SECOND THRESHOLD IN WEAK INTERACTIONS

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(*Received January 7, 1977*)

The point of view that weak interactions must have a second threshold below 300 — 600 GeV is developed. Above this threshold new physics must come in. This new physics may be the Higgs system, or some other nonperturbative system possibly having some similarities to the Higgs system. The limit of large Higgs mass is thought to be relevant in this context. Radiative corrections proportional to m^2 and $\ln m^2$, m being the Higgs mass, are calculated. Contemplation of the theory in the limit of large Higgs mass suggests that the "new physics" may contain breakdown of μ-e universality and other than V–A neutrino interactions already at relatively low energies.

1. *Introduction*

The work of 't Hooft on the renormalization of gauge theories [1] has resulted in a fundamental change in theoretical elementary particle physics. Gauge theories of weak and e.m. interactions [2] have become credible[1], and the so-called standard model [3] including a color gauge theory of strong interactions is now very popular. The observation of neutral currents [4], as well as the apparent experimental verification [5] of the prophetic paper of Gaillard, Lee and Rosner [6] concerning charm [7] is most encouraging.

In spite of these successes we must be careful to maintain an objective attitude. What can be concluded given that neutral currents and charm exist as required? From a phenomenological point of view we can say that the data fit a current-current type model, where the currents satisfy an algebra as required by a gauge theory. But we have no direct evidence for the existence of vector bosons, and the Higgs mechanism is experimentally totally unverified. The vector boson hypothesis may perhaps be verified with the new accelerators

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[1] Obviously, a theory with unknown calculational rules can hardly be credible. Such was the state of affairs before the work of Ref. [1].

are thus led to the study of gauge theories as a function of the Higgs mass, in the limit of large Higgs mass.

Actually, the situation is quite complicated. It has been shown that in a massive Yang–Mills theory without mass terms for the leptons there are no non-renormalizable divergences at the one loop level [8]. This situation is to all practical purposes unchanged if a lepton mass is introduced, provided that the lepton mass is small with respect to the vector boson mass. Therefore the increase in radiative corrections as a function of increasing Higgs mass must come from two or more loop diagrams, and is thus suppressed by an additional factor of at least g^2 (g = coupling constant, presumably $g^2/4\pi^2 \sim 1/45$). In this sense the new physics is screened off. However, inspection of the Higgs sector shows, that for large Higgs mass we essentially obtain a strong interaction theory. Thus for very large Higgs mass we have a strongly interacting system, screened off from direct observation through a factor g^2. Studying this is very much like studying strong interactions using exclusively leptons. It is obvious that the study of strong interactions through radiative corrections on pure lepton systems is a rather futile enterprise.

On the other hand, a strongly interacting system will in general have bound states that may well be quite low in mass compared to the elementary constituents (i.e. the Higgs particles in our case)[2]. Such low mass states would have striking properties: they would be evident in neutrino physics, and they could perhaps result in explicit breakdown of $\mu - e$ universality. Here there is an extremely interesting domain of speculation, and one may even be tempted to identify a strongly interacting Higgs system with the usual strong interactions. We will not enter into this any further, for the time being, but wish to emphasize here only that one should always be very watchful for other than V – A currents in neutrino reactions at very high energy, as well as a possible breakdown of $\mu - e$ universality, perhaps even at relatively low energies.

There is one exception to the screening rule mentioned above, and that is in radiative corrections to coupling constants and masses. Such quantities are considered to be renormalized in the above mentioned result. Clearly, such corrections are unobservable, unless the same coupling constant is supposed to appear at different places. Indeed, it turns out that the coupling constant g of the vector boson to the leptons is affected differently from the three W vertex coupling constant, as a function of large Higgs mass. There is consequently an order $g^2 \ln m$ effect to the three W vertex relative to the W-lepton vertex (m = Higgs mass).

Similarly, in the Weinberg model, the vector boson mass is related to the Higgs-lepton coupling. This relation turns out to have $g^2 m^2$ radiative corrections.

In the rest of the paper we will substantiate the various statements made above, restricting ourselves to a somewhat simplified model of weak interactions. This model is essentially the Weinberg model in the limit of zero weak mixing angle θ_w and zero electric charge. We believe that this is quite adequate to the purposes of this article. Incidentally, there is no interesting radiative correction to the ratio of neutral to charged vector boson mass in the Weinberg model, at least as a function of the Higgs mass.

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[2] In particular such seems to be the case in the usual Higgs system, as shown by preliminary investigations by G. Passarino and the author.

The fermion mass term can be rewritten

$$m_e(\bar{e}e) = \frac{m_e}{2}(\bar{l}_-(1-\tau^3)l_+) + \frac{m_e}{2}(\bar{l}_+(1-\tau^3)l_-) \tag{2.4}$$

which makes its transformation properties more transparent.

In the following the above Higgs-less model will be designated as the massive Yang–Mills model.

3. *One loop divergencies*

As argued in the introduction the massive Yang–Mills model as described in the previous section is non-renormalizable and therefore can given an accurate prescription of the data only up to some cut-off energy E_t. Radiative corrections will blow up if E_t is made too high. A study of the infinities of the model will reveal where the radiative corrections will be sensitive to this cut-off E_t. The techniques for doing this are those of Ref. [8]; the only modification is that due to the fermion mass term. We will sketch the analysis for one closed loop, and indicate at the end what happens for two or more closed loops.

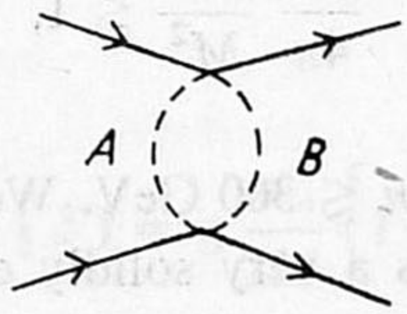
Consider thus the Lagrangian (2.2), with the fermion mass term written in the form (2.4). To this we apply an operator gauge transformation of the form (2.3) (Bell–Treiman transformation) involving a set of scalar fields A^a divided by the vector boson mass M. To first order in the fields A^a the Lagrangian (2.2) becomes

$$\mathscr{L} \to \mathscr{L} + MW^a_\mu \partial_\mu A^a + \frac{im_e g}{4M}(\bar{l}_-(1-\tau^3)A^a\tau^a l_+) - \frac{im_e g}{4M}(\bar{l}_+ A^a\tau^a(1-\tau^3)l_-). \tag{3.1}$$

As shown in Ref. [8] this leads to a Ward identity which now involves also vertices of the type $A(\bar{l}l)$. Further Ward identities are obtained by an iterative procedure, and in the second step one will meet vertices obtained from the above by a further gauge transformation involving a set of fields B

$$g\varepsilon_{abc}B^b W^c_\mu \partial_\mu A^a - \partial_\mu B^a \partial_\mu A^a$$
$$+ \frac{m_e g^2}{8M^2}(\bar{l}_-(1-\tau^3)A^a\tau^a B^b\tau^b l_+) + \frac{m_e g^2}{8M^2}(\bar{l}_+ B^b\tau^b A^a\tau^a(1-\tau^3)l_-). \tag{3.2}$$

We will not repeat the analysis of Ref. 8, and note only that in one loop diagrams in the so-called unitary gauge the $k_\mu k_\nu$ term in the vector boson propagator can be removed at the expense of introducing vertices of the form as given by the above terms. The latter two terms correspond to non-renormalizable type vertices, and they generate the non-renormalizable part of the massive theory at the one loop level. Specifically, we have the diagram for fermion-fermion scattering



It is logaritmically divergent, implying a behaviour of the form

$$g^4 \frac{m_e^2}{M^4} \ln E_t \tag{3.3}$$

as a function of the threshold E_t. This is the only infinity apart from corrections to masses and coupling constants. Because of the appearance of the ratio of the lepton to the vector boson mass this contribution is extremely small and no sensible limit onto E_t results.

For diagrams with two loops one encounters vertices obtained by two further gauge transformations on the terms (3.2). Then other processes get $\ln E_t$ terms, and the fermion process gets a contribution proportional to E_t^2, but multiplied by an extra factor g^2. One gets, generally, for the fermion-fermion amplitude a series of the form

$$g^2 \frac{m^2}{M^4} \left\{ \ln E_t + g^2 \frac{E_t^2}{M^2} + g^4 \frac{E_t^4}{M^4} + \dots \right\}.$$

The same type of contribution will arise for any process, except that there will be more factors g^2, as the divergencies appear only in higher order. For instance, in the vector boson propagator one will have a contribution of the form

$$g^4 k^4 \left\{ \ln E_t + g^2 \frac{E_t^2}{M^2} + g^4 \frac{E_t^4}{M^4} + \dots \right\},$$

which is a momentum dependence to the fourth power, normally not occurring in a renormalizable theory. However, there is at least a factor g^4 here, because it occurs for the first time at two loop level.

What can be concluded from this? If there had been a contribution of the form $g^2 E_t^2/M^2$ at the one loop level we would certainly have obtained an interesting limit for E_t, perhaps E_t less than a few times the vector boson mass. As it is we cannot say very much, except that we may expect considerable corrections if $g^2 E_t^2/M^2 \gtrsim 1$. But then perturbation theory breaks down, and our calculational methods are insufficient. Some segment of the theory becomes a strong interaction type theory. We then have the situation as described in the introduction. In such a case mass-spectra are more interesting, from an experimental point of view.

On the basis of the above we would like to conclude, tentatively, that the Higgs mass is less than the value which makes perturbation theory break down. Thus we require

$$\frac{g^2}{4\pi} \frac{m^2}{M^2} \lesssim 1,$$

where m = Higgs mass. This gives $m \lesssim 300$ GeV. We emphasize that this must be considered as an indication, and not as a very solidly established number.

4. *Radiative corrections to the vector boson mass*

In the Weinberg model the Higgs system contains only one multiplet, and due to this limitation there are some specific relations concerning masses and coupling constants. These relations are

$$M_0^2 = \frac{M^2}{\cos^2 \theta_w}, \tag{4.1}$$

$$g_{ze} = g \frac{m_e}{2M}. \tag{4.2}$$

In here M_0 and M are the neutral and charged vector boson masses, g_{ze} is the Higgs--electron Yukawa coupling constant, and m_e = electron mass. Note that the weak mixing angle θ_w can be determined from the structure of the neutral currents.

The relations (4.1) and (4.2) need not be true in general, so deviations to these relations may be due to more complicated Higgs structures, or radiative corrections. We will consider the second possibility, and compute the lowest order radiative corrections in leading order with respect to the Higgs mass. It will be shown that there are g^2m^2/M^2 type radiative corrections to the relation (4.2), such that g_{ze} is larger than deduced from (4.2). The relation (4.1) suffers no correction, which liberates us from the need to work in the full Weinberg model, and we restrict ourselves to the simplified model with zero θ_w and zero electric charge.

The computation of the radiative corrections to the vector boson mass and g_{ze} is straight-forward. No g^2m^2/M^2 terms appear in the radiative corrections to m_e or g. Limiting ourselves to terms proportional to m^2 (the Higgs mass squared) relatively few one-loop diagrams survive. They are listed in Appendix B. The relation (4.2) suffers two corrections, namely a correction to M, and a correction to g_{ze} due to the Higgs particle wave function renormalization. The vector boson propagator becomes

$$\frac{1}{(2\pi)^4 i} \frac{1}{k^2+M^2(1+\delta)}, \qquad \delta = \frac{g^2}{128\pi^2} \frac{m^2}{M^2}.$$

The Z-propagator

$$\frac{1}{(2\pi)^4 i} \frac{1}{1-\delta'} \frac{1}{k^2+m'^2},$$

where m' is the radiative corrected mass of the Higgs particle, and δ' is given by

$$\delta' = \frac{g^2m^2}{16\pi^2M^2}\left(\tfrac{3}{2} - \frac{\pi\sqrt{3}}{4}\right).$$

The $Z\bar{e}e$ coupling constant becomes

$$g \frac{m_e}{2M} \frac{1}{\sqrt{1-\delta}} \simeq g \frac{m_e}{2M}(1+\tfrac{1}{2}\delta') = g \frac{m_e}{2M}\left\{1 + \frac{g^2m^2}{32\pi^2M^2}\left(\tfrac{3}{2} - \frac{\pi\sqrt{3}}{4}\right)\right\}$$

If we write $\bar{g}_{ze}$ and $\bar{M}$ for the coupling constant and mass inclusive the above radiative correction, then we have

$$\frac{\bar{g}_{ze}}{g}\cdot\frac{2\bar{M}}{m_e} = (1+\tfrac{1}{2}\delta)(1+\tfrac{1}{2}\delta') \simeq 1+\tfrac{1}{2}\delta+\tfrac{1}{2}\delta' = 1+\frac{g^2m^2}{32\pi^2M^2}\left(\tfrac{13}{8}-\frac{\pi\sqrt{3}}{4}\right),$$

which is larger than one. If g^2m^2/π^2M^2 becomes of the order one then the calculation is meaningless because contributions of higher order in g become of order one also.

The conclusion of the above is this: in the limit of large Higgs mass the coupling of the lepton system to the Higgs system suffers large corrections. Alternatively, one can say that the vector boson mass gets a large contribution relative to the lepton mass. This view leads us to suppose that perhaps the vector boson mass arises mainly from radiative corrections, and this automatically raises the interest of any process that is supposedly sensitive to the structure of the vector boson. In particular we may think of the $3W$ vertex and further WWX vertices, where X stands for any combination of particles, and notably involving Higgs particles. We will not investigate the latter, because the main interest of such a vertex will be the case of very high Higgs mass, so that bound states arise. It may well be that in that case the vector boson becomes a major tool for studying the Higgs system.

5. *Radiative corrections to the WWW vertex*

The radiative corrections to the WWW vertex are proportional to $g^2 \ln m^2/M^2$. Of course, as g^2m^2/π^2M^2 becomes of order 1 we get further corrections of order g^2 due to higher order diagrams. Again, we must think of a series of the type mentioned in Section 3. An accurate measurement of the WWW coupling constant relative to the W-lepton coupling constant may be extremely difficult, but in principle we have here a truly weak radiative correction of order α. In the unfortunate case that the Higgs mass is larger than 200 GeV but not so large that low lying bound states appear then this measurement may well be the only clue to the Higgs system.

As shown in Appendix C the vector boson propagator obtains a wave function renormalization amounting to a propagator factor

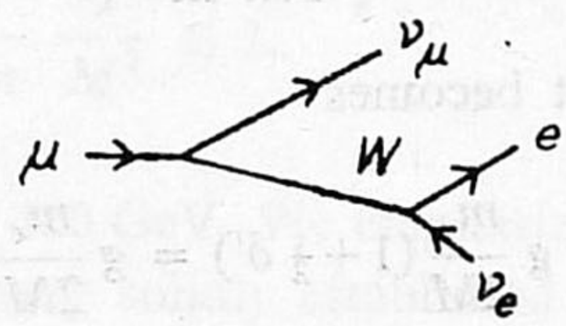
$$\frac{1}{1-\varepsilon}, \qquad \varepsilon = \frac{g^2}{192\pi^2}\ln m^2.$$

The $3W$ vertex, inclusive its lowest order radiative corrections proportional to $\ln m^2$

$$g(1+\varepsilon'), \qquad \varepsilon' = -\frac{g^2}{128\pi^2}\ln m^2 = -\tfrac{3}{2}\varepsilon.$$

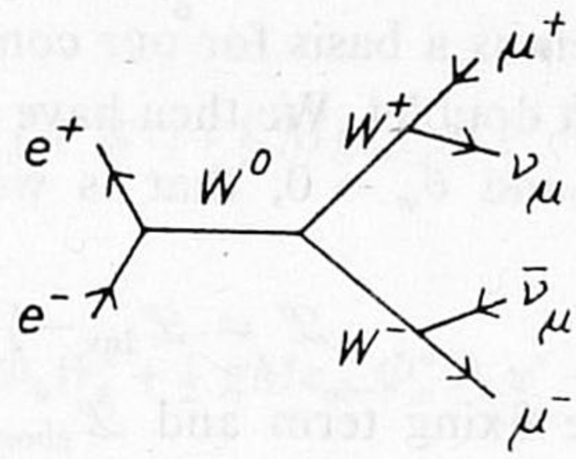
The radiative corrections to a process like μ-decay are

$$\frac{1}{1-\varepsilon}$$



The radiative corrections to $e^+e^- \to \mu\nu + \mu\nu$ are

$$(1+\varepsilon')\left(\frac{1}{1-\varepsilon}\right)^3$$



Thus in μ decay the experimental coupling constant is

$$g_{\exp,\mu} = g(1+\tfrac{1}{2}\,\varepsilon)$$

while the $3W$ coupling constant is unchanged

$$g_{\exp,3W} = g(1+\varepsilon')\left(\frac{1}{1-\varepsilon}\right)^{3/2} = g(1-\tfrac{3}{2}\,\varepsilon+\tfrac{3}{2}\,\varepsilon) = g.$$

In the second process mentioned above one thus must measure a correction $(1-\frac{1}{2}\varepsilon)$ in amplitude. Unfortunately, the interesting factor $\ln m^2$ appears in ε with the small coefficient

$$\frac{g^2}{192\pi^2} = \frac{1}{48\pi}\,\frac{g^2}{4\pi} \simeq \tfrac{1}{150}\cdot\tfrac{1}{45} \approx 10^{-4}.$$

6. *Conclusions*

From the foregoing we can draw the following conclusions:

(*i*) Somewhere around or before 300 — 600 GeV new physics must necessarily appear.

(*ii*) This new physics may be the Higgs system as employed in the various models. If the Higgs mass is less than 200 GeV (which we take as the practically reachable limit of energy for an e^+e^- machine) then it can perhaps be observed directly [11]. If the Higgs mass is between 200 and 600 GeV then detection and/or investigation seems extremely difficult, if not hopeless. If the Higgs mass is larger than 600 GeV we may hope for the existence of low lying bound states, that may be detected if they are below 200 GeV.

(*iii*) The new physics may be something beyond perturbation theory (e.g. bound states). Even if this new physics is not a Higgs system we nevertheless expect that many features will be like in the Higgs system. In particular, the large Higgs mass case may be an attractive simulation for that case, because for large mass also bound states arise. The physical phenomena arising in that case are plentiful and very interesting; however the analysis has not yet reached a stage in which more precise statements can be made. But it seems that breakdown of $\mu-e$ universality, as well as other then V−A interactions in neutrino experiments would be likely consequences, among others. It will be very difficult to substantiate these speculations, because the calculation of physical effects from a strongly interacting Lagrangian is far from a well established procedure. In any case, it is interesting and stimulating to know that potentially the new physics needed by weak interactions could plausibly involve effects normally taken to be absent.

APPENDIX A

The model chosen as a basis for our computations is the model of Ref. 1, Section 6 enlarged with a lepton doublet. We then have essentially the Weinberg model in the simultaneous limit $e \to 0$ and $\theta_w \to 0$, that is weak interactions without electromagnetism The Lagrangian is

$$\mathscr{L} = \mathscr{L}_{\text{inv}} - \tfrac{1}{2} C^2 + \mathscr{L}_{\text{ghost}},$$

where C is the gauge fixing term and $\mathscr{L}_{\text{ghost}}$ is the Faddeev–Popov ghost Lagrangian. Further

$$\begin{aligned}\mathscr{L}_{\text{inv}} = {} & -\tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \tfrac{1}{2} M^2 W^2 - \tfrac{1}{2} (\partial_\mu Z)^2 - \tfrac{1}{2} m^2 Z^2 \\ & -\tfrac{1}{2} (\partial_\mu \psi^a)(D^a_\mu \psi) + \tfrac{1}{2} g W^a_\mu (Z \partial_\mu \psi^a - \psi^a \partial_\mu Z) \\ & -\tfrac{1}{8} g^2 W^2_\mu (\psi^2 + Z^2) - \tfrac{1}{2} g M W^2 Z - \alpha M g Z (\psi^2 + Z^2) \\ & -\tfrac{1}{8} \alpha g^2 (\psi^2 + Z^2)^2 - \beta \left[\tfrac{1}{2}(Z^2 + \psi^2) + \frac{2M}{g} Z \right] - M \psi^a \partial_\mu W^a_\mu \\ & -(\bar{l} \gamma^\mu \partial_\mu l) - m_e (\bar{e} e) + \frac{ig}{8} (\bar{l} \gamma^\mu (1 + \gamma^5) \tau^a l) W^a_\mu \\ & -\frac{g m_e}{2M} Z (\bar{e} e) - \frac{g m_e}{4M} \psi_a \{ \bar{l} (\varepsilon_{abc} \tau^b s^c + i \tau^a \gamma^5 - i s^a \gamma^5) l \}.\end{aligned}$$

In here

$$W^2 = W^a_\mu W^a_\mu, \quad \psi^2 = \psi^a \psi^a, \quad G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g \varepsilon_{abc} W^b_\mu W^c_\nu,$$

$$D^a_\mu \psi = \partial_\mu \psi^a + g \varepsilon_{abc} W^b_\mu \psi^c, \quad \alpha = \frac{m^2}{4M^2},$$

m = Higgs mass, Z = physical Higgs particle, ψ = unphysical Higgs ghost, m_e = electron mass, $l = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ = lepton doublet, τ^a = Pauli spin matrices. The quantity s is a spurion, with $s^1 = s^2 = 0$ and $s^3 = 1$. β is a constant to be adjusted so that the total tadpole contribution disappears. In lowest order $\beta = 0$.

For the gauge fixing term we take

$$C^a = -\partial_\mu W^a_\mu + M \psi^a,$$

which is the 't Hooft gauge. The transformation properties of the various fields under infinitesimal gauge transformations Λ are.

$$W^a_\mu \to W^a_\mu + g \varepsilon_{abc} \Lambda^b W^c_\mu - \partial_\mu \Lambda^a,$$

$$\psi^a \to \psi^a + \tfrac{1}{2} g \varepsilon_{abc} \Lambda^b \psi^c - \tfrac{1}{2} g Z \Lambda^a - M \Lambda^a,$$

$$Z \to Z + \tfrac{1}{2} g \Lambda^a \psi^a,$$

$$l_+ \to \left(1 - \frac{i}{2} g \Lambda^a \tau^a\right) l_+, \quad l_- \to l_-, \quad l_\pm = \tfrac{1}{2}(1 \pm \gamma^5) l.$$

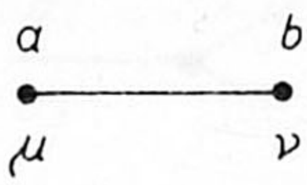
The choice of C and the transformation properties of the W and ψ fields imply the transformation property

$$C^a \to C^a + \partial^2 \Lambda^a - M^2 \Lambda^a - g\varepsilon_{abc}\partial_\mu(\Lambda^b W^c) + \tfrac{1}{2} M g \varepsilon_{abc} \Lambda^b \psi^c - \tfrac{1}{2} M g Z \Lambda^a.$$

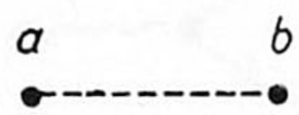
The ghost Lagrangian is then

$$\mathscr{L}_{\text{ghost}} = \Phi_a^*(\partial^2 - M^2)\Phi_a + g\varepsilon_{abc}\partial_\mu \Phi_a^* \Phi_b W_\mu^c + \tfrac{1}{2} g M \varepsilon_{abc} \Phi_a^* \Phi_b \psi^c - \tfrac{1}{2} g M \Phi_a^* \Phi_a Z.$$

The Feynman rules corresponding to this Lagrangian can now be written down

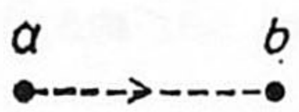


$$\frac{\delta_{ab}\delta_{\mu\nu}}{k^2 + M^2 - i\varepsilon}$$ W-propagator



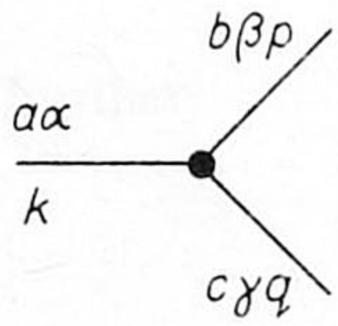
$$\frac{\delta_{ab}}{k^2 + M^2 - i\varepsilon}$$ Higgs ghost propagator

$$\frac{1}{k^2 + m^2 - i\varepsilon}$$ Physical Higgs propagator

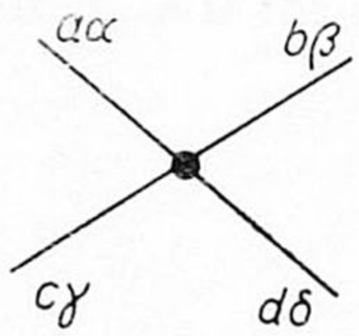


$$\frac{\delta_{ab}}{k^2 + M^2 - i\varepsilon}$$ F–P ghost propagator

$$\frac{-i\gamma k + m_l}{k^2 + m_l^2}$$ lepton propagator
$m_l = m_e$ for electron, $= 0$ for neutrino

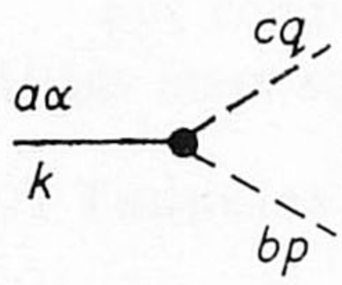


$$-ig\varepsilon_{abc}\{\delta_{\alpha\gamma}(k-q)_\beta + \delta_{\beta\gamma}(q-p)_\alpha + \delta_{\alpha\beta}(p-k)_\gamma\}$$
Yang–Mills three W vertex

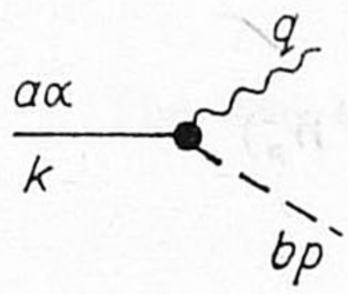


$$-g^2\{\varepsilon_{gdc}\varepsilon_{gba}(2\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta})$$
$$+\varepsilon_{gdb}\varepsilon_{gca}(2\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\gamma\beta} - \delta_{\alpha\gamma}\delta_{\beta\delta})\}$$

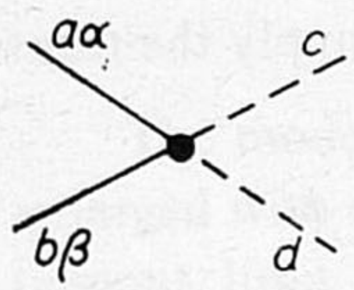
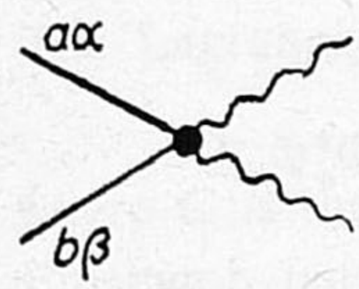
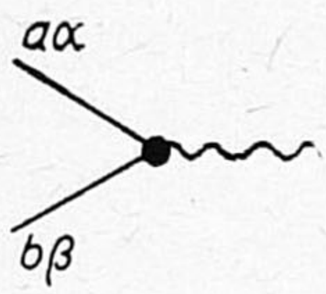
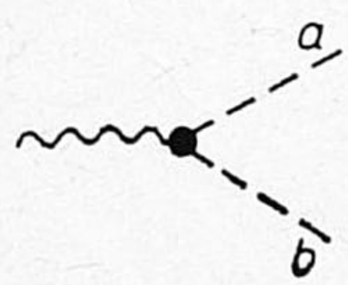
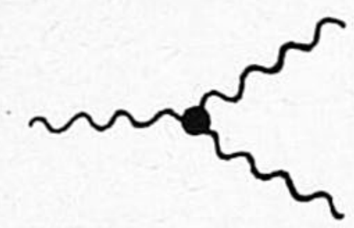
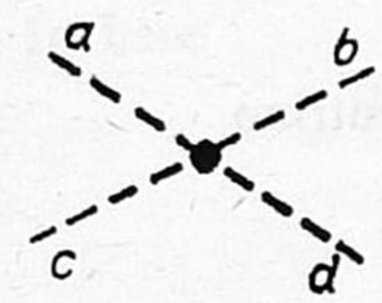
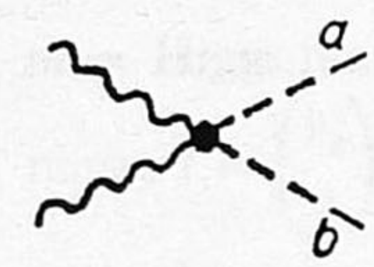
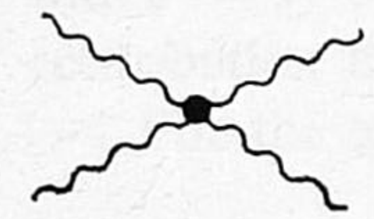
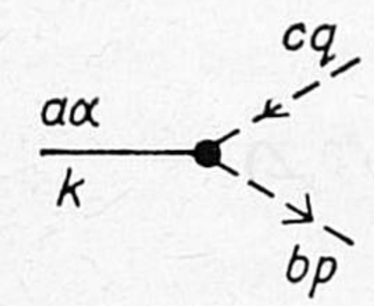
Yang–Mills four W-vertex

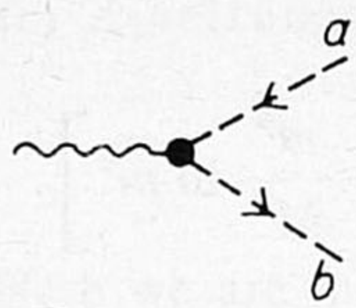


$$\frac{i}{2} g\varepsilon_{abc}(p-q)_\alpha \qquad \text{from} \qquad \frac{g}{2}\varepsilon_{abc} W_\mu^a(\partial_\mu \psi^b)\psi^c$$

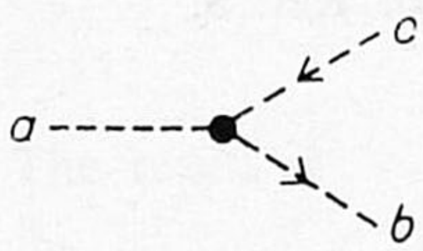


$$\frac{i}{2} g\delta_{ab}(p-q)_\alpha \qquad \frac{g}{2} W_\mu^a(Z\partial_\mu \psi^a - \psi^a \partial_\mu Z)$$

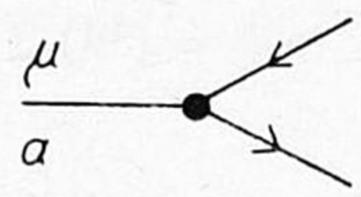
	$-\frac{1}{2}\, g^2\delta_{ab}\delta_{cd}\delta_{\alpha\beta}$	$-\frac{1}{8}\, g^2 W^a_\mu W^a_\mu \psi^b\psi^b$
	$-\frac{1}{2}\, g^2\delta_{ab}\delta_{\alpha\beta}$	$-\frac{1}{8}\, g^2 W^a_\mu W^a_\mu Z^2$
	$-gM\delta_{ab}\delta_{\alpha\beta}$	$-\frac{1}{2}\, gMW^a_\mu W^a_\mu Z$
	$-2\alpha Mg\delta_{ab}$	$-\alpha MgZ\psi^2$
	$-6\alpha Mg$	$-\alpha MgZ^3$
	$-\alpha g^2(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})$	$-\frac{1}{8}\,\alpha g^2\psi^a\psi^a\psi^b\psi^b$
	$-\alpha g^2\delta_{ab}$	$-\frac{1}{4}\,\alpha g^2\psi^a\psi^a Z^2$
	$-3\alpha g^2$	$-\frac{1}{8}\,\alpha g^2 Z^4$
	$-\beta$	$-\frac{1}{2}\,\beta Z^2$
	$-\beta\delta_{aa'}$	$-\frac{1}{2}\,\beta\psi^2$
	$-\dfrac{2M}{g}\,\beta$	$-\dfrac{2M}{g}\,\beta Z$
	$+i\kappa g\varepsilon_{abc}p_\alpha$	$-\kappa g\varepsilon_{abc}\Phi^{*a}\partial_\mu(\Phi^b W^c_\mu)$



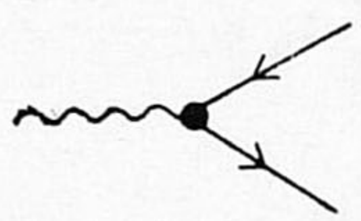
$$-\tfrac{1}{2}\,Mg\delta_{ab} \qquad -\tfrac{1}{2}\,Mg\Phi_a^*\Phi_a Z$$



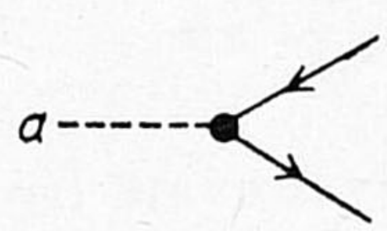
$$\tfrac{1}{2}\,Mg\varepsilon_{abc} \qquad \tfrac{1}{2}\,Mg\varepsilon_{abc}\Phi_a^*\Phi_b\psi^c$$



$$\frac{ig}{8}\,\gamma^\mu(1+\gamma^5)\tau^a$$



$$-\frac{gm_e}{2M}\cdot\tfrac{1}{2}\,(1-s^a\tau^a)$$



$$-\frac{gm_e}{4M}\,(\varepsilon_{abc}\tau^b s^c + i\tau^a\gamma^5 - is^a\gamma^5)$$

lepton couplings

From the experiment we infer

$$\frac{g^2}{4\pi} = \frac{e^2}{4\pi}\,\frac{1}{\sin^2\theta_w} \approx \tfrac{1}{45},$$

where we use

$$\sin^2\theta_w = 0.32 \pm 0.05.$$

Further

$$M^2 = \frac{1400}{\sin^2\theta_w}\ \mathrm{GeV}^2, \qquad M = 66 \pm 6\ \mathrm{GeV}.$$

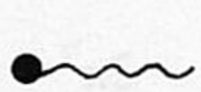
If $\theta_w \neq 0$ then the neutral vector boson has a mass of $66/\cos\theta_w = 80$ GeV.

APPENDIX B

One-loop radiative corrections

For completeness we list all lowest order radiative corrections proportional to the Higgs mass squared

(*i*) Tadpoles



The first diagram gives

$$-\frac{2M}{g}\,(2\pi)^4 i\beta.$$

The constant β must be chosen such that the total is zero. This gives

$$(2\pi)^4 i\beta = -\frac{3g^2 i}{8}\left[\left(m^2+\frac{m^4}{M^2}\right)\left(\frac{2\pi^2}{n-4}-\Delta-\pi^2\right)+\pi^2 m^2 \ln M^2+\pi^2\frac{m^4}{M^2}\ln m^2\right].$$

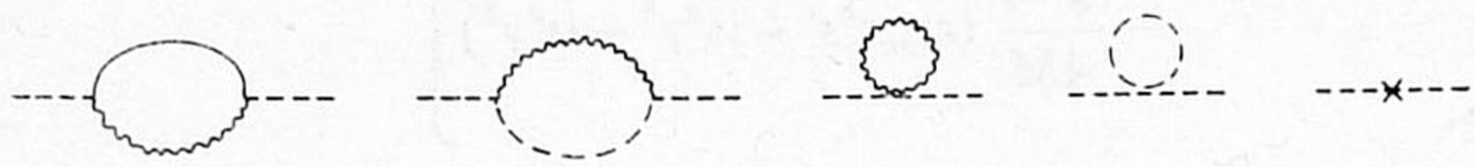
Note that these diagrams carry a combinatorial factor, see Appendix of Ref. 8.

(*ii*) W-selfenergy

$$\Sigma_W = -g^2\frac{i\pi^2}{8}m^2\delta_{\mu\nu}$$

The second diagram has a combinatorial factor 1/2.

(*iii*) ψ-selfenergy



$$\Sigma_\psi = -g^2\frac{i\pi^2}{8}\frac{m^2k^2}{M^2}$$

(*iv*) $\psi - W$ transition

Since this is only one diagram we give the complete calculation, as an example

$$-ig^2\alpha M\int d_nq\,\frac{(k+2q)_\mu}{((q+k)^2+m^2)(q^2+M^2)}$$

$$\simeq -\frac{ig^2m^2}{4M}\int d_nq\,\frac{(k+2q)_\mu}{(q^2+m^2)(q^2+M^2)}\left\{1-\frac{2qk+k^2}{q^2+m^2}\right\}$$

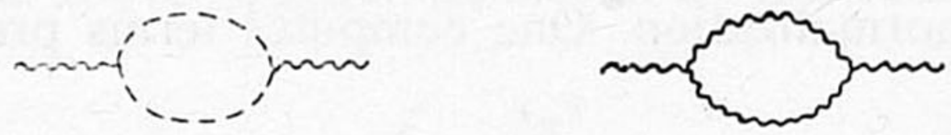
$$\simeq -\frac{ig^2}{4}\frac{m^2}{M^2}\int d_nq\,\frac{1}{(q^2+m^2)(q^2+M^2)}\left\{k_\mu-k_\nu\frac{4q_\mu q_\nu}{q^2+m^2}\right\}$$

$$\simeq -\frac{ig^2}{4}\frac{m^2}{M^2}k_\mu\int d_nq\left[\frac{1}{(q^2+m^2)(q^2+M^2)}-\frac{4}{n}\frac{1}{(q^2+m^2)^2}\right]$$

$$= -\frac{ig^2}{4}\frac{m^2}{M}ik_\mu\left[-\frac{2\pi^2}{n-4}+\Delta+\pi^2-\pi^2\ln m^2-\frac{4}{n}\left(-\frac{2\pi^2}{n-4}+\Delta-\pi^2\ln m^2\right)\right]$$

$$= -\frac{ig^2}{4}\frac{m^2}{M}ik_\mu\left[\pi^2-\frac{\pi^2}{2}\right] = -g^2\frac{i\pi^2}{8}\frac{m^2}{M}ik_\mu$$

(*v*) Z-selfenergy



Here we are interested in the wave function renormalization. We must develop

$$\Sigma_Z(k^2) = \Sigma_Z(-m^2)+(k^2+m^2)\Sigma_Z'(-m^2)+O((k^2+m^2)^2).$$

The result is

$$\Sigma_Z(k^2) = \tfrac{3}{8}\, g^2 \frac{m^4}{M^2}\left[-\frac{2i\pi^2}{n-4}+i\Delta-i\pi^2\left(\ln M^2-2-\ln\frac{M^2}{m^2}-3\frac{M^2}{m^2}+2\frac{M^2}{m^2}\ln\frac{M^2}{m^2}\right)\right]$$

$$+\tfrac{9}{8}\, g^2 \frac{m^4}{M^2}\left[-\frac{2i\pi^2}{n-4}+i\Delta-i\pi^2\left(\ln m^2-2+\frac{\pi}{\sqrt{3}}\right)\right]$$

$$+g^2\frac{m^4}{M^2}(k^2+m^2)\left[\frac{3i\pi^2}{8m^2}+\frac{9i\pi^2}{8m^2}\left(1-\frac{2\pi}{3\sqrt{3}}\right)\right]+O((k^2+m^2)^2).$$

(*vi*) Slavnov Taylor identities

A check on these results may be obtained by two S–T identities:

where the double line in the first diagram stands for multiplication with ik_μ. The right hand side has no m^2 terms. Thus we must have

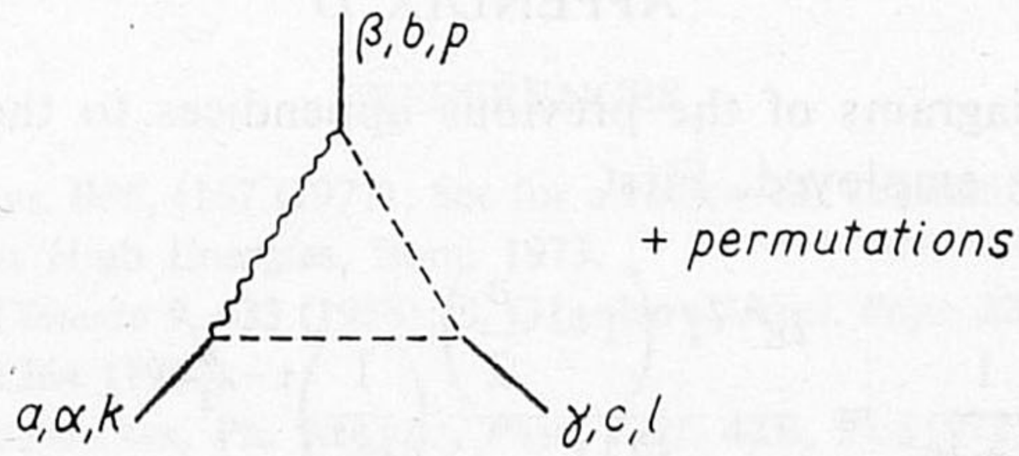
$$ik_\mu\left(-g^2\frac{i\pi^2}{8}m^2\delta_{\mu\nu}\right)\frac{\delta_{\nu\alpha}}{k^2+M^2}+M\left(g^2\frac{i\pi^2}{8}\frac{m^2}{M}ik_\nu\right)\frac{\delta_{\nu\alpha}}{k^2+m^2}=0.$$

Similarly, the $\psi-W$ and $\psi\psi$ results are related.

APPENDIX C

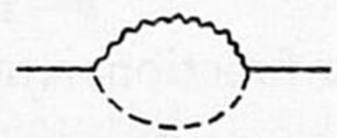
Coupling constant corrections

We compute up to $\ln m^2$. First there are the triangle diagrams:



$$\Gamma_{WWW} = \frac{g^3}{8}\,\varepsilon_{abc}\left(-\frac{2\pi^2}{n-4}+\Delta+\tfrac{3}{2}\pi^2-\pi^2\ln m^2\right)\{\delta_{\alpha\gamma}(k-l)_\beta+\delta_{\beta\gamma}(l-p)_\alpha+\delta_{\alpha\beta}(p-k)_\gamma\}.$$

The constant ε mentioned in Section 3 follows after division by $(2\pi)^4 i$. Further there is the W-wave function renormalization. One computes terms proportional to k^2 of one diagram only.

$\Sigma_W =$ 

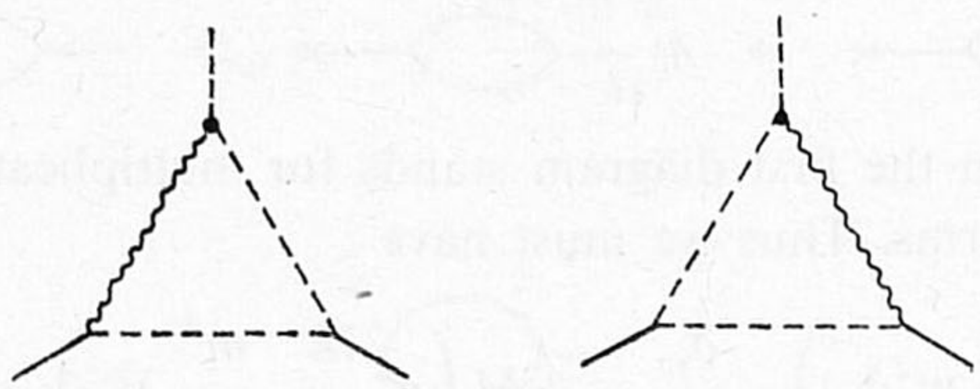
Neglecting constant terms and terms of higher order in k we get

$$\Sigma_W(k^2 \text{ part}) = \frac{ig^2}{12} k_\mu k_\nu \left(-\frac{2\pi^2}{n-4} + \Delta - \pi^2 \ln m^2 + \tfrac{4}{3}\pi^2 \right)$$
$$- \frac{ig^2}{12} \delta_{\mu\nu} k^2 \left(-\frac{2\pi^2}{n-4} + \Delta - \pi^2 \ln m^2 + \tfrac{5}{12}\pi^2 \right).$$

The part proportional to $\ln m^2$ is

$$\frac{ig^2}{12}(k^2\delta_{\mu\nu} - k_\mu k_\nu)\pi^2 \ln m^2.$$

Also here the results may be checked on a Slavnov–Taylor identity. This is more complicated than in the previous cases and requires the calculation of the $WW\psi$ vertex corrections



The diagrams give

$$\Gamma_{\psi WW} = \frac{i\pi^2 g^3}{24M} \ln m^2 \varepsilon_{abc}\{(k^2 - l^2)\delta_{\alpha\gamma} - \tfrac{1}{2} p_\alpha (k-l)_\gamma - \tfrac{1}{2}(k-l)_\alpha p_\gamma\},$$

which is a multiple of the WWW vertex multiplied by p_β. We leave it to the reader to check the relevant S–T identity.

APPENDIX D

To compute the diagrams of the previous appendices to the order required some simple tricks have been employed. First

$$\int d_n q \frac{1}{q^2 + m^2} = \frac{i\pi^{n/2}\Gamma\left(1 - \frac{n}{2}\right)}{\Gamma(1)} \left(\frac{1}{m^2}\right)^{1-\frac{n}{2}}$$
$$= im^2 \left(\frac{2\pi^2}{n-4} - \Delta - \pi^2 + \pi^2 \ln m^2 \right) + O(n-4). \tag{D1}$$

In here Δ contains various pieces coming from the development of $\pi^{n/2}$ and $\Gamma(1-n/2)$ around $n = 4$. In fact, the above equation defines Δ. By differentiation with respect to m^2

$$\int d_n q \frac{1}{(q^2+m^2)^2} = i\left(-\frac{2\pi^2}{n-4} + \Delta - \pi^2 \ln m^2\right), \tag{D2}$$

$$\int d_n q \frac{1}{(q^2+m^2)^3} = \frac{i}{2}\pi^2 \frac{1}{m^2}. \tag{D3}$$

Further, by explicit calculation

$$\int d_n q \frac{1}{(q^2+m^2)(q^2+M^2)} = -\frac{2i\pi^2}{n-4} + i\Delta + i\pi^2$$

$$-\pi^2 i\left\{\frac{m^2}{m^2-M^2}\ln m^2 - \frac{M^2}{m^2-M^2}\ln M^2\right\}. \tag{D4}$$

Subsequent results obtain through differentiation with respect to m^2 or M^2. The integrals encountered in the diagrams have momentum dependence. The trick is to move this momentum dependence to the Higgs propagator and then to develop. For example, to compute the ψ-selfenergy one must compute the following integral up to order $1/m^2$

$$\int d_n q \frac{1}{(q^2+M^2)((q-k)^2+m^2)} = \int d_n q \frac{1}{(q^2+M^2)(q^2+m^2)}\left\{1 - \frac{k^2-2kq}{q^2+m^2} + \dots\right\}.$$

The term kq gives zero, after symmetrical integration. We then have integral (D4) and k^2 times the integral obtained from (D4) by differentiation with respect to m^2.

For the Z selfenergy diagrams, where $k^2 = -m^2$ must be taken one can not apply the above, but there the computation is straightforward and conventional.

For the triangle diagrams one proceeds as follows. The non-Higgs propagators are taken together by means of the Feynman trick. That reduces the problem to the type mentioned above. After working out the terms of the required order in m^2 there is no trouble in doing the Feynman parameter integration.

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