The Higgs System [1)]

M. Veltman [2)]

University of Michigan

Ann Arbor, MI 48109

and

DESY, Hamburg, F.R.G.

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1) These lectures were also presented at the Instituut-Lorentz for Theoretical Physics, Leiden, Netherlands

2) Humboldt award recipient

1 Introduction

The standard model of weak, electromagnetic and strong interactions stands today virtually unchallenged. The model is renormalizable, thus may be considered internally consistent. In principle observable quantities can be calculated to arbitrary precision, and compared with experiment. While relatively few radiative corrections are accessible to present day experiment, those that have been measured agree with the theory.

Yet there is a deep feeling of frustration among many theorists. The standard model accomodates most observable facts, but leaves many unexplained. Why three families? Why the particular symmetry structure? Etc. In addition there are many ad hoc parameters, so many that no one seriously believes them all to be fundamental: the particle masses, the coupling constants are all unexplained free parameters. We do not even know if the neutrino mass is fundamentally zero or just very small.

To add insult to injury there is a whole segment of the theory that has not even been observed or even directly tested. This is the Higgs sector. In the next decade experimental physics will enter this domain, first indirect, later direct. It will be very difficult but at the same time, in all likelihood, very interesting. It is necessary here to explain what is meant by "interesting".

If the Higgs particle turns out to exist as conventionally described, with a reasonably low mass (say less than 200 GeV) then that closes the standard model from a mathematical point of view. It is then quite conceivable that new physics, not contained in the standard model, is way beyond reach of any accelerator imaginable today. Humanity might in that case never get an answer to the questions posed above. This is a distinct possibility. It is not an "interesting" possibility.

On the other hand, the Higgs sector might well contain essential clues to the further understanding of matter. To support this view consider the following points.

- Within the standard model there are two ways for any particle to obtain mass. One of them is through the interaction with the Higgs field. It now happens that all known particles obtain their mass exclusively through the Higgs system.

- If we insist that all particles obtain their mass through one and the same Higgs system then parity non-conservation follows almost automatically. The argument is not airtight, but nonetheless this is the first time that there is at least some argument pointing to violation of parity.

- Any theoretical speculation based on taking seriously the vacuum structure as generated by the Higgs system has failed: monopoles, axions are examples.

- At the same time the Higgs system conflicts with precisely the weak point of the theory of gravity, namely the observed vanishing of the cosmological constant.

Theoretically, the Higgs field functions in the standard model as an ultraviolet cut-off. Without that field the Yand-Mills interactions are non- renormalizable, i.e. uncontrolably divergent. The long range, i.e. low energy part of the Higgs system brings us in great difficulty (the cosmological constant). It is only natural to guess that somewhere the extrapolation from the ultraviolet to the infrared breaks down. If so, that is what one expects to see when investigating the Higgs sector. This is "extremely interesting".

Most of the above must be classified as daydreaming or speculation. But there is enough in it to stimulate research in the Higgs sector. Such research, for a theorist, tends to be seemingly dull: at this moment because of lack of imagination one cannot do much more then try to calculate effects due to the Higgs system in order to allow comparison with experiment. This confrontation turns out to be quite difficult and will require a huge experimental effort.

In these notes we will attempt to discuss the Higgs system in a systematic but not necessarily complete way.

2 Some history

The idea of mass generation through interaction with a non-empty vacuum appears to have been mentioned explicitly for the first time in a fundamental paper by Schwinger [1] in 1957. This paper describes what we now call the linear σ-model. That model will be described amply further on. Here we will quote a few lines from this paper.

On page 416: "... a coupling of the form illustrated by ... will produce effective mass terms for each field through the action of the vacuum fluctuations of the other fields".

On page 423: "As a field which is a scalar under all operations in the three- dimensional isotopic space and in space time, $\phi_{(0)}$ has a nonvanishing expectation value in the vacuum. Although unable to affix the value implied by the strong interactions with heavy fermions, one could at least anticipate that $< \phi_{(0)} >$ would have the magnitude of nucleon masses, and thus a suitable μ-meson mass constant might emerge from $g_\mu < \phi_{(0)} >$ without requiring a particularly large coupling constant g_μ".

On page 424: "... in which we again use the σ-field to remove three- dimensional internal symmetries and produce masses for charged particles".

Schwinger's paper contains many more quotable lines, but let us leave it at this. The whole paper is manifestly something created in the middle of a search process. The vector bosons are there, spontaneous symmetry breaking is there. Yet the standard model as we know it now is agonizingly not there. We know that Schwinger made much more progress here, as acknowledged by S. Glashow at various occasions.

The σ-model as invented by Schwinger is precisely the simplest possible Higgs sector of the standard model. Schwinger used the σ to give nucleons a mass, and in fact his σ seems like a scalar, strongly interacting meson of about, say, 500 MeV. Thus his σ was not at all like the Higgs of the standard model. The model however is strictly the same.

The next fundamental paper in this context is the 1960 paper of Gell-Mann and Levy [2]. In this paper the σ-model and symmetry breakdown are linked to PCAC. The understanding that spontaneous symmetry breakdown leads to a partially conserved current has been of great importance for the subsequent developments.

In the sixties the σ-model was studied by many authors. In particular we may mention here the lectures of B. Lee at Cargese in 1970 [3]. It is our understanding that this paper, and of course the two fundamental ones mentioned above, were the essential basis for the discovery of the renormalizability of spontaneously broken gauge theories [4].

The name "Higgs" system owes its existence to the work of P. Higgs [5], who discovered that spontaneous symmetry breakdown (i.e. a non-empty vacuum) may be used to generate masses for vector bosons. Englert and Brout did much the same, in fact slightly earlier [6]. This is different from the case of fermion mass generation because massive vector bosons have one degree of freedom more than mass-less vector bosons. In Higg's approach that one degree of freedom was shuffled from a scalar field to the vector field. This mechanism now known as the Higgs mechanism owes its existence to certain developments in solid state physics due to Anderson [7]. It would carry us too far to untangle the precise historical development of this line of research. Let us just mention here the paper of Kibble [8], where the work of Higgs is extended to non-Abelian theories, and in fact the complete Higgs system of the standard model is constructed. That work was known to Weinberg [9] who made Kibble's model realistic by adding in electromagnetism and weak mixing as advocated by Glashow [10].

3 The original Higgs model

3.1 Lagrangian and Feynman rules

The simplest example of a Higgs system is the model introduced by Higgs in which one single neutral vector boson obtains mass through spontaneous symmetry breakdown. The model describes electromagnetism with a massive photon. The starting point is a Lagrangian describing a massless photon interacting with a scalar charged particle and its antiparticle. The scalar field develops a vacuum expectation value, and the vector boson acquires a mass. One scalar degree of freedom disappears, that is converts to a vector boson degree of freedom, and the other remains as a physical particle.

The Lagrangian is:

$$L = -\frac{1}{4}F_{\mu v}F_{\mu v} - (D_\mu K)^* D_\mu K - \mu K^* K - \frac{\lambda}{2}(K^* K)^2$$

with

$$\begin{aligned} F_{\mu v} &= \partial_\mu A_v - \partial_v A_\mu \\ D_\mu &= \partial_\mu - igA_\mu \end{aligned}$$

This Lagrangian is invariant under the gauge transformation

$$\begin{aligned} K &\rightarrow e^{-ig\Lambda} K \\ A_\mu &\rightarrow A_\mu - \partial_\mu \Lambda \end{aligned}$$

with Λ a function of space time, $\Lambda = \Lambda(x)$. This Lagrangian may be rewritten:

$$L = -\frac{1}{4}F_{\mu v}F_{\mu v} - (D_\mu K)^* D_\mu K - \frac{\lambda}{2}(K^* K - f_0^2)^2 + \frac{\lambda}{2} f_0^4$$

with $f_0^2 = -\mu/\lambda$. For this model to make sense λ must be positive. Else for high values of the field K the energy becomes arbitrarely negative. Remember that classically $L = T - V$, where V is the potential energy.

If f_0 is real positive, i.e. μ negative then K develops a vacuum expectation value. In lowest order that is the value for which, with constant K, the Lagrangian reaches an extremum. Obviously this happens for $K = f_0$, and we write

$$< 0|K|0 >= \sqrt{-\frac{\mu}{\lambda}}$$

If we now substitute

$$K = K + f_0$$

the Lagrangian becomes:

$$\begin{aligned} L = & - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - (D_\mu K)^* D_\mu K - g^2 A_\mu^2 f_0 (K + K^*) \\ & - igA_\mu f_0 \partial_\mu (K - K^*) - \frac{\lambda}{2}(K^* K + f_0 K^* + f_0 K)^2 \\ & - g^2 f_0^2 A_\mu^2 + \frac{\lambda}{2} f_0^4 \end{aligned}$$

Since the field K is complex we may rewrite it as a combination of two fields:

$$K = \frac{1}{\sqrt{2}}(H + i\varphi)$$

where $H(x)$ and $\varphi(x)$ are real fields. A gauge transformation of K implies a transformation of H and φ. Before the shift $K = K + f_0$ the infinitesinal form of the gauge transformation was:

$$K \rightarrow (1 - ig\Lambda)K$$

Taking into account the shift that becomes

$$K \rightarrow K - ig\Lambda K - ig\Lambda f_0$$

In terms of H and φ:

$$H(x) \rightarrow H(x) + g\Lambda\varphi(x)$$

$$\varphi(x) \rightarrow \varphi(x) - g\Lambda H(x) - g\Lambda f_0\sqrt{2}$$

The transformation of the field $\varphi(x)$ involves the field independent part $g\Lambda f_0\sqrt{2}$. This shows that φ is unphysical.

In terms of φ and H the Lagrangian becomes:

$$\begin{aligned} L = & - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - (D_\mu K)^* D_\mu K - gM\ A_\mu^2 H \\ & + MA_\mu\partial_\mu\varphi - \frac{\lambda}{2}(\frac{1}{2}\varphi^2 + \frac{1}{2}H^2 + \sqrt{2}f_0 H)^2 \\ & - \frac{1}{2}M^2 A_\mu^2 + \frac{\lambda}{2}f_0^4 \quad ; \qquad M \equiv gf_0\sqrt{2} \end{aligned}$$

The photon mass term is now explicit. To this Lagrangian the gauge fixing term and the Faddeev-Popov Lagrangian must be added. We choose the Lorentz gauge:

$$\begin{aligned} C &= -\partial_\mu A_\mu + M\varphi \\ L_{gf} &= -\frac{1}{2}C^2 = -\frac{1}{2}\partial_\mu A_\mu^2 + M\partial_\mu A_\mu \varphi - \frac{1}{2}M^2\varphi^2 \end{aligned}$$

The $F-P$ Lagrangian follows by subjecting C to an infinitesimal gauge transformation:

$$C \to C + \partial^2\Lambda - gMH(x)\Lambda - M^2\Lambda$$

This implies the $F-P$ Lagrangian

$$L_{FP} = \chi^*\partial^2\chi - M^2\chi^*\chi - gM\chi^*\chi H$$

The photon field A_μ has no interaction with the $F-P$ ghost χ.

This model has 3 independent parameters: g, λ and μ, or g, λ and M, with $f_0 = M/g\sqrt{2}$. The mass term of the Higgs field H is

$$-\lambda f_0^2 H^2 \equiv -\frac{1}{2}m^2 H^2$$

and we identify the Higgs mass m by

$$m^2 = 2\lambda f_0^2 = \frac{\lambda M^2}{g^2}$$

We will use the parameter m instead of λ, the relation being

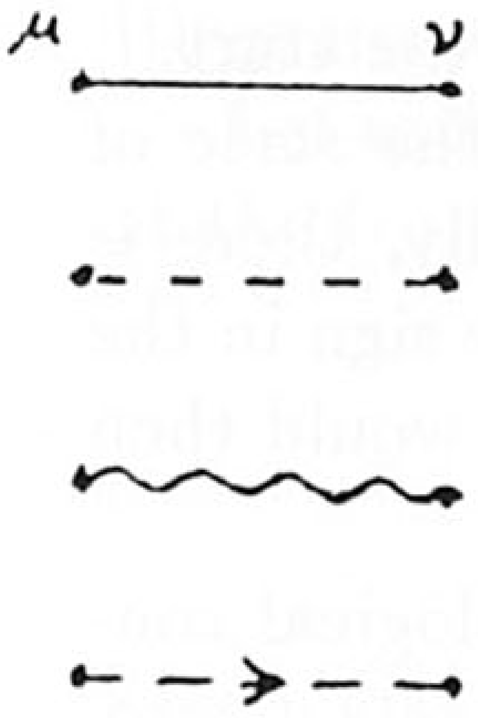
$$\lambda = \frac{m^2 g^2}{M^2}$$

In terms of these parameters the Lagrangian including gauge fixing and $F-P$ terms is:

$$\begin{aligned} L = &- \frac{1}{2}(\partial_\mu A_v)^2 - \frac{1}{2}M^2 A_\mu^2 - \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}M^2\varphi^2 \\ &- \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}m^2 H^2 - gA_\mu(\varphi\partial_\mu H - H\partial_\mu\varphi) - \frac{g^2}{2}A_\mu^2(H^2+\varphi^2) \\ &- gM\ A_\mu^2 H - \frac{m^2 g^2}{8M^2}(\varphi^2 + H^2)^2 \\ &- \frac{m^2 g}{2M}\ H(\varphi^2 + H^2) + \frac{m^2 M^2}{8g^2} \\ &- \partial_\mu\chi^*\partial_\mu\chi - M^2\chi^*\chi - gM\chi^*\chi H \end{aligned}$$

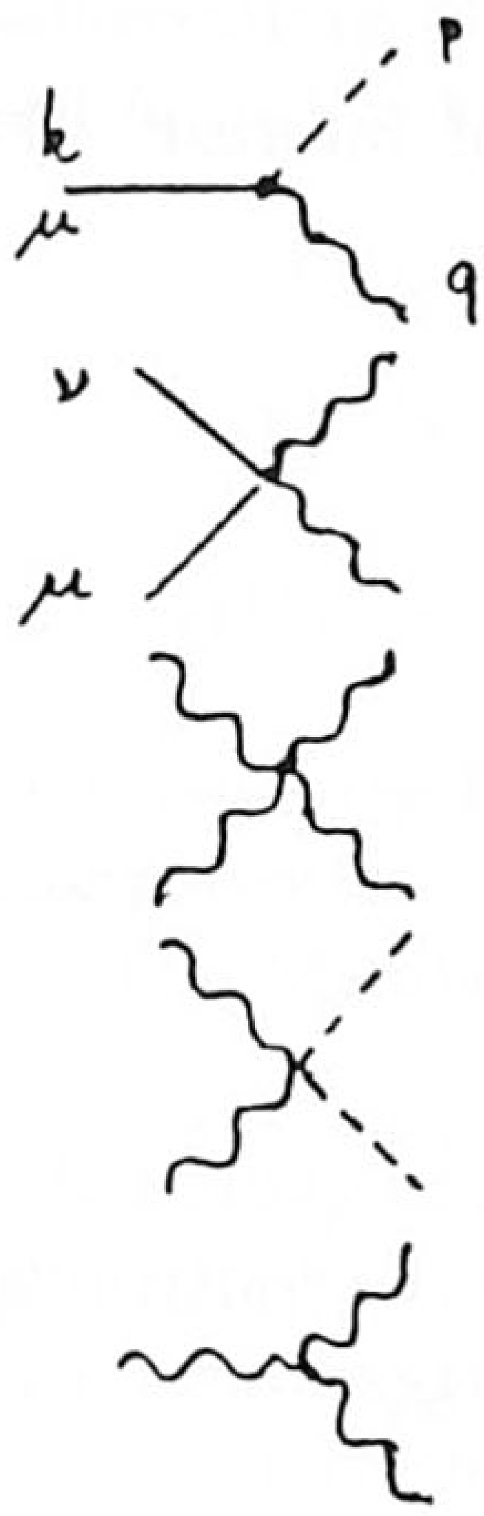
The Feynman rules corresponding to this Lagrangian can now be written down.

Propagators:



$\frac{\delta_{\mu\nu}}{k^2+M^2-i\epsilon}$	Vector particle (massive photon).
$\frac{1}{k^2+M^2-i\epsilon}$	Higgs ghost φ .
$\frac{1}{k^2+m^2-i\epsilon}$	Higgs particle.
$\frac{1}{k^2+M^2-i\epsilon}$	Faddeev-Popov ghost.

Vertices:



$ig(p-q)_\mu$	$-2gM\delta_{\mu\nu}$
$-2g^2\delta_{\mu\nu}$	$-2g^2\delta_{\mu\nu}$
$-3\ \frac{m^2g^2}{M^2}$	$-3\ \frac{m^2g^2}{M^2}$
$-\ \frac{m^2g^2}{M^2}$	$-\ \frac{m^2g}{M}$
$-3\ \frac{m^2g}{M}$	$-\ gM$
× $\frac{m^2M^2}{8g^2}$	Cosmological constant.

3.2 Cosmological constant [11]

The Lagrangian of the Higgs model contains the field independent constant:

$$C \equiv \frac{m^2M^2}{8g^2}$$

Gravitational interactions are commonly introduced through a number of constructions such as replacing ∂_μ by the appropriate derivative D_μ containing the gravitational field $g_{\mu\nu} \equiv \delta_{\mu\nu} + \kappa h_{\mu\nu}$. Furthermore the Lagrangian must be given the overal factor $\sqrt{\det(g_{\mu\nu})}$, involving the determinant of the metric tensor. At this point the addition of constant to the Lagrangian is of physical consequence; the coefficient of the term that contains no other field dependance other than $\sqrt{\det g}$ is the cosmological constant. It gives rise to the cosmological member in Einstein's theory of gravity. A non-zero value implies a curved Universe in the absence of matter. The cosmological constant defines the curvature of the vacuum. In Einstein's theory there is really no reason to suppose that the cosmological constant is zero; here in the Higgs model we see that field theory generates such a constant even if there was none at the start.

Experimentally the Universe is known to be very flat, certainly on the scale of values as relating to a possible Higgs contributions. Of course, formally, there is nothing wrong: we could have introduced the same constant with a minus sign in the original Lagrangian, and after the shift in the Higgs field the total result would then be zero.

To put things in all generality: Einstein's theory contains the cosmological constant as a free parameter. One customarily worries not too much and just sets it zero, as experiment requires. The trouble is now that field theory modifies that constant with an a priori unknown amount. Moreover, further investigation shows that this constant is also affected by radiative corrections. It is hard to see why then the final result, as deduced from experiment, is so very close to zero.

To put this in more explicit terms some numbers might be of interest. The constant κ is:

$$\kappa = 5.8 \times 10^{-22}\ MeV^{-1}$$

The experimental limit on the cosmological constant is:

$$\kappa^2 C < 10^{-57}\ \mathrm{cm}^{-2}$$

or

$$C < 0.23 \times 10^{-35}\ MeV^4 = (1.23 \times 10^{-9}\ MeV)^4$$

The value of the cosmological constant generate by the Higgs system as quoted above is way out of range for any reasonable value of m, M and g. Typically the contribution of the Higgs system is of the order of $(50\ 000\ MeV)^4$, assuming Higgs mass m and vector boson mass M to be of order $50\ GeV = 50\ 000\ MeV$.

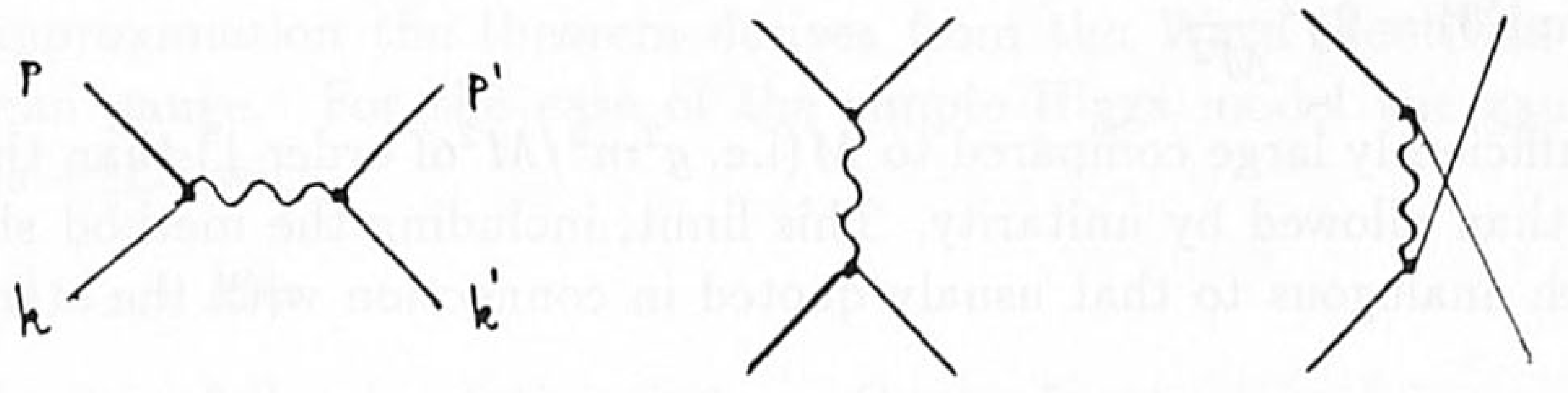
Of all explanations suggested so far one can only say that one or more of them may be correct.

3.3 Unitarity limit, heavy Higgs

The Higgs model contains only one vector boson, and three completely arbitrary parameters g, m and M. The vector boson may be like the photon; it can be coupled to other charged particles in the customary way, involving the e.m. coupling constant e. In this abelian model there is no relation between the coupling constants e and g. In particular one may take the limit $g = 0$, and nothing survives of the Higgs system except a photon mass term and a cosmological constant (infinite, to be sure, but who cares about that?). This shows that just giving a mass to the photon is not destroying renormalizability.

Another approach is to consider the behaviour of the theory as a function of the Higgs mass m [12]. The theory with a finite photon mass M and no Higgs is renormalizable. The theory with finite photon mass M and finite Higgs mass m is also renormalizable. The celebrated decoupling theorem [13] claims then that the limit of $m = \infty$ should be the same as the theory without a Higgs particle. In view of the fact that certain vertex couplings contain m^2, and thus become large as the Higgs mass becomes large, application of this theorem is questionable.

Let us first consider photon-photon scattering. The following diagrams contribute at the tree level:



All momenta are taken to be ingoing. All diagrams involve Higgs exchange. The external vector boson lines imply polarization vectors, and we specialize to longitudinal polarizations. In the center of mass system with $\vec{p}$ and $\vec{k}$ along the third axis and $\vec{p}'$ and $\vec{k}'$ in the 1-3 plane one has:

$$k = (0, 0, k_\ell, ik_0) \qquad k' = -(k_\ell \sin\theta, 0, k_\ell\cos\theta, ik_0)$$

$$e(k) = \frac{1}{M}(0, 0, k_0, ik_\ell)$$

$$e(k') = -\frac{1}{M}(k_0 \sin\theta, 0, k_0\cos\theta, ik_\ell)$$

and similarly for p and p' differing from the above by a minus sign for the spatial components.

The Mandelstam variables s, t and u are defined by:

$$s = -(p + k)^2 \qquad t = -(p + p')^2 \qquad u = -(k + p')^2$$

The relation

$$s + t + u = 4M^2$$

holds. The variable s is related to the total energy in the centre of mass system; $s = E_{\rm cm}^2 = 4k_0^2$.

The three diagrams can be evaluated readily. One finds:

$$\begin{aligned} \mathrm{Ampl}(0) \; &+ \; \frac{g^2}{-s+m^2}\left(\frac{s^2}{M^2} - 4s + 4M^2\right) \\ &+ \; \frac{g^2}{-t+m^2}\left(\frac{t'^2}{M^2} - 4t' + 4M^2\right) \\ &+ \; \frac{g^2}{-u+m}\left(\frac{u'^2}{M^2} - 4u' + 4M^2\right) \end{aligned}$$

where t' and u' are different from t and u:

$$t' = \frac{1}{2}(1 - \cos\theta)s = -t + 0(M^2)$$

$$u' = \frac{1}{2}(1 + \cos\theta)s = -u + 0(M^2)$$

For fixed s, t, u the amplitude goes to zero for large Higgs mass m. However, consider now the case that $m >> M$, but fixed, and then take the limit $s \to \infty$. To clearly see what happens we specialize to the case of scattering in the forward direction, i.e. $\cos\theta = 1$. Then $t = 0$ and $u = -s + 4M^2$. In the limit $M^2 << m^2 << s$ one finds:

$$\mathrm{Ampl}(0) = 2g^2 \frac{m^2}{M^2}$$

If m is sufficiently large compared to M(i.e. $g^2m^2/M^2$of order 1) than the amplitude is larger than allowed by unitarity. This limit, including the method shown here is very much analogous to that usualy quoted in connection with the standard model [14].

As is clear from the above the limit of large Higgs mass is quite delicate. The longitudinal four-photon amplitude remains small for energies below the Higgs mass, but for higher energies the amplitude reaches a maximum that grows with m^2. What is the limit really? More precisely, what happens with the one-loop radiative corrections? Such corrections can roughly be understood as the square of the tree amplitude integrated over all intermediate energies. We then have two effects that tend to cancel each other (as $m \to \infty$ the region where the amplitude is substantial decreases, while the amplitude increases in that region).

A complete calculation in the limit $M^2 << s, t, u << m^2$ shows that a result remains. For the longitudinal photon-photon scattering amplitude the one loop radiative corrections grow quadratically with s and t:

$$\mathrm{Ampl}(1) = \frac{i\pi^2 g^4}{M^4}(st + s^2 + t^2) \cdot \left(\frac{2}{3} + 9B_0\right)$$

with

$$B_0 = \frac{\pi}{\sqrt{3}} - 2$$

Note that $st+s^2+t^2 = (u^2+s^2+t^2)/2$. This calculation is actually quite complicated, and no details will be given here. A similar calculation, but for a more realistic Higgs sector will be discussed later.

The above result requires calculation also of tadpoles, two- and three point function. These quantities are needed for the renormalization procedure. While various $\log(m^2)$ terms appear in the process, they are completely renormalized away in the case of the four-photon amplitude. It should be emphasized that the above result was obtained in the limit $M^2 << s, t << m^2$.

Thus for large m^2 unitarity is "violated" for energies such that $g^2 s/M^2 \geq 1$. Even if the tree amplitude is well within bounds (in fact it is vanishingly small if $m^2 \to \infty$), the one loop radiative corrections violate the unitarity bound. The above expression for this amplitude suggest that perhaps a partial summation of diagrams in the s, t and u channels take care of the problem. That however is pure speculation.

3.4 Equivalence theorem [15]

The equivalence theorem refers to a very useful identity applicable to calculations of amplitudes involving longitudinally polarized vector bosons. The statement is that

that at high energies a longitudinally polarized vector boson behaves like the Higgs ghost. Since amplitudes with in(out)-going Higgs ghosts are usually much easier to calculate the theorem is often quite useful.

Since the Higgs ghost is a gauge dependent object, the equivalence theorem is likewise gauge sensitive. The 't Hooft-Feynman gauge is in fact understood.

In first approximation the theorem derives from the Ward identities in the 't Hooft-Feynman gauge. For the case of the simple Higgs model the gauge fixing Lagrangian is $-\frac{1}{2}C^2$ with

$$C = -\partial_\mu A_\mu + M\varphi$$

The Ward identities follow by stating that any Greens function involving one or more field combinations C as external lines is zero. Two steps are then needed to arrive at the theorem:

i The polarization vector $e_\mu(k)$ of a longitudinally polarized vector boson with momentum k is proportional to k_μ neglecting terms of order M/k:

$$e_\mu(k) = \frac{k_\mu}{M} + O\left(\frac{M}{k}\right)$$

ii An S-matrix element is not immediately equal to a Greens function. A procedure for the external lines involving wave function renormalization, and the mass-shell limit must be carried through. Whatever differences there are with respect to this procedure for vector bosons and Higgs ghosts must be adjusted for [16]. In practice this amounts rather simply to the following: forget any selfenergy insertions in the external lines and attach the proper wave function renormalization factors for vector bosons and Higgs ghosts respectively. In short, the S-matrix elements defined as usual can be used. This recipe requires that the gauge choice is strictly as stated, and that this gauge is maintained. See for example the two-loop calculation of radiative corrections to the ρ-parameter [17].

Thus, from i, the desired equality will be true only up to terms that go to zero as the energy of the vector boson becomes large.

The equivalence theorem can be used to calculate or estimate high energy behaviour of amplitudes involving longitudinally polarized vector bosons. The point is that the individual components of the polarization vector of transversally polarized vector bosons are of order 1, while, as shown above, the polarization vector corresponding to longitudinal polarizations has components that grow with the energy. For a vector boson moving with momentum k_ℓ along the third axis the transversal polarization vectors are of the form

$$e^t_\mu = (a, b, 0, 0) \quad , \qquad a^2 + b^2 = 1$$

and the longitudinal is:

$$e^\ell_\mu = \frac{1}{M}(0, 0, k_0, ik_\ell) \quad , \quad k_0 = \sqrt{k_\ell^2 + M^2}$$

One has

$$\begin{aligned} e^{\ell}_{\mu} &= \frac{k_\mu}{M} + \delta^v_\mu \\ \delta^v_\mu &= (o, o, \delta^v, -i\delta^v) \\ \delta^v &= \frac{1}{M}(k_0 - k_\ell) = \frac{M}{2k_\ell}\left(1 + O\left(\frac{M^2}{k_\ell^2}\right)\right) \end{aligned}$$

Note that δ^v_μ is a light like vector, i.e. $\delta^v_\mu \delta^v_\mu = 0$. Furthermore $k_\mu \delta^v_\mu = M$. From the expression for δ^v_μ it follows that

$$e^{\ell}_{\mu} = \frac{k_\mu}{M} + O\left(\frac{M}{k_\ell}\right).$$

The equivalence theorem must be applied with great care. First note that the equivalence of longitudinally polarized vector bosons and Higgs ghosts is true only in leading order in the energy of the vector boson, as shown explicitly above. In view of the fact that the S-matrix elements involved are Lorentz invariant this raises some questions. Consider any process involving only one vector boson. No matter which polarization, by means of a suitable Lorentz transformation the vector boson may be transformed to a longitudinally polarized vector boson of large momentum (first go to the restframe, then boost along the polarization direction[1]). In the process the amplitude is invariant, but a transversal vector boson may become a longitudinal one. It follows that the leading behaviour in terms of the vector boson momentum for the longitudinal case must somehow cancel, i.e. the factor k_μ/M must effectively behave as a constant (or less) for large k_μ. But then the neglected part (δ^v in the equations above) may be of the same order of magnitude as the k_μ/M part, and there is no meaningful equality.

Thus for processes involving only one longitudinally polarized vector boson of large momentum k_μ the amplitude is down by a factor of order M/k as compared to what might be inferred when replacing the polarization vector by k_μ/M. Indeed, as can be seen explicitly in simple examples, in case of only one vector boson the equivalence theorem is not valid. An example within the standard model including fermions is the decay of the neutral vector boson into a neutrino-antineutrino pair. At the tree level the Higgs ghost is not even coupled to such a pair.

The above "argument" fails if there are more vector bosons involved. The reason is that the Lorentz transformations mentioned may turn a transversal vector boson into a longitudinal one, but at the same time the other vector boson may be transformed from a longitudinal to a transversal one. Furthermore, the polarization vector components may become of order k_ℓ^2/M^2 after a boost. As an example consider the dotproduct of two polarization vectors (longitudinal) in the centre of mass. It is equal to $(-k_0^2 - k_\ell^2)/M^2$. A Lorentz transformation that brings one vector boson to rest will make the components of the polarization vector of the other to something of order k_ℓ^2/M^2, so that the dotproduct remains invariant.

[1] A note of caution however: a vector with components of order k_ℓ/M may be transformed into a vector with components of order k_ℓ^2/M^2.

The above arguments are somewhat sloppy, and must be seen as indicative only of the fact that usually there are cancellations. However there is another fact. In a gauge theory couplings of vector bosons are generally such that the high energy behaviour of longitudinally polarized vector bosons is damped. That must be in order for the theory to be renormalizable. Thus if the energies of all vector bosons are very large compared to all masses (including the Higgs mass) then there must be cancellations. Therefore also in that case the equivalence theorem is not of any use. It might still be valid, but the amplitude involving longitudinally polarized vector bosons is of the same order of magnitude as that for transversally polarized vector bosons.

There is still an interesting region were the equivalence theorem is effective. That is when

- At least two vector bosons are involved.

- The energy of all vector bosons is large with respect to the vector boson mass M but small with respect to some other mass, notably for example the Higgs mass.

The second condition can perhaps be sharpened. If there is one mass m_x in the theory such that in the limit $m_x \to \infty$ the theory becomes non-renormalizable then one might expect that there will be no complete cancellation if the energies are below that mass. This is true for the Higgs mass. Another example where this might happen is the standard model, if the top quark mass is large.

It is clear that application of the equivalence theorem is non-trivial. For example, in case of vector boson-vector boson scattering there is, in the limit $M \ll$ Energies $\ll m$ (= Higgs mass) a cancellation at the tree level, but not as much as needed to reduce the longitudinal cross section to the level of a transversal cross section. The equivalence theorem turns out to be correct, and useful. On the one loop level there appears to be no cancellation, and the theorem works and is very useful indeed.

A non-proven rule of thumb might be this: suppose some process involving longitudinal vector bosons must be considered. Assume the validity of the equivalence theorem. Compute the Higgs ghost amplitude. If the result is larger (for large energies) then might be expected for amplitudes involving only transversal vector bosons then chances are that the theorem works.

It should be noted that the replacement of the polarization vector by the corresponding four momentum ($e_\mu \to k_\mu/M$) is not an essential approximation. To all intents and purposes this replacement may be taken as exact. The point is that things can be reformulated such that this approximation can be avoided. Consider as an example an amplitude involving two vector bosons. It can be written in the form:

$$A^{ij}(p,k) = e^i_\mu(k) e^j_\nu(p) A_{\mu\nu}$$

where the polarization vectors are explicitly shown. The quantity $A_{\mu\nu}$ contains whatever else is involved besides external vector bosons. The interest is in the high energy behaviour of the amplitude, for k_0 and p_0 large with respect to the vector boson mass M. The supposition is that leading behaviour obtains for longitudinal polarizations, which is true. However things may be formulated as follows. Consider the amplitude squared, and sum over all vector boson polarizations:

$$\sum_{ij}\left|A^{ij}\right|^2 = \left(\delta_{\mu\alpha} + k_\mu k_\alpha/M^2\right)\left(\delta_{\nu\beta} + p_\nu p_\beta/M^2\right) A_{\mu\nu} A^*_{\alpha\beta}$$

The leading behaviour now follows by considering only the $k_\mu k_\alpha$ and $p_\nu p_\beta$ terms. The cancellation issue remains however: if there is a cancellation in that part then the $\delta_{\mu\alpha}$ and $\delta_{\nu\beta}$ parts cannot be ignored.

The above expression can be worked out completely using Ward identities. The work is largely the same as that needed to show that the unitary gauge (no Higgs and Faddeev-Popov ghosts, vector boson propagator numerator of the form $\delta_{\mu\nu}+k_\mu k_\nu/M^2$) is equivalent to the 't Hooft-Feynman gauge (Higgs and F-P ghosts, propagator of the form $\delta_{\mu\nu}$). It remains then to demonstrate what piece is leading in the limit of large energies.

4 The σ-model

4.1 The model and its symmetries

Schwinger's σ-model contains four scalar particles. We will denote them by σ and $\varphi_i, i = 1,2,3$. There are several ways to write the Lagrangian, each with their advantages and disadvantages. The most direct form is this:

$$L = -\frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}\partial_\mu\varphi_i\partial_\mu\varphi_i - \frac{\mu}{2}(\sigma^2+\varphi_i^2) - \frac{\lambda}{8}(\sigma^2+\varphi_i^2)^2$$

The fields σ and φ_i are real. There is evidently an 0(4) symmetry, manifest if we write φ_4 for σ. That is a 6 parameter symmetry. Customarily one writes this as an 0(3) symmetry in which the φ_i transform into themselves and σ is invariant:

$$\begin{aligned}\varphi_i &\to \varphi_i + \varepsilon_{ijk}\Lambda_j\varphi_k\\ \sigma &\to \sigma\end{aligned}$$

This is actually the infinitesimal form, involving three quantities Λ_i and the completely antisymmetric Weyl tensor ε. In addition then there is a symmetry mixing σ and the φ_i (infinitesimal form):

$$\begin{aligned}\varphi_i &\to \varphi_i - \sigma\bar\Lambda_i\\ \sigma &\to \sigma + \varphi_i\bar\Lambda_i\end{aligned}$$

The first symmetry is often seen as an isospin symmetry, and in fact it is just that in the standard model. The symmetry is a global symmetry, with the Λ and $\bar\Lambda$ independent of space and time.

Another way to write the Lagrangian is by means of a complex two component quantity K:

$$L = -\partial_\mu K^\dagger \partial_\mu K - \mu K^\dagger K - \frac{1}{2}\lambda (K^\dagger K)^2$$

with

$$K = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma & + & i\varphi_3 \\ -\varphi_2 & + & i\varphi_1 \end{pmatrix}$$

The manifest invariance here is an $SU(2)$ invariance, involving three paramters Λ^L:

$$K \to K - \frac{i}{2}\Lambda_i^L \tau^i K$$

where the τ^i are the usual Pauli spin matrices:

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Working out this transformation one finds:

$$\sigma \quad \to \sigma + \frac{1}{2}\Lambda_i^L \varphi_i$$
$$\varphi_i \quad \to \varphi_i + \frac{1}{2}\left(-\sigma \Lambda_i^L + \varepsilon_{ijk}\Lambda_j^L \varphi_k\right)$$

This shows that the symmetry is a combination of the previous two symmetries, with $\Lambda_i = \bar{\Lambda}_i = \frac{1}{2}\Lambda_i^L$. In addition there is manifestly a $U(1)$ symmetry:

$$K \to e^{-i\Lambda^0} K$$

or, in infinitesimal form:

$$\sigma \quad \to \sigma + \Lambda^0 \varphi_3$$

$$\varphi_1 \quad \to \varphi_1 + \Lambda^0 \varphi_2$$
$$\varphi_2 \quad \to \varphi_2 - \Lambda^0 \varphi_1$$
$$\varphi_3 \quad \to \varphi_3 - \Lambda^0 \sigma$$

This symmetry follows by taking $\Lambda_3 = -\bar{\Lambda}_3 = -\Lambda^0$, all other Λ and $\bar{\Lambda}$ zero. The remaining symmetry is not manifest.

The third way to write the σ-model Lagrangian involves the matrix Φ:

$$\Phi = \sigma \tau^0 + i\varphi_i \tau^i$$

with τ_0 the 2×2 unit matrix:

$$\tau^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In detail:

$$\Phi = \begin{pmatrix} \sigma + i\varphi_3 & i\varphi_1 + \varphi_2 \\ i\varphi_1 - \varphi_2 & \sigma - i\varphi_3 \end{pmatrix}$$

Note that the first colum of Φ is like the previous K. The trace of Φ is real. The Lagrangian is now:

$$\begin{aligned} L = - & \frac{1}{4}Tr(\partial_\mu \Phi^\dagger \partial_\mu \Phi) - \frac{\mu}{2}\left[\frac{1}{2}Tr(\Phi^\dagger\Phi)\right] \\ - & \frac{\lambda}{8}\left[\frac{1}{2}Tr(\Phi^\dagger\Phi)\right]^2 \end{aligned}$$

The manifest symmetries are now two $SU(2)$ symmetries:

$$\begin{aligned} \Phi \rightarrow G\Phi \qquad & G = \exp\left(\frac{i}{2}\Lambda_i^L \tau^i\right) \\ \Phi \rightarrow \Phi H \qquad & H = \exp\left(\frac{i}{2}\Lambda_i^R \tau^i\right) \end{aligned}$$

involving the six quantities Λ^L and Λ^R. In infinitesimal form:

$$\begin{aligned} \Phi \rightarrow \Phi & - \frac{i}{2}\Lambda_i^L \tau^i \Phi \\ \Phi \rightarrow \Phi & - \frac{i}{2}\Lambda_i^R \Phi \tau^i \end{aligned}$$

G and H are often called the left and right $SU(2)$ symmetry respectively. In terms of the σ and φ these transformations amount to

$$\begin{aligned} \sigma \rightarrow \sigma & + \frac{1}{2}\left(\varphi_i \Lambda_i^L + \varphi_i \Lambda_i^R\right) \\ \varphi_i \rightarrow \varphi_i & - \frac{1}{2}\sigma\left(\Lambda_i^L + \Lambda_i^R\right) + \frac{1}{2}\varepsilon_{ijk}\left(\Lambda_j^L - \Lambda_j^R\right)\varphi_k \end{aligned}$$

This is like the very first transformations mentioned with

$$\begin{aligned} \Lambda & = \frac{1}{2}\left(\Lambda^L - \Lambda^R\right) \\ \bar{\Lambda} & = \frac{1}{2}\left(\Lambda^L + \Lambda^R\right) \end{aligned}$$

The $U(1)$ symmetry in connection with the σ-model in K notation obtains from $\Lambda_3^R = \Lambda^0$, all other Λ zero.

The K-notation is often used when writing down the Lagrangian of the standard model using the σ-model as Higgs sector. The reason is that the manifest $SU(2)$ and $U(1)$ symmetries in that notation are precisely those of the vector boson and fermion part of the model. The full symmetry structure is then somewhat hidden, and is often referred to as "accidental symmetry".

The σ-model can be rewritten such as to make the vacuum expectation manifest. Using the "direct" Lagrangian we write:

$$L = -\frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(\partial_\mu \varphi_i)^2 - \frac{\lambda}{8}\left[\sigma^2 + \varphi_i^2 + \frac{2\mu}{\lambda}\right]^2 + \frac{\mu^2}{2\lambda}$$

Like in the Higgs model λ must be positive to have the energy bounded from below. If $\mu/\lambda < 0$ then there is a vacuum expectation value. At this point, given the full $O(4)$ invariance one may rotate the φ_i and the σ such that only σ is non-zero. Making the shift

$$\sigma \to \sigma + \sqrt{-\frac{2\mu}{\lambda}} \equiv \sigma + f_0$$

the part between square brackets in the Lagrangian takes the form:

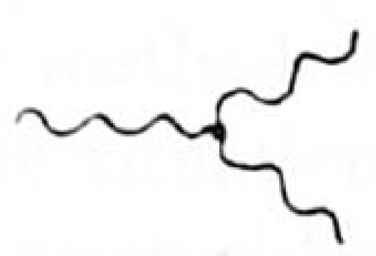
$$-\frac{\lambda}{8}\left[\sigma^2 + 2\sigma f_0 + \varphi_i^2\right]^2$$

$$= -\frac{\lambda f_0^2}{2}\sigma^2 - \left[4 f_0 \sigma \left(\sigma^2 + \varphi_i^2\right) + \left(\sigma^2 + \varphi_i^2\right)^2\right] \cdot \frac{\lambda}{8}$$

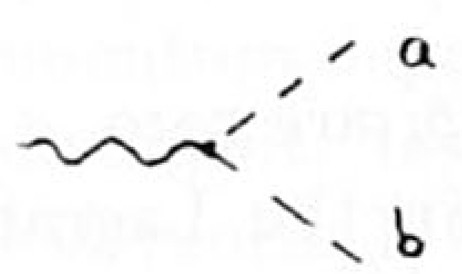
Considering the quadratic term it follows that the σ-particle has the mass

$$m^2 = \lambda f_0^2 ,$$

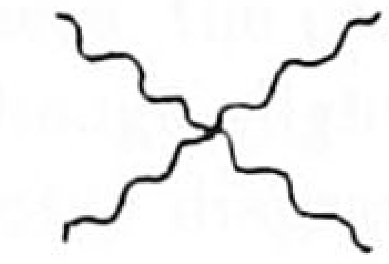
while the φ_i remain massless. They are called the Goldstone bosons. The Feynman rules for this model are simple. The vertices are:



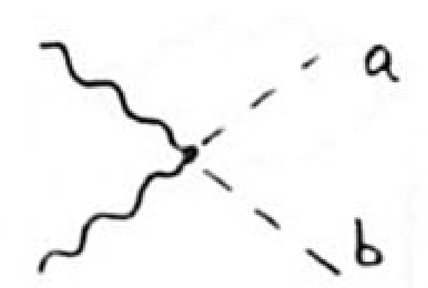
$-\frac{3m^2}{f_0}$



$-\frac{m^2}{f_0}\delta_{ab}$



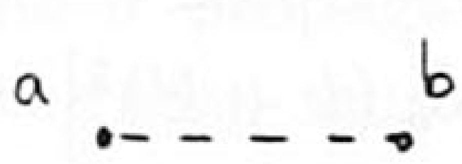
$-3\,\frac{m^2}{f_0^2}$



$-\frac{m^2}{f_0^2}\delta_{ab}$

$-\frac{m^2}{f_0^2}\left(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}\right)$

The propagators:



$\frac{\delta_{ab}}{k^2 - i\epsilon}$

$\frac{1}{k^2 + m^2 - i\epsilon}$

4.2 The non-linear σ-model as limit of the σ-model

The Lagrangian of the σ-model, in case of spontaneous symmetry breakdown (σ with vacuum expectation value) is:

$$L = -\frac{1}{2}\left(\partial_\mu \sigma\right)^2 - \frac{1}{2}\left(\partial_\mu \varphi_i\right)^2 - \frac{m^2}{8 f_0^2}\left[\sigma^2 + \varphi_i^2 + 2\sigma f_0\right]^2$$

with

$$f_0 = \sqrt{-\frac{2\mu}{\lambda}}\ , \quad m^2 = \lambda f_0^2$$

Consider now the limit of large m for fixed f_0. Then the last term dominates, and the Lagrangian reaches an extremum if the expression inside square brackets is zero. That fixes a relation between the σ-field and the φ_i, and in fact the σ-field disappears as an independent field. Thus:

$$\sigma^2 + 2\sigma f_0 + \varphi_i^2 = 0$$

$$\sigma = -f_0\left\{1 \pm \sqrt{1 - \varphi_i^2/f_0^2}\right\}$$

The minus sign must be taken as σ should be zero if the φ_i are zero.

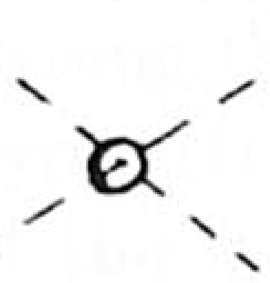
Substituting this solution in the Lagrangian results in the Lagrangian for the non-linear σ-model:

$$L_{n\ell} = -\frac{1}{2}\left(\partial_\mu \varphi_i\right)^2 - \frac{1}{8 f_0^2} \cdot \frac{1}{1 - \varphi_i^2/f_0^2}\left(\partial_\mu (\varphi_i)^2\right)^2$$

Expanding the denominator:

$$L_{n\ell} = -\frac{1}{2}\left(\partial_\mu \varphi_i\right)^2 - \frac{1}{8 f_0^2}\left(\partial_\mu \varphi^2\right)^2\left[1 + \frac{\varphi^2}{f_0^2} + \frac{\varphi^4}{f_0^4} + \cdots\right]$$

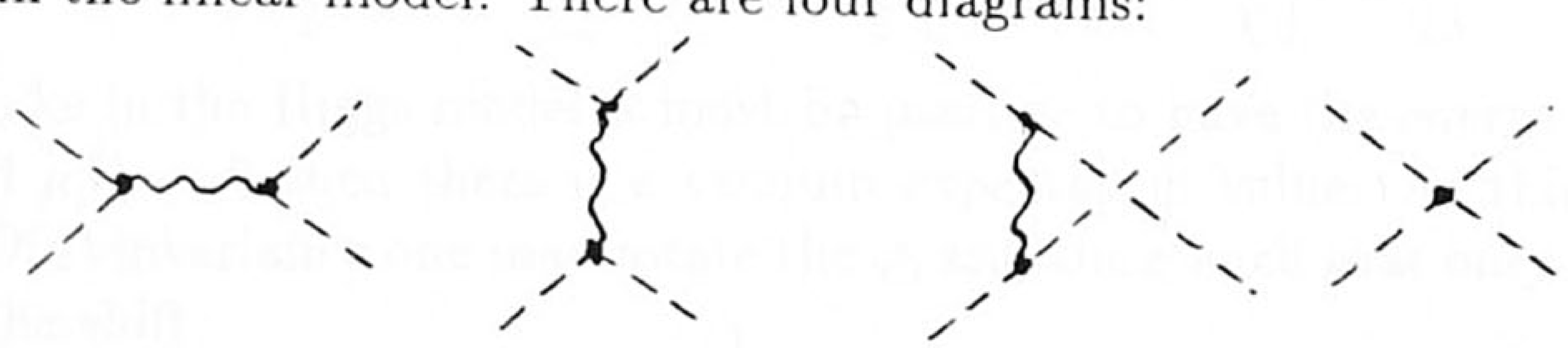
This is a non-polynomial Lagrangian. It describes a triplet of massless particles interacting with each other. In terms of Feynman rules this model has an infinite number of vertices involving $4, 6, 8 \cdots$ φ-lines. The 4-vertex is:



$$-\frac{1}{f_0^2}\left[\delta_{ab}\delta_{cd}(p+k)^2 + \delta_{ac}\delta_{bd}\left(k + p'\right)^2 + \delta_{ad}\delta_{bc}(k + k')^2\right]$$

where a, b, c, d and k, p, p', k' refer to the isospin indices and the momenta of the particles (all momenta taken to be ingoing).

It is actually of interest to see if the linear model, in the limit of large mass m, becomes the non-linear model also in terms of diagrams. Consider $\varphi - \varphi$ scattering in the linear model. There are four diagrams:



The Feynman rules have been given before. The first diagram gives:

$$\frac{m^4}{f_0^2}\delta_{ab}\delta_{cd}\,\frac{1}{(p+k)^2+m^2}$$

Taking the limit of large m^2 one must actually develop the propagator and not simply make it equal to $1/m^2$:

$$\frac{m^4}{f_0^2}\,\delta_{ab}\delta_{cd}\,\frac{1}{m^2}\left(1-\frac{(p+k)^2}{m^2}\right)$$

Doing the same for the second and third diagram and summing these expressions:

$$\frac{m^4}{f_0^2}\Big[\tfrac{1}{m^2}\delta_{ab}\delta_{cd}+\tfrac{1}{m^2}\delta_{ac}\delta_{bd}+\tfrac{1}{m^2}\delta_{ad}\delta_{bc} \\ -\tfrac{(p+k)^2}{m^4}\delta_{ab}\delta_{cd}-\tfrac{(k+p')^2}{m^4}\delta_{ac}\delta_{bd}-\tfrac{(k+k')^2}{m^4}\delta_{ad}\delta_{bc}\Big]$$

Adding the expression for the four φ vertex of the linear model gives precisely the four φ vertex of the non-linear model as given above.

This same method works for all tree diagrams. However, considering diagrams with loops the situation becomes more complicated. The development of the σ-propagator as given above may no longer be valid. The reason is that this development introduces momentum dependence in the numerator, and the resulting loop integrals may become divergent. Indeed, from the point of view of momentum dependence a dependence of the form $1/k^2$ is replaced by k^2.

As it happens, the propagator expansion can still be carried through for one- loop diagrams, although slightly modified [12]. As an example consider the very simplest case, φ-selfenergy diagrams:

The expression corresponding to the first diagram is:

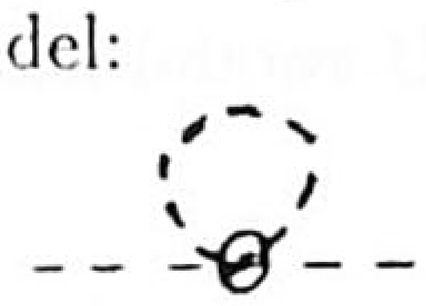
$$\frac{m^4}{f_0^2}\int d_4p\frac{1}{(p^2-i\varepsilon)/(p+k)^2+m^2-i\varepsilon)}$$

Note that the σ propagator contains dependence on the external momentum k in addition to the loop momentum p. The development is now as follows:

$$\frac{1}{(p+k)^2+m^2}=\frac{1}{p^2+m^2}\left\{1-\frac{2pk+k^2}{p^2+m^2}+\frac{(2pk+k^2)^2}{(p^2+m^2)^2}+\cdots\right\}$$

Note now that from the point of view of large momentum p the series is still well behaved.

Evaluating the integral and taking the limit of large m^2 is trivial; adding the expression corresponding to the second diagram one obtains the same as from the single diagram in the non-linear model:



The crucial point is really the propagator expansion. That is what corresponds to taking the limit of large m directly in the Lagrangian.

For two-loop diagrams no such procedure works. Inevitably any propagator development leads to more and more divergent expressions. And in fact, by actual calculation, it is found that the limit of the linear model is not simply the non-linear model [17]. The problem has not yet been understood in all generality.

4.3 Higgs sector of the standard model

The vector bosons of weak interactions must now be coupled to the particles of the σ-model. As usual an $SU(2) \times U(1)$ symmetry of the vector boson Lagrangian is assumed. Thus there are four vector bosons, namely a triplet $B^a_\mu, a = 1,2,3$ and a singlet B^0_μ. The vector boson Lagrangian is:

$$\begin{aligned} L_w &= -\frac{1}{4}\left(G^a_{\mu\nu}\right)^2 - \frac{1}{4}\left(G^0_{\mu\nu}\right)^2 \\ G^a_{\mu\nu} &= \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\varepsilon_{abc} B^b_\mu B^c_\nu \\ G^0_{\mu\nu} &= \partial_\mu B^0_\nu - \partial_\nu B^0_\mu \end{aligned}$$

It will be convenient to introduce a new notation for the B_μ. Define the 2×2 matrices b_μ and c_μ:

$$b_\mu = -\frac{i}{2} B^a_\mu \tau^a \qquad c_\mu = -\frac{i}{2} B^0_\mu \tau^x \qquad (\tau^x)^2 = \tau^0$$

where the τ^a are the Pauli spinmatrices as given before, τ^0 is the 2×2 unit matrix, and τ^x is not yet specified. Explicitly:

$$b_\mu = -\frac{i}{2}\begin{pmatrix} B^3 & B^1 - iB^2 \\ B^1 + iB^2 & -B^3 \end{pmatrix}_\mu \quad c_\mu = -\frac{i}{2}\begin{pmatrix} \alpha B^0 & 0 \\ 0 & \beta B^0 \end{pmatrix}_\mu \quad \alpha^2 + \beta^2 = 1$$

The Lagrangian can be rewritten in terms of b and c:

$$L_w = -\frac{1}{2} Tr\left(b_{\mu\nu} b_{\mu\nu}\right) - \frac{1}{2} Tr\left(c_{\mu\nu} c_{\mu\nu}\right)$$

involving the 2×2 matrices $b_{\mu\nu}$ and $c_{\mu\nu}$:

$$\begin{aligned} b_{\mu\nu} &= \partial_\mu b_\nu - \partial_\nu b_\mu + g[b_\mu, b_\nu] \\ c_{\mu\nu} &= \partial_\mu c_\nu - \partial_\nu c_\mu \end{aligned}$$

The Lagrangian is invariant under the $SU(2)$ transformations:

$$b_\mu \to S b_\mu S^\dagger + \frac{1}{g} S \partial_\mu S^\dagger$$

with the 2×2 matrix S of the form:

$$S = \exp\left(-\frac{i}{2} g \lambda_a \tau^a\right)$$

where the three λ_a are three functions of space-time. It is left to the reader to check that the $b_{\mu\nu}$ transform as

$$b_{\mu\nu} \to S b_{\mu\nu} S^\dagger .$$

The proof uses that S is unitary, i.e. $S^\dagger = S^{-1}$. Similarly for c_μ and $c_{\mu\nu}$:

$$\begin{aligned} c_\mu &\to c_\mu + \frac{i}{2}\lambda_0 \tau^x \\ c_{\mu\nu} &\to c_{\mu\nu} \end{aligned}$$

Let us now return to the σ-model in Φ matrix notation. The manifest symmetries are an $SU(2)$-left and an $SU(2)$-right. To make these symmetries local (i.e. invariance also if the Λ^L and Λ^R are space-time dependent) one needs two triplets of vector bosons. Let us denote the fields by

L^a_μ left $SU(2)$ vector fields

R^a_μ right $SU(2)$ vector fields

At some point we will identify the B^a_μ with the L^a_μ or R^a_μ or some combination of them.

The locally invariant Lagrangian is obtained by replacing the derivative $\partial_\mu \Phi$ by a covariant derivative $D_\mu \Phi$:

$$D_\mu \Phi = \partial_\mu \Phi + g l_\mu \Phi - g' \Phi r_\mu$$

where now l_μ and r_μ are matrices:

$$\begin{aligned} l_\mu &= -\frac{i}{2} L^a_\mu \tau^a \\ r_\mu &= -\frac{i}{2} R^a_\mu \tau^a \end{aligned}$$

If (see section on σ-model symmetries):

$$\Phi \to G \Phi H^\dagger$$

then it is not difficult to show that

$$D_\mu \Phi \to G (D_\mu \Phi) H^\dagger$$

provided

$$\begin{aligned} l_\mu &\to G l_\mu G^\dagger + \frac{1}{g} G \partial_\mu G^\dagger \\ r_\mu &\to H r_\mu H^\dagger + \frac{1}{g'} H \partial_\mu H^\dagger \end{aligned}$$

using $G^\dagger = G^{-1}$ and $H^\dagger = H^{-1}$. It then follows that

$$Tr\left[(D_\mu \Phi)^\dagger D_\mu \Phi \right]$$

is invariant.

If now σ develops a vacuum expectation value then the vector bosons get a mass. Performing the shift $\sigma \to \sigma + f_0$ and keeping in Φ only the terms containing f_0 provides us with the vector field mass terms. Thus write:

$$\Phi = f_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

so that

$$D_\mu \Phi = (g l_\mu - g' r_\mu) f_0 = -\frac{i}{2} f_0 \left[g L^a_\mu - g' R^a_\mu \right] \tau^a$$

and

$$\begin{aligned} L_M &= -\tfrac{1}{4} Tr \left[(D_\mu \Phi)^\dagger D_\mu \Phi \right] = \\ &= -\tfrac{1}{8} f_0^2 \left(g L^a_\mu - g' R^a_\mu \right)^2 \end{aligned}$$

If we write

$$g' = gs/c$$

where s and c are the sine and cosine of some angle θ_w then:

$$L_M = -\frac{1}{8} f_0^2 \frac{g^2}{c^2} \left(c L^a_\mu - s R^a_\mu \right)^2$$

This shows that there are three field combination $cL - sR$ that obtain a mass while the field combinations

$$s L^a_\mu + c R^a_\mu$$

remain massless.

Identifying the vector bosons of weak interactions with the above is pretty obvious. The three massive combinations must be taken. The photon can then be identified with one of the three massless combinations, for which we take the third component. There are no other massless candidates in nature, so we must assume that R^1_μ and R^2_μ are non-existent. That implies a breaking of the $SU(2)$-right symmetry.

The result is the following identification:

$$L^a_\mu = B^a_\mu \qquad\qquad R^3_\mu = B^0_\mu$$

The mass terms are now:

$$\begin{aligned} \frac{1}{2} M^2 \left(B^1_\mu \right)^2 &- \frac{1}{2} M^2 \left(B^2_\mu \right)^2 - \frac{1}{2} \frac{M^2}{c^2} \left(c B^3_\mu - s B^0_\mu \right)^2 \\ M^2 &= \frac{1}{4} f_0^2 g^2 \end{aligned}$$

Introducing the notation W^a_μ for the physically observed vector bosons we have:

$$W^1_\mu = B^1_\mu \ , \quad W^2_\mu = B^2_\mu \ , \quad W^3_\mu = c B^3_\mu - s B^0_\mu$$

and the photonfield:

$$A_\mu = sB^3_\mu + cB^0_\mu$$

In terms of the W the mass terms are

$$-\frac{1}{2}M^2\left(W^1_\mu\right)^2 - \frac{1}{2}M^2\left(W^2_\mu\right)^2 - \frac{1}{2}M_0^2\left(W^3_\mu\right)^2$$

with $M_0 = M/c$. Defining

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right)^2 \quad , \quad W^0_\mu = W^3_\mu$$

this becomes:

$$-M^2 W^+_\mu W^-_\mu - \frac{1}{2}M_0^2\left(W^0_\mu\right)^2$$

In the literature W^0_μ is often called the Z^0. In this model the neutral and charged vector boson masses are related.

The gauge symmetry of the model is as follows. The vector boson fields transform as (infinitesimal form):

$$B^a_\mu \rightarrow B^a_\mu + g\varepsilon_{abc}\Lambda^L_b B^c_\mu - \partial_\mu \Lambda^L_a$$

$$B^0_\mu \rightarrow B^0_\mu - \partial_\mu \Lambda^0$$

In terms of the matrices b_μ and c_μ as defined before we must chose $\tau^x = \tau^3$, and the above transformations correspond to:

$$b_\mu \rightarrow G b_\mu g^\dagger + \frac{1}{g} G \partial_\mu G^\dagger$$

$$c_\mu \rightarrow H c_\mu H^\dagger + \frac{1}{g'} H \partial_\mu H^\dagger$$

$$\begin{aligned} G &= \exp\left(-\frac{i}{2}g\Lambda^L_a\tau^a\right) \\ H &= \exp\left(-\frac{i}{2}g'\Lambda^0\tau^3\right) \\ g' &= gs/c \qquad\qquad s = \sin\theta_w \qquad\qquad c = \cos\theta_w \end{aligned}$$

4.4 Isospin

Referring to the section on symmetries of the σ-model it may be noted that the Λ-transformations leave the σ invariant. The Λ transformation is equivalent to a simultaneous Λ^L and Λ^R transformation with $\Lambda = \Lambda^L = -\Lambda^R$.

Consider now the model including vector bosons ignoring electromagnetism. This amounts to setting $s = \sin\theta_w = 0$ and $c = \cos\theta_w = 1$, while removing B^0_μ. Since there are no vector bosons R_μ associated with $SU(2)$-right we have a local $SU(2)$-left invariance (Λ^L) and a global $SU(2)$-right invariance (Λ^R). Because the σ is invariant under the simultaneous $\Lambda^L = -\Lambda^R$ transformation it follows that even after the substitution $\sigma = \sigma' + f_0$ this transformation leaves the Lagrangian invariant. This global invariance will be called isospin invariance. The isospin transformation properties of the various fields are (infinitesimal form):

$$\begin{aligned}
\sigma &\to \sigma \\
\varphi_i &\to \varphi_i + \varepsilon_{ijk}\Lambda_j\varphi_k \\
W^a_\mu &\to W^a_\mu + \varepsilon_{abc}\Lambda_b W^c_\mu
\end{aligned}$$

Note that, ignoring electromagnetism, the W fields are the same as the B- fields.

This isospin invariance has as immediated consequence that the masses of the three vector bosons are equal. Thus isospin symmetry is responsible for the relation between neutral and charged vector boson mass.

The addition of electromagnetism in the form of B^0_μ breaks the isospin symmetry because $SU(2)$-right is broken. Thus electromagnetism breaks isospin symmetry. Furthermore the mixing between B^3 and B_0 affects the neutral vector boson mass by a factor $\cos\theta_w$. This modifies the relation between the masses to [18]:

$$\rho \equiv \frac{M^2}{M_0^2 c^2} = 1 \ .$$

Radiative corrections due to e.m. interactions will change the values of ρ, as they break isospin. When considering the fermions, whose masses must also be generated using coupling to the σ field further isospin breaking effects will be introduced. Also these effects will, through radiative corrections, affect the value of the ρ-parameter.

4.5 Fermion masses, parity

There are two ways to assign a mass to the fermions:

- by hand, by simply adding a mass term to the Lagrangian
- through the Higgs system. Coupling the fermions to the σ field will, after the shift $\sigma \to \sigma + f_0$, give rise to a mass term.

It must now be noted that a mass-less fermion has two independent degrees of freedom, namely left and right handed polarized states. Any Lorentz transformation transforms these states into themselves. This is not true for massive fermions. One can then, by a Lorentz transformation, go to the rest system of a fermion, and then by means of a space rotation change the direction of polarization.

It follows then that if fermions are given a mass "by hand" then left and right handed fermions must be coupled in the same way to the vector bosons in order to have gauge invariance. Parity would be conserved, contrary to experiment. Thus parity non-conservation forces mass generation of the fermions through the Higgs mechanism.

Employing the σ-model as Higgs sector also the converse is true. If fermion masses are to be generated through the vacuum expectation value of the σ-field then necessarily parity is violated. This can be seen as follows.

Let there be given a fermion doublet $\psi_a, a = 1, 2$. The left and right handed fields ψ^L and ψ^R are defined by

$$\psi_a^L = \frac{1}{2}\left(1+\gamma^5\right)\psi_a$$
$$\psi_a^R = \frac{1}{2}\left(1-\gamma^5\right)\psi_a$$

Under Lorentz transformations they transform into themselves.

A mass term is of the form:

$$m\bar{\psi}\psi = m\left(\bar{\psi}^L\psi^R + \bar{\psi}^R\psi^L\right) ,$$

thus connecting ψ^L and ψ^R as expected. To generate such a mass term via the Higgs mechanism we must introduce a coupling between ψ^L, ψ^R and Φ. This coupling must be $SU(2)$-left invariant (the G-transformations), which is the gauge symmetry of the model. One cannot make bothe ψ^L and ψ^R to behave as doublets under $SU(2)$-left because one cannot make an invariant coupling with three doublets (ψ^L, ψ^R and Φ from the left). Thus necessarily either ψ^L or ψ^R must in fact be invariant under $SU(2)$-left. We will take this to be ψ^R.

Adding electromagnetism, i.e. invariance under Λ_3^R, is simple. Making ψ^R a doublet with respect to $SU(2)$-right it is then trivial to write a coupling that is both $SU(2)$-left and $SU(2)$-right invariant:

$$L_{fm} = -g_f\bar{\psi}_a^L\Phi_{ab}\psi_b^R \ + \ \text{h.c.} \ .$$

Since the full SU(2)-right is respected this coupling also respects isospin. If all fermion mass terms were of this form then in fact the masses within a doublet would be equal. This becomes obvious substituting for Φ its vacuum expectation value $f_0\delta_{ab}$:

$$L_{fm} \to -g_f f_0\left(\bar{\psi}_1^L\psi_1^R + \bar{\psi}_2^L\psi_2^R\right) \ + \ \text{h.c.}$$

The mass generated is determined by the arbitrary parameter g_f and the σ vacuum expectation value f_0.

However, full invariance under global SU(2)-right is not a requirement. Only the gauged $SU(2)$-left, and the gauged rotation around the third axis of $SU(2)$-right must be rigorously respected. Since

$$\exp\left(\frac{i}{2}\Lambda_3\tau^3\right)$$

commutes with τ_3 it follows that the fermion mass term

$$L_{fm} = -g_f \bar{\psi}_a^L \Phi_{ab} \left(1 + \eta\tau^3\right)_{bc} \psi_c^R + \text{h.c.}$$

is acceptable. The parameter η is arbitrary. If η is non-zero isospin is broken and the masses within a doublet are different, as can be seen by substituting f_0 for Φ:

$$\begin{aligned} L_{fm} \rightarrow & - g_f f_0 \bar{\psi}_b^L \left(1 + \eta\tau^3\right)_{bc} \psi_c^R \\ & = - g_f \left\{(1+\eta)\bar{\psi}_1^L \psi_1^R + (1-\eta)\bar{\psi}_2^L \psi_2^R\right\} \end{aligned}$$

The above reasoning suggest that under the $U(1)$ invariance of the standard model, which in the Higgs sector of the standard model relates to the transformation

$$\Phi \rightarrow \phi H^\dagger = \Phi \exp\left(\frac{i}{2} g' \Lambda^0 \tau^3\right)$$

the left handed fermion ψ^L is invariant while ψ^R transforms as a rotation around the third axis:

$$\psi^R \rightarrow \exp\left(-\frac{i}{2} g' \Lambda^0 \tau^3\right) \psi^R .$$

However, the above mass term has another manifest symmetry, whereby all ψ^R and ψ^L obtain the same phase factor:

$$\psi^L \rightarrow \exp\left(-\frac{i}{2} \tilde{\Lambda} \tau^0\right) \psi^L$$

$$\psi^R \rightarrow \exp\left(-\frac{i}{2} \tilde{\Lambda} \tau^0\right) \psi^R$$

One can actually identify the $U(1)$ invariance with any addition of the above, for example (infinitesimal form):

$$\psi^L \rightarrow \psi^L - \frac{i}{2} \lambda \Lambda^0 \psi^L$$

$$\psi^R \rightarrow \psi^R - \frac{i}{2} \lambda \Lambda^0 \psi^R - \frac{i}{2} g' \Lambda^0 \tau^3 \psi^R$$

with arbitrary λ.

It is interesting to check the coupling of the photon field A_μ to the fermions. The invariant Lagrangian is:

$$-\bar{\psi}^L D_\mu \psi^L - \bar{\psi}^R D_\mu \psi^R$$

$$\begin{aligned} D_\mu \psi^L &= \partial_\mu \psi^L + g b_\mu \psi^L - \frac{i}{2} \lambda B_\mu^0 \psi^L \\ D_\mu \psi^R &= \partial_\mu \psi^R - \frac{i}{2} \lambda B_\mu^0 \psi^R - \frac{i}{2} g' B_\mu^0 \tau^3 \psi^R \end{aligned}$$

The e.m. coupling emerges if we write

$$b_\mu = -\frac{i}{2} s A_\mu \tau^3 \qquad\qquad B^0_\mu = c A_\mu$$

with s and c as before, i.e. sine and cosine of the weak mixing angle. Inserting this:

$$D_\mu \psi^L \ : \qquad -\frac{i}{2} A_\mu (g s \tau^3 + \lambda c) \psi^L$$

$$D_\mu \psi^R \ : \qquad -\frac{i}{2} A_\mu (g' c \tau^3 + \lambda c) \psi^R$$

Remembering that $g' = gs/c$ we see that ψ^L and ψ^R couple identically to the photon field. As a consequence parity is conserved in e.m. interactions.

If we define the electric charge e as

$$e = gs = g \ \sin \theta_w$$

then the above expressions show that the two members of the isospin doublet differ by e in charge. The central charge of the doublet is determined by λ, a free parameter.

4.6 Discussion

Making the following basic assumptions:

i the gauge symmetry is $SU(2) \times U(1)$;

ii the Higgs sector is as the linear σ-model;

iii all masses, notably the fermion masses, derive from the Higgs mechanism;

iv fermions are either $SU(2)$ doublets or singlets;

then, as a consequence:

a there are three massive and one massless vector boson.

b there is a relation between the charged and the neutral vector boson masses: $\rho = M^2/M_0^2 c^2 \ = 1 \ + $ Radiative corr.

c parity is violated in weak interactions.

d parity is conserved in e.m. interactions.

Actually, assumption iv might be weakened. The point is that it is not easy to generate fermion masses using an $SU(2)$ doublet Higgs if the fermions are $SU(2)$ triplets or higher. In other words, to a large extend iv is probable a consequence of i-iii.

Considering the above an important question presents itself. To what extend are the conclusions a-d dependent on assumption ii, i.e. the choice of the σ-model for the Higgs sector?

5 Other Higgs systems

5.1 Higher representations [18]

The σ-model in K notation is most suitable as starting point of the discussion. There is then a complex doublet K transforming as follows:

$$K \to GK$$

with

$$G = \exp\left(-\frac{i}{2}\tau^a \Lambda^a\right) \quad , \qquad a = 1 - 3 \; .$$

The required $U(1)$ symmetry is simply a phase factor:

$$K \to \exp\left(-\frac{i}{2}\Lambda^0\right) K$$

Discussing higher multiplets is done most easily using a tensor notation.

Let there now be given an n-multiplet

$$K^{ij\cdots}$$

with K completely symmetric in all n upper indices. Every index transforms with G. The coupling to the vector bosons is given by:

$$(D_\mu K)^{ij\cdots} = \partial_\mu K^{ij\cdots} + g b^i_{\mu i'} K^{i'j\cdots} + g b^j_{\mu j'} K^{ij'\cdots} + \cdots$$

where b_μ is the vector boson matrix introduced before.

Let now K develop a vacuum expectation value. Assume $K^{11\cdots}$ to be a non-zero constant K_0 while all other components (at least one index 2) are zero. The tensor $D_\mu K$ is non-zero if all indices are 1, and in that case it is equal to:

$$(D_\mu K)^{11\cdots} = ng\left(-\frac{i}{2}\right) B^3_\mu K_0$$

The only other non-zero components are those that have one index equal to 2, the others 1. These components are all equal:

$$(D_\mu K)^{211\cdots} = (D_\mu K)^{121\cdots} = g\left(-\frac{i}{2}\right)\left(B^1_\mu + iB^2_\mu\right) K_0$$

It follows that

$$(D_\mu K)^\dagger (D_\mu K) = \frac{n^2 g^2}{4} B^3_\mu B^3_\mu K_0^2 + \frac{ng^2}{4}\left\{\left(B^1_\mu\right)^2 + \left(B^2_\mu\right)^2\right\} K_0^2$$

Only if $n = 1$ (the doublet case) are the masses equal. In the general case, if the charged boson mass is M then the neutral vector boson mass is $M\sqrt{n}$.

Introducing the $U(1)$ symmetry and the vector boson B^0_μ produces the mass terms as above, except B^3_μ is replaced by $\left(cB^3_\mu - sB^0_\mu\right)/c$ as before. Thus the photon is also massless here (there is no massterm involving $sB_\mu^3 + cB_\mu^0$), and the neutral boson mass gets a mixing factor c.

The case shown above is not the general case. The reason is that one cannot in general bring the vacuum expectation value into a form in which only the component with all indices one is non-zero. The group $SU(2) \times U(1)$ is not big enough for that. To illustrate this consider the case of a two tensor. In all generality one can write:

$$K^{ij} = (u^a\tau^a + iv^a\tau^a)^i_k\,\varepsilon^{jk} \quad , \qquad a = 1,2,3$$

where u and v are two real three dimensional vectors.

Multiplying K by a phase factor $\cos\alpha + i\sin\alpha$ results in a transformation of u and v:

$$u \to u\cos\alpha - v\sin\alpha$$

$$v \to u\sin\alpha + v\cos\alpha$$

The angle α may be chosen such that the new u and v are orthogonal. Next, noting that $SU(2)$ transformations of K correspond to an $O(3)$ transformation of u and v one can arrange things such that u and v are of the form

$$u = (a, 0, 0) \quad v = (0, b, 0)$$

This is the best one can do. With such an u and v the tensor K takes the form:

$$K^{11} = a + b \quad K^{22} = -a + b \quad K^{21} = K^{12} = 0$$

Considering now $D_\mu K$ as shown above but with an additional $U(1)$ piece

$$-ig'B_\mu^0 K^{ij}$$

one may now compute $(D_\mu K)^\dagger(D_\mu K)$ for the tensor K shown above, with the result

$$\begin{aligned}(D_\mu K)^\dagger(D_\mu K) &= 2g^2b^2B_1^2 + 2g^2a^2B_2^2 + 2g^2(a^2+b^2)B_3^2 \\ &+ 2g'^2(a^2+b^2)B_0^2 + 8gg'abB_0B_3\end{aligned}$$

The notation $B_a \equiv B_\mu^a$ has been used.

There is mixing between B_0 and B_3. The mass values of the mixtures are obtained by considering the eigenvalues of the matrix

$$\begin{pmatrix} g^2(a^2+b^2) & 2gg'ab \\ 2gg'ab & g'^2(a^2+b^2) \end{pmatrix}$$

Let us call the eigenvalues m_1^2 and m_2^2. They are non-zero in general. Thus there are then four massive vector bosons, with masses g^2b^2, g^2a^2, m_1^2 and m_2^2. There being no mass zero vector boson one must conclude that in this case the photon obtains a mass.

There are some cases in which a zero mass value obtains, namely if a or b are zero, or if $a = b$. In the latter case the determinant of the above matrix is zero. A zero value for the constant g' also gives a zero mass. In such a case the $U(1)$ boson

B^0_μ is not coupled to the Higgs system; no mixing between B^0_μ and B^3_μ results, and the would-be photon $A_\mu = B^0_\mu$ will not even couple to the vector bosons.

There are many other difficulties with such a Higgs sector. The higher multiplets cannot be used to generate fermion masses. Thus masses need then to be put by hand, implying parity conservation for the $SU(2)$ interactions.

The only way out is to introduce a Higgs doublet in addition. By arranging things such that both the doublet and the higher multiplet(s) have a vacuum expectation value one can achieve a model with parity violation and without a specific relation between neutral and charged vector boson masses. There is no guarantee that the photon remains massless.

Another dangerous pitfall is the occurrence of massless physical Higgs bosons. Often in models with more than one Higgs multiplet there will be more Goldstone bosons then needed (i.e. more than 3). The reason is the occurrence of several $U(1)$ symmetries [18]. This problem occurs also if more than one doublet is used. The notorious axion is an example of such a Goldstone. In that case two Higgs doublets are used to eliminate strong CP violation [19].

5.2 Additional doublets

A possibility that cannot be ruled out sofar is to have two or more σ-model type Lagrangians. Each σ-model would produce masses satisfying the isospin relation $\rho = 1$. This of course assuming that all models have the same $g' = gs/c$, which is not dictated by anything. Only if $SU(2)$-right is really a gauged symmetry (with somehow two of the vector bosons corresponding to that hidden) then the coupling constants g' would have to be the same.

Consider now the case of two Higgs doublets. Let us assume that both have the same g', i.e. that they have the same properties under $SU(2)$-right. Under this assumption the most general renormalizable Higgs Lagrangian can be written down. The K-notation will be used:

$$\begin{aligned} L = & - (D_\mu K)^\dagger)(D_\mu K) - (D_\mu L)^\dagger(D_\mu L) + A(K^\dagger K) + B(L^\dagger L) + C(K^\dagger K)^2 \\ & + D(L^\dagger L)^2 + E(K^\dagger K)(L^\dagger L) + F(K^\dagger L)(L^\dagger K) + A'(K^\dagger L)(K^\dagger L) \\ & + B'(K^\dagger K)(K^\dagger L) + C'(K^\dagger L)(L^\dagger L) + \text{ h.c.} \end{aligned}$$

The part h.c. stands for the hermitean conjugate of those terms that are not hermitean by themselves. The terms $A' - C'$ are allowed only if L and K have the same properties under $U(1)$. The coefficients $A - F$ must be real, and $A' - C'$ may be complex.

A model of this type may generate T-violation in two distinct ways. First, trivially, if the coefficients $A' - C'$ are not real then we may expect T-violation, whose manifestation in the observed data would probably very small as it requires the explicit involvement of both doublets. Secondly, the system may develop spontaneous time reversal breakdown, i.e. a non-real vacuum expectation value for one of the doublets [20].

All these matters are quite speculative, and not particularly enlightening. However, there is the problem of strong CP violation due to instantons [21]. Such CP

violation, in all probability much larger than allowed by the data, can be suppressed using a Higgs system with two doublets, with two independent (but otherwise identical!) $U(1)$ symmetries. That would be as the above Lagrangian with the coefficients $A' - C'$ zero. Such a Lagrangian will generate a zero mass physical Goldstone, that however would acquire a small mass due to instanton effects.

Here the interest is on the vector boson masses. Thus consider the vacuum expectation values of K and L. By a suitable $SU(2)$-left transformation the second component of K may be made zero, and a subsequent $U(1)$ transformation makes that component real. This is of course as before, in the single doublet case. Note that both symmetries are valid including the vector bosons.

At this point no specific form will be assumed for the vacuum expectation value of L. Thus the vacuum expectation values of K and L are taken to be:

$$K_0 = \begin{pmatrix} s_0 \\ 0 \end{pmatrix} \qquad L_0 = \begin{pmatrix} a \\ b \end{pmatrix}$$

with real s_0 and complex a and b. Now consider $D_\mu K$ and $D_\mu L$ defined as before:

$$D_\mu K = \partial_\mu K + g b_\mu K - \frac{i}{2} g' B^0_\mu K$$

with as usual the 2×2 matrix b:

$$b_\mu = -\frac{i}{2} B^a_\mu \tau^a$$

and similarly for L. Notably the same g' is used, implying identical $U(1)$ properties for K and L. Inserting now the expressions for K_0 and L_0 gives:

$$\begin{aligned}(D_\mu K_0)^\dagger D_\mu K_0 + (D_\mu L_0)^\dagger D_\mu L_0 &= \frac{1}{2}\Big\{ M^2 B_1^2 + M^2 B_2^2 + M^2 B_3^2 + \frac{g'^2}{g^2} M^2 B_0^2 \\ &+ B_0 B_3 \left(2 \frac{g'}{g} M^2 - 2g' g b b^* \right) \\ &+ g g' B_0 B_1 (a b^* + a^* b) + i g g' B_0 B_2 (a b^* - b^* a) \Big\}\end{aligned}$$

where

$$M^2 = \frac{1}{2} g^2 \left(a a^* + b b^* - s_0^2 \right)$$

The vector boson mass matrix is:

$$\begin{pmatrix} M^2 & 0 & 0 & a_1 \\ 0 & M^2 & 0 & a_2 \\ 0 & 0 & M^2 & a_3 \\ a_1 & a_2 & a_3 & M'^2 \end{pmatrix}$$

where

$$a_1 = \frac{1}{2} gg'(ab^* + a^*b)$$
$$a_2 = \frac{1}{2} igg'(ab^* - a^*b)$$
$$a_3 = \frac{g'}{g} M^2 - g'gbb^*$$
$$M' = \frac{g'}{g} M$$

The eigenvalue equation corresponding to this mass matrix is:

$$(M^2 - \lambda)^2 \left\{ \lambda^2 - \lambda M^2 \left(1 + \frac{g'^2}{g^2} \right) + g^2 g'^2 bb^* s_0^2 \right\} = 0$$

There are two eigenvalues M and two other eigenvalues, generally non-zero. Thus the two doublet model will in general produce a photon with non-zero restmass. Only the special case $b = 0$ (or $s_0 = 0$) produces one zero vector boson mass.

This must be considered a severe drawback of the two doublet model.

To counteract strong CP violation the Higgs Lagrangian must contain two doublets with two independent $U(1)$ invariances. Thus terms involving $(K^\dagger L)$ or $(L^\dagger K)$ must not appear (these are the terms with coefficients $A' - C'$ in the Lagrangian is the beginning of this sector). Otherwise the $U(1)$ properties of K and L are taken to be the same. Again, in general, a non-zero photon mass results; let us however assume that by accident there is a zero vector boson mass, i.e. assume that the vacuum expectation values of K and L are of the form

$$K_0 = \begin{pmatrix} s_0 \\ 0 \end{pmatrix} \qquad L_0 = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

with in general complex a. Since now $SU(2)$-left and two $U(1)$ symmetries are spontaneously broken there are in general 3 + 1 + 1 Goldstone's, which is one more then can be absorbed in the vector boson system. One, the axion, remains as a physical particle.

The way to solve such a system is as follows. Substitute in the Lagrangian the shifted multiplets:

$$K = \begin{pmatrix} s_0 + \sigma + i\varphi_3 \\ -\varphi_2 + i\varphi_1 \end{pmatrix} \qquad L_0 = \begin{pmatrix} a + \sigma' + i\varphi_3' \\ -\varphi_2' + i\varphi_1' \end{pmatrix}$$

The resulting Lagrangian will have terms linear in the fields (tadpole terms). The constants s_0 and a are then to be fixed by the requirement that all tadpole terms disappear. Here:

$$s_0^2 = (2AD - BE - BF)/\mathrm{Det}$$
$$|a|^2 = (2BC - AE - AF)/\mathrm{Det}$$
$$\mathrm{Det} = (E + F)^2 - 4CD$$

The vector boson masses are not under consideration here, and the invariance under $U(1)$ applied to L may be used, i.e. a may be taken to be real. It is then easy to see that neither φ_3 or φ'_3 has a mass term. Some mixture of φ_1 and φ'_1 as well as some mixture of φ_2 and φ'_2 has no mass term either.

It is noteworthy that if $F = 0$ then the $SU(2)$ invariance of K and L are independent. In that case all φ and φ' turn out to be massless, i.e. there are then six Goldstone's.

5.3 Discussion

The use of higher Higgs multiplets, or more than one doublet has the immediate disadvantage that in general no zero mass vector boson remains. In other words, the observed zero photon mass is then an "accident". For this reason alone these schemes are very unattractive. In addition the experimentally observed near equality $\rho \simeq 1$ shows no indication of any complications in the Higgs sector.

It is therefore our view that CP violation is not to be attributed to the use of more than one Higgs multiplet. Also the axion idea must be rejected. Indeed, no such thing has ever been seen.

6 Higgs hunting

6.1 Introduction

One fact clearly emerges from the preceeding discussion: the σ-model as Higgs sector explains several striking facts, and any complication of that sector tends to weaken the model. Therefore we will stick to that model. The outstanding question is then: is this σ-model truly realized in nature, or is it a temporary approximate description of a much more complicated situation? To put this question in more practical terms: what is the mass of the Higgs boson?

It may well be that at some time in the future the Higgs will be produced and observed in some experiment. This may happen if its mass is not too high, say less than 300 GeV. A critical part of such a discovery would be to establish that the particle couples to all massive particles with a strength proportional to the mass of those particles. An extensive discussion of this possibility exists in the literature, and will not be presented here.

Another way to search for the Higgs is through radiative corrections. This is not an easy way. Probably such radiative corrections will be presented in terms of a few numbers, and those numbers present a collective of many effects of which the Higgs effect is only one. For example, radiative corrections to the ρ-parameter include sizable effects due to the fermions, notably the top quark. The value of the top quark mass needs to be known with some precision or else the uncertainty in that will be as large as a possible Higgs induced contribution. It may well be that q.c.d. effects must be taken into account. It is quite conceivable that for some time the ρ-parameter will be our only view on the Higgs sector. Very probably more detailed calculations need then to be made.

Since without the Higgs the theory is non-renormalizable it is to be expected that certain radiative corrections become large for large Higgs mass. Measuring such corrections might be a way to establish an upper limit on the Higgs mass. Unfortunately, it has been shown that without a Higgs the theory is still renormalizable, or rather quasi-renormalizable, including one loop radiative corrections [22]. Thus one loop effects are not likely to explode for large Higgs mass. This has been advocated under the name of "screening". In practice this amounts to the fact that one loop radiative corrections grow at most like $\ell n(m^2)$ for large m [23]. The essentials of that investigation will be shown in the next section.

Finally the absence of the Higgs may be studied directly by considering vector-boson scattering. This amplitude grows with energy untill Higgs exchange comes into effect, i.e. untill the energy is comparable to the Higgs mass. The keyword "unitarity limit" is often used in this context [14]. Some aspects of that question will be discussed in this chapter.

6.2 Screening. Radiative corrections

In this section we will concentrate on those radiative corrections that become large with large Higgs mass. In other words, the interest is on the leading effects in the limit of large Higgs mass m.

At this point the precise Feynman rules of the standard model are needed. They can be found in [24]. For most purposes in this section we can do with the simplified standard model, which is the standard model but without electromagnetism, i.e. $\sin\theta_w = 0$. The Feynman rules for that model are given in appendix A. They include interactions of the Higgs with a hypothetical scalar particle U, introduced in order to detect sensitivity to such particles.

If the Higgs mass becomes large certain interactions become strong, i.e. factors associated with certain vertices become large. The factor $r = m^2/4M^2$ in appendix A carries that dependence. Note that only vertices involving either the Higgs or the Higgs ghosts have such a factor (the U will not be considered here). Therefore these vertices will not occur in tree diagrams or one loop diagrams having only vector bosons or fermions as external lines. That is already a major part of the aforementioned screening.

The only way then that Higgs mass dependence occurs in one loop diagrams is through the Higgs propagator. We must therefore analyze the structure of the expressions corresponding to diagrams involving one or more Higgs particles.

A general one loop diagram gives rise to an integral of the form:

$$\int d_4q \, \frac{(q^2)^\lambda}{(q^2+M_1^2)\left((q+k_1)^2+M_2^2\right)\cdots\left((q+k_i)^2+m^2\right)\left((q+k_j)^2+m^2\right)\cdots}$$

Using Feynman's trick the non-Higgs propagators can be taken together and also the Higgs propagators seperately. The integral obtained will be of the form:

$$\int d_4q \, \frac{(q^2)^\lambda}{(q^2+M^2)^j\left((q+k)^2+m^2\right)^k}$$

where M^2 is some linear combination of masses other than the Higgs mass. Strickly speaking one has a linear combination of dotproducts of external momenta in addition to m^2 in the above, but as we are interested in effects for large m^2 they have been ignored. It is thus assumed that all external momenta are small with respect to m.

Writing

$$q^2 = (q^2 + M^2) - M^2$$

as much as needed one can eliminate the q^2 dependence in the numerator. We therefore can limit ourselves to an integral of the form

$$\int d_4 q \, \frac{1}{(q^2 + M^2)^j \, ((q+k)^2 + m^2)^k}$$

Next we may expand:

$$\begin{aligned} \frac{1}{(q+k)^2 + m^2} &= \frac{1}{q^2 + m^2 + 2qk + k^2} \\ &= \frac{1}{q^2 + m^2} \left\{ 1 - \frac{2qk + k^2}{q^2 + m^2} + \left(\frac{2qk + k^2}{q^2 + m^2} \right)^2 + \cdots \right\} \end{aligned}$$

It follows that we can limit ourselves to the study of an integral of the type

$$\int d_4 q \, \frac{1}{(q^2 + M^2)^j (q^2 + m^2)^k}$$

The above expansion is valid if such integrals become small for sufficiently large k, which happens to be the case for large m.

Integrals of this type can be readily calculated. The leading behaviour can be estimated easily. In general, if $j > 2$ then the integral behaves like $(m^2)^{-k}$. In other words, if the integral is convergent without the Higgs propagators one may make the approximation $q^2 + m^2 \sim m^2$. If $j = 2$ the integral will be logarithmically divergent if no Higgs propagators were present; the leading behaviour is like $(m^2)^{-k} \ell n(m^2)$.If $j = 1$ the behaviour is like $(m^2)^{-k+1}$ with possibly a factor $\ell n(m^2)$. If $j = 0$ one has (using dimensional regularization):

$$\begin{aligned} \int d_n q \, \frac{1}{(q^2 + m^2)} &= i\pi^2 \left\{ \frac{2m^2}{n-4} - m^2 + m^2 \ell n(m^2) \right\} \\ \int d_n q \, \frac{1}{(q^2 + m^2)^2} &= i\pi^2 \left\{ -\frac{2}{n-4} - \ell n(m^2) \right\} \\ \int d_n q \, \frac{1}{(q^2 + m^2)^3} &= \frac{1}{2} \, \frac{i\pi^2}{m^2} \end{aligned}$$

The general rule is this: consider the dimensionality of the integral. That determines the highest power of m^2 that can appear. Thus integrals with the dimension of a $(\text{mass})^2$ behave maximally like m^2. These are quadratically divergent integrals ($j = 0$, $k = 1$ in the above). Dimensionless integrals (logarithmically divergent) behave like $\ell n(m^2)$. The rest goes to zero as $m^2 \to \infty$. For completeness here the result for $j = k = 1$:

$$i\pi^2\left\{-\frac{2}{n-4}+1+\frac{1}{m^2-M^2}\left(M^2\ell n(M^2)-m^2\log\ell n(m^2)\right)\right\}$$

The conclusion is quite simple. For any expression involving Higgs propagators consider only logarithmic and quadratic divergent parts. They behave like $\ell n(m^2)$ and m^2.

Since m^2 and $\ell n(m^2)$ occur associated with (unobservable) divergencies it is not obvious that these terms can be seen. The obvious place to look is for differences (ratio's) of such expressions where the divergencies cancel out but hopefully some m^2 or $\ell n(m^2)$ remains. The only quadratically divergent integrals occur for vector boson self energies. Therefore the ρ-parameter, essentially the ratio of vector boson masses, is the obvious candidate for large Higgs mass dependence. Unfortunately, while m^2 terms appear in the vector boson self-energies they cancel out in the ratio. Only $\ell n(m^2)$ remaines. That is then the final part of the screening statement: no observable m^2 dependence in any one-loop radiative correction.

The $\ell n(m^2)$ correction to the ρ-parameter can be calculated. It is [24]:

$$\rho=1-\frac{3GM^2}{8\pi^2}\frac{s^2}{c^2}\ell n\ \frac{m^2}{M^2}$$

where M = charged vector boson mass and G is the fermi coupling constant

$$G=\frac{1.02\times10^{-5}}{\sqrt{2}m_p^2}\qquad m_p\ \text{proton mass.}$$

As usual, s and c are the sine and cosine of the weak mixing angle. Even with $m\sim 3000$ GeV the correction is only 0.4%.

Since at this point no other mass ratio's are predicted by the standard model there is no other possibility for m^2 dependence.

Other divergent integrals occurring in the standard model arise in vertex type diagrams. Since all vertices basically involve the same coupling constant a $\ell n(m^2)$ dependence may be observed by comparing vertices. This indeed occurs, and comparing the three vector boson vertex with a vector boson- fermion vertex (as occurring for example in μ-decay) shows an observable $\ell n(m^2)$ correction. Such effects are part of the radiative corrections to $e^+e^-\to WW$ [23].

Analyzing two loop integrals is much more complicated than the one loop effects. As a general rule it appears that if $\ell n(m^2)$ occurs at the one loop level for some observable quantity, then m^2 dependence occurs in two loop effects [17]. If m is of order of 1000 GeV such two-loop effects are of the same order as the one-loop effects. Perturbation theory becomes useless. Hence the statement that one has strong interactions if the Higgs is heavier than 1 TeV = 1000 GeV. At the same time strong interactions of a somewhat different type occur in vector boson scattering. That is the subject of the next section.

6.3 Unitarity limit

In a renormalizable theory amplitudes must not grow indefinitly as a function of the momenta of the particles. The standard model including the Higgs system is renormalizable, and amplitudes behave as should be. Without the Higgs the theory is non-renormalizable, and indeed certain amplitudes, at the tree level, do not behave correctly in this sense. For a finite but large Higgs mass such amplitudes behave badly up till energies of the order of that mass. For still higher energies behaviour as in a renormalizable theory re-appears. Thus certain amplitudes reach a maximum for energies of the order of the Higgs mass. If, at the tree level, such amplitudes become of order one then that generally implies a breakdown of perturbation theory. One loop diagrams, that can sloppily be understood as the product of two tree diagrams, become of the same order of magnitude as the tree diagrams.

The elastic scattering amplitude for longitudinally polarized vector bosons is perhaps the most instructive, and also most practical case. Considering the dependence on s, the centre off mass energy squared, the following happens:

- Individual diagrams grow as s^2.
- The Yang-Mills structure of the tree and four vector boson vertices leads to certain cancellations, and growth as s results.
- Diagrams involving Higgs exchange give further cancellations, and the total behaves as a constant.

It is interesting to note that absence of a Yang-Mills structure can be observed quite before a missing Higgs. That aspect will not be discussed any further here.

First the tree-level situation will be considered. We will restrict ourselves to the simplified model with the Feynman rules as in appendix A. Also no fermions are considered. Then isospin invariance holds, which simplifies things considerably.

Thus consider two ingoing longitudinally polarized vector bosons in the centre of mass. The momenta and polarization vectors are taken to be:

$$k = (0,0,k_\ell,ik_0) \qquad p = (0,0,-k_\ell,ik_0)$$
$$e(k) = \tfrac{1}{M}(0,0,k_0,ik_\ell) \quad e(p) = \tfrac{1}{M}(0,0,-k_0,ik_\ell)$$

There are two outgoing longitudinally polarized vector bosons. By convention momenta are always taken to be ingoing and we write:

$$k' = -\left(k_\ell \sin\theta, 0, k_\ell\cos\theta, ik_0\right)$$

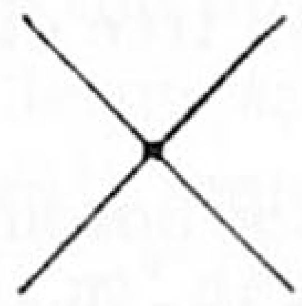
$$p' = -\left(-k_\ell \sin\theta, 0, -k_\ell\cos\theta, ik_0\right)$$

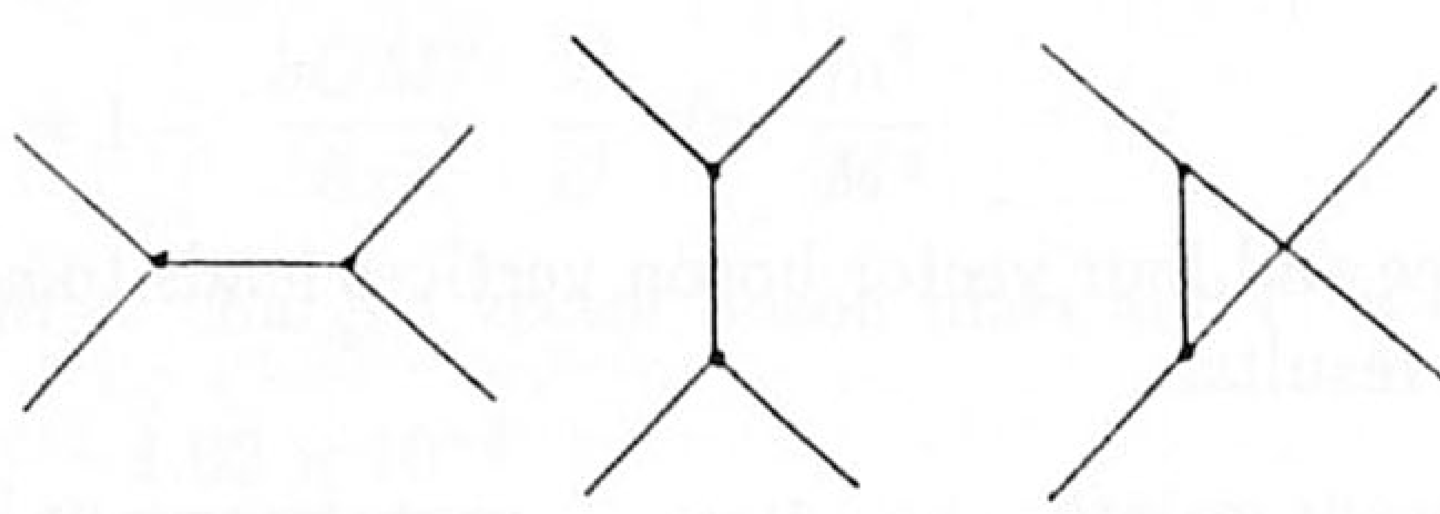
$$e(k') = -\frac{1}{M}\left(k_0\sin\theta, 0, k_0\cos\theta, ik_\ell\right)$$

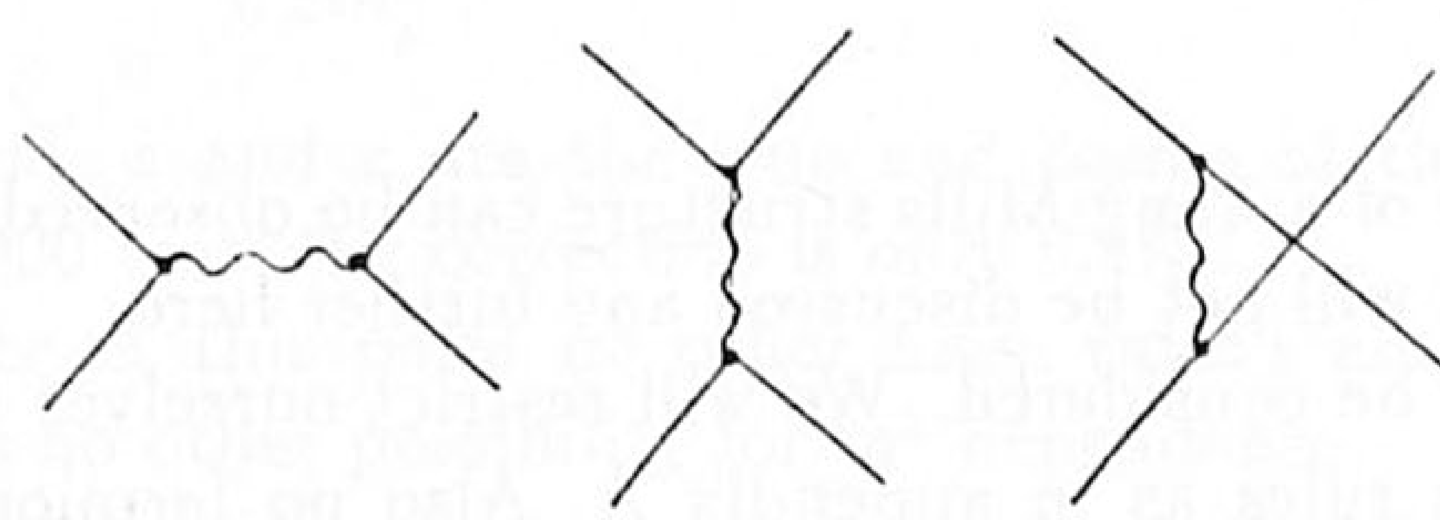
$$e(p') = -\frac{1}{M}\left(-k_0\sin\theta, 0, -k_0\cos\theta, ik_\ell\right)$$

The angle θ is the scattering angle in the centre of mass system. The coordinate system has been chosen such that the incoming vector bosons have momenta along the third axis while the outgoing bosons are in the 1-3 plane.

Obviously the individual components of the polarization vectors can become very large for large $s = 4k_0^2$. Roughly speaking the polarization vectors behave like $\sqrt{s}$. Consider now the Feynman rules given in appendix A. The $WWWW$ vertex is momentum independent, as is the WWH vertex, while the WWW vertex is linear in the momenta. A propagator behaves as $1/s$ (or $1/t$ or $1/u$). It is easily seen then that

i  behaves as s^2

ii  behaves as s^2

iii  behaves as s

Since the total must (and indeed does) behave as a constant for large s we deduce that the s^2 behaviour of diagrams i and ii must cancel, while the remaining behaviour proportional to s must cancel against that of diagrams iii.

This is precisely what happens. Denoting the isospin states of the vector boson with momentum k by a (simarly b to p, c to p' and d to k') the leading behaviour of diagram i is:

$$\text{i}: \quad \delta_{ab}\delta_{cd}\left(\frac{1}{2}st - \frac{1}{4}s^2 + \frac{1}{2}t^2\right)/M^2 + O(s)$$

There are terms proportional to $\delta_{ac}\delta_{bd}$ and $\delta_{ad}\delta_{bc}$, but those are not shown. A factor $g^2(2\pi)^4 i$ has been omitted. Diagrams i and ii combined show the behaviour:

$$\text{i} + \text{ii}: \quad \delta_{ab}\delta_{cd}\,\frac{s}{4M^2}$$

Finally Higgs exchange:

$$\text{iii}: \quad \delta_{ab}\delta_{cd}\,\frac{s^2}{4M^2}\cdot\frac{1}{-s+m^2}$$

Developing the Higgs propagator:

$$\frac{1}{-s+m^2} = -\frac{1}{s} - \frac{m^2}{s^2}$$

and noting that there is no other m^2 dependence the total result obtains

$$\underline{\text{i}} + \underline{\text{ii}} + \underline{\text{iii}} = -i(2\pi)^4 g^2 \delta_{ab}\delta_{cd}\,\frac{m^2}{4M^2} + (m - \text{independent constant})$$

The unitarity limit of ref. [14] is based on this constant. Roughly speaking, if

$$g^2\frac{m^2}{4M^2} \sim 1$$

then higher order effects become important.

Considerations concerning the unitarity limit show the precise moment where the tree level amplitude is larger than allowed by unitarity. But in general no one takes the trouble to say by how much, i.e. how large higher order effects must be in order to restore unitarity. Here we refer to the fact that the complete theory is unitary by construction. Furthermore, even if tree level unitarity is not violated it still might be that higher order corrections are sizable. Therefore a calculation of the one loop radiative corrections to this amplitude is of interest. This calculation has been done in the limit $M^2 \ll s,t,u \ll m^2$ and the result is [26]:

$$\begin{aligned}\text{One loop :}\quad & \frac{i\pi^2 g^4}{M^4}\,\delta_{ab}\delta_{cd}\left\{-\frac{st}{36} - \frac{t^2}{36} + \frac{5s^2}{144}\right.\\ +\quad & \frac{9s^2}{32}\,B_0 - \frac{s^2}{32}\,\ell n\frac{s}{m^2} - \left(\frac{st}{96}+\frac{t^2}{48}\right)\ell n\frac{t}{m^2}\\ -\quad & \left.\left(\frac{su}{96}+\frac{u^2}{48}\right)\ell n\left(\frac{u}{m^2}\right)\right\}\\ +\quad & \delta_{ac}\delta_{bd}\,\{s\to u,\ t\to s,\ u\to t\}\\ +\quad & \delta_{ad}\delta_{bc}\,\{s\to t,\ t\to u,\ u\to s\}\end{aligned}$$

with $B_0 = \pi/\sqrt{3} - 2$.
The terms involving $\ell n(m^2)$ have been reported before [27], [28]. In this last ref. also other approximations then the one considered here are discussed.

The ratio of the one-loop result to the tree level amplitude may now be considered. Keeping only terms containing logarithms one finds for the case $t = -s$ (thus $u \sim 0$):

$$R = -\frac{\alpha_w s}{\pi M^2}.\frac{1}{24}\ell n\frac{s}{m^2}$$

for the $\delta_{ab}\delta_{cd}$ channel. In here $\alpha_w = g^2/4\pi \simeq 1/30$. Again, this result has been deduced in the approximation $M^2 \ll s \ll m^2$. For $M \sim 80$ GeV, $s \sim (400\ \text{GeV})^2$ and $m \sim 2000$ GeV $= 2$ TeV one finds

$$R = 0.04$$

Thus while the Higgs mass is substantially above the unitarity limit of ref. [14] the effect is still quite small at 400 GeV. Note that the result of ref. [14] obtains in the limit $s \gg m^2$, which is a somewhat unrealistic limit to consider if the aim is to find a Higgs or to set a limit on the Higgs mass.

If the energy increases the ratio R increases quadratically with that energy. Thus at 800 GeV effects of order 0.16 can be expected.

The conclusion is therefore that if the Higgs mass is very heavy, i.e. well above 1 TeV, then very large deviations of the tree amplitude will become manifest for energies well above 500 GeV.

The question now is this: how can the WW scattering amplitude be measured. This process has been the subject of extensive research, reported in the literature, for example refs. [15] and [29] and references quoted therein.

6.4 Multi vector boson production

As the polarization vector of a longitudinally polarized vector boson has components growing linearly with energy it might be anticipated that amplitudes involving more and more vector bosons are ever so much rising in magnitude as a function of energy untill the energy is of the order of the Higgs mass. This turns out not to be the case, as will be discussed now.

As before the approximation $M \ll$ energies $\ll m$ will be understood. Moreover we will use the equivalence theorem to estimate the leading behaviour [30]. Only tree diagrams are considered.

Amplitudes with four, five etc. external Higgs ghost lines will be estimated. First the case of four external lines. There are 4 diagrams, in fact already shown in section 4.2. The crucial point now is to understand the mechanism whereby the leading behaviour in terms of s, t, u for large m^2 arises. Consider the first diagram, Higgs exchange in the s-channel. The corresponding expression is:

$$4r^2 M^2 \delta_{ab}\delta_{cd}\,\frac{1}{-s+m^2}\,, \qquad r = \frac{m^2}{4M^2}$$

Developing the propagator:

$$\frac{1}{-s+m^2} \simeq \frac{1}{m^2}\left(1+\frac{s}{m^2}+\cdots\right)$$

one obtains

$$\delta_{ab}\delta_{cd}\left\{\frac{m^2}{4M^2}+\frac{s}{4M^2}\right\}$$

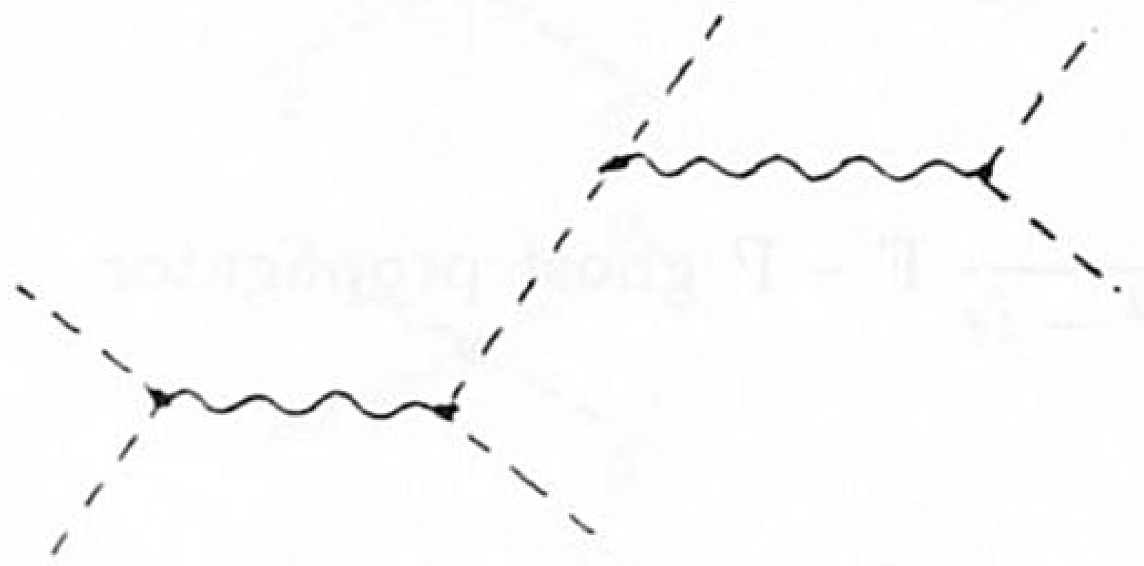
The first term cancels against a similar piece in the four φ vertex. Therefore the leading behaviour is

$$\delta_{ab}\delta_{cd}\,\frac{s}{4M^2}\,,$$

a result obtained before (diagrams i + ii of section 6.3). This in fact verifies the equivalence theorem for the four W case.

The crucial trick is to keep track of m^2 dependence. Counting factors r and Higgs propagators suffices to establish leading behaviour. All energy dependence derives from the expansion of the Higgs propagators. This procedure is precisely like the work involved going from the linear to the non-linear σ-model. That model is quite trivial in its energy dependence: all vertices having four or more external lines have at most a dependence as (energy)^2. Connecting two such vertices maintains that behaviour in view of an extra propagator behaving like (energy)$^{-2}$.

Consider now a process involving 6 external φ-lines. The maximum number of factors r involving the least number of Higgs propagators obtains using the $\varphi\varphi H$ vertex. A typical diagram:



The behaviour of this diagram for large m^2 is:

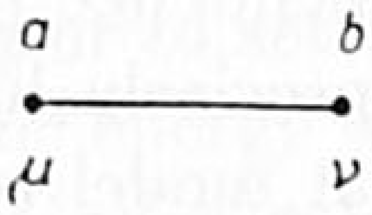
$$(r)^4 \cdot \left(\frac{1}{m^2}\right)^2 \simeq \frac{m^8}{m^4} = m^4$$

This implies behaviour like s^2. However, there is one φ-propagator, and the total behaviour is like s.

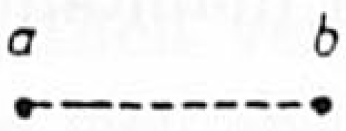
The same reasoning can be extended to diagrams with more external φ-lines. The general conclusion is that at the tree level diagrams with arbitrarely many φ-lines behave at most like (energies)^2. If the equivalence theorem holds, at least insofar as necessary to estimate the leading behaviour for large energy then the same holds for diagrams with arbitrarely many external longitudinally polarized vector bosons.

To verify the validity of the equivalence theorem insofar used here one must also establish leading behaviour for diagrams involving φ-lines and other lines, including W lines, but not longitudinally polarized. There is no particular difficulty following the method outlined above. Note that only longitudinally polarized W's give factors $\sqrt{s}$ associated with the external lines.

Appendix A: Feynman rules.

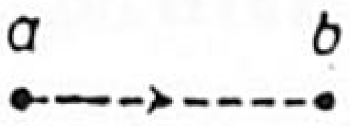


$$\frac{\delta_{ab}\delta_{\mu\nu}}{k^2 + M^2 - i\epsilon}$$ W-propagator



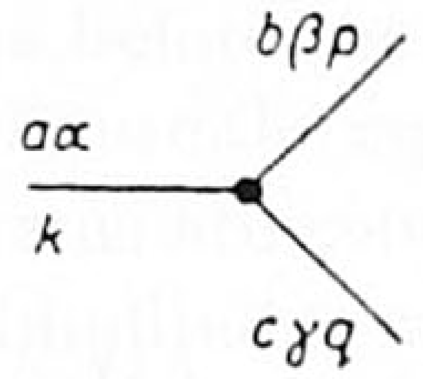
$$\frac{\delta_{ab}}{k^2 + M^2 - i\epsilon}$$ Higgs ghost propagator

$$\frac{1}{k^2 + m^2 - i\epsilon}$$ Physical Higgs propagator



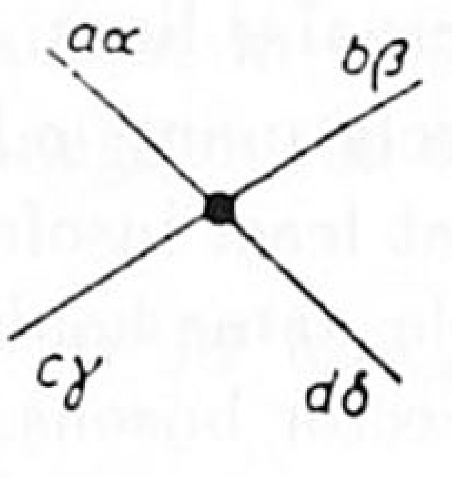
$$\frac{\delta_{ab}}{k^2 + M^2 - i\epsilon}$$ F - P ghost propagator

$$\frac{1}{k^2 + m_u^2 - i\epsilon}$$ U - particle propagator



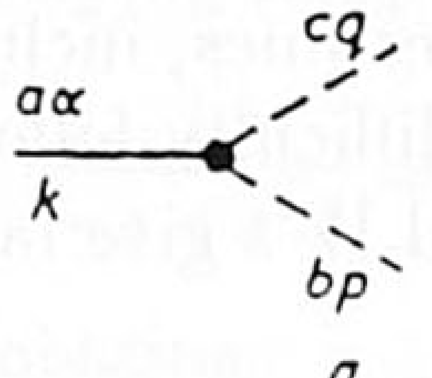
$$-ig\epsilon_{abc}\left\{\delta_{\alpha\gamma}(k-q)_\beta + \delta_{\beta\gamma}(q-p)_\alpha + \delta_{\alpha\beta}(p-k)_\gamma\right\}$$

Yang-Mills three W vertex

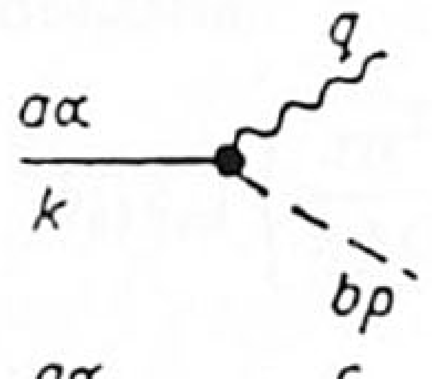


$$-g^2\Big\{\epsilon_{gdc}\epsilon_{gba}\Big(2\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta}\Big)$$
$$+\epsilon_{gdb}\epsilon_{gca}\Big(2\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\gamma\beta} - \delta_{\alpha\gamma}\delta_{\beta\delta}\Big)\Big\}$$

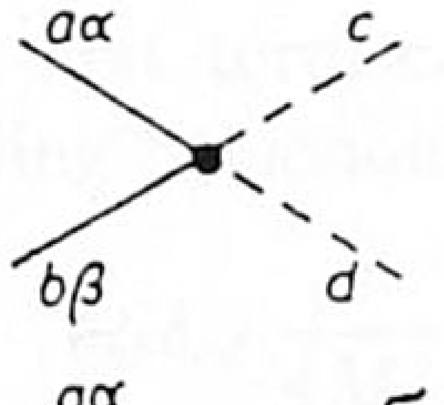
Yang-Mills four W-vertex



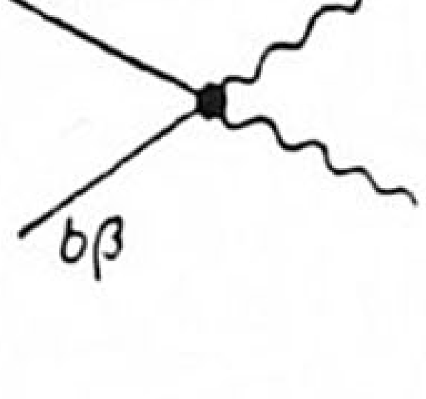
$$\frac{i}{2}g\epsilon_{abc}(p-q)_\alpha$$



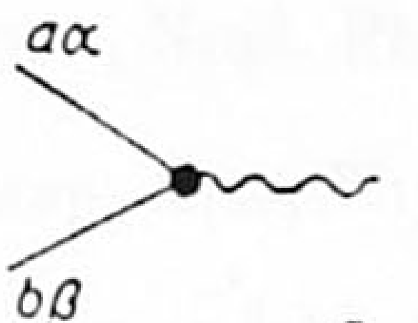
$$\frac{i}{2}g\delta_{ab}(p-q)_\alpha$$



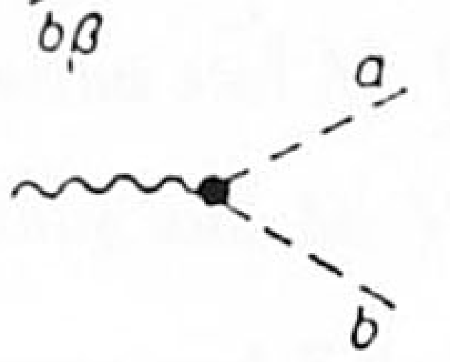
$$-\frac{1}{2}g^2\delta_{ab}\delta_{cd}\delta_{\alpha\beta}$$



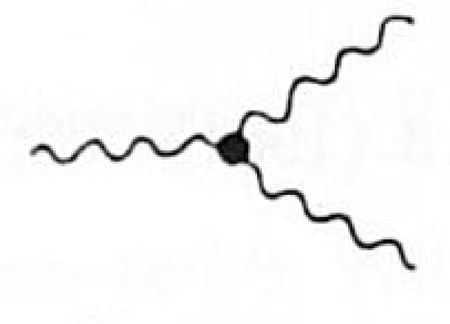
$$-\frac{1}{2}g^2\delta_{ab}\delta_{\alpha\beta}$$



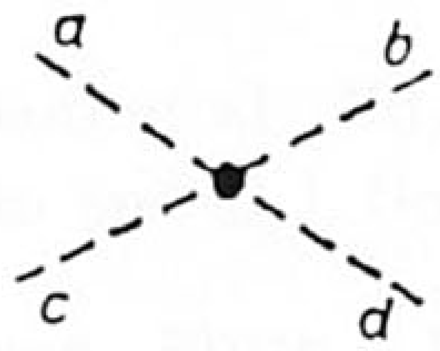
$$-g\,M\delta_{ab}\delta_{\alpha\beta}$$



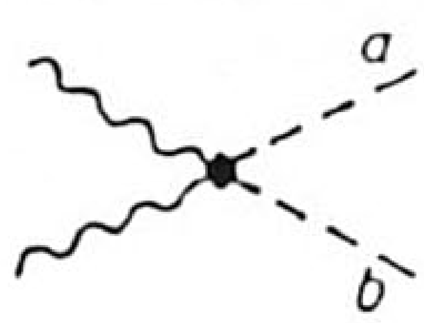
$$2rMg\delta_{ab}, \qquad r = \frac{m^2}{4M^2}$$



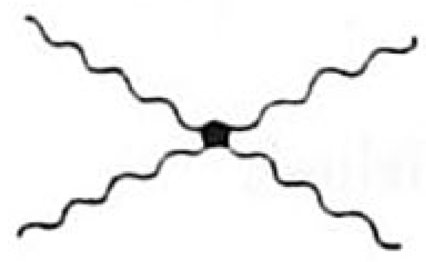
$$-6rMg$$



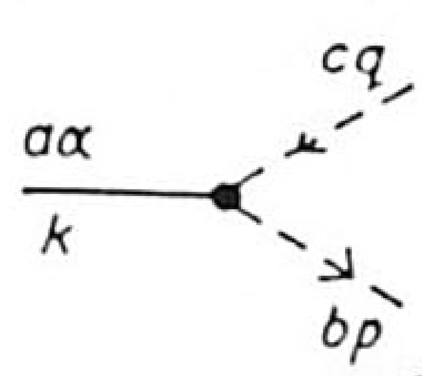
$$-rg^2(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$$



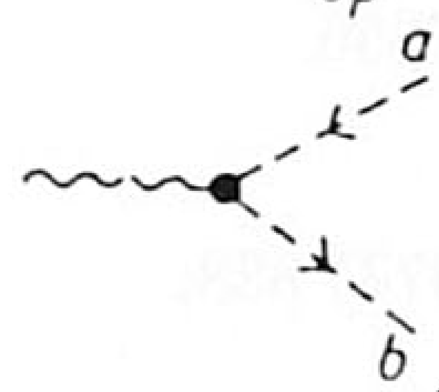
$$-rg^2\delta_{ab}$$



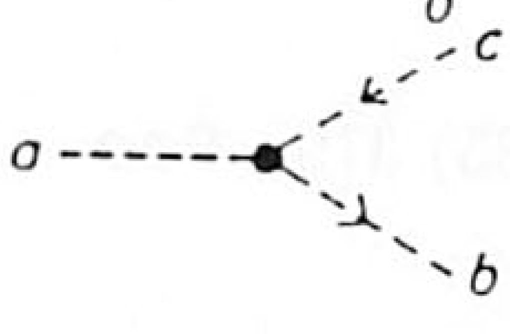
$$-3rg^2$$



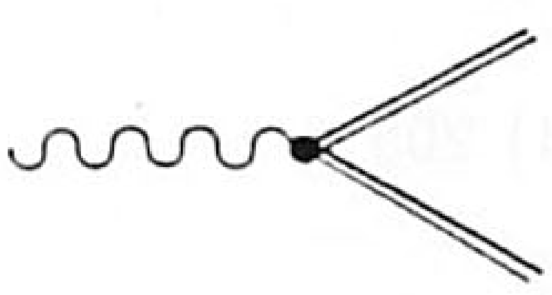
$$+i\ \ g\epsilon_{abc}p_\alpha$$



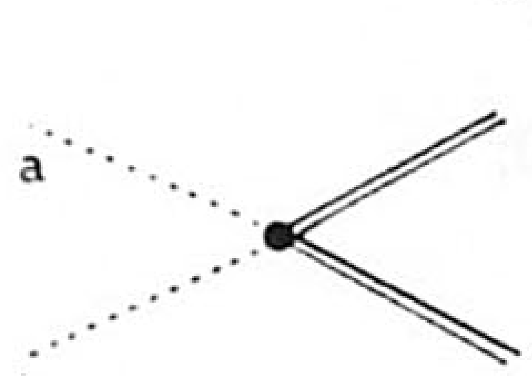
$$-\frac{1}{2}Mg\delta_{ab}$$



$$\frac{1}{2}Mg\epsilon_{abc}$$



$$-2rMg_ug$$



$$-r\delta_{ab}g_ug^2$$

$$-rg_ug^2, \qquad r = \frac{m^2}{4M^2}$$

References

[1] J. Schwinger, Annuals of Physics **2** (1957) 407.

[2] M. Gell-Mann and M. Levy, Il Nuovo Cim. XVI (1960) 705.

[3] B. Lee, Cargese Lectures 1970.

[4] G. 't Hooft, Nucl. Phys. **B35** (1971) 579.

[5] P.W. Higgs, Phys. Lett. **12** (1964) 132; Phys. Rev. Lett. **13** (1964) 508; Phys. Rev. **145** (1966) 1156.

[6] F. Englert and R. Brout, Phys. Rev. Lett. **13** (1964) 321.

[7] P.W. Anderson, Phys. Rev. **112** (1958) 1900.

[8] T. Kibble, Phys. Rev. **155** (1967) 1554.

[9] S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.

[10] S. Glashow, Nucl. Phys. **22** (1961) 579.

[11] A.D. Linde, JETP Lett. **19** (1974) 183.
M. Veltman, Rockefeller Univ. preprint, May 1974, unpublished;
Phys. Rev. Lett. **34** (1975) 777.

[12] M. Veltman, Acta Phys. Pol. **B8** (1977) 475.

[13] K. Symanzik, Comm. Math. Phys. **34** (1973) 7;
T. Appelquist and J. Carazzone, Phys. Rev. **D11** (1975) 2856.

[14] D. Dicus and V. Mathur, Phys. Rev. **D7** (1973) 3111.
B. Lee, C. Quigg and H. Thacker, Phys. Rev. Lett. **38** (1977) 888;
Phys. Rev. **D16** (1977) 1519.

[15] M. Chanowitz and M.K. Gaillard, Nucl. Phys. **B261** (1985) 379. See also ref. 30.

[16] Y.P. Yao and C.P. Yuan, Phys. Rev. **D38** (1988) 2237.

[17] J. Van Der Bij and M. Veltman, Nucl. Phys. **B231** (1984) 205.

[18] D.A. Ross and M. Veltman, Nucl. Phys. **B59** (1975) 135.

[19] R. Peccei and H. Quinn, Phys. Rev. Lett. **38** (1977) 1440;
Phys. Rev. **D16** (1977) 1791.
F. Wilczek, Phys. Rev. Lett. **40** (1978) 279.

[20] T.D. Lee, Phys. Rev. **D8** (1973) 1226.

[21] V. Baluni, Phys. Rev. **D19** (1979) 2227.
R. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Lett. **88B** (1979) 123.

[22] M. Veltman, Nucl. Phys. **B7** (1968) 637.

[23] M. Veltman, Acta Phys. Pol. **B8** (1977) 475.

[24] G. Passarino and M. Veltman, Nucl. Phys. **B160** (1979) 151.

[25] M. Lemoine and M. Veltman, Nucl. Phys. **B164** (1980) 445.

[26] M. Veltman and F.J. Yndurain, Michigan preprint, March 1989.

[27] R. Akhoury and Y.-P. Yao, Phys. Rev. **D25** (1982) 3361.

[28] S. Dawson and S. Willenbrock, Phys. Rev. Lett. **62** (1989) 1232.

[29] Abbasabadi et al., Phys. Rev. **D38** (1988) 2770.
W. Repko and C.J. Suchyta, Phys. Rev. Lett. **62** (1989) 859.

[30] H. Veltman, Phys. Rev. D41 (1990) 2294