

ON THE RELATION BETWEEN THE FORM-FACTOR RATIO IN $K_{\mu 3}$ DECAY
AND THE SCALAR COUPLING CONSTANT IN μ CAPTUREJ. J. J. KOKKEDEE and M. VELTMAN
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Received 11 November 1964

In a recent experiment [1] evidence has been found for the $\pi^+\pi^-$ decay of K^0_2 , the direct implication of which would be the CP violation (or, equivalently, if the CPT theorem still holds, violation of time-reversal invariance) of the weak interactions. In this connection new interest has been raised in the magnitude of the parameter ξ (defined below) occurring in the matrix elements for $K_{\mu 3}$ decay. The general form of the matrix element of the meson current for this decay is

$$\langle \pi | J_\lambda | K \rangle = G \{ F_1(q^2) (p+k)_\lambda + F_2(q^2) q_\lambda \}, \quad (1)$$

where $F_1(0) = 1$, and where $q = k - p$ with k and p the four-momenta of K and π respectively. The parameter ξ is defined by:

$$\xi = F_2(q^2)/F_1(q^2). \quad (2)$$

If time-reversal violation is to be observed in this decay, ξ must be non-zero and have a non-vanishing imaginary part. Especially in Cabibbo's scheme of time-reversal violation [2] ξ is purely imaginary in the limit of exact SU_3 . As is well known, a non-zero value of $\text{Im } \xi$ will entail a non-zero polarization of the muon perpendicular to the decay plane (the plane of p and k).

In this note we wish to make a connection between ξ and the scalar coupling constant ξ' in the nucleonic current in muon capture. The matrix element of the baryon current for μ capture can be written as

$$\langle n | J_\lambda^V + J_\lambda^A | p \rangle,$$

where J_λ^V and J_λ^A are the vector and axial nucleonic currents. Here we are interested in the vector and scalar parts (not the magnetic part) of the matrix element of the vector current:

$$G' [\bar{u}(n) \{ \gamma_\lambda + i \xi' q_\lambda / m_\mu \} u(p)]. \quad (3)$$

In this expression m_μ is the muon mass and q_λ the difference of proton and neutron momenta. From μ capture one obtains an upper limit for ξ' [3]. One finds:

$$2 \text{Re } \xi' + |\xi'|^2 < 0.4 \quad (4)$$

Thus if ξ' is real, $|\xi'| < 0.2$; if ξ' is imaginary $|\xi'| < 0.6$. This is a rather small value as compared, for instance, with the value of seven for the corresponding quantity in the matrix element for the axial current.

We now remark that in the framework of the eightfold way any exchanged system giving rise to ξ' in μ capture has a partner in an octet which may contribute to ξ in $K_{\mu 3}$ decay. To obtain an approximate relation between the two parameters we introduce the following model.

Let us assume that the main contributions to ξ and ξ' arise from the exchange of scalar mesons K' and π' , respectively, which are assumed to be members of the same octet. Let us moreover identify K' with the κ (725) resonance, no π' has been observed as yet. Then we may estimate ξ from ξ' , or vice versa, provided we know how the K' and π' couple to the $(K\pi)$ and (np) systems. Some information about these couplings may be obtained from the model introduced by Coleman and Glashow [4], who use the scalar meson octet mentioned above as principal agent, via the tadpole mechanism, in the production of SU_3 breaking effects. In comparing actual experimental results with their work one may conversely arrive at a Lagrangian for interaction of scalar mesons with pseudoscalar mesons and baryons that contains, among others, the terms *

* S. Coleman and S. L. Glashow [4], take the interaction Lagrangian to be of the form

$$\mathcal{L} = d \text{Tr} \{ \bar{B} B S + \bar{B} S B \} + f \text{Tr} \{ \bar{B} B S - \bar{B} S B \} + g m_{K'} \text{Tr} \{ P P S \},$$

in which \bar{B} , B , S and P are the antibaryon, baryon, scalar and pseudoscalar meson octets, respectively, represented as 3×3 matrices; d , f and g , are dimensionless coupling parameters. The mass formulae obtained by them are functions of d , f and g . Working back one finds $f/d = -3$ and $f/g = \frac{2}{3}$. Writing out the traces in the above expression of \mathcal{L} one arrives at (5). Note, that since the mass and width of K' are known, we can calculate, $g_{\pi K K'}$, and hence d , f and g .

$$\frac{1}{2\sqrt{2}} g \{ 1.2 (np) \pi^+ - m_{K^+} (K^\pm \pi^0) K^{\mp} \} \quad (5)$$

From this we find for $q^2 = 0$ and for the case of K^\pm decay, with the help of

$$G = G' \operatorname{tg} \theta / \sqrt{2}$$

(this follows from the conserved vector current hypothesis); $\operatorname{tg} \theta$ is the ratio between the coupling constants for strangeness-changing and strangeness-conserving decays, where θ is the Cabibbo angle [5],

$$\xi = \frac{\xi'}{1.2} \frac{m_{\pi^+}^2}{m_{\mu} m_{K^+}}$$

which holds for general complex values of ξ . The same result is valid for K^0 and \bar{K}^0 decay.

The limits for ξ given by the existing experi-

mental data are $|\xi| \lesssim 1$, if ξ is real, and $|\xi| \lesssim 3$, if ξ is purely imaginary. Taking $m_{\pi^+} \simeq 570 \text{ MeV}$ (see ref. 4), relation (6) then predicts for ξ' the corresponding upper limits: $|\xi'| \lesssim 0.2$ and $|\xi'| \lesssim 0.6$ respectively, which happen to be the same as the limits given by (4).

We thank Professors J. Prentki and N. Cabibbo for reading the manuscript.

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