THEORETICAL ASPECTS OF NEUTRINO PHYSICS

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In discussing the theoretical aspect of neutrino reactions it is useful to split the subject into four parts:

i) The elastic process (octet-octet transitions);
ii) the quasi-elastic process (octet-decuplet transitions);
iii) the inelastic processes ($\nu + n \to n + \pi \ldots + \mu^-$);
iv) general tests (Adler theorem, time reversal etc.).

Each of these seem to be more or less experimentally accessible, thus it is useful to establish what can be learned about weak interactions from any of them - in the framework of present theory, of course.

I. THE ELASTIC PROCESS

In Fig. 1 the relevant processes are indicated. ν processes go according to the arrow, $\bar{\nu}$ processes in the opposite way. As is clear, an up to now unobserved reaction

$$\bar{\nu} + p \to \Sigma^0 + \mu^+$$

may occur. For $Q^2 = 0$ (Q^2 = four-momentum transfer squared) this reaction provides a test of the $\Delta I = 1/2$ rule against the $\Sigma^- \to n + \ell + \nu$ process. The validity of SU_3 in weak interactions may be tested in studying the form factors of the processes. This matter is theoretically completely unclear. Let us review the situation. The most general form for the matrix element of the baryon current for a process of the type (Fig. 2)

$$\nu + N \to \mu^- + N'$$

may be written as:

$$J_\alpha = \bar{u}(N') \Big\{ G_\nu \gamma^\alpha + \frac{\mu}{2m} \sigma^{\alpha\beta} Q_\beta + iA \frac{Q_\alpha}{m_\mu} + G_A \gamma^\alpha \gamma^5 + \frac{B}{2m} \sigma^{\alpha\beta} Q_\beta \gamma_5 + ib \frac{Q_\alpha}{m_\mu} \gamma_5 \Big\} u(N) ,$$

where m = mass of the incident nucleon, m_μ = muon mass, $\sigma^{\alpha\beta} = i/2(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha)$, $Q_\beta = (p_N - p_{N'})_\beta$ = four-momentum transfer, u(N) and $\bar{u}(N')$ are spinors belonging to initial and final nucleons. G_ν, μ, A, G_A, B, b are scalar functions of the three possible invariants m^2, M^2 (M = mass of final nucleon) and Q^2. If G parity holds A and B are zero.

Present day knowledge may be summarized as follows:

a) The CVC theory predicts G_V and μ for arbitrary Q^2 from those measured in electron scattering experiments. The scarce evidence in the ν process up to now seems to be in accordance with this. Of course, no form factors are known for strangeness changing currents. They are, in principle, related through the Cabibbo scheme, with unknown deviations through SU_3 breaking (K^* exchange instead of p etc.).

b) Experimental knowledge of the magnetic-moment term is very scarce. It seems to be consistent with a coupling 3D + 2F for the total magnetic moment, and further evidence in this direction is highly desirable. As the magnetic term contributes very strongly for non-zero Q^2 this might be within experimental possibilities.

c) The axial form factor G_A has been measured to some extent in the past neutrino experiments. The amazing fact emerging from this seems to be that this form factor behaves very similar to the vector form factor. G_V. Now in G_V the state of lowest mass is the two-pion state, in G_A we have the three-pion exchange (G parity). Moreover, there is a strong resonance (ρ) contributing to G_V of rather low mass (763 MeV), while no such resonance is known for the axial form factor. The same remarks as quoted in a) for $\Delta S \neq 0$ processes hold here.

d) The induced pseudoscalar term (although very interesting in view of the Goldberger-Treiman relation) contributes very little to the cross-section and is, therefore, probably out of experimental range. This is unfortunate because the establishment of an F/D ratio would be extremely interesting.

Rough production cross-sections may be obtained by using the Clebsch-Gordan coefficients of SU_3, and neglecting mass differences (i.e. setting $\sigma_{tot} \sim G_V^2 + G_A^2$) (numbers of M.M. Block, Phys.Rev. Letters 12, 263, 1964 table):

$$\left.\begin{array}{ll} \bar{\nu} + p \rightarrow n + \mu^+ & 100 \\ \left.\begin{array}{ll} \bar{\nu} + n \rightarrow \Sigma^- + \mu^+ & 2.5 \\ \bar{\nu} + p \rightarrow \Sigma^0 + \mu^+ & 1.25 \end{array}\right\} \Delta I = \tfrac{1}{2} & \\ \bar{\nu} + p \rightarrow \Lambda + \mu^+ & 4 \end{array}\right\} SU_3$$

II. THE QUASI-ELASTIC PROCESS

In Fig. 3 the relevant processes are indicated. Obviously we are dealing here with a completely new domain. The only other source of information on these processes is the photoproduction of N^* on nucleons, giving some coefficients in the $N - N^*$ current at $Q^2 = 0$. But it is expected soon that information on form factors will come from the electro-production of N^*.

The most general form of the $N - N^*$ current is (same notation as before):

$$J_\alpha = \Big[\bar{u}_\beta(N^*) \Big(\Big\{ a_1 \; \delta_{\beta\alpha} + \frac{Q_B}{m+M} \left(i a_2 \, \gamma^\alpha + \frac{i a_3}{m+M} \; \sigma_{\alpha\lambda} Q_\lambda + \frac{a_4}{m+M} \, Q_\alpha \right\} \gamma^5$$

$$+ b_1 \; \delta_{\beta\alpha} + \frac{Q_B}{m+M} \left(i b_2 \, \gamma^\alpha + \frac{i b_3}{m+M} \; (p_n + p_{n'})_\alpha + \frac{b_4}{m+M} \, Q_\alpha \right) u(N) \Big] \; ,$$

$a_1 \ldots b_4$ are again functions of the masses and Q^2. G parity does not give any restriction on this process itself, but it connects neutrino and anti-neutrino processes. Thus, G parity may be tested. The coefficient b_1 is, through the Goldberger-Treiman relation, related to the widths of the N^* resonances. Finally, as there is no such thing as F/D ratio, SU_3 uniquely predicts the strangeness changing production ratios [again from Block, neglecting kinematics: $\sigma_{tot} \sim (SU_3 \text{ coeff})^2$]

$\bar{\nu} + n \to N^{*-} + \mu^+$	100	$\Delta I = 1$	SU_3
$\bar{\nu} + p \to N^{*o} + \mu^+$	3.3		
$\bar{\nu} + n \to Y_1^{*-} + \mu^+$	2.5	$\Delta I = \frac{1}{2}$	
$\bar{\nu} + p \to Y_1^{*o} + \mu^+$	1.25		

III. THE INELASTIC PROCESSES

No theory is known for inelastic processes. Probably in the future something might be done with respect to the higher baryon resonances. For strangeness changing processes we know nothing, unless some specific schemes emerge (as, say, the 70 in SU_6). Conversely, neutrino physics may help in understanding that situation.

Furthermore, we have the possibility that peripheral processes contribute in an appreciable way. If so, further progress in understanding the inelastic processes is possible, for instance, one may learn about meson-vector meson-lepton coupling.

IV. GENERAL TESTS

As yet two interesting possibilities seem to exist for the study of $\nu, \bar{\nu}$ reactions on complicated systems (nuclei). The first one is the (Adler) tests for CVC and Goldberger-Treiman relations and, further, the test for time-reversal invariance.

a) The Adler test

Adler has observed that for muons produced in the forward direction, the lepton factor will be proportional to the momentum transfer Q_α (provided that m and M are not equal, as then Q will be simply zero, while the lepton trace is not). Thus, the cross-section becomes proportional to

$$\frac{\partial J^V_\alpha}{\partial x_\alpha} + \frac{\partial J^A_\alpha}{\partial x_\alpha} .$$

According to the CVC hypothesis $\partial J^V_\alpha / \partial x_\alpha$ is zero (apart from electromagnetic corrections). Furthermore, $\partial J^A_\alpha / \partial x_\alpha$ is, according to the Goldberger-Treiman relation, proportional to the pion cross-section for pions with energy momentum Q_α. Thus

$$\sigma_\nu(\text{forward}) \sim \sigma_\pi(Q^2, M') ,$$

where Q^2 and M' denote the dependence on Q_α. This pion cross-section may be obtained by extrapolating the known cross-section

$$\sigma_\pi(-m_\pi^2, M')$$

to the region needed above. Here one encounters two effects:

- effects of angular momentum barriers,
- nuclear effects.

The effects of angular momentum may be estimated theoretically on the example of N* production. The nuclear effects have been discussed by Bell. At first sight the remarkable point is that one expects neutrino cross-sections to be proportional to the number of nucleons in the nucleus, thus to A, whereas the pion cross-section is proportional to $A^{2/3}$. Extrapolating the pion cross-section to the region of interest for neutrino processes leads to the appearance of a term proportional to A in addition to the original piece proportional to $A^{2/3}$, and to test the PCAC hypothesis one must separate this $A^{2/3}$ term.

b) T invariance

If $K_2^0 \to 2\pi$ is a T-violating effect, and if $K_{\mu 3}$ shows up nothing (as expected from CVC) then test of time-reversal violation in the axial current in neutrino-induced processes becomes inevitable.

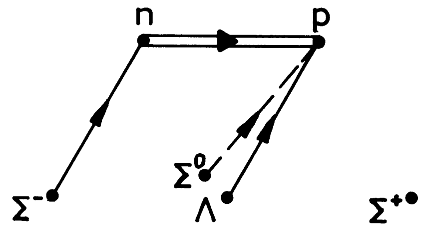

Fig. 1

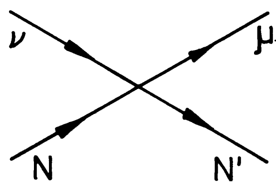

Fig. 2

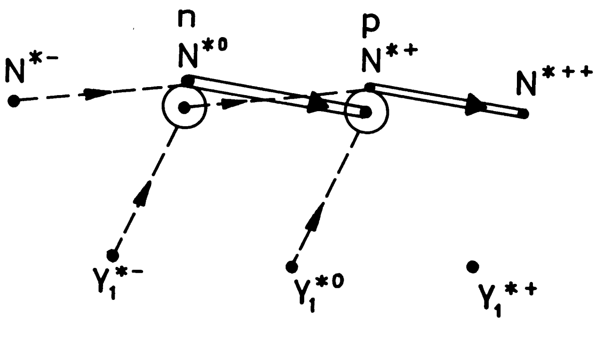

Fig. 3