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I. Theoretical aspects of high energy neutrino interactions

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1. Introduction

In recent years high energy neutrinos produced at the large accelerators have been used to investigate the properties of weak interactions.† As a result we know now that there are at least two kinds of neutrinos, and that an eventual intermediate vector boson is heavier than 2 GeV. In addition, the conventional theory of weak interactions has been tested in a larger domain, and found to be in reasonable agreement with experiment; in particular, strange particle production does not exceed appreciably what is predicted by Cabibbo's theory, which may be interpreted as further evidence against the older universal Fermi interaction theory.

Thus the situation at this moment is quite satisfactory, as far as the established notions on weak interactions are concerned. We may now ask to what extent high energy neutrino physics may be used as a tool to extend our knowledge of the weak interactions.

Here we must distinguish between testing the usual theory in a larger domain, and obtaining information in order to build out the theory. In §§ 2 and 3 we will discuss the possibilities of testing some features of the present theory in elastic and inelastic reactions at target nucleons as for instance through the process

$$\bar{\nu}+p \rightarrow \Lambda+\mu^+,$$

while in § 4 some tests of a general character are mentioned. In § 5 we explore the possibility of using neutrinos in order to obtain rather detailed information on the PCAC hypothesis (Gell-Mann & Levy 1960; Nambu 1960; Adler 1965).

2. The elastic process

The following processes may be studied experimentally (we assume all neutrinos are muon-neutrinos):

$$\nu+n \rightarrow p+\mu^-, \qquad \bar{\nu}+p \rightarrow \Lambda+\mu^+,$$
$$\bar{\nu}+p \rightarrow n+\mu^+, \qquad \bar{\nu}+n \rightarrow \Sigma^-+\mu^+,$$
$$\bar{\nu}+p \rightarrow \Sigma^0+\mu^+,$$

If we accept the notion of $\mu-e$ universality then all except the third reaction are known and have been studied in processes like

$$\Lambda \rightarrow p+e+\nu.$$

Of course only small values of momentum of the lepton combination occur here.

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Thus the above reactions may be studied to extend our knowledge of these processes to higher momentum transfer, while observation of the third process may be used to test the $\Delta I = \frac{1}{2}$ rule at zero momentum transfer by comparison with the

$$\Sigma^- \to n + e^- + \bar{\nu}$$

rate. To be more specific, the most general form of the matrix-element of the baryon current for a process
$$\nu + N \to \mu^- + N'$$
is
$$J_\alpha = \bar{u}(N')\left\{G_v(Q^2)\,\gamma^\alpha + \frac{\mu(Q^2)}{2m}\,\sigma^{\alpha\beta}Q_\beta + G_A(Q^2)\,\gamma^\alpha\gamma^5 + ib(Q^2)\,\frac{Q_\alpha}{m_\mu}\,\gamma^5\right\}u(N),$$
where we assume absence of currents of the second kind. Further: m is the mass of the incident nucleon and m_μ the muon mass, $\sigma^{\alpha\beta} = \frac{1}{2}(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha)$, $Q_\beta = (p_N - p_{N'})_\beta$ = four momentum transfer to the lepton system, and $u(N)$ and $\bar{u}(N')$ are spinors corresponding to initial and final nucleon.

G_v, μ, G_A and b are functions of m^2, M^2 (= mass of final nucleon) and Q^2. The masses m and M do not vary appreciably in the above processes, and it is mainly the dependence on Q^2 that may be investigated. In the Cabibbo theory of leptonic processes these functions are mutually related, for all Q^2. Here we must note that the relation itself, containing as a rule two arbitrary coefficients, may depend on Q^2, i.e. the two arbitrary coefficients may be functions of Q^2. For instance, a nonzero charge distribution for the neutron observed in electron scattering experiments would, through the CVC hypothesis, indicate a deviation for nonzero Q^2 of the pure F-coupling for G_v which holds supposedly at zero Q^2. This fact makes comparison of the theory with experiment somewhat illusory, at least for the time being.

It would be very interesting if the function $b(Q^2)$ could be measured, as this could, for the strangeness changing current, provide a test of the PCAC hypothesis for the axial current with $\Delta S = 1$:
$$\partial_\mu J^A_\mu = icK.$$
For this purpose the process
$$\bar{\nu} + p \to \Lambda + \mu^+, \quad \Lambda \to p + \pi^-$$
might be very suitable in the not too distant future. Note that this process allows analysis of the Λ-spin, and has only charged particles as secondaries. Perhaps it might be attacked with other than bubble chamber methods. The cross-section is expected to be 1 to 4 % of the cross-section for the process
$$\bar{\nu} + p \to n + \mu^+.$$

3. The inelastic process

The 'quasi-elastic' process
$$\nu + p \to N^{*++} + \mu^-$$
might well be the first that will be explored in some detail. The most general form of the hadron current is

$$J_\alpha = \Big[\bar{u}_\beta(N^*)\Big(\Big\{a_1\,\delta_{\beta\alpha} + \frac{Q_\beta}{m+M}\,(ia_2\,\gamma^\alpha) + \frac{ia_3}{m+M}\,\sigma_{\alpha\lambda}\,Q_\lambda + \frac{a_4}{m+M}\,Q_\alpha\Big\}\,\gamma^5$$
$$+\,b_1\,\delta_{\beta\alpha} + \frac{Q_\beta}{m+M}\,(ib_2\,\gamma^\alpha) + \frac{ib_3}{m+M}\,(p_N + p_{N*})_\alpha + \frac{b_4}{m+M}\,Q_\alpha\Big)\,u(N)\Big];$$

$a_1 \dots b_4$ are again functions of m^2, M^2 and Q^2. The coefficient b_1 is, through PCAC, related to the N^* width, if one assumes small b_2 and b_3 (for which some arguments may be given). Again, the angular distribution of the N^* decay products may provide us with a substantial amount of information concerning the coefficients a_i and b_i. As this has been discussed quite extensively in the literature (Bell & Berman 1962; Berman & Veltman 1965) we do not go any further into this.

As regards other inelastic processes the theory is very incomplete, and unless it turns out to be possible to apply peripheral models it is not clear what one can do with eventual data on these processes. But maybe the experiment will provide us with some new insight.

4. General tests

There are three tests of a general nature that might be feasible in the near future. Consider the process

$$\nu + A \rightarrow \mu^- + B.$$

Three proposed tests are:

(i) Test of CP-invariance through measurement of the μ-polarization in the above process, with A and B any system of strongly interacting particles (Berman & Veltman 1964; see also Bell 1965).

(ii) Test of CVC. It has been shown by Adler (1964, 1965) that for a μ in the forward direction the vector current cannot contribute. Therefore parity violating effects should not occur for such configurations, and this may be tested.

(iii) Test of PCAC Adler (1964, 1965) has also shown that for a μ in the forward direction the cross-section may be related to some appropriate pion cross-section. Whereas there are some problems if A is a nucleus (Bell 1964; Weinberg 1966), in the case that A is a proton this test is rather clean. Maybe a re-evaluation of experimental possibilities on this point is in order.

From these three tests especially (i) and (ii) may be of practical interest in the not too distant future. In all cases a rather higher event rate than achieved in the past experiments is necessary. Test (iii) requires knowledge of the neutrino spectrum; in both (ii) and (iii) it is, from a theoretical point of view, still not clear what is the meaning of 'forward direction'. In fact, if the state B is a state of high angular momentum the 'forward region' with respect to test (ii) and (iii) might be uninterestingly small. The only way out would be a complete analysis of the B-system, a not too amusing prospect.

5. CVC and PCAC hypothesis

Recently it has become clear that certain second-order processes may be useful to explore the properties of the currents.

By second-order process we mean a process involving, in addition to hadrons other than pions (or kaons), at least two of any of the following:

lepton-pair; photon; pion; kaon.

For definiteness we will concentrate on the second-order process

$$\bar{\nu} + p \rightarrow n + \mu^+ + \gamma.$$

This process is just about as rare as the previously mentioned process $\bar{\nu}+p\rightarrow\Lambda+\mu^+$, but has the disadvantage of having two neutral secondaries. Anyway, we will discuss only the theoretical aspects of this process.

The lepton current is coupled to a current composed of a vector and an axial vector part. CVC and PCAC restrict these currents, and we want to know these restrictions up to first order in e, the e.m. coupling constant. In the following we neglect the process where the γ is emitted by the muon.

Customarily one has for vector and axial currents $\mathbf{J}^v_\mu$ and $\mathbf{J}^A_\mu$ respectively (the **arrow** refers to isospin):

$$\partial_\mu \mathbf{J}^v_\mu(x) = 0, \tag{5·1}$$

$$\partial_\mu \mathbf{J}^A_\mu(x) = \mathrm{i}c\boldsymbol{\phi}(x), \tag{5·2}$$

where one identifies the field $\boldsymbol{\phi}(x)$ with the pion field (Gell-Mann & Levy 1960; Nambu 1960; Adler 1965). This is certainly correct if one considers matrix-elements of this equation for values of energy-momentum that are near those of a physical pion. The big question was and is what happens away from the 'pion-pole', and possibly the e.m. field can be used as a tool to explore this.

To see this we ask what happens if e.m. effects are taken into account. In a very large class of models the effect of e.m. interactions can be accounted for by the recipe of replacing ∂_μ by $\partial_\mu - \mathrm{i}eA_\mu$ when acting on a charged quantity, and we shall assume this principle also. Derivatives may occur in a number of places in (5·1) and (5·2); if we have a simple model like the quark model only the ones explicitly indicated in (5·1, 2) occur and one has, identifying $\boldsymbol{\phi}(x)$ with the pion field (for a more complete discussion of these equations see Veltman 1966):

$$\partial_\mu \mathbf{J}^v_\mu(x) = \mathrm{i}e\mathbf{A}_\mu \times \mathbf{J}^v_\mu, \tag{5·3}$$

$$\partial_\mu \mathbf{J}^A_\mu(x) = \mathrm{i}c\boldsymbol{\pi}(x) + \mathrm{i}e\mathbf{A}_\mu \times \mathbf{J}^A_\mu. \tag{5·4}$$

Here $\mathbf{A}$ is an isovector whose 1, 2 components are zero. The third component of the second term on the right-hand side of (5·4) is thus zero; in this model there are thus no radiative corrections on the third component of equation (5·2).

There are reasons to believe that equation (5·4) is incorrect, as it predicts, as shown by Sutherland (1966) that the e.m. decay

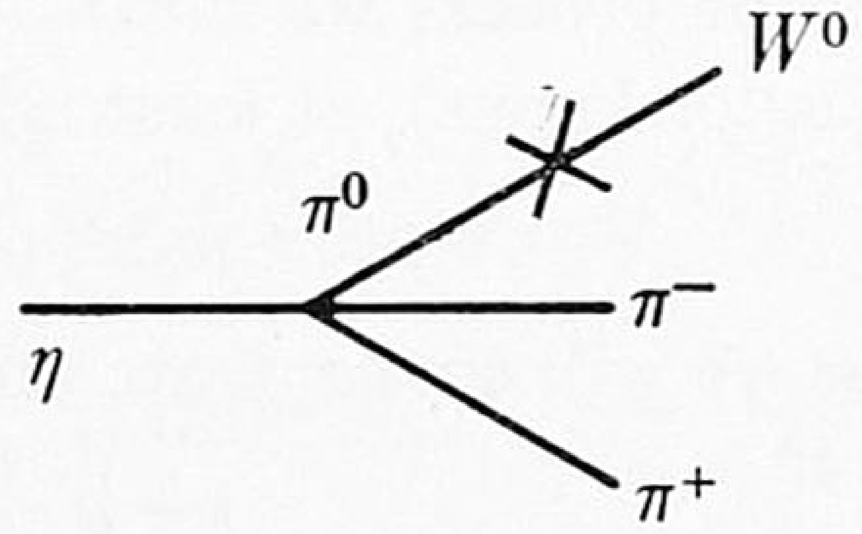
$$\eta \rightarrow \pi^+\pi^-\pi^0$$

is forbidden. To see this we consider the matrix-element of the third component of the axial current between an η and $\pi^+\pi^-$ (corresponding to an η decaying into a $\pi^+\pi^-$ and a hypothetical axial vector boson W^0). Denoting the four-momenta of η, π^+, π^- and W^0 by k, p, p' and q respectively we may write for the Fourier transform of this matrix-element:

$$\langle \pi^+\pi^- | J^{A0}_\mu | \eta\rangle = c_1 q_\mu + c_2 p_\mu + c_3 k_\mu + dg\,\frac{q_\mu}{q^2+m_\pi^2}.$$

Here the last factor arises from the sequence

$$\eta \rightarrow \pi^+\pi^-\pi^0 \rightarrow \pi^+\pi^- W^0$$



and is the most singular contribution around $q = 0$. g is the $W-\pi$ coupling constant. Note that diagrams like $\eta \to \pi^+\pi^- \to \pi^+\pi^- W^0$ are forbidden. Keeping the mass of η, π^+ and π^- fixed the coefficients c_1, c_2, c_3 and d are functions of the invariants (qk), q^2 and $(p-p', k)$. The first two are in the η-rest system directly related to $W^0(\pi^0)$ energy and mass respectively. We will need this matrix-element for small values of q and make an expansion around $q = 0$ keeping only the lowest order terms, i.e. we write

$$d(q^2, qk) \approx d_1 + (qk)\, d_2$$

and take for c_1, c_2, c_3 the value for $q = 0$.

From this we obtain for the matrix-element of the divergence of the current:

$$-\mathrm{i}\,\langle \pi^+\pi^- \,|\partial_\mu J_\mu^{A0}|\, \eta\rangle = c_1 q_2 + c_2(pq) + c_3(kq) + g\left\{d_1 + (qk)\, d_2 - \frac{m_\pi^2}{q^2+m_\pi^2}(d_1 + (qk)\, d_2)\right\}.$$

On the other hand, the corresponding matrix-element of the right-hand side of (5·4) is (remember that $A_\mu \times J_\mu^A$ does not contribute here):

$$c\,\langle \pi^+\pi^- \,|\pi^0|\, \eta\rangle = \frac{c}{q^2+m_\pi^2}\langle \pi^+\pi^- \,|j|\, \eta\rangle,$$

where j is the pseudo scalar current to which the π^0 couples, measured by studying $\eta \to \pi^+\pi^-\pi^0$. This process is the same as already occurring above and giving rise to the pole term. One has

$$\langle \pi^+\pi^- \,|j|\, \eta\rangle = d(q^2, (qk), (p-p', k)),$$

where q is now the π^0 four-momentum. From experiment one knows, for $q^2 = -m_\pi^2$:

$$d\{-m_\pi^2, (qk), (p-p', k)\} \approx d_1\left(1 - \frac{qk}{2(qk)_m}\right),$$

where $(qk)_m$ is the minimum value of (qk), equivalent to maximum π^0 energy. No dependence on $(qk)^2$ or $(p-p', k)^2$ or higher is observed, and we will assume this expression to be independent of q^2 also. Thus, with (5·4)

$$q^2 c_1 + (pq)\, c_2 + (kq)\, c_3 + g\left\{d_1 + (qk)\, d_2 - \frac{m_\pi^2}{q^2+m_\pi^2}(d_1 + (qk)\, d_2)\right\}$$
$$= \frac{c}{q^2+m_\pi^2}\left\{d_1 - \frac{qk}{2(qk)_m} d_1 + \frac{q^2+m_\pi^2}{m_\pi^2}\alpha\right\}$$

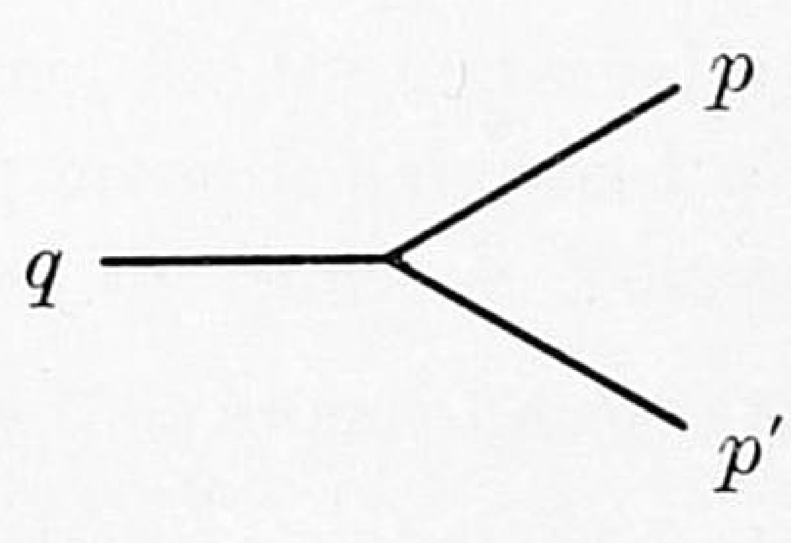
where α is assumed to be small with respect to d_1. We must note that this rather arbitrary assumption has its only basis in the fact that other quadratic terms, like $(qk)^2$, are very unimportant. We then conclude from the above equation:

$$gd_1 = \alpha c / m_\pi^2,$$

which gives $d_1 = -\alpha$, in contradiction with the assumption made above.

Similarly, the processes

$$\pi^0 \to 2\gamma, \quad \omega \to \pi^0\gamma$$



are forbidden. In fact, we have an extension of Adler's consistency condition on strong interactions (Adler 1965): any process containing a π^0 must be zero in the limit of zero π^0 four momentum, even when e.m. effects are taken into account, provided that no other than one-pion poles occur.

Thus there is some doubt concerning equation (5·4). Let us now go back to (5·2) and assume some other form for $\boldsymbol{\phi}(x)$:

$$\partial_\mu \mathbf{J}_\mu^A(x) = \mathrm{i}c_1\,(x)\boldsymbol{\pi} + \mathrm{i}c_2\,\partial_\mu \boldsymbol{\pi}(x) \times \mathbf{J}_\mu^v(x).$$

Without any conflict with existing tests of PCAC (for example the Goldberger–Treiman relation) the effect of the coefficient c_2 may be, say, $0{\cdot}25c_1$. But switching on e.m. interactions gives

$$\partial_\mu \mathbf{J}_\mu^A(x) = \mathrm{i}c_1\,\boldsymbol{\pi}(x) + \mathrm{i}c_2\,\partial_\mu \boldsymbol{\pi}\,(x) \times \mathbf{J}_\mu^v(x) + \mathrm{i}e\mathbf{A}_\mu \times \mathbf{J}_\mu^A - ec_2(\mathbf{A}_\mu \times \boldsymbol{\pi}) \times \mathbf{J}_\mu^v.$$

The third component of the last term has nonzero matrix-elements between η and $\pi^+\pi^-$, and $\eta \to 3\pi$ is now allowed.

From the above it may be seen that the e.m. field can be used as a tool to detect the 'smoothness' of the field $\boldsymbol{\phi}(x)$ in the relation (5·2). The reaction

$$\bar{\nu} + p \to n + \mu^+ + \gamma$$

is sensitive to terms as discussed above. Related to this, is the very interesting

$$\gamma + N \to \pi + N \quad \text{and also} \quad \pi \to \mu\nu\gamma.$$

From a theoretical point of view we have now a way to precise the meaning of the PCAC relation; a nonsmooth $\boldsymbol{\phi}(x)$ gives rise to extra e.m. terms as compared to the simple case (5·4). It seems that tests for this kind of terms may provide us with a deeper insight in the fundamental properties of strongly interacting particles.

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