Vol. 25(1994) *ACTA PHYSICA POLONICA B* No 12

THE SCREENING THEOREM AND THE HIGGS SYSTEM*

M. Veltman

Department of Physics, University of Michigan
Ann Arbor, MI 48109, USA

and

NIKHEF-H
Amsterdam, Netherlands

(Received August 4, 1994)

The screening theorem is discussed, and its consequences for the Higgs system are exhibited. Next the present situation with respect to possibilities for the Higgs system is discussed.

PACS numbers: 11.10. Gh, 11.15. Bt

1. The screening theorem

Long ago it was shown that Yang–Mills theories with massive vector bosons, but without a Higgs, are one loop "renormalizable" [1]. The quotes are put because, certainly at that time when no good regularization scheme was available, renormalizability was not an easy thing to establish. In the paper quoted what was meant was that the divergencies encountered up to one loop were as in a renormalizable theory:

self-energies at most quadratically divergent;
three-point functions at most linearly divergent;
four-point functions at most logarithmically divergent;
no divergencies in five- or higher point functions.

Of course, the strict definition of renormalizability is that the divergencies can be absorbed in the available constants. Thus, the divergencies must

---

* Presented at the XXXIV Cracow School of Theoretical Physics, Zakopane, Poland, June 1–10, 1994.

satisfy the same relations as the constants in the Lagrangian, that is the divergencies must satisfy certain relations as required by the gauge symmetry of the theory. The four-point divergencies of the Higgs-less theory do not satisfy that criterion, but the self-energy divergencies can be absorbed in mass and wave function renormalization.

Consider now a truly renormalizable Yang–Mills theory with Higgs. The case mentioned above, no Higgs, can presumably be derived by taking the limit $m \to \infty$, where m is the Higgs mass. It follows that at one loop in this limit:

2-point function behaves as m^2;
3-point function behaves as m;
4-point function behaves as $\ln m$.

If we are to detect the Higgs from radiative corrections then we must find a correction that behaves at least as m^2 (like for instance the radiative correction due to a heavy top-quark). Else it will be too small and too insensitive to the Higgs mass to be useful as a tool for establishing that Higgs mass. From the above we see that such a correction must be sought in self-energy diagrams. These Higgs mass dependent terms are however precisely as terms that can be absorbed in the vector boson masses. Such "renormalizable" terms are then observable only if not all masses are free parameters. Thus the only way to have an observable m^2 type radiative correction is a relationship between the vector boson masses, which then could be broken by these m^2 radiative corrections.

Is there such a relation between masses in the Standard Model? There is, but only if we restrict ourselves to the simplest Higgs system, the complex doublet Higgs. Then

$$\rho \equiv \frac{M^2}{c^2 M_0^2} = 1 + \mathcal{O}(g^2).$$

In here M and M_0 are the charged and neutral vector boson masses, and $c = \cos\theta_{\mathrm{w}}$ with θ_{w} the weak mixing angle. Thus there could be terms of the form:

$$\rho = 1 + a_0 g^2 \frac{m^2}{M^2} + \ldots$$

As it happens, sadly, the coefficient a_0 turns out to be zero [2]. The conclusion is that up to one loop, in the Standard Model with a doublet Higgs, the only observable radiative corrections that grow with the Higgs mass are logarithmic. This is what has become known as the "screening theorem".

Another conclusion is that in models where ρ is not necessarily fixed (more complicated Higgs sector containing other than Higgs doublets) this ρ parameter will probably receive large corrections proportional to m^2. That depends on the model, it is not a well investigated subject. It makes it all the more unlikely that $\rho = 1$ by accident; the experimental observation that $\rho \approx 1+$(Top-correction) therefore strongly suggests that there are Higgs doublets only.

What happens if $m \geq 1$ TeV? Radiative corrections are still small, being logarithmic. Remarkably, tree amplitudes for WW scattering can show strong effects, but that will not be discussed here. See [3] for our view on the subject.

2. Symmetries of the Higgs system

It seems therefore to be opportune to investigate the symmetries of the doublet Higgs system, also known as the linear σ-model. This is the one that is commonly used in the Standard Model. The Lagrangian of the linear σ-model is:

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi_i)^2 - \frac{\mu}{2}\varphi_i^2 - \frac{\lambda}{8}(\varphi_i^2)^2 ,$$

with real fields φ_i, $i = 1 \ldots 4$. This Lagrangian is obviously invariant for rotations in four-dimensional real space, O(4):

$$\varphi_i \to O_{ij}\varphi_j , \quad \text{with} \quad O^{-1} = \widetilde{O} .$$

The symmetry O(4) is a six-parameter symmetry, as may be seen easily. Usually one writes σ for φ_4. Then O(4) is like O(3) and a (real) "Lorentz-transformation":

$$\begin{aligned} \varphi_i &\to \varphi_i + \epsilon_{ijk}\Lambda_j\varphi_k \\ \sigma &\to \sigma . \end{aligned}$$

We are considering here infinitesimal transformations with infinitesimal Λ. The above is the O(3) part, much like isospin. The other part:

$$\begin{aligned} \varphi_i &\to \varphi_i - \sigma\bar{\Lambda}_i \\ \sigma &\to \sigma + \varphi_i\bar{\Lambda}_i . \end{aligned}$$

The O(3) part is like the ordinary three-dimensional rotations, and in fact that is known under the name of isospin. Under the isospin transformations the φ_i behave as an isospin triplet. Written this way the σ-model can be used as an effective model describing the pions and their interactions, with the σ some resonance way up. That has received considerable attention, and has met with certain successes. The "Equivalence theorem" relates longitudinal vector bosons to the φ_i of the Higgs sector; this suggests a similarity between longitudinal vector boson scattering and pion scattering, because also the latter is reasonably well described by a σ-model. How useful this line of reasoning is remains to be seen. See Ref. [3].

There are two alternative ways of writing the linear σ-model. The symmetry remains of course the same, but in these other notations other symmetries, part of O(4), become manifest. Here the second way of writing the σ-model, in a way that makes it more directly applicable as Higgs sector of the Standard Model, as it exhibits explicitly the necessary SU(2)×U(1) symmetry:

$$\mathcal{L} = -\partial_\nu K^\dagger \partial_\nu K - \mu K K^\dagger - \tfrac{1}{2}\lambda \left(KK^\dagger\right)^2 ,$$

with

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma + i\varphi_3 \\ -\varphi_2 + i\varphi_1 \end{pmatrix} .$$

This is manifestly SU(2) invariant:

$$K \rightarrow K - \tfrac{i}{2}\Lambda_i^L \tau^i K .$$

In here the τ_i are the Pauli spin matrices. This transformation of K corresponds to a certain transformation of φ and σ. The corresponding transformation is:

$$\sigma \rightarrow \sigma + \tfrac{1}{2}\Lambda_i^L \varphi_i ,$$

$$\varphi_i \rightarrow \varphi_i + \tfrac{1}{2}\left(-\sigma\Lambda_i^L + \epsilon_{ijk}\Lambda_j^L \varphi_k\right) .$$

This corresponds to the O(4) transformation with $\Lambda_i = \bar{\Lambda}_i = \frac{1}{2}\Lambda_i^L$. The U(1) symmetry for which the Lagrangian in the K-form is invariant:

$$K \rightarrow \mathrm{e}^{-i\Lambda^0} K .$$

This corresponds to $\Lambda_3 = -\bar{\Lambda}_3 = -\Lambda^0$.

There is a third way to write the σ-model Lagrangian that exhibits explicitly other symmetries. Define the 2×2 matrix Φ:

$$\Phi = \begin{pmatrix} \sigma + i\varphi_3 & i\varphi_1 + \varphi_2 \\ i\varphi_1 - \varphi_2 & \sigma - i\varphi_3 \end{pmatrix} = \sigma\tau^0 + i\varphi_j \tau^j .$$

Here τ^0 is the 2×2 unit matrix. The σ model Lagrangian can be written in terms of Φ:

$$\mathcal{L} = -\frac{1}{4}\,\mathrm{Tr}\left(\partial_\nu \Phi^\dagger \partial_\nu \Phi\right) - \frac{\mu}{2}\left[\frac{1}{2}\,\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right] - \frac{\lambda}{8}\left[\frac{1}{2}\,\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 .$$

Now the obvious invariances are:

$$\Phi \rightarrow G\Phi \qquad G = \exp\left(-\tfrac{i}{2}\Lambda_i^L \tau^i\right)$$

$$\Phi \rightarrow \Phi H^\dagger \qquad H = \exp\left(-\tfrac{i}{2}\Lambda_i^R \tau^i\right)$$

We could of course have used H instead of $H^\dagger$, but this is more convenient. G and H are called $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$. With infinitesimal Λ^L and Λ^R:

$$\sigma \to \sigma + \tfrac{1}{2}\left(\varphi_i \Lambda_i^L - \varphi_i \Lambda_i^R\right)$$

$$\varphi_i \to \varphi_i - \tfrac{1}{2}\sigma\left(\Lambda_i^L - \Lambda_i^R\right) + \tfrac{1}{2}\epsilon_{ijk}\left(\Lambda_i^L + \Lambda_i^R\right).$$

From this we find the relation to the O(4) symmetry:

$$\Lambda_i = \tfrac{1}{2}\left(\Lambda_i^L + \Lambda_i^R\right)$$

$$\bar{\Lambda}_i = \tfrac{1}{2}\left(\Lambda_i^L - \Lambda_i^R\right).$$

From this we learn that transforming with equal Λ^L and Λ^R is the isospin transformation discussed in the beginning. One often writes $\mathrm{SU}(2)_L + \mathrm{SU}(2)_R$.

The latter way of writing the σ-model exhibits the full six parameter symmetry. It is also straightforward to identify part of that with the SU(2)×U(1) symmetry required for the Standard Model, and therefore it is probably the best way of writing this model.

In coupling the vector bosons to the σ-model one may actually accommodate 6 vector bosons, three with each SU(2). For the moment we will not yet argue which of these are to be the vector bosons of weak interactions and the photon, but just take the six of them and see what happens. Thus we assume the existence of 6 vector bosons:

L_ν^a	$a = 1,2,3$	left SU(2) bosons
R_ν^a	$a = 1,2,3$	right SU(2) bosons.

Define the 2×2 matrices ℓ_ν and r_ν:

$$\ell_\nu = -\tfrac{i}{2} L_\nu^a \tau^a \qquad r_\nu = -\tfrac{i}{2} R_\nu^a \tau^a .$$

Now the actual coupling of these vector bosons to the σ and φ of the σ-model. This is achieved by replacing $\partial_\nu \Phi$ by $D_\nu \Phi$:

$$\partial_\nu \Phi \to D_\nu \Phi = \partial_\nu \Phi + g \ell_\nu \Phi - g' \Phi r_\nu .$$

Remember, Φ, ℓ and r are all 2×2 matrices. The coupling constants g and g' may be chosen independently. With this substitution the global (*i.e.* with space-time independent parameters Λ) $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry becomes a local (with space-time dependent Λ) symmetry under which also the L and R transform, in fact as ordinary triplets much like the pions. Actually, under the isospin transformation (with equal Λ^L and Λ^R) the R and L each behave indeed as a triplet. It is worthwhile to keep an eye on that symmetry; if it would survive there would be isospin symmetry and the vector bosons in a given triplet would all have the same mass. This indeed happens, except that, as we will see, electromagnetism breaks this symmetry. The masses will not be equal but there remains a relation, which is precisely what we want to understand.

3. Vector boson masses

So, rather then going in all kinds of details we will simply find the mass matrix for this model. The Φ field develops a vacuum expectation value, and applying the available symmetry this vacuum expectation value can be put in diagonal form. We may write:

$$\Phi_0 = f_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

Any constant matrix can be transformed into this by some well chosen G and H transformation. The relevant term in the Lagrangian is the term containing $D_\nu \Phi$. Inserting this constant Φ:

$$D_\nu \Phi_0 = -\tfrac{i}{2} f_0 \left(g L^a_\nu - g' R^a_\nu \right) \tau^a ,$$

and the vector boson mass term is:

$$\begin{aligned} \mathcal{L}_M &= -\tfrac{1}{4} \,\mathrm{Tr} \left(D_\nu \Phi_0^\dagger D_\nu \Phi_0 \right) \\ &= -\tfrac{1}{8} f_0^2 \frac{g^2}{c^2} \left(c L^a_\nu - s R^a_\nu \right)^2 . \end{aligned}$$

We introduced the weak mixing angle, writing $g' = g\frac{s}{c}$, with $s = \sin\theta_\mathrm{w}$ and $c = \cos\theta_\mathrm{w}$. There are three terms in this Lagrangian ($a = 1, 2, 3$). Thus three vector bosons can get a mass. We now identify the three massive bosons of the weak interactions and the photon with the L and R. Since there are six L and R two must be absent. We assume R^1 and R^2 to be non-existent. That will ruin the $\mathrm{SU}(2)_R$ symmetry, but that cannot be helped. The $\mathrm{SU}(2)_L$ symmetry remains, as must be in order to keep the weak Lagrangian $\mathrm{SU}(2)\times\mathrm{U}(1)$ invariant. We identify:

$$\begin{aligned} W^{1,2}_\nu &= L^{1,2}_\nu \quad \text{with} \quad W^\pm = \frac{1}{\sqrt{2}} \left(W^1_\nu \mp i W^2_\nu \right) \\ Z^0_\nu &= W^3_\nu = c L^3_\nu - s R^3_\nu . \end{aligned}$$

With this:

$$\mathcal{L}_M = -M^2 W_\nu^+ W_\nu^- - \tfrac{1}{2} M_0^2 \left(Z_\nu^0\right)^2 .$$

There is a fourth combination, orthogonal to the Z_0, namely $A_\nu = sL_\nu^3 + cR_\nu^3$. This field, not occurring in the mass term, has no mass, and we identify it with the photon. Given that there are four vector bosons, at least one must remain massless with this Lagrangian, because the mass term accommodates only three vector bosons. It is a prediction of this particular Higgs sector that there is one massless vector boson, as indeed observed. That is a strong point in favour of this σ-model Higgs sector. Moreover, the three masses are not independent. In the limit that $g' = 0$, *i.e.* $c = 1$ and $s = 0$, the Z^0 mass becomes the same as the W mass. If g' is zero then we never note the non-existence of R^1 and R^2 and the σ model keeps its global $\mathrm{SU}(2)_R$. Since $\mathrm{SU}(2)_L$ is not only a global but even a local symmetry we have global $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ symmetry. Therefore isospin (global $\mathrm{SU}(2)_L + \mathrm{SU}(2)_R$) holds, the three vector bosons form an isospin multiplet and their masses are equal. This global symmetry is often called the custodial symmetry, a particularly bad name. If $g' \neq 0$ ($\sin\theta_\mathrm{w} \neq 0$) then the isospin symmetry disappears. As luck has it a relation between the vector boson masses remains, namely

$$M_0 = M/c \approx M\left(1 + \frac{\theta_w^2}{2!}\right) ,$$

which makes the ρ-parameter equal to one. It is often stated that the custodial symmetry forces

$$\rho = \frac{M^2}{c^2 M_0^2} = 1 ,$$

but the correct statement is that this symmetry gives $M_0 = M + \mathcal{O}(\theta_\mathrm{w})$, not involving a coupling constant.

Anyway, here we have a mass relation, and there could have been radiative corrections to this proportional to the square of the Higgs mass m, but as it happens they are not there. Altogether the model so far fits wonderfully with experiment: not only is the ρ-parameter found to be very close to one, but moreover the deviation from one can be understood as a radiative correction involving a top quark. This latter correction is proportional to the top mass squared [4]. There is no visible large Higgs correction, in accordance with the screening theorem.

4. Other Higgs systems

What happens if a more complicated Higgs sector is used? First, using anything else but the σ-model type Higgs Lagrangians, *i.e.* SU(2) doublets (we refer here to the K notation), destroys immediately the mass relation that makes the ρ-parameter one in lowest order, with calculable (and indeed observed) radiative corrections. But one could add more doublets, each of which would give a contribution to the masses such that ρ remains one. However, the other prediction, namely that there is one massless vector boson, disappears. It must be forced. Let us elaborate on that for a moment. Suppose two σ-model Lagrangians, which we will examine using the K notation. The first K field, K^1, will have a vacuum expectation value that can, by using the available symmetry, be put in the form

$$K_0^1 = \begin{pmatrix} s_0 \\ 0 \end{pmatrix} .$$

The vacuum value of the second K can *a priori* be anything. Less symmetry is now available to transform it to some preferred form, because we must keep K_0^1 to the form chosen. The result will be

$$K_0^2 = \begin{pmatrix} a \\ b \end{pmatrix} .$$

In words, the two vacuum expectation values are not necessarily aligned. They would be aligned if b were zero, which is in general not the case. The vector boson mass matrix is then:

$$\begin{pmatrix} M^2 & 0 & 0 & a_1 \\ 0 & M^2 & 0 & a_2 \\ 0 & 0 & M^2 & a_3 \\ a_1 & a_2 & a_3 & M'^2 \end{pmatrix} ,$$

with

$$\begin{aligned} a_1 &= \tfrac{1}{2} g g' (ab^* + a^* b) \\ a_2 &= \tfrac{1}{2} i g g' (ab^* - a^* b) \\ a_3 &= \tfrac{g}{g'} M^2 - g g' b b^* \\ M^2 &= \tfrac{1}{2} g^2 \left(a a^* + b b^* - s_0^2 \right) \\ M' &= \tfrac{g'}{g} M \end{aligned}$$

The eigenvalue equation for eigenvalues κ corresponding to this matrix is:

$$\left(M^2 - \kappa \right)^2 \left[\kappa^2 - \kappa M^2 \left(1 + \frac{g'^2}{g^2} \right) + g^2 g'^2 b b^* s_0^2 \right] = 0 .$$

This has four non-zero solutions for κ except when $b = 0$. The prediction of zero photon mass disappears unless somehow b happens to be zero.

This may be understood also in a more general way. In the Higgs doublet model there are four degrees of freedom (σ and the three φ). Since there remains always at least one physical Higgs at most three can be reshuffled to become longitudinal vector bosons and thus give mass to three vector bosons. Thus if there are four vector bosons, one remains massless. However, if we add another Higgs doublet then there are six degrees of freedom available. Something must then be done to keep the photon massless.

There are two cases when more than one Higgs doublet is needed.

— Peccei-Quinn [5] solution to the strong CP violation problem[6];
— Supersymmetry.

An additional problem is that if there is more than one Higgs multiplet there is often an additional symmetry that will be broken spontaneously, and then gives a zero mass physical Higgs particle. In the Peccei-Quinn case that is called the axion. Despite considerable efforts nothing of the kind has been seen.

The situation with respect to supersymmetry was investigated by Diaz-Cruz and Mendez [7]. Their conclusions: the Higgs system will in general generate a photon mass except for the minimal supersymmetric extension of the Standard Model. However, even for this minimal SUSY-SM the problem appears in the slepton-squark sector.

Nobody knows how to avoid these problems in a natural way. Thus, on the basis of these arguments we propose:

— Exit Peccei-Quinn mechanism and axions;
— Exit supersymmetry.

But this leaves us with a big problem: strong CP violation. To solve that needs some really drastic measures.

REFERENCES

[1] M. Veltman, *Nucl. Phys.* **B7**, 637 (1968).
[2] M. Veltman, *Acta Phys. Pol.* **B8**, 475 (1977).
[3] H. Veltman, M. Veltman, *Acta Phys. Pol.* **B22**, 669 (1991).
[4] M. Veltman, *Nucl. Phys.* **B123**, 89 (1977).
[5] R. Peccei, H. Quinn, *Phys. Rev. Lett.* **138**, 1440 (1977); *Phys. Rev.* **D16**, 1791 (1977); F. Wilczek, *Phys. Rev. Lett.***40**, 279 (1978).
[6] V. Baluni, *Phys. Rev.* **D19**, 2227 (1979); R. Crewther, P. Di Vecchia, G. Veneziano, E. Witten, *Phys. Lett.* **88B**, 1223 (1979).
[7] J.L. Diaz-Crus, A. Mendez, *Nucl. Phys.* **380**, 39 (1992).