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RESEARCH ARTICLES

ON THE MASS OF THE GRAVITON†

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§(1): *INTRODUCTION AND SUMMARY*

The relation between the mass of the π-meson and the range of the Yukawa force is well known. The same relation has been often used (e.g. see [*1*]) to find a small upper value for the mass of the graviton. It is our purpose to show that for a spin 2 particle, such as the graviton, the relation between the mass of the particle and the interaction which it mediates is richer than for a spin 0 particle. The richer relation allows one to conclude, on the basis of experiment, that the mass of the graviton must be zero, and not just small (for an earlier and more detailed paper see [*2*]).

For spin 1 particles already an obvious discontinuity exists in that a finite mass spin 1 particle has 3 different states of polarization, whereas a zero mass spin 1 particle has 2 such states. However, for massive quantum electrodynamics in the limit of small mass the "third" state decouples and there is no discontinuity in the results predicted by the Feynman rules. In the case of the simplest Feynman diagram for massive gravitons, a similar decoupling occurs for the states of spins ±1 in the direction of motion, but not for the state of spin 0 in the direction of motion. This leads to the fact that the results predicted for massive gravitons in the limit of small mass do not agree with those for zero mass. This shows up in the bending of light by the sun, in the time

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delays observed in radar echos from the inner planets, and in the perihelion precession of Mercury. For the first two of these effects the massive, but small mass, theory gives 3/4 of the results of the massless theory. For the perihelion precession one finds in the limit of small mass 2/3 of the result predicted by the massless theory.

§(2): *MASSIVE AND MASSLESS GRAVITONS*

Consider the scattering of two material bodies involving the exchange of one graviton.

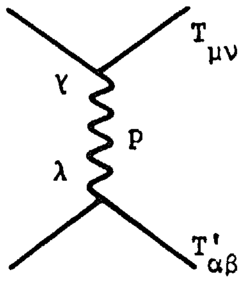

Figure 1

This process is described by two energy-momentum tensors $T_{\mu\nu}$ and $T'_{\alpha\beta}$, and if p is the four momentum vector of the exchanged graviton we have up to first order in λ:

$$p_\mu T_{\mu\nu} = p_\nu T_{\mu\nu} = p_\alpha T'_{\alpha\beta} = p_\beta T'_{\alpha\beta} = 0, \qquad (1)$$

which is nothing but the Fourier transform of the usual condition of the vanishing of the four divergence of the energy-momentum tensor of matter.

With the graviton which is known to have spin 2 there is associated a propagator, and we have to find this propagator for both mass-less and massive gravitons.

Consider first massive gravitons. For a graviton of energy-momentum p_μ there are five polarization states described by five polarization tensors $e^i_{\mu\nu}$, $i = 1,..,5$, $e_{\mu\nu} = e_{\nu\mu}$, $p_\mu e_{\mu\nu} = p_\nu e_{\mu\nu} = 0$ and $e_{\mu\mu} = 0$. The latter three conditions guarantee that the graviton contains no disguised spin 0 or 1 part. In the graviton rest-frame these five independent $e^i_{\mu\nu}$ may be taken to be:

$$\left(\frac{2}{3}\right)^{\frac{1}{2}} \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

In figure 1 gravitons of all polarization may be exchanged. Of interest in connection with the condition of unitarity is the sum over polarizations:

$$\sum_{i=1}^{5} e^{i}_{\mu\alpha} e^{i}_{\alpha\beta} = \begin{cases} \frac{1}{2}(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha}) - \frac{1}{3}\,\delta_{\mu\nu}\delta_{\alpha\beta} & \mu,\nu \\ & \alpha,\beta \neq 4 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Note that the factor 1/3 of the last term agrees with the requirement $e^{i}_{\mu\mu} e^{i}_{\alpha\beta} = 0$. Equation (3) can be made valid in any coordinate system if we make everywhere the replacement:

$$\delta_{\lambda x} \to \delta_{\lambda x} + p_\lambda p_x / M^2, \tag{4}$$

where M is the graviton mass. In figure 1 the p terms do not contribute due to the property (1) of the energy-momentum tensor. The massive graviton propagator may therefore be taken to be

$$\frac{\frac{1}{2}(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha}) - \frac{1}{3}\,\delta_{\mu\nu}\delta_{\alpha\beta}}{p^2 + M^2}\,. \tag{5}$$

For mass-less gravitons the situation is different. Evidently we cannot go to the rest frame of the graviton. In the frame where the three-momentum is along the third axis

$$p_\mu = (0,0,p,ip) \quad p = |\vec{p}|$$

there are two independent polarization vectors describing the two helicity states:

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The sum over polarization is now:

$$\sum_{i=1}^{5} e^{i}_{\mu\nu} e^{i}_{\alpha\beta} = \begin{cases} \frac{1}{2}(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha} - \delta_{\mu\nu}\delta_{\alpha\beta}) & \mu,\nu \\ \alpha,\beta \neq 3,4 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Defining $\bar{p}_\mu$ by:

$$\bar{p}_\mu = (0,0,-p,ip),$$

the expression in other frames is obtained through the replacement, in (6):

$$\delta_{\lambda x} \to \delta_{\lambda x} - \frac{\bar{p}_\lambda p_x + p_\lambda \bar{p}_x}{(p\bar{p})} \tag{7}$$

but again the p-terms are of no importance. The mass-less graviton propagator may thus be taken to be:

$$\frac{\frac{1}{2}(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha} - \delta_{\mu\nu}\delta_{\alpha\beta})}{p^2} \tag{8}$$

There is indeed a discrete difference between the massive and mass-less cases (5) and (8). This difference is observable. Let us first enquire as to the magnitude of the coupling constant λ in figure 1. In case that $T_{\mu\nu}$ and $T'_{\alpha\beta}$ are slow moving massive objects we must get Newton's law. In that case $T_{\mu\nu}$ and $T'_{\alpha\beta}$ are to good approximation only non-zero for $\mu = \nu = \alpha = \beta = 4$. For this value of the indices the numerators in (5) and (8) are 2/3 and 1/2 respectively. Thus, if we want in both cases to reproduce Newton's law we must take the coupling constant λ_m^2 for the massive case 3/4 of that of the mass-less case

$$\lambda_m^2 = \frac{3}{4}\lambda_0^2.$$

In the case that one of the two T's represents a light ray and the other a slow moving massive body, the situation is different. A mass-less object is represented by a traceless energy-momentum tensor:

$$T_{\mu\mu} = 0.$$

In that case (5) and (8), differing only by a term of the form $\delta_{\mu\nu}\delta_{\alpha\beta}$, give equal results. However, the coupling constants are different, and we find that the deflection of a light ray near the sun in the massive theory is 3/4 of the deflection in the mass-less theory (for recent experimental results see [3]).

The assumption that the photon has zero mass is not essential. One obtains a similar result if one replaces the photon by a fast moving massive body. For such a body the first two terms in (5) and (8) dominate.

For the time delays in radar echos from the planets [4] the prediction of the small mass theory is again 3/4 of that of the massless theory. Finally, for the perihelion motion the massive theory gives a result which is 2/3 of that predicted by Einstein†. Hence, we conclude, on the basis of experiment that the mass of the graviton is not just small, but must be zero.

Let us clarify the decoupling of the "third" state which occurs for massive photons in the limit of small mass. After that we investigate what happens to the decoupling in the case of massive gravitons. Consider a massive photon with linear momentum in the z-direction. The three states of polarization of such a photon can be characterized by 3 perpendicular spatial unit vectors in the rest frame of that photon: $\ell^{(1)}(k)$, $\ell^{(2)}(k)$, $\ell^{(3)}(k)$. These vectors can be obtained from the unit vectors in the x, y and z directions, ℓ_1, ℓ_2, ℓ_3, by a Lorentz-transformation ("boost") in the z-direction. This boost only affects ℓ_3, i.e. $\ell_1 = \ell^{(1)}(k)$, $\ell_2 = \ell^{(2)}(k)$. Consider a given source of massive photons, $j(k)$, satisfying $k_\mu j_\mu(k) = 0$. The last condition means that in the rest frame of k the time component of j is 0, i.e. j can be written $j = \sum_i \ell^{(i)}(k) j^{(i)}(k)$. The creation of a photon with polarization i is described by the appropriate component $j^{(i)}$ in the rest frame of k. Consider next a limiting point k' on the forward light cone in the direction of the z-axis. As one lets k approach k', by letting the mass of the photon approach zero, the velocity, v, of the "boost" mentioned before approaches 1. If one assumes that the components $j_\mu(k)$ are given, finite, and continuous at k' in k, then one obtains in that limit, as v approaches 1: $j^{(1)}(k) = j_1(k')$, $j^{(2)}(k) = j_2(k')$, $j^{(3)}(k) = \sqrt{1-v^2}\, j_3(k)$. In other words $j^{(3)}(k)$ approaches zero by Lorentz-contraction, and the photons of zero spin in the forward direction decouple from the source $j(k)$. For massive quantum electrodynamics the decoupling occurs not just for the simple Feynman diagram of figure 1, but for the results in any order given by the Feynman diagrams. This is, however, a consequence of the interaction of quantum electrodynamics, for the Yang Mills theory such a decoupling does not happen already for the diagrams of second order [2].

Consider next a source $T(k)$ for massive gravitons, satisfying $k_\mu T_{\mu\nu}(k) = 0$, assume again that the components $T_{\mu\nu}(k)$ are continuous near a similar point k' on the light cone. By an argument analogous to that for massive photons, one finds that in the limit of zero mass the source decouples from the last 2 polarizations of (2). This decoupling, by Lorentz-contraction, does not happen for

† This was calculated by using a method similar to that used in [5]. For experimental results see [6].

the first polarization of (2). This happens because the first tensor of (2) has x and y components which are not affected by the boost. The first polarizations of (2) correspond to spin zero in the direction of motion. Hence, besides the states of spin ±2 in the forward direction of motion, the spin zero state in the direction of motion remains coupled to the source T in the limit of zero mass. This is the reason for our results.

To conclude we wish to make three remarks. The first remark is that addition of a scalar graviton to the massive theory makes the disagreement with the massless theory worse, because this results in the addition of a term

$$\frac{a\delta_{\mu\nu}\delta_{\alpha\beta}}{p^2 + M^2}, \quad a > 0,$$

to the propagator (5).

The second remark concerns the coupling of the graviton to the energy-momentum tensor $T_{\mu\nu}$, which satisfies (1). One can make the discontinuity disappear by coupling the massive graviton instead of to $T_{\mu\nu}$ to a different $B_{\mu\nu}$ which satisfies (1) up to first order in the mass of the graviton [7]. However, whereas the construction of $T_{\mu\nu}$ is easy, and local, for a material body, the construction of such a $B_{\mu\nu}$ is nonlocal and involves the graviton as well as the body.

The third remark concerns the fact that in using results from quantum field theory we have assumed space-time to be flat, at least asymptotically. It can be shown that this assumption does no harm for the local experiments which we discussed [8].

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