of the pore pressure at the potential shear plane gives the exceedance of the critical condition. The subsequent changes after failure in the hydrological and mechanical behaviour of the system are not considered and feedback mechanisms that influence the occurrence of landslides are ignored. All in all, the model outcome gives an idea of the system response in terms of landslide susceptibility to the changing environmental conditions, with all the other factors remaining constant. For the 1- to 2-D approaches, the inferred changes in the hydrological behaviour do not surpass the domain of the modelled slope. The variability within the catchment and induced changes in the spatial and temporal distribution of landslide activity are most times neglected altogether.

Under consideration of these limitations it must be concluded that in order to evaluate the temporal and spatial alterations in landslide susceptibility under changing environmental conditions simple, physically based hydrological models are needed. These models should be modest in their aim and in their data requirements. This demand for modesty originates from the trade-off between the model’s capability to simulate the real world adequately on the one side and the uncertainty that is included in the model and in the model input on the other (Hillel, 1986). All models should be applied with care and their results interpreted with scrutiny. After all, no model can predict nor assess changes beyond its original scope.

The validity of the model assumptions and boundary conditions determine the model’s flexibility under the changing conditions. This excludes the use of entirely empirical models because they are based on unspecified sources of variability within the present system. In spite of this severe shortcoming of empirical models, no model can escape from a certain amount of empirism. The simplification of the real world system leads to a neglect of factors that will be compensated by adjustment of the parameters that are included in the calibration of the model. For this reason, no model will be right in an absolute manner (Konikow & Bredehoeft, 1992), but represents rather a possible realisation of the actual system (Beven, 1989). This raises fundamental questions on the possibility to predict the behaviour of geomorphic systems at all (Haff, 1996). This limitation can be overcome by adopting a probabilistic approach in which deterministic processes on the underlying scales are replaced by random variables.
In its application, the coupled hillslope model unites partly contradictory demands on the spatial and temporal scale. The aim is to predict landslide activity over relatively long timespans and large areas. At this scale of consideration, the model must incorporate the maximum of spatial and temporal detail at a minimum of costs in terms of parameterisation and computation time. The required temporal detail of the model is dependent on the rainfall characteristics. In Mediterranean environments, this rainfall distribution is erratic and characterised by extreme rainfall events. Therefore, the temporal resolution should comply with the present rainfall distribution and with the inferred conditions from downscaled climate change scenarios. Because of the resulting short-term fluctuations in soil moisture content, the model describes transient flow instead of a static or quasi-static condition. Only for a limited number of special boundary conditions can flow under transient conditions be described by analytical functions. In the majority of cases, only numerical solutions, in which the process-domain is divided in finite increments of space and time, provide the flexibility to cover the constantly changing conditions. The hydrological component of the coupled hillslope model is for this reason based on an explicit finite difference approximation of a forward numerical scheme. The discretisation of the slope geometry has, subsequently, implications for the numerical stability of the model. Within an explicit numerical scheme, the travel distance of any flux can not exceed the distance of one spatial increment for one timestep. Thus, the choice of the spatial resolution confines the possible temporal scale at which the model can operate. A decreased spatial resolution reduces the ability of the model to represent the topographical effects on the hydrology and on the slope stability adequately (Zhang & Montgomery, 1994; Evans, 1998). In contrast, a finely grained topography needs smaller timesteps to maintain numerical stability (Bates et al., 1998).

The demand for high spatial detail is based on the topographical control of the convergence of subsurface flow and slope stability (Montgomery & Dietrich, 1994; Carrara et al., 1995). At the same time, the spatial resolution should also represent the spatial distribution of the important parameters—for example land use— with sufficient detail (Grayson et al., 1992). This means that a distributed model is required rather than a local, 1- or 2-D approach. The detail that can be included in the 3-D model is defined by the resolution of the available data. Therefore a compromise must be found between the resolution of the digital elevation or terrain model (DEM or DTM), the spatial distribution of soil and land use properties, the temporal resolution of the available meteorological records or downscaled climate scenarios and the intended period over which the model should be applied.

The issue of model parameterisation is also related to the spatial resolution of the DEM. Distributed modelling assumes homogeneous conditions for every elemental area or volume of the DEM. Because of the required homogeneity, parameter values must be interpolated spatially. These homogeneous conditions can either be derived from a generalised set of parameter values, from a Monte Carlo simulation, from geostatistical interpolation or a combination of these. Regardless of the type interpolation, the variability within the cell is ignored.
For model scales that surpass the process scale of the original sample scale, it may be necessary to increase the support of the sample artificially. A possible solution to the discrepancy between the model scale and the sample support is to replace the field measurements by effective values, which are obtained by optimisation of the model performance. The disadvantage of this procedure with respect to predictive modelling is that parameters might lose their physical meaning and, consequently, a part of their predictive capability.

Ideally, the interpolated parameter values provide a priori knowledge with a minimum of unexplained variance (Grayson et al., 1992). If parameters possess a strong spatial correlation or depend on a spatial attribute like soil type or land use, this information should be included in the interpolation. This improves the accurateness of the spatial prediction on the one hand and avoids that it is lumped as white noise whilst aggregating data at a lower resolution (Heuvelink & Pebesma, 1999).

Extensive testing of the model is a prerequisite for the use of any model as a predictive tool. The discrepancy between the deterministic relationships for soils obtained under controlled laboratory conditions and the highly variable processes in nature led Beven (1989, page 161) to classify distributed physically based models as lumped conceptual models. In spite of his criticism, the calibration of the model within the limits of the natural variability of the parameter set provides an internal test for the adequacy of the constituent equations that are applied in the model. Validation over a historical database gives an estimate of the predictive quality of the model for future use and, with the calibration, of the prediction uncertainty (Beven, 1989; Beven & Binley, 1992). The exceptional and often catastrophic nature of landslide events stresses the need for model validation: since the occasional triggering conditions are underrepresented in most calibration sets, a calibrated model could represent the normal conditions adequately but might fail as a predictor of the extreme events. The validation set, which, stretching over a longer period, will include more extreme conditions, provides then an additional benchmark for the performance of the calibrated model. It should be noted that, because of the uncertainty in the parameterisation, which arises from the natural variability, measurement errors and support of the field measurements, no physically based model can provide absolute answers albeit its deterministic nature.

The uncertainty in the model performance stresses the necessity of a parsimonious parameter set for which the values can be obtained from field measurements with a certain feasibility (Hillel, 1986). It also avoids mutually dependent parameters (Beven, 1989). Their presence may result in non-unique solutions during calibration. Although these solutions have the same goodness-of-fit between the observed and simulated behaviour, their existence is in contradiction to the supposed singularity of nature. Additionally, the predictive uncertainty associated with large parameter sets will dwindle the possibility of discerning the response of the system under changing environmental conditions (Nandakumar & Mein, 1997). The crux is to prevent over-parameterisation and to reduce the indeterminacy in the process of calibration even if detail has to be sacrificed to model robustness. Or, as Beven (1989, page 165) puts it: “It is surely easier to use physical reasoning to calibrate the residence time parameter of a linear store than it is to obtain a correct combination of interacting parameters.”
Based on these considerations, the coupled hillslope model has been developed. The use of simple and linear relations has been favoured when possible. The effects of land use have been captured in static parameters that are attributed to the vegetation cover. Although this excludes the dynamics in the interactions between soil, water and vegetation, the approach is simple and finds its justification in the circuitous hydrological relations between land use and slope stability.

All model parameters can be obtained from field measurements. The eventual values of the parameters can be calibrated to optimise the model performance and validated to determine its predictive ability. The exact procedure on the calibration and validation of the model is treated in the chapters on model implementation (Chapter 6 & 7).

The resulting model is distributed but its intended spatial scale of operation is the hillslope. The temporal scale of operation of the model is bound to the temporal rainfall patterns and ranges from hours to days. In the following sections, time increments of one day are used consistently.

The STARWARS and PROBSTAB model components use the embedded meta-language of the PCRaster GIS package (Van Deursen, 1995; Van Deursen et al., 1996). This language allows the development of dynamic models, or scripts, that simulate a process or system over time but the overall structure is rather simple. It excludes, for example, the use of iterations other than the main (time) loop so no complex numerical solutions can be employed. In contrast with this disadvantage is the flexibility that the GIS environment offers to accommodate spatial and temporal variable input and output with readily available commands. The GIS is raster-based what results in the use of cells of equal horizontal dimensions. The dynamic model uses time increments of equal length.

3.2.2 Modelling of slope hydrology

A complication in the understanding of the causal relation between the triggering rainfall events and landslide occurrence is the fact that it is obscured by the antecedent moisture conditions. Percolation through the unsaturated zone attenuates the response of the groundwater level to a large rainfall event. The importance of the antecedent net precipitation increases when the rainfall distribution becomes more erratic in time, as is the case in Mediterranean areas.

The aim of the hydrological model –simulating the spatial and temporal occurrence of critical pore pressures- stipulates that the delay and loss of percolation in the unsaturated zone are included in the model. Therefore, the model must describe the saturated and the unsaturated transient flow in the vertical and lateral directions.

In the model, the response of the groundwater to the net rainfall is direct. The saturated and unsaturated zones are taken as freely draining. Hence, the model describes only unconfined groundwater levels. The response of the groundwater is imposed on a constant groundwater level or generated over a semi-impervious lithological boundary that restricts the direct loss of soil moisture into the deeper strata. In the latter case, the resulting groundwater is a perched level, for example over the underlying bedrock. Although in theory the model is capable of simulating the response of deeper groundwater, only the latter case of perched groundwater layers is considered here. In this case, vertical flow is stagnating over the lithic contact between soil and bedrock.

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The hydrological component describes the transient saturated and unsaturated flow as a function of the elevation potential only, neglecting the matric potential for the flow in the unsaturated zone. As a consequence, percolation is limited to gravitational vertical flow only. Over the saturated zone, the piezometric head defines the lateral flow. The rationale for the omission of the matric potential in the unsaturated flow is that it is generally considered of small importance in those instances when freely drainable water is available, i.e. when the soil moisture content is above the field capacity. When the moisture content is below field capacity, the flux under influence of the matric potential becomes negligible because of the sharply reduced unsaturated hydraulic conductivity at low moisture contents. The same holds even more strongly for the unsaturated lateral flow that is driven by matric potential only. Philip (1991) concluded on the basis of 2-D analytical solution of the Richards’ Equation that, for a planar slope, the unsaturated lateral flux is negligible for all inclinations below 30°. Even if the lateral unsaturated flux is substantial, the large vertical gradient directs it effectively towards the saturated zone (Jackson, 1992; Van Asch et al., 2001). Therefore, resulting fluxes under the gradient of the matric potential are small when compared to those of the vertical and lateral gravitational flows.
Figure 3.2: Flow chart of the model structure of STARWARS – surface and vegetation interactions. Horizontal lines signify positive decisions.
Figure 3.3: Flow chart of the model structure of STARWARS (cont.) – unsaturated and saturated zones
In most soils, the soil properties decrease with depth because of the different intensity of soil forming processes (Beven, 1982). These differences in the degree of weathering and in biological activity will result in distinct soil horizons. In order to simulate the resulting fluxes over such a differentiated soil profile above the semi-impervious lithic contact, it has been discretised into several layers. In Figures 3.2 to 3.3, a simplified scheme and a flow chart for the model are given.

For these layers, all storage and fluxes are given in units of waterslice. The advantage of this approach is that it facilitates computation as it can be converted by means of the maximum storage into the relative degree of saturation. The use of the relative degree of saturation has the advantage that it is the basis for the calculation of the percolation in the unsaturated zone. By definition, the relative degree of saturation, $\theta_E$, is

$$\theta_e = \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}}. \quad (3.1)$$
In Equation 3.1, $\theta$ denotes the volumetric moisture content, $\theta_{\text{sat}}$ the saturated moisture content and $\theta_{\text{res}}$ the residual moisture content, all in units of $(m^3 \cdot m^{-3})$. The saturated moisture content is usually set to the porosity and the residual moisture content to a fraction of this maximum. The relative degree of saturation varies by definition between zero and unity when the soil is respectively at its residual and its saturated moisture content.

In Equation 3.1, the numerator and the denominator represent the actual and the maximum amount of drainable pore water. Consequently, the effective degree of saturation can be expressed in units of waterslice if both are multiplied by the thickness of the unsaturated zone above the groundwater table

$$
\theta_E = \frac{D_{\text{unsat}}}{D_{\text{unsat}} \theta_{\text{sat}} - \theta_{\text{res}}} = \frac{\text{StorMat}}{D_{\text{unsat}} (\theta_{\text{sat}} - \theta_{\text{res}})},
$$

in which $D_{\text{unsat}}$ is the thickness in (m) and StorMat is the actual storage in $(m \cdot m^3 \cdot m^{-3})$ of the unsaturated zone. Once the initial relative degree of saturation is known, the actual volumetric moisture content is not required any more but can be obtained at any time by the formula

$$
\theta = \theta_{\text{res}} + \theta_E (\theta_{\text{sat}} - \theta_{\text{res}}).
$$

For each model layer $z$, the vertical, unsaturated matric flow, $Perc$, is based on the unsaturated hydraulic conductivity, $k(\theta_E)$. This unsaturated hydraulic conductivity is calculated from the soil water retention curve (SWRC), and the saturated hydraulic conductivity, $k_{\text{sat}}$. Both the saturated and the unsaturated hydraulic conductivity are defined in units of $(m \cdot d^{-1})$.

For the calculation of the unsaturated hydraulic conductivity, the mathematical formulation of the SWRC by Farrel and Larson (1972) has been used. This relation has been preferred above the well-known Van Genuchten (1980) model for two reasons. First of all, it reduces the number of parameters that describe the SWRC from three to two. The goodness-of-fit of Farrel & Larson's model is in general as good as the more complex Van Genuchten-model, especially if the number of parameters in the latter is reduced for mathematical expediency (Nielsen & Van Genuchten, 1985). Secondly, Farrel & Larson's relation is an exponential function. This can be considered as an improvement in model robustness when compared to the more complex relation of Van Genuchten. The model for the SWRC of Farrel and Larson is empirical, based on analogies in thermo-dynamics. The original model was defined in terms of $\theta/\theta_{\text{sat}}$. Rewritten in terms of $\theta_E$, it is given by

$$
|h| = h_A \exp\left[\alpha (1-\theta_E)\right],
$$

where $|h|$ is the absolute value of matric suction in (m), $h_A$ is the absolute matric suction that corresponds to the air entry value in (m) and $\alpha$ is the dimensionless slope of the log-linear relationship between $\ln(|h|)$ and $(1-\theta_E)$. This relation holds whenever the matric suction exceeds the air entry value.

The unsaturated hydraulic conductivity is calculated from the capillary analogy of Millington & Quirk (1959, 1961). According to their definition, the relative unsaturated
hydraulic conductivity, \( k(\theta_E) \) is an analogous function of filled capillaries representing the filled pores at different suction levels. Based on the SWRC of Farrel & Larson, the relative unsaturated hydraulic conductivity becomes

\[
k_r(\theta_E) = \theta_E \cdot \frac{\exp(2\alpha \theta_E) - 2\alpha \theta_E - 1}{\exp(2\alpha) - 2\alpha - 1},
\]

with the parameter of the tortuosity, \( \tau \), set to 4/3 (-). The \( k_r(\theta_E) \) ranges from zero at the residual moisture content to unity at complete saturation and is dimensionless. The absolute unsaturated hydraulic conductivity, \( k(\theta_E) \), is subsequently obtained by multiplying the relative unsaturated hydraulic conductivity with \( k_{sat} \) (m·d\(^{-1}\)).

Given the thickness of the unsaturated zone and the relative degree of saturation at the beginning of the timestep, the percolation is proportional to the travel time of soil moisture over the unsaturated zone. With the vertical gradient of gravitational flow set at unity, the travel time is defined by the ratio of the depth of the unsaturated zone, \( D_{unsat} \), over the unsaturated hydraulic conductivity (m·m\(^{-1}\)·d\(^{-1}\)) for each model layer. The percolation is then set to the proportional loss of the unsaturated storage for layer \( z \), StorMat, over the time increment \( \Delta t \)

\[
Perc = StorMat \cdot \frac{k(\theta_E) \cdot \Delta t}{D_{unsat}}.
\]

The loss of percolation to the groundwater table or over the semi-impervious lower boundary has consequently the dimensions of (m) waterslice and can be deducted directly from the drainable storage in the unsaturated zone and used to recalculate the resulting \( \theta_E \) for the next timestep.

In the instances that the groundwater level does not reach into the model layer \( z \), the percolation is calculated over the total depth of the layer and passed to the underlying unit. If the groundwater reaches the surface, the rate of percolation is zero and the vertical input is directly given by the evapotranspiration or the net precipitation. If the groundwater table reaches into the model layer \( z \), the thickness of the unsaturated zone, \( D_{unsat} \), is given by the depth to the groundwater table. An ever-increasing groundwater level will lower the thickness of the unsaturated zone and shorten the travel time for percolating soil moisture. This increases the likelihood that the fraction of Equation 4.6 exceeds unity and, drainage of the pore space would be complete. To prevent this unrealistic situation, the fraction is used to recalculate the degree of saturation first. This degree of saturation is subsequently used to project the saturated hydraulic conductivity at the end of the timestep. If the expected \( \theta_E = 0 \), it is set to an arbitrary value. The new unsaturated hydraulic conductivity is then used to calculate the average for the current timestep as the geometric mean, i.e. \( k(\theta_E) = \sqrt[k(\theta_E)]{k(\theta_E)^\prime \cdot k(\theta_E)^\prime\prime} \). This average of the unsaturated hydraulic conductivity is then reused in Equation 3.6.

The use of the average unsaturated hydraulic conductivity also reduces the percolation close to the phreatic surface and is a surrogate for the influence of the matric potential in the vertical and for the effect of a capillary fringe. However, this conceptual approach is much simpler and reduces the problem of numerical instability if the model has to be applied over longer temporal scales. Moreover, since the model neglects the influence of
the matric potential, it does not suffer from the discontinuity in hydrological behaviour that arises from the air entry value of the SWRC of Equation 3.4. The calculation of the percolation supposes that, for a raster cell, the percolation can be represented by the unsaturated matric flow under effective parameter values. However, for those cases that a clear dual porosity network appears to exist, an option is included to transfer a fraction of the rainfall excess directly to the lithic contact within one timestep. This will simulate the bypass flow through macropores or fissures in the regolith.

Currently, the model includes three layers that can represent the different strata over the depth of the soil mantle. Based on available knowledge, the parameterisation of the model can use spatial averages or use distributed values for the parameter values. If more geological detail or a more gradual profile of soil moisture is required, the number of model layers can be expanded. However, it must be realised that in the former case the number of parameters will increase and that in both cases the time increments has to be decreased to maintain numerical stability.

The problem of numerical stability also applies for the lateral outflow over the saturated zone. This flux is determined by the piezometric head and the effective $k_{sat}$ and the condition must be satisfied that the flux density for a cell does not exceed the cell length over the timestep (Equation 3.10). Since both are based on a simple, explicit numerical solution, the saturated and the unsaturated flow must comply to the Courant criterion, i.e. $V_k << \Delta x/\Delta t$.

In the model, the numerical solution uses a forward finite difference scheme for the discretisation of time, meaning that the fluxes in the present timestep are calculated from the state variables as obtained at the end of the previous one.

The generation of a perched groundwater table is in nature dependent on the loss of percolation across the semi-impervious lithic contact and is simulated in the model by imposing a boundary condition. If the substratum is considered impervious, no water is lost and a perched water table will form instantaneously. If this is not the case, an alternative boundary condition must be applied: at the lithic contact water is lost into an infinite fourth layer, for example bedrock. The lower boundary condition is a combination of a state-controlled, or Dirichlet-type, and a flux-controlled, or Neumann-type, boundary condition. For the fourth infinite layer, the matric suction or relative degree of saturation can be set to a fixed level, $|h|_{BC}$ or $\theta_{BC}$. Both values are interchangeable by means of the SWRC. This results in a constant unsaturated hydraulic conductivity at the base of each element, $k(\theta_{BC})$, in (m·d$^{-1}$). Per timestep the loss to the fourth infinite store is dependent on this value and, if a saturated zone is present, on the hydraulic conductivity of the layer overlying the contact. In order to simulate the effect of the hydrostatic pressure over the contact, the height of the water level, $WL = \Sigma D_{sat}(z)$, is divided by the total soil depth, $Z = \Sigma D(z)$, and the geometric mean is used to obtain the bulk $k_{sat}$ of the horizontal layers (Kutilek & Nielsen, 1994):

$$Perc_{BC} = \sqrt{k(\theta_{BC})k_{sat}WL/Z} \Delta t,$$

in units of (m) (Figure 3.1). The suffix BC has been omitted as all parameters except $WL$ and $Z$ refer to the fourth, infinite layer.
If no groundwater exists, the vertical loss into the fourth infinite store is dependent on $k(\theta_{E\ BC})$ only. This implies that the percolation from the basal layer must exceed this loss to generate groundwater. The lower boundary conditions determines for this reason the response of the hydrological system and this is in line with the physical reasoning that, in combination with the antecedent moisture conditions, above average rainfall is required to trigger landslides. The predominance of the lower boundary condition turns it in a suitable parameter for model calibration. The choice of the lower boundary condition to optimise model performance is also warranted by the fact that the characteristics of the substratum are usually less known and that it, on the temporal scales under consideration, can be presumed to be unrelated to land use and meteorological conditions.

In the model, the lower boundary condition is specified as a fixed value for the matric suction, $|h|_{BC}$ in (m). This parameter is preferred since the matric suction is continuous over space whereas the relative degree of saturation depends on the local conditions of the SWRC. Thus, $|h|_{BC}$ has to be defined after which the $\theta_{E\ BC}$ and $k(\theta_{E\ BC})$ are calculated with incorporation of any spatially varying attributes of the SWRC and the saturated hydraulic conductivity.

The lateral outflow over the saturated zone, $Q_{sat}$, is given by the piezometric gradient, $i$. This gradient is defined by the absolute elevation of the phreatic surface. This absolute elevation is obtained by summing the elevation of the lithic contact in the cell with the height of the water level, $WL$ (both in m). Over a window of 3x3 cells (Figure 3.5), the maximum slope of the phreatic surface, $\tan(\alpha)$, is obtained for each timestep. The cell that coincides with this maximum slope is designed as the drainage direction for the saturated lateral outflow from the central cell. This type of map, in which the flow is directed in one of the eight cardinal directions, is named the local drainage direction map (LDD). The piezometric gradient, $i$, for the central cell under consideration is given by the difference in elevation over the slope parallel distance, $\Delta x / \cos(\alpha)$. Over the 3x3 window, the gradient is thus equivalent with the sine of $\alpha$.

![Figure 3.5: Cardinal directions of Local drainage direction map (LDD). To cells without drainage direction, pits, the value 5 is attributed.](image)
Along the LDD, the lateral outflow, $Q_{\text{sat}}$, from the central cell to the downward cell passes the common height of the saturated zone and travels with the apparent velocity of the saturated lateral conductivity, $k_{\text{lateral}}$. Both are defined as the arithmetic average between the central cell $x$ and the downstream cell $x+1$. For the calculation of the lateral saturated hydraulic conductivity, $k_{\text{lateral}}$, the bulk saturated hydraulic conductivity over the entire water table over the lithic contact is used, i.e.

$$\hat{k}_{\text{sat}} = \frac{\sum D_{\text{sat}} \cdot k_{\text{sat}}}{WL}.$$  \hfill (4.8)

The average saturated lateral conductivity in the downstream direction is then

$$k_{\text{lateral}} = \frac{1}{2} \left( \hat{k}_{\text{sat}}(x) + \hat{k}_{\text{sat}}(x+1) \right)$$  \hfill (4.9)

and has units of $(\text{m} \cdot \text{d}^{-1})$.

Likewise, the height of the common saturated zone is averaged over the water levels. The resulting lateral flux over the saturated zone is given by

$$Q_{\text{sat}} = k_{\text{lateral}} \frac{1}{2} (WL(x) + WL(x+1)),$$  \hfill (4.10)

which has units of $(\text{m}^2 \cdot \text{d}^{-1})$. The use of the lateral saturated hydraulic conductivity makes the necessity that all model layers are of equal depth, superfluous.

For each cell the storage of the saturated zone is balanced for the outgoing and incoming fluxes. For the lateral flow, the change in storage, given in $(\text{m})$, is obtained by multiplying the net lateral flow with the time increment and dividing it over the cell length in the direction of the flow. Within a timestep, water is added to the saturated storage by the net percolation from the unsaturated zone and from bypass flow, if any occurred. The saturated storage is diminished by the loss over the semi-impervious lithic contact and by evapotranspiration. The latter loss is only substantial when the groundwater table is close to the surface (see below).

The budget for the saturated zone returns the groundwater level at the beginning of the next timestep. The inherent convergence of the LDD leads to a concentration of $Q_{\text{sat}}$ in the downstream direction at the junctions of the LDD. As long as the model is run at timesteps that are small enough to secure numerical stability, this will not be a problem and, because the piezometric gradient is used, the flow paths will adjust themselves to the evolving situation. As a result, the groundwater on the slopes will tend to be more diffuse. Albeit its more complex nature, this is an advantage, for it avoids the rigour of models in which the flow is only driven by the topographical gradient (Dupuit-Forcheimer approximation – Beven & Kirkby, 1979; Lambe et al., 1998). Although this may be a valid assumption in many cases, the method employed here is more flexible as it is able to reproduce the effects of local anomalies in the hydraulic conductivity and soil depth. The cost for the extra detail appears in the additional computational load and must be weighed against the spatial and the temporal resolution of the model. If either of these is reduced, the approaches will become more similar.
In the model, each cell is reported for which numerical stability for the saturated zone can not be maintained during the simulation. To prevent budget errors, the lateral and vertical fluxes are scaled to the available storage. The losses are summed and compared to the available storage after the incoming percolation and the bypass flow have been added. When the total loss exceeds the storage, its fluxes are reduced to match the available budget. After the local changes are balanced, the saturated lateral flow $Q_{\text{sat}}$ is routed over the LDD to obtain the water level for the next timestep.

Any saturated storage that is in excess of the maximum storage of the soil, is removed as exfiltration. This exfiltration is passed over the LDD of the topographical surface. In the nearest downstream cell it is treated as surface detention in the next timestep and available for evaporation. This exfiltration is of little importance and it can not be routed realistically for the temporal scales under consideration. In the model, the user has the option to evacuate it directly from the entire area.

After a layer has become completely saturated, subsequent drainage may lower the groundwater level under its upper boundary. If the relative degree of saturation of the overlying layer is used as the initial value for the draining layer. In case that total saturation of the soil profile were to occur, the initial value has to be defined arbitrarily. In the model, this value is taken as the relative degree of saturation at field capacity, the amount of water the soil retains against the forces of gravity after 24 hours of drainage. For temporal resolutions shorter than one day, a value is used that is interpolated linearly between saturation and field capacity. For timesteps of more than one day, the field capacity itself is used. The choice of the $\theta_E$ at field capacity is free but for convenience the absolute matric suction is used as input, from which the $\theta_E$ is calculated. As for the definition of the lower boundary condition, this avoids the troublesome definition of the relative degree of saturation from spatially varying soil properties (porosity and SWRC).

In addition to the processes of percolation and saturated, lateral flow, STARWARS also includes infiltration and evapotranspiration. These last mentioned processes are determined to a large extent by land use and vegetation and they define the boundary condition that is imposed at the surface of the soil. The upper boundary condition for the model is flux-controlled and concerns the meteorological input at the surface consisting of the precipitation and the evapotranspiration. Both values are given as daily totals, which can be split over smaller time increments if this is required to maintain numerical stability. Alternatively, more detailed records can be included when available. In the latter case, a better infiltration module may be needed. Currently, the infiltration capacity is defined as a ratio, $k_0$, of the saturated hydraulic conductivity of the top layer. Infiltration excess occurs if the rain that reaches the surface exceeds the infiltration capacity.

The amount of rainfall that is available for infiltration is limited by the evapotranspiration on one hand and by interception on the other. All rainfall is subject to interception by the vegetation, which is included in the model as a canopy store of finite capacity. Interception is here defined as the gross interception (Zinke, 1965). This is the amount of water that is detained by the canopy and lost to evaporation. This loss is proportional to the gross precipitation rate, $P$, over a certain time span. All water that is not intercepted by the canopy, will be passed to the surface. A part of this water will be intercepted by the vegetation and litter that cover the soil and is not available for infiltration. However,
in the model this loss is expected to be included by the evapotranspiration of the soil and is not covered by canopy interception. This choice is based on the notions that the canopy interception is easier to measure in the field and that the conditions of the vegetation can vary independently at the two levels over time (e.g. in the case of deciduous forests).

In the hydrological component of the model, the evaporation loss of the interception at the surface is accommodated by the actual evapotranspiration.

For the canopy, the precipitation balance is

\[ C = (1 - p) \int P dt - \int D dt - \int E dt, \]  

(3.11)

where \( p \) is the fraction of not intercepted rainfall, \( P \) is the rainfall intensity, \( D \) is the drainage rate from leaves and stem, \( E \) is the rate of evaporation and \( C \) is the quantity of water detained on the canopy (Aston, 1979). All rates are in units of waterslice over time and defined over a unit area of the projected canopy.

The first two terms of Equation 3.11 define the amount of water that reaches the surface. In practice, the distinction between the direct throughput, conditioned by the proportion \( p \), and the drip of the foliage can not be separated. So, both fluxes are lumped into one quantity, the throughfall \((P_t)\). The term of \( D \) then comprises only the concentrated routing of water along the stem and branches, the stemflow \((P_s)\). Written as a budget of the sum of these terms, the gross interception loss of the canopy, \( I_c \), becomes

\[ \Sigma I_c = \Sigma P - \Sigma P_t - \Sigma P_s. \]  

(3.12)

Thus, the net rainfall, \( \Sigma P_n \), that is available for infiltration equals \( \Sigma P - \Sigma I_c = \Sigma P_t + \Sigma P_s \).

All interception, which is not indirectly passed on to the topsoil, can be lost due to evaporation. Since the maximum storage capacity, \( C_{\text{max}} \), is finite, the effective rainfall intensity, including the loss due to evaporation, determines the moment at which saturation is reached. Only when saturation is reached, the drainage process, \( D \), will become fully operative (Rutter et al., 1971). If the canopy does not reach saturation, no drainage will occur and the total gross interception equals a proportion \((1-p)\) of the integrated intercepted rainfall (Equation 4.11; Lankreijer et al., 1992). Because both the time to saturation and the drainage processes are related to the effective rainfall intensity, most interception equations have the general appearance of a curvilinear relation that is bounded by \( C_{\text{max}} \).

The description of the interception is simplified if the evaporation during the rainfall is neglected. This is a reasonable assumption as the rate of evaporation is strongly reduced when the temperatures are low and the relative humidity is high. A simple conceptual model is that of Merriam (1960) that defines the detained water at a given time as an exponential function of the maximum storage capacity and the accumulated rainfall

\[ C = C_{\text{max}} \left[ 1 - \exp( - (1 - p) \Sigma P / C_{\text{max}} ) \right]. \]  

(3.13)

Aston (1979) replaced the fraction of intercepted rainfall, \((1-p)\), by an empirical coefficient \( k \), increasing the number of parameters to two. For six out of eight Eucalyptus species, he found a close agreement between this empirical parameter and the physically rationalised value \((1-p)\).

In Chapter 5, the performance of Merriam’s model and that of a linear regression are evaluated for inclusion in the model. Both functions are described by two parameters. For
the linear regression these are the slope and intercept which give the rainfall dependence and the maximum storage capacity respectively. For Merriam’s model, the parameters are the maximum storage capacity, \( C_{\text{max}} \), and the factor \((1-p)\) (Equation 3.13). Because the fraction \( p \) specifies the amount of direct throughfall, it is also referred to as the *free throughfall coefficient*. When not used as a fitting parameter, it can be estimated from measurements of the leaf area index, the ratio of the leaf surface over the projected canopy area \( (LAI, \text{m}^2\cdot\text{m}^{-2}) \).

Because most interception models are event-based, their performance will be poor on the temporal scale of days. This limited skill is explained by the fact that the sum of rainfall provides no information on the rainfall intensity and that the discretisation leads to an artificial division of prolonged rainfall events. This limitation will reduce the general interception function effectively to an invariable maximum storage capacity. The definite form of the interception model is based on the findings of Section 5.2.2. To some extent, the functionality of all interception models will be limited by the natural variability in rainfall intensity, by the temporal differences in vegetation conditions and by the spatial variability of vegetation (open stands).

After the interception has been deducted, the net rainfall is passed to the surface and stored as surface detention. Both interception and surface detention are subject to evaporation. The evapotranspiration that is used as input for the model is the reference potential evapotranspiration, \( ET_0 \). It is entered separately as a timeseries into the model and can consist of observed or calculated values. In this case, Penman’s equation has been used to calculate it, which is treated in detail in Section 4.3.

Because of the importance of the evapotranspiration in the Mediterranean, a detailed module to simulate the loss to evapotranspiration has been incorporated in the model. In the model, any interception can be lost directly to the potential reference evapotranspiration over the current timestep. The deduction from the surface detention affects only the excess from the previous timestep, after which the surface detention over the present time step, becomes available for infiltration. If the new surface detention exceeds the infiltration capacity, the excess is stored and becomes available for direct evaporation in the next timestep.

Any remaining reference potential evapotranspiration, \( ET_0 \), may be lost from the water that is stored in the soil. However, a limited soil water availability and the presence of vegetation may reduce this potential rate. In the model, simple, dimensionless scaling factors or *crop factors*, \( k_c \), relate this reference potential evapotranspiration, \( ET_0 \), to the actual evapotranspiration for specific vegetation covers. The decline of the actual evapotranspiration, \( ET_{\text{Act}} \), with decreasing water storage in the soil is simulated by scaling the remainder of the potential evapotranspiration with the ratio between actual and the maximum storage of an element

\[
ET_{\text{act}} = k_c \cdot ET_0 \cdot \left( \frac{\sum \text{StorMat} + \sum \text{StorSat}}{\sum \text{StorMax}} \right),
\]

where StorMat, StorSat and StorMax represent respectively the unsaturated, saturated and the maximum storage in units of waterslice (m).
The actual loss due to evapotranspiration is proportional to the available storage in each layer. Over the unsaturated zone, the loss to evapotranspiration is divided over sink terms, $S_z$, for every layer, $z$. The rate of evapotranspiration in each layer is calculated with the aid of the gradient, this being the slope of evapotranspiration over the depth of the unsaturated zone (Figure 3.6). At the same time, the amount of evapotranspiration of each unsaturated layer is limited by the difference in the relative degree of saturation between the layer under consideration, $z$, and the overlying layer, $z-1$. Bearing in mind the implicit assumption that the relative degree of saturation is a substitute for the continuity of matric suction in the unsaturated zone, this condition assures that the soil moisture content deeper in the soil can not decrease below that of the top layer due to evapotranspiration, as is usually the case in nature.

![Figure 3.6: Linear decrease of evapotranspiration from the unsaturated matrix. Maximum evapotranspiration from layer $z$ is limited by the relative degree of saturation of the overlying layer.](image)

### 3.2.3 Modelling of slope stability

On slopes, the soil above a potential shear plane is subject to a driving force, or demand ($D$), that includes the downslope component of the own weight of the soil and any additional loads acting on it. Movement is resisted by the reaction force of the mobilised shear strength. The mobilised shearing resistance is finite and can be considered as the capacity, $C$, of the soil to resist failure. Failure is to occur as soon as the demand exceeds the capacity. This principle is the basis for the limiting equilibrium approach, which forms the basis for the slope stability assessment in the coupled hillslope model; the
maximum shearing resistance that can be mobilised is equal to the disturbing forces at imminent failure (hence the name limiting equilibrium).

With the limiting equilibrium method, the stability can be expressed as the ratio or the difference between the capacity and the demand. The first measure is known as the safety factor, $F$, 

$$F = \frac{C}{D}.$$  

(3.15)

At $F= 1$, failure is imminent and the situation critical. When $F< 1$, the slope should theoretically have failed. The difference is known as the safety margin, $SM$, 

$$SM = C - D.$$  

(3.16)

For critical situations, $SM$ equals zero whilst positive values indicate stable conditions. The safety factor is dimensionless whereas the safety margin has the units of the demand and capacity, which can be defined as forces, moments or stresses. It is the most convenient to define the conditions at the potential shear plane as stresses, the average of the inter-particle forces over an elementary area., i.e. $f= \Sigma F/A$.

The limiting equilibrium approach only considers yielding by plastic failure. The maximum shearing resistance can then be described by the Mohr-Coulomb failure criterion 

$$\tau_f = c + \sigma \tan \phi,$$  

(3.17)

where $\tau_f$ is the shearing resistance at failure ($F= 1$), $c$ is the cohesion $\sigma$ is the total normal stress and $\phi$ is the angle of internal friction. The latter is defined as an angle whilst the other variables have units of stress ($N\cdot m^2$ or kPa). The contribution of the cohesion is considered constant and consists of all inter-particle bonds such as those between clay minerals or cementation by salts or metal oxides.

The frictional component of the shearing resistance varies with the average inter-particle stress normal to the potential shear plane, $\sigma$. If the weight of the soil, $W$, is the only load acting at the potential shear plane, the total normal stress for an area of unit width and length is given by 

$$\sigma = W \cdot \cos^2 \beta,$$  

(3.18)

where $\beta$ is the slope angle. $W$ equals $Z\cdot \gamma \cdot 1 \cdot m^2$, respectively the depth (m) and the bulk density of the soil (kN·m$^{-3}$). Under the same assumptions, the disturbance is given by the shear stress, $\tau$, 

$$\tau = W \cdot \sin \beta \cos \beta.$$  

(3.19)

When part of the imposed load is carried by the water that is present in the pores, the frictional component of the shear resistance depends on the effective normal stress, $\sigma'$, which is the total normal stress reduced with the pore pressure, $u$ 

$$\sigma' = \sigma - u.$$  

(3.20)
The use of effective conditions is indicated by primed symbols for the normal stress and the shear strength parameters, \( c' \) and \( \phi' \). Under the assumption that the occurring groundwater levels are unconfined, the effective stress is affected by the buoyancy of the particles below the groundwater level, so

\[
\sigma' = [(Z - WL) \cdot \gamma + WL \cdot \gamma'] \cos^2 \beta ,
\]

(3.21)

where WL represents the groundwater height above the shear surface and, respectively, \( \gamma \) and \( \gamma' \) are the moist and buoyant bulk unit weight (kN·m\(^{-3}\)).

For the calculation of the slope stability, the model uses the same schematisation of the soil mantle over a lithic contact as depicted in Figure 3.1. The depth of the potential shear plane determines the weight acting on it. In the field, such potential shear planes include planes of structural weaknesses as bedding planes or horizons in the soil profile, like lower boundary of the root zone or the lower end of the weathering profile. When the depth of landslides is defined by such structural weaknesses, their form can be described as planar or translational, and their length will be much greater than their depth (Crozier, 1973). For this type of landslides the curvature of the shear plane is negligible and inclined almost parallel to the topographical surface. This approaches an situation in which the soil mantle is of infinite length. Under homogeneous soil conditions, it can then be argued that, at equilibrium, all forces acting on the upslope part of an element are balanced by downslope forces that are equally large but opposite in sign. This slope stability model is known of the infinite slope model (Nash, 1987). In a raster-based GIS this has the advantage that the interactions between cells can be neglected and the calculated stability is dependent on the local cell attributes only. This one-dimensional approach will make the stability model less dependent on the actual cell size and the error that otherwise results from adding generalised attributes over multiple cells, is avoided. It allows a tight coupling of the model with the GIS environment which is impossible to achieve with 2- and 3-D models (Van Asch et al., 1993). A loose coupling, in which the output of the hydrological model is exported to a 2-D or 3-D slope stability model, is possible but requires a high resolution DEM to avoid discretisation errors in the landslide morphology. The usefulness of this option is limited when the stability has to be assessed over larger areas and when no landslides or landslide prone areas are distinguished in advance. The data transfer and the recurring calculations make a slope stability assessment by loosely coupled 2-D or 3-D models on a regional scale unwieldy.

In the infinite slope model, the local values of the demand and the capacity govern the stability. On natural slopes, these quantities are primarily influenced by the own weight of the soil above the potential shear plane (Equation 3.17 and 3.19). The stability is assessed in terms of effective stress, with the frictional component of the shear strength set to \((\sigma-u) \tan(\phi')\). To determine the pore pressure, the assumptions of the infinite slope model impose the condition that the seepage is parallel to the shear plane and topographic surface. As an alternative to Equation 3.21, the pore pressure that is needed for the calculation of the effective normal stress is given by

\[
u = \gamma \cdot WL \cdot \cos^2 \beta ,
\]

(3.22)
where WL represents the groundwater height above the shear surface, $\gamma_w$ is the density of water (kN·m$^{-3}$) and $\beta$ is the inclination (Figure 3.7).

Figure 3.7: Definition of the infinite slope model for translational slides

If no groundwater is present, the matric suction provides an additional interstitial force that adds to the shear strength. According to the model of Fredlund (Fredlund, 1987), this force is additional to the frictional shear strength and not an intrinsic part of it. Over a unit area, it can be represented as an apparent cohesion term $\Delta c'$

$$\Delta c' = |h| \tan \phi^b,$$

which is positive and consists of the absolute matric suction, $|h|$ (kPa), and $\phi^b$, the friction angle that describes the increase in shear strength at higher values of suction.

Both can be included in the equation of the infinite slope model. With stability expressed by the factor of safety, $F$, this equation is derived by substituting Equations 3.17, 3.19, 3.20 and 3.23 into Equation 3.15. This results in

$$F = \frac{c' + \Delta c' + [(Z - WL) \cdot \gamma + WL \cdot \gamma_s] \cos^2 \beta \tan \phi^b}{[(Z - WL) \cdot \gamma + WL \cdot \gamma_s] \sin \beta \cos \beta},$$

where $c'$ and $\Delta c'$ are respectively the true and the apparent cohesion, $\phi'$ is the angle of internal friction, $Z$ is the depth to the potential shear plane, $\beta$ is the slope angle, WL is the water level above this plane and $\gamma$, $\gamma_s$ and $\gamma'$ are respectively the moist, saturated and buoyant bulk densities.

The slope stability assessment is deterministic and requires the input of the soil depth, $Z$, and the other parameters. The model has the option to calculate the critical depth, at which $F = 1$, but for the present case the depth has been fixed to that of the third layer and the lithic contact is taken as the potential shear plane (Section 5.7). The hydrologic input consists of the absolute matric suction, $|h|$, and the groundwater height, WL, which stem from the hydrological model component STARWARS.
The safety factor and the safety margins are deterministic measures for the slope stability. They predict instability if the driving forces exceed the available resistance locally. On this scale, i.e. for a slope or cell, the safety factor and the safety margin can be used to predict instability. However, these measures are poor estimators of landslide activity on a regional scale because they are linear scaling methods that relate the actual demand to the potential capacity (Christian & Beacher, 1999). Since both do vary with local conditions, identical values of F or SM do not provide information on the amount of capacity that is required to ensure stability and can not be compared. This limits their usefulness if landslide activity must be forecasted under hypothetical conditions. Moreover, the deterministic measures of F and SM are unable to explain any discrepancy between the observed landslide occurrence and simulated failure as they ignore local variations in the capacity and the demand.

These problems can be partly overcome by the probability of failure, which reflects the likelihood that the demand exceeds the available capacity given the variability in the variables of Equation 3.24. This probability of failure is directly comparable and the probabilistic approach thus provides a better statistic to compare landslide activity on the one hand, and addresses the problem of natural variability on the other.

The probability of failure can be envisaged as the overlap between the distributions of the capacity and demand (Lee et al., 1983; Figure 3.8). Mathematically, the probability that the true safety factor is smaller than unity, $P(F \leq 1)$, for known distributions of the capacity, $C$, and demand, $D$, is given by

$$P(F \leq 1) = \int G_C(D)g_D(D)dD,$$

(3.25)

where $G_C$ and $g_D$ are respectively the cumulative probability function of the capacity and the probability density function of the demand.

The distributions are defined by the mean and variance of the capacity and the demand. Assuming that both the demand and capacity are normally distributed, the probability of failure is given by (Lee et al., 1983)

$$P(F \leq 1) = 1 - G_F \left( \frac{M[C] - M[D]}{\sqrt{V[C] + V[D]}} \right),$$

(3.26)

where $G_F$ denotes the cumulative probability of the standard normal distribution, $G_F(z)$, and $M[\cdot]$ and $V[\cdot]$ denote respectively the arithmetic mean and variance of the demand and the capacity.

A probabilistic approach can be followed to calculate the mean and variance of the capacity (Mood et al., 1974; Mulder, 1991). The mean and the variance of the sum of two independent variables is given by

$M[a_1x + a_2y + a_3] = a_1M[x] + a_2M[y] + a_3,$

(3.27A)

$V[a_1x + a_2y + a_3] = a_1^2V[x] + a_2^2V[y] + a_3^2.$

(3.27B)
where $a_1$, .. $a_5$ denote constants and $x$ and $y$ are variables with arithmetic average $M[\cdot]$ and the variance $V[\cdot]$.

The mean and variance for the product of two independent variables is given by

$$M[x \cdot y] = M[x] \cdot M[y], \quad V[x \cdot y] = M[x]^2 \cdot V[y] + M[y]^2 \cdot V[x] + V[x] \cdot V[y].$$

These rules (Equations 3.27 and 3.28) can be applied to the infinite slope model of Equation 3.24. For the capacity, given by the Mohr-Coulomb Failure Criterion, the mean and variance become

$$M[C] = M[c'] + M[\sigma'] \cdot M[\tan \phi'], \quad V[C] = V[c'] + M[\sigma']^2 \cdot V[\tan \phi'] + M[\tan \phi']^2 \cdot V[\sigma'] + V[\tan \phi'] + V[\sigma'].$$

The terms of $c'$ and $\sigma'$ can be expanded by use of the theorems to include the variability in the underlying variables (e.g. $\tan \phi$, $\gamma$, $z$ and $u$ or WL). The same holds for the demand, although the variability in the demand is generally smaller than that in the capacity. The probabilistic theorems also permit the inclusion of other distributions than the normal or Gaussian distribution if the parameters that describe them are converted to the arithmetic mean and the variance, $M[\cdot]$ and $V[\cdot]$. Successively, the probability of failure can be calculated using 3.26 assuming a normal distribution of the factor of safety.

The choice of the normal distribution is not evident for natural variables as the limits of the integral of Equation 3.25 can range from $-\infty$ to $\infty$. In reality, many natural variables and the factor of safety will be censored as the shearing resistance can not be negative. This limits the range of the integral of $F$ in Equation 3.25 from 0 to $\infty$. If the variability is small, the error due to the fraction $F<0$ of the area under the normal curve will be negligible. When it is large, however, another distribution may be needed. Commonly
used statistical distributions that may restrict the distribution to the range \( F \geq 0 \) are the uniform and the lognormal distribution. Especially the lognormal distribution has been used to describe the shear strength parameters of \( c' \) and \( \phi' \) and the factor of safety (e.g. Lumb, 1970; Mulder, 1991). If the mean and variance of the safety factor are calculated directly, the probability of failure can be calculated for any probability density function \( g_F \)

\[
P(F \leq 1) = \int_{-\infty}^{1} g_F(F) dF.
\]

(3.30)

This solution is more flexible than the approach of Equation 3.25 as only the distribution of \( F \) has to be assumed. The distributions of the variables that are included as sources of uncertainty in \( F \) are considered to be known from field measurements. For the normal distribution, the probability of failure is then given by the integration of the standard normal curve for the range \(-\infty < F \leq 1\), where the z-score is given by

\[
z = \frac{x - M[F]}{\sqrt{V[F]}}.
\]

(3.31)

For the lognormal distribution, the z-score is based on the logtransform \( y = \ln(F) \) and takes the form

\[
z = \frac{\ln(x) - M[y]}{\sqrt{V[y]}}.
\]

(3.32)

In both cases is \( x \) the value of the safety factor for which the probability of failure is determined, i.e. \( x = 1 \).

The probability density function \( g_F(z) \) for the standard normal curve is

\[
g_F(z) = \frac{1}{\sqrt{\pi} \lambda} \exp\left(-\frac{1}{2} z^2\right),
\]

(3.33)

where \( \lambda = \sqrt{V[F] \cdot 2\pi} \) for the normal distribution, and \( x\sqrt{V[y] \cdot 2\pi} \) for the lognormal distribution.

The integrated value of \( g_F(z) \) is usually determined numerically. In PCRaster, however, iterations outside the main dynamic loop are not possible. Alternatively, a parameterised function is used which approximates the cumulative standard normal curve by

\[
G_F(z) = \frac{1}{2} + \frac{\tan^{-1}\left(z \cdot \left(C_1 + C_2 \cdot z^2\right)\right)}{\pi},
\]

(3.34)

where \( C_1 \approx 1.253 \) and \( C_2 \approx 0.579 \). This function deviates slightly from the true probability at the extremes of the curve but the difference is negligible (< 0.01).

The slope stability model component uses Equation 3.32 with Equations 3.30 and 3.31 to determine the probability of failure for the normal and the lognormal distribution of \( F \).
The probabilistic assessment is straightforward but can involve many variables for which the distributions have to be specified. The conversion from the parameters from distributions that are not normally distributed to the arithmetic mean and variance involves some error. From this point of view, it would be better to use a Monte Carlo simulation that offers more flexibility to establish the probability of failure for a cell (Christian et al., 1994). However, with the coupling of the slope stability assessment to the hydrological model component, a deterministic approach is more appropriate for validation purposes as the similarity in the simulated and actual hydrological triggering mechanisms must be assessed. For this reason, a probabilistic approach for the stability assessment has been followed in which the probabilistic element has been restricted to the shear strength parameters. These are the cohesion, and the angles of internal friction, $\phi'$ and $\phi^b$. The variability in the bulk density has been ignored as it has a only a very limited influence on the calculated slope stability (Section 7.2). The added information does not outweigh the complexity it introduces in the calculations.

Because only the intrinsic soil properties are included in the probabilistic slope stability assessment, it merely assesses the variability in the susceptibility (Section 7.3). The hydrological input into the stability analysis is kept deterministic and assesses the worst-case scenario over the period considered, both over the validation period and for the hypothetical conditions of the scenarios. This choice is based on the dependence of slope failure on critical pore pressure conditions and for ease of comparison of simulated landslide sensitivity under the current and hypothetical environmental conditions, which is the main objective of this study.

### 3.3 Model implementation

In a raster GIS, such as PCRaster in which the coupled hillslope model is embedded, calculations take place on the level of the individual cells. Therefore, all parameters must be specified at the level of the individual cell. Since it is not feasible to specify the input for the individual cell, some form of generalisation or interpolation is required. Dependent on the assumptions that are made on the spatial distribution of the parameters, the input can be defined as spatially constant, linked to known spatial attributes (e.g. land use), or estimated by geostatistical techniques. PCRaster supports these different options. Parameters can be entered as constants in the model script or included in tables, which relate the parameter value to a spatial attribute. The program GSTAT provides geostatistical interpolation techniques that are fully integrated with PCRaster (Pebesma & Wesseling, 1998).

If a parameter is dynamic, it has to be specified for every moment in time. Spatially distributed parameters have to be provided as stacks of maps, with one map for every timestep. If the spatially distribution can be ignored or simplified to several units, timeseries can be used to enter dynamic model input.

Model output can also be generated in the form of maps and timeseries. Spatial information is well represented by maps, but often difficult to analyse over a longer period. For this purpose, the condensed information of timeseries, which give the temporal information for a limited number of points, is more suited.
For the application of the coupled model, the overall structure defines that the hydrological model component precedes the stability assessment. So, the hydrological model component needs the input of the precipitation and the reference potential evapotranspiration over time. Because of the precedence of the hydrological model component, it also requires initial values for some of its state variables. These state variables are subsequently changed dynamically in the simulation of the hydrological processes. This contrasts them against the constant parameters of the model that are used throughout a model run to schematise the topography and soil profile and to parameterise the constituent equations (Figure 3.9).

The schematisation of the topography is based on the DEM, and the depth of the different layers above the semi-impervious lithic contact. This schematisation is identical for the hydrological and the slope stability model component and has been used to specify the input for these modules. For the topsoil, the influence of the vegetation on the hydrological processes of interception, infiltration and evapotranspiration will be linked to land use units that are considered as internally homogeneous.
Table 3.1: Model in- & output of the coupled hillslope model for hydrology (STARWARS) and stability (PROBSTAB)

<table>
<thead>
<tr>
<th>Model component:</th>
<th>Hydrology – STARWARS</th>
<th>Stability – PROBSTAB</th>
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<tr>
<td><strong>Model Input</strong></td>
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<td>Schematisation</td>
<td>• High resolution DEM (m)</td>
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<td></td>
<td>• Layer depth D(z) (m)</td>
<td></td>
</tr>
<tr>
<td>Constant parameter values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global boundary conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Matric suction for lower boundary condition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Matric suction at field capacity, 1st layer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Residence* of surface detention</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Fraction of bypass flow*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global – land use dependent Evapotranspiration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Crop factor</td>
<td>k_c (-)</td>
<td></td>
</tr>
<tr>
<td>Infiltration</td>
<td>K_0 (-)</td>
<td></td>
</tr>
<tr>
<td>Interception</td>
<td>C_max (m)</td>
<td>p (-)</td>
</tr>
<tr>
<td>• Max. storage capacity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Direct throughfall ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer-dependent**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Saturated hydraulic conductivity**</td>
<td>k_sat (m·d⁻¹)</td>
<td></td>
</tr>
<tr>
<td>• Porosity</td>
<td>r_A (m)</td>
<td></td>
</tr>
<tr>
<td>• Air entry value**</td>
<td>α (-)</td>
<td></td>
</tr>
<tr>
<td>• SWRC slope**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All parameters can be considered as layer and land use dependent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dynamic input – All timesteps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Reference potential evapotranspiration</td>
<td>ET₀ (m·d⁻¹)</td>
<td>W (m · d⁻¹)</td>
</tr>
<tr>
<td>• Precipitation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial conditions - state variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Groundwater level</td>
<td>WL (m)</td>
<td>θ (m³·m⁻³)</td>
</tr>
<tr>
<td>• Volumetric soil moisture content</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model Output</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maps and Timeseries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Groundwater level</td>
<td>WL (m)</td>
<td>F (-)</td>
</tr>
<tr>
<td>• Volumetric soil moisture content</td>
<td>θ (m³·m⁻³)</td>
<td>P_Y (-)</td>
</tr>
<tr>
<td>• Critical depth</td>
<td>Z_c = 1 (m)</td>
<td></td>
</tr>
<tr>
<td>All parameters of the top layer can be considered as land use dependent</td>
<td></td>
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</tr>
</tbody>
</table>

*: not considered (by default all water is transferred over the LDD and no bypass flow occurs)

**: also required for lower boundary condition

***: optional if the potential shear planes do not coincide with the layer boundaries
Of the model layers, $z$, only the first layer is taken as land use dependent in the hydrological model component. For these layers, input per layer consists of the parameters for the porosity, SWRC and the saturated permeability. These parameters are listed in Table 3.1. Per layer, the hydrological model thus requires the 5 parameters mentioned in Equations 3.3 and 3.4 and $k_{sat}$. When the tortuosity is set to the constant value of $4/3$, as used by Millington & Quirk (1959, 1961), this number reduces to 4 parameters, of which the SWRC characteristics $h_A$ and $\alpha$ are mutually dependent. So, for $z$ model layers, the required number of parameters becomes $4z$, and this may increase if, the upper layer, is divided over $n$ land use units. Since only the first layer is taken as land use dependent, the number of required parameters becomes $4(z-1)+4n$.

The hydrological model component requires several parameters for the definition of the upper and lower boundary conditions. For a semi-impervious lower boundary, one parameter is needed, the matric suction $|h|_{BC}$, to define the unsaturated hydraulic conductivity, $k(\theta_{E BC})$, of the infinite store under the lithic contact. As intended, the SWRC and the $k_{sat}$ are considered as identical to that of the overlying layer, thus trading three parameters for one general assumption.

For the upper boundary conditions of the hydrological model component, four parameters are needed to define the flux-controlled interactions between land use and soil moisture at the surface (Table 3.1). These parameters are the fractions and constants defining actual evapotranspiration, infiltration capacity and interception. Dependent on the number of land use classes, this adds $4n$ constants to the list of parameters. The last parameters, which can be specified in the model, define the residence of surface detention and the bypass flow (respectively Recharge and Bypass; Section 3.2.2). The first defines the fraction of the surface detention that is routed over the LDD of the topographical surface. The remainder is transferred to the next timestep. The second factor defines the fraction of the surface detention that is passed directly as bypass flow to the lithic contact. They can be used to simulate the presence of a fissure network in unstable areas but are not applied in this study. All surface detention is routed and available for evapotranspiration and infiltration in the downstream pixel during the next timestep while all vertical flow towards the lithic contact is confined to matric pores.

To illustrate the sample requirements of the hydrological model component STARWARS the following example is given. For a schematisation of three layers and four land use classes, ($z=3$, $n=4$), $4(z-1) + 4n + 4n= 40$ values are needed to parameterise the processes above the lithic contact. If $|h|_{BC}$ is considered as constant and the only unknown for the lower boundary condition and the fractions of recharge and bypass flow are specified, 3 more parameters are needed. This brings the total for this configuration to 43 parameters, or 41 if the latter two are ignored.

To this number, the initial values for the state variables must still be added. Initial values must be specified for the initial groundwater level and for the surface detention at the start of any new simulation. Per layer, the volumetric soil moisture content, $\theta_z$, has to be specified, from which the matric suction and the degree of saturation can be derived. Therefore, $2+z$ initial values must be added to the total number of parameters. All in all, this leads to the rather disappointing conclusion that, even for a simple model, the data
requirements are high: 47 parameters are needed when the morphology is not included. Distinct parameters for the different land use units are the main source for the large data requirements. Ideally, the in- & output will be defined according to the relevant detail for the intended use. In reality, data availability will restrict the available options.

The probabilistic stability component, PROBSTAB, is less demanding and requires the specific input of the shear strength parameters and of the dry bulk density. For the calculation of the probability, the variance for these parameters must be specified as well (Table 3.1).

If the potential shear planes do not coincide with the layer boundaries, the depth of the potential unstable soil mantle can be included. If the stability has to be calculated for a layered soil profile, the total number of parameters equals 4z.

For the calculation of the effective normal stress and the shear stress, the input of the groundwater level and the soil moisture for each timestep is required. Likewise, the matric suction is needed to calculate its contribution to the unsaturated strength, but this variable is interchangeable with the soil moisture through the SWRC. In the coupled hillslope model, thus z+1 dynamic variables are derived from the hydrological component.

The output of the stability assessment typically consists of the degree of safety, in this case of the safety factor, and the probability of failure over time. Together with the preceding output of the hydrological model component, the output of the coupled model comprises

- Groundwater levels;
- Soil moisture content for z layers;
- Average or expected factor of safety at specified locations, E[F], for example the base of z layers;
- Probability of failure at the above specified locations;
- The critical soil depth for which F= 1 (Z_{F=1}).

All output is basically composed of stacks of maps, reported at each timestep. The option exists to limit the output to maps that are generated at pre-set timesteps, or to limit the output to a continuous timeseries for predefined locations in the area of interest. Because of the tight coupling and the topographical control on landslide occurrence, both models are bound to the same spatial scale. The temporal scale for the stability assessment is not necessarily confined to the high resolution that the hydrological component dictates. It can be run at a coarser temporal resolution, using the aggregated output of the hydrological model component. This option is desirable if one wants to assess the most critical situation that is defined by the highest pore pressures for a given period (worst case scenario). In most cases, however, it is more economic if the temporal scales of both components coincide. The results must then be analysed and aggregated over the scales of interest afterwards.
3.4 Concluding remarks

Before the model is applied, it is helpful to enumerate its limitations and possibilities. They define, above all, the potential of the model to evaluate the impact of changing environmental conditions on landslide sensitivity.

The coupled hillslope model is physically based and is in principle capable to simulate the landslide activity under present and hypothetical environmental conditions. Changes in landslide activity, however, are only reflected in the temporal sensitivity as the susceptibility is not changed for the scenarios. The meaning of these changes depend entirely on the agreement of the model with the present processes and the extent to which the importance of these processes remain unchanged. This demands that the validity of the model is assessed. Moreover, it implies that, even for a valid physically based model, the environmental changes that can be imposed on the model must be moderate and based on the present situation.

The model validity is affected by operational and model errors. Operational limitations concern rounding errors and data limitations that originate from the discretisation of the model. Model input may also be affected by the available data, which will be presented and discussed in the chapter on data acquisition and model parameterisation (Chapter 5). A manifest limitation of the scenarios is the incapability to capture all changing environmental conditions into one valid scenario. The definition of scenarios including consistent change in all related, environmental factors, requires knowledge that in all cases will surpass the available information for the construction and implementation of the model. In general, only the broad outlines of the present situation and its heterogeneity are known. As a consequence, the predefined scenarios are more or less artificial for only the changes in a limited number of environmental conditions are inferred while all others factors remain constant (Chapters 4 and 8).

Discretisation and simplification put a further restraint on the potential of the model. Because the scope of the model stretches beyond the event-scale, runoff can not be included realistically in the model. Over time increments of days, runoff can cover large distances whereas this is limited to a mere cell size of several meters in the simulation. If runoff is considerable or surface erosion has to be assessed, another approach becomes compulsory.

Similar limitations originate from the use of distinct model layers and from the definition of the relative degree of saturation at field capacity. The latter, used to initiate drainage after complete saturation, supposes a linear decrease in moisture, regardless of the elapsed time, and simplifies the initial conditions to one general value for every situation. The use of model layers poses a specific lower limit to the response time. With three model layers, the transfer of water from the surface to the lithic contact lasts at least three time steps. Depending on the duration, the response is more or less retarded. Consequently, the optimum model configuration may change if different time increments are used.

Oversimplification also affects the land use characteristics that are incorporated in the model as the spatial and temporal variability is lumped over larger units or periods. Such land use dependent parameters will be adequate descriptors for the mean land use conditions but over short periods and on local scales they may be in error and influence
the capability of the model to predict the occurrence of actual landslides. For evaluation purposes, time-dependent relations could replace the constant land use parameters. Sidle (1992) used such an approach to model the temporal variation of shear strength under the impact of timber harvesting. For the hydrological properties, however, these relations are difficult to quantify and not always clearly discernible against the background noise. For this reason, the simpler approach has been favoured, in spite of the ensuing limitations.

A clear model limitation is the static nature of the slope stability model. The model does not simulate changes in landslide susceptibility, which could arise from adaptations in the morphology and the soil properties of a slope. This simplification is valid under the assumption that changes in the temporal sensitivity precede those in the susceptibility (Chapter 1).

Still, the infinite slope mode offers a simple means to assess the stability but other measures of instability than the traditional measures of the factor of safety or the safety margin are required as the main interest lies in the assessment of landslide sensitivity on a regional scale and over longer period. The probability of failure is a good alternative that takes the local variability into account and its values can be compared and analysed on a regional scale. The eventual accuracy of the infinite slope model is determined by the degree to which the underlying assumptions are met and the confidence by which the actual topography and soil properties can be represented on the chosen spatial resolution. This can be evaluated from a comparison of the observed activity with simulated failure, which is covered in Chapter 7.

The uncertainty that results from the model structure and its implementation poses, in combination with the oversimplified relationships, limitations to the reliability of the model outcome. To evaluate the consequences of these limitations, the model should be tested on its sensitivity, before an attempt at calibration and validation is made. From the sensitivity analysis, some preliminary ideas on the performance of some optimisation parameters can be obtained. Sensitivity analyses of the hydrological model component and of the stability assessment are considered in Chapters 6 & 7.