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MASSIVE YANG-MILLS FIELDS

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Abstract: Two problems are studied in the paper: (i) the relation between Lagrangian and Feynman rules if the Lagrangian contains derivative couplings and/or vector meson fields and (ii) the behaviour of certain two closed loop diagrams in the perturbation theory of Yang-Mills fields. With respect to (i), using an extremely simple regularisation procedure, the canonical formalism is shown to give directly manifestly covariant Feynman rules. With respect to (ii), a certain set of Feynman rules involving ghost particles is known to be valid for diagrams of the Yang-Mills theory with no or one closed loop. It is shown however that with the same set of rules the imaginary part of the two-loop self-energy diagrams receives a non-zero contribution of the unphysical three-ghost intermediate state.

1. INTRODUCTION

In a recent paper [1] perturbation theory of massive Yang-Mills fields has been investigated. In the first instance one obtains a set of Feynman rules (set 1) involving a vector-boson propagator, a three-boson and a four-boson vertex. It was shown that the S-matrix diagrams with no or one closed loop obtained with these rules give contributions equal to those obtained with a different set (set 2) of Feynman rules involving a "ghost" particle that appears only in closed loops. This is very interesting since power counting of diagrams obtained with set 2 indicate a renormalizable theory ‡.

The derivations of that paper however contain a gap. For a given Lagrangian, Feynman rules were assumed rather than derived; and as is well known in the case that a Lagrangian contains derivatives and/or vector meson fields the derivation via the canonical formalism is not straightforward. Although for a considerable set of cases, notably the Lagrangians studied in sect. 2 of this paper, Feynman rules were established by Lee and Yang [2],

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‡ D. Boulware, to be published in Annals of Physics has studied the question of renormalizability with different methods, and concludes that the theory is non-renormalizable.

the case of interest to us seems not to have been studied in a satisfactory way. Thus it was not at all clear that the Feynman rules used had anything to do with the Lagrangians considered.

In this paper we investigate two aspects of the theory. First we will demonstrate that the Feynman rules used were the correct ones for the given Lagrangians; this we will do by employing a very simple trick that results in that the usual canonical formalism leads directly to the desired manifestly covariant set of Feynman rules, whether or not derivatives and/or vector fields are present. Secondly we investigate the two-loop self-energy diagrams of set 2 and show that the imaginary part receives a non-zero contribution from the three-ghost intermediate state. Since we have not computed the contribution of the two-ghost intermediate state, we have not yet a conclusive proof that set 2 is wrong for diagrams with two closed loops because a cancellation could occur. Such a type of cancellation would be totally different from the type encountered in the one-closed-loop case (where phase space was never involved) and for this reason it seems not very plausible to us that set 2 is correct for two or more closed loops.

In sect. 2 we will very briefly describe the canonical troubles and demonstrate how to obtain Feynman rules in case of a Lagrangian containing scalar fields and derivative couplings. No essential difficulties present themselves in case of vector fields; that case is treated in appendix A. In sect. 3 we review the Feynman rules, set 1 and set 2, and give a simple example that shows how for instance unitarity is maintained for set 2 despite the appearance of a ghost closed loop. The two-loop self-energy diagram is discussed in sect. 4. Appendix B finally gives a short review of the method of indefinite metric which we use quite extensively.

2. INTERACTION LAGRANGIAN WITH DERIVATIVES

Consider a Lagrangian density of the following type:

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu \psi)^2 - \tfrac{1}{2}m^2\psi^2 + \mathcal{L}_{\text{int}}(\partial_\mu \psi, \psi). \tag{2.1}$$

Here ψ represents a set of n scalar fields:

$$\psi = \begin{pmatrix} \psi_1(x) \\ \vdots \\ \psi_n(x) \end{pmatrix}.$$

In the canonical formalism one normally proceeds as follows. Momenta Π_i are defined by

$$\Pi_i = \frac{\delta \mathcal{L}}{\delta \partial_0 \psi_i}. \tag{2.2}$$

This equation gives the Π_i as functions of the ψ_i and $\partial_\mu \psi_i$. Next one must invert this equation in the sense that the $\partial_0 \psi_i$ must be found as function of the Π_i and ψ_i (and eventually their spatial derivatives). If $\mathcal{L}_{\text{int}}$ contains derivatives one obtains in general rather involved expressions for the $\partial_0\psi_i$. Further one defines a Hamiltonian $\mathcal{H} = \Pi_i\partial_0\psi_i - \mathcal{L}$ substituting everywhere these expressions for the $\partial_0\psi_i$; there results an interaction Hamiltonian which depends in a rather complicated way on the momenta Π_i and fields ψ_i. Perturbation theory in the interaction representation consists of employing $\mathcal{H}_{\text{int}}$ where the Π_i, ψ_i (and their spatial derivatives) are replaced by their free field values $\partial_0\psi_i^0, \psi_i^0$. Since $\mathcal{H}_{\text{int}}$ is not simply related to $\mathcal{L}_{\text{int}}$, the vertices in the resulting Feynman rules are not simply related to $\mathcal{L}_{\text{int}}$. On the other hand, the time-ordered product of two $\partial_0\psi_i^0$ contains certain contact terms and it was demonstrated by Lee and Yang [2] that a certain rearrangement of the perturbation series leads to both vanishing of these contact terms and those vertices which are not directly suggested by the structure of $\mathcal{L}_{\text{int}}$. Briefly, the complications occurring in solving the equation for the $\partial_0\psi_i$ cancel finally against the contact terms in the propagators for the time derivatives of the field.

Thus, if we could somehow manage to get rid of the propagator contact terms, we might expect that all the spurious vertices in the interaction Hamiltonian disappear. This is indeed what happens.

To get rid of the contact terms we add a set of scalar fields φ_i of mass M, negative metric and positive energy (see appendix B). In the interaction Lagrangian we replace everywhere $\partial_\mu\psi_i$ by $\partial_\mu\psi_i + \partial_\mu\varphi_i$, and in the end we take the limit $M \to \infty$, thus removing the φ from physics. The time-ordered product for the combination $\partial_\mu\psi_i + \partial_\mu\varphi_i$ contains no contact terms, those due to $\partial_\mu\psi_i$ cancelling against those coming from $\partial_\mu\varphi_i$.

The interaction Hamiltonian deduced from this Lagrangian is now indeed found to be very much like the interaction Lagrangian, as in the case of no derivatives. To see this consider the modified Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu\psi)^2 - \tfrac{1}{2}m^2\psi^2 + \tfrac{1}{2}(\partial_\mu\varphi)^2 + \tfrac{1}{2}M^2\varphi^2 + \mathcal{L}_{\text{int}}(\dot\psi + \dot\varphi, \partial_a\psi + \partial_a\varphi, \psi). \qquad (2.3)$$

In here the index a takes the values 1, 2, 3. Further

$$\dot\psi = \partial_0\psi = i\partial_4\psi\,,$$

$$\dot\varphi = \partial_0\varphi = i\partial_4\varphi. \qquad (2.4)$$

The momenta Π_ψ and Π_φ are defined by:

$$\Pi_{\psi_i} = \frac{\delta\mathcal{L}}{\delta\dot\psi_i} = \dot\psi_i + \frac{\delta\mathcal{L}_{\text{int}}}{\delta\dot\psi_i} = \dot\psi_i + \frac{\delta\mathcal{L}_{\text{int}}}{\delta(\dot\psi_i + \dot\varphi_i)}\,,$$

$$\Pi_{\varphi_i} = -\dot\varphi_i + \frac{\delta\mathcal{L}_{\text{int}}}{\delta(\dot\psi_i + \dot\varphi_i)}\,. \qquad (2.5)$$

Solving these equations for $\dot\psi_i$ and $\dot\varphi_i$ would still be complicated, however we need only to know the combination $\dot\psi + \dot\varphi$. Subtracting the second from the first equation we find:

$$\dot{\psi}_i + \dot{\varphi}_i = \Pi_{\psi_i} - \Pi_{\varphi_i}. \tag{2.6}$$

We can now write down the Hamiltonian. For the free part of the Hamiltonian we obtain:

$$\mathcal{H}_{\text{free}} = \Pi_\psi \dot{\psi} + \Pi_\varphi \dot{\varphi} - \tfrac{1}{2}(\dot{\psi}+\dot{\varphi})(\dot{\psi}-\dot{\varphi}) + \text{terms}, \tag{2.7}$$

where "terms" stands for terms not containing Π, $\dot{\psi}$ or $\dot{\varphi}$. Using eq. (2.6) only this reduces to:

$$\mathcal{H}_{\text{free}} = \tfrac{1}{2}\Pi_\psi(\dot{\psi}+\dot{\varphi}) + \tfrac{1}{2}\Pi_\varphi(\dot{\psi}+\dot{\varphi}) + \text{terms}$$

$$= \tfrac{1}{2}\Pi_\psi^2 - \tfrac{1}{2}\Pi_\varphi^2 + \text{terms}.$$

Since $\mathcal{L}_{\text{int}}$ depends only on $\dot{\psi} + \dot{\varphi}$ the whole Hamiltonian can be written down:

$$\mathcal{H} = \tfrac{1}{2}\Pi_\psi^2 - \tfrac{1}{2}\Pi_\varphi^2 + \tfrac{1}{2}(\partial_a\psi)^2 + \tfrac{1}{2}m^2\psi^2 - \tfrac{1}{2}(\partial_a\varphi)^2 - \tfrac{1}{2}M^2\varphi^2 + \mathcal{H}_{\text{int}},$$

with

$$\mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{int}}(\Pi_\psi - \Pi_\varphi, \partial_a\psi + \partial_a\varphi, \psi).$$

Since the free field values of Π_ψ and Π_φ are $\partial_0\psi$ and $-\partial_0\varphi$ respectively, we see that $\mathcal{H}_{\text{int}}$ in the interaction representation is just minus the interaction Lagrangian. In the resulting Feynman rules one has no spurious vertices and furthermore one has the propagator

$$\langle 0 | T(\partial_\mu\psi_i(x) + \partial_\mu\varphi_i(x), \partial_\nu\psi_j(x') + \partial_\nu\varphi_j(x')) | 0\rangle$$

$$= \frac{\delta_{ij}}{i(2\pi)^4}\int d_4k\, e^{ik(x'-x)}\left\{\frac{k_\mu k_\nu}{k^2+m^2-i\epsilon} - \frac{k_\mu k_\nu}{k^2+M^2-i\epsilon}\right\}.$$

In the limit $M\to\infty$ the second part does not contribute. For completeness, note:

$$\langle 0 | T(\partial_\mu\psi_i(x), \partial_\nu\psi_j(x')) | 0\rangle$$

$$= \delta_{ij}\left\{i\delta_4(x-x')\,\delta_{\mu4}\,\delta_{\nu4} + \frac{1}{i(2\pi)^4}\int d_4k\, e^{ik(x'-x)}\,\frac{k_\mu k_\nu}{k^2+m^2-i\epsilon}\right\}.$$

The same procedure works if vector-meson fields are present. In addition to the work done above one must also treat the complication of a dependent field, the fourth component of the vector field [3]. However, apart from an extra partial integration with respect to the spatial variables x_a ($a = 1, 2, 3$) no further tricks are required. Details are given in appendix A.

Here we must make a remark. What we have done is in fact another application of the regulator method of Pauli and Villars [4]. Also here it might be that there are other regulator methods, which might even lead to different results for the S-matrix in case that convergence problems exist. We mention in this context the extra vertex obtained by Lee and Yang [2] in the case of a photon coupled to vector bosons with a non-zero anomalous magnetic moment. What method must be preferred in such cases may be dictated by the symmetries of the theory; otherwise it becomes a matter of personal taste.

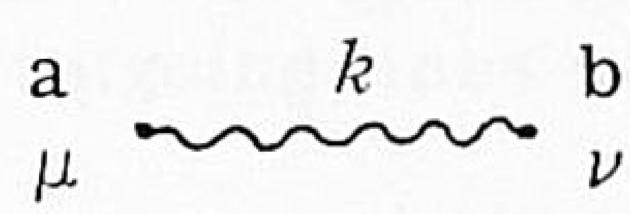
3. FEYNMAN RULES

The Lagrangian describing the massive Yang-Mills fields is:

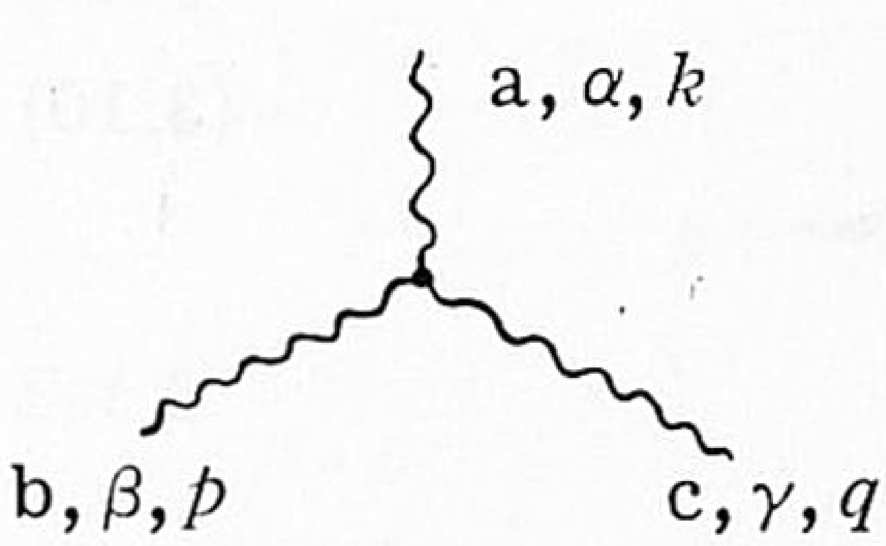
$$\mathcal{L} = -\tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \tfrac{1}{2} M^2 W^a_\mu W^a_\mu , \tag{3.1}$$

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon_{\rm ade} W^d_\mu W^e_\nu . \tag{3.2}$$

We have specialised to the case of three different vector-meson fields, $a = 1, 2, 3$. The corresponding Feynman rules are:

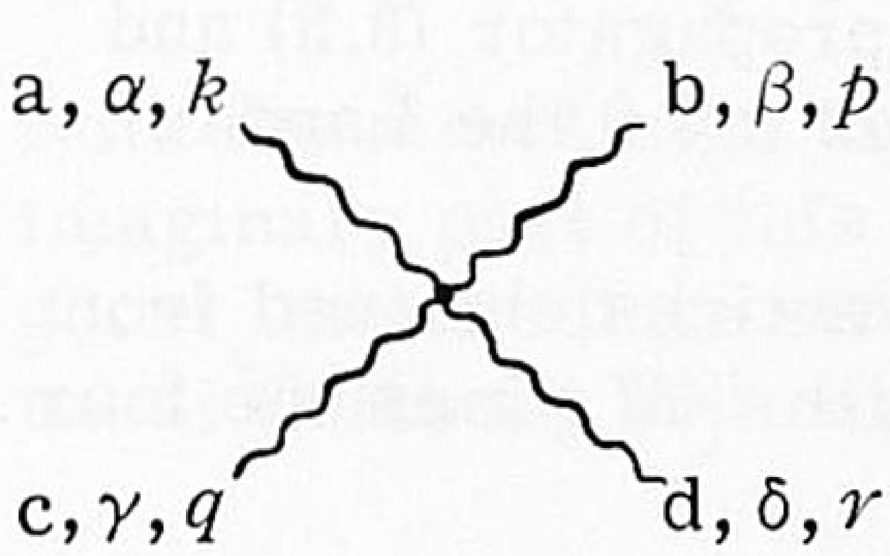


$$\delta_{\rm ab} \frac{\delta_{\mu\nu} + k_\mu k_\nu / M^2}{k^2 + M^2 - i\epsilon} , \tag{3.3}$$



$$-ig\epsilon_{\rm abc}\{\delta_{\alpha\gamma}(k-q)_\beta + \delta_{\beta\gamma}(q-p)_\alpha + \delta_{\alpha\beta}(p-k)_\gamma\} , \tag{3.4}$$

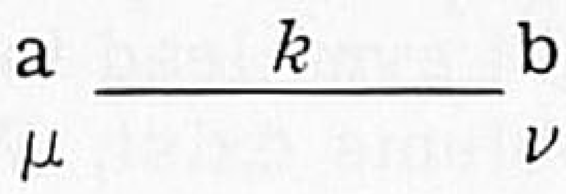
$p + k + q = 0$



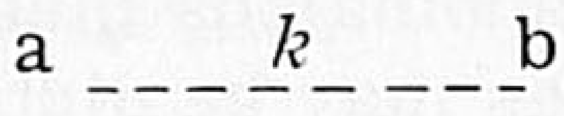
$$-g^2[\epsilon_{\rm gdc}\epsilon_{\rm gba}\{2\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma} - \delta_{\alpha\beta}\delta_{\gamma\delta}\}$$
$$+ \epsilon_{\rm gdb}\epsilon_{\rm gca}\{2\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\gamma\beta} - \delta_{\alpha\gamma}\delta_{\beta\delta}\}] , \tag{3.5}$$

$p + k + q + r = 0$

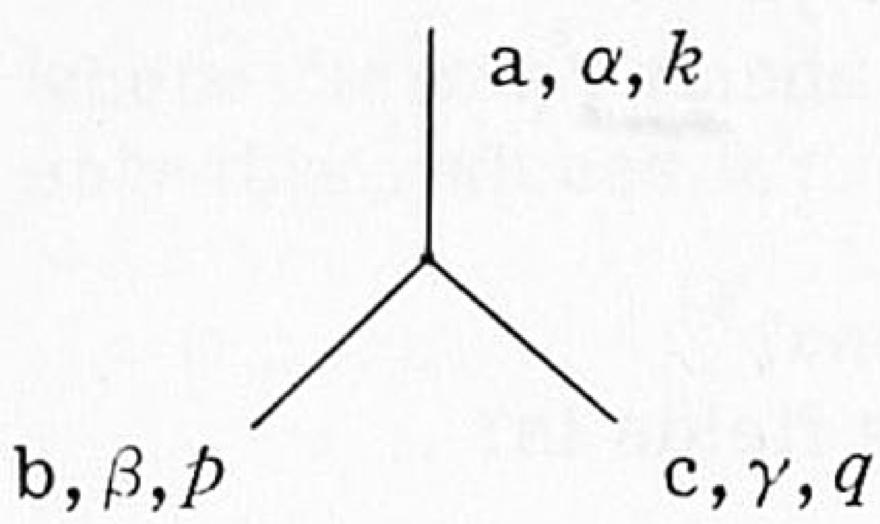
This is what we have called set 1. In ref. [1] it has been shown that for diagrams with no or one closed loop there exists another set, called set 2, giving identical results for the S-matrix elements. These rules involve a ghost particle whose mass was taken to be zero, although for the proofs given any mass which is a multiple of the W-meson mass could have been taken. We give the rules of set 2, taking the mass of the ghost particle to be κ times the W-mass.



$$\delta_{ab}\left(\frac{\delta_{\mu\nu} + (\kappa^2 - 1)\, k_\mu k_\nu/(k^2 + \kappa^2 M^2 - i\epsilon)}{k^2 + M^2 - i\epsilon}\right), \tag{3.6}$$

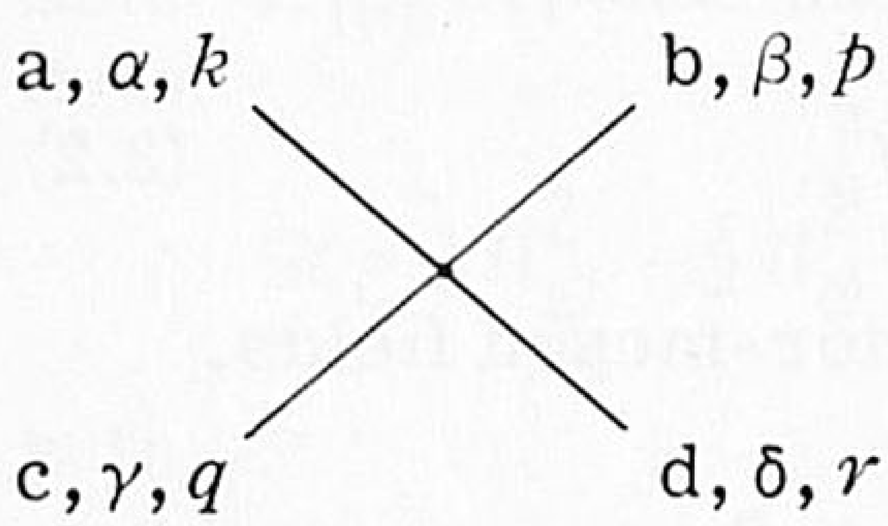


$$-\delta_{ab}\,\frac{1}{k^2 + \kappa^2 M^2 - i\epsilon}, \tag{3.7}$$



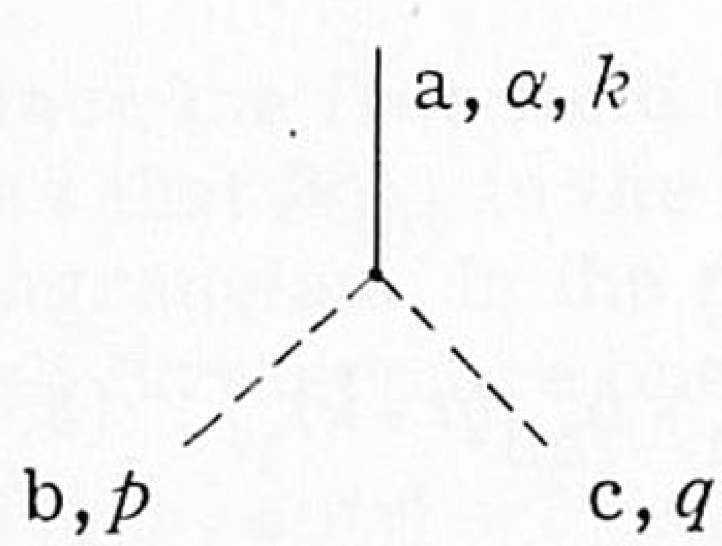
$p + k + q = 0$

$$-ig\,\epsilon_{abc}\left\{\delta_{\alpha\gamma}(k-q)_\beta + \delta_{\beta\gamma}(q-p)_\alpha + \delta_{\alpha\beta}(p-k)_\gamma\right\}, \tag{3.8}$$



$p + k + q + r = 0$

$$-g^2\left[\epsilon_{gdc}\,\epsilon_{gba}\left\{2\,\delta_{\alpha\gamma}\,\delta_{\beta\delta} - \delta_{\alpha\delta}\,\delta_{\beta\gamma} - \delta_{\alpha\beta}\,\delta_{\gamma\delta}\right\}\right.$$
$$\left. + \,\epsilon_{gdb}\,\epsilon_{gca}\left\{2\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} - \delta_{\alpha\delta}\,\delta_{\gamma\beta} - \delta_{\alpha\gamma}\,\delta_{\beta\delta}\right\}\right], \tag{3.9}$$



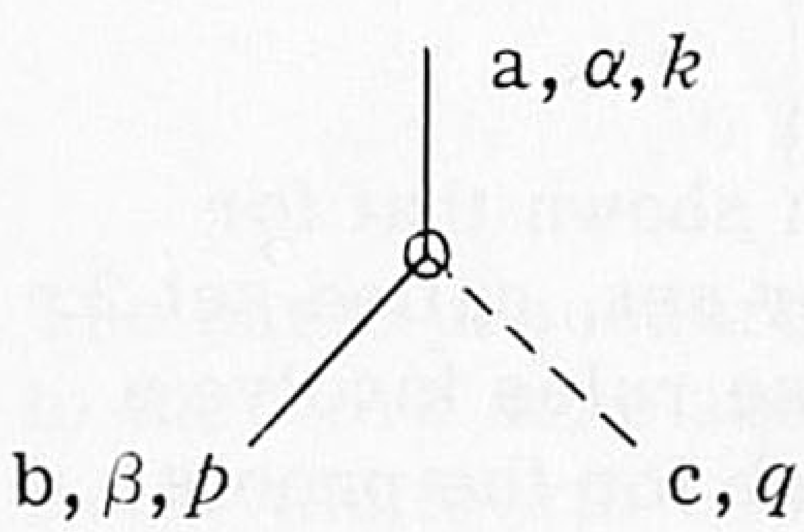
$p + k + q = 0$

$$\tfrac{1}{2}\, ig\,\epsilon_{abc}\,(p-q)_\alpha . \tag{3.10}$$

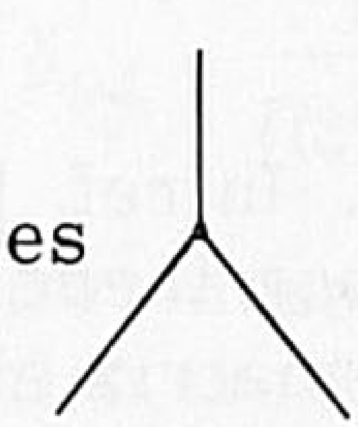
Factor -1 for every closed loop of an even number of ghost propagators.

Note that the propagator (3.6) is the sum of the W-propagator (3.3) and $k_\mu k_\nu/M^2$ times the ghost propagator (3.7). If one takes $\kappa = 0$ the Landau gauge results; for $\kappa = 1$ one has the Feynman gauge.

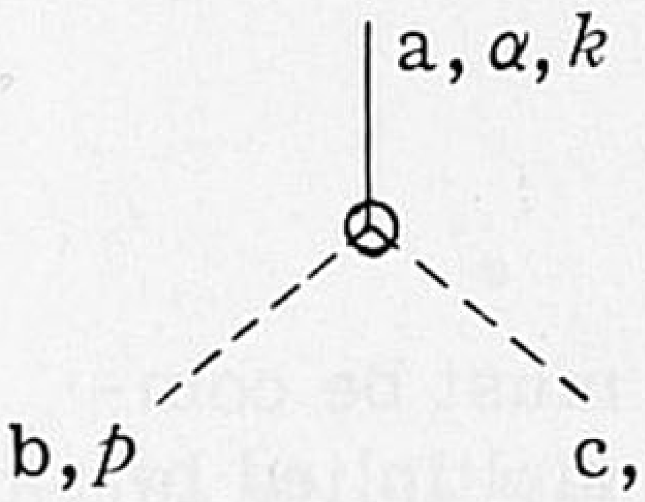
For further reference we introduce some further vertices obtained from expressions (3.8), (3.9) and (3.10) by multiplication with i/M times the four momentum of one of the lines. For example:



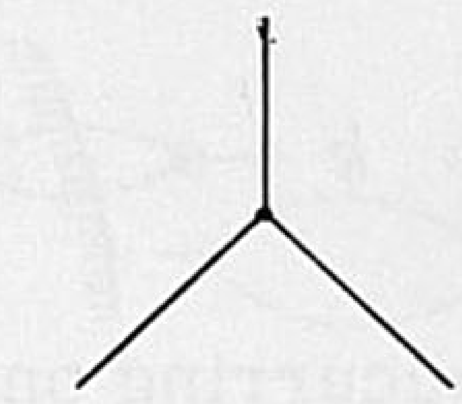
$= \dfrac{i}{M}\, q_\gamma$ times



$$= -\frac{g}{M}\,\epsilon_{abc}\left[(p^2 - k^2)\,\delta_{\alpha\beta} - p_\alpha p_\beta + k_\alpha k_\beta\right], \tag{3.11}$$

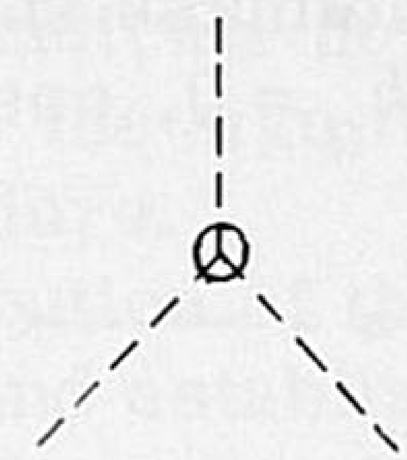


$$= \frac{i}{M} q_\gamma \cdot \frac{i}{M} p_\beta .$$



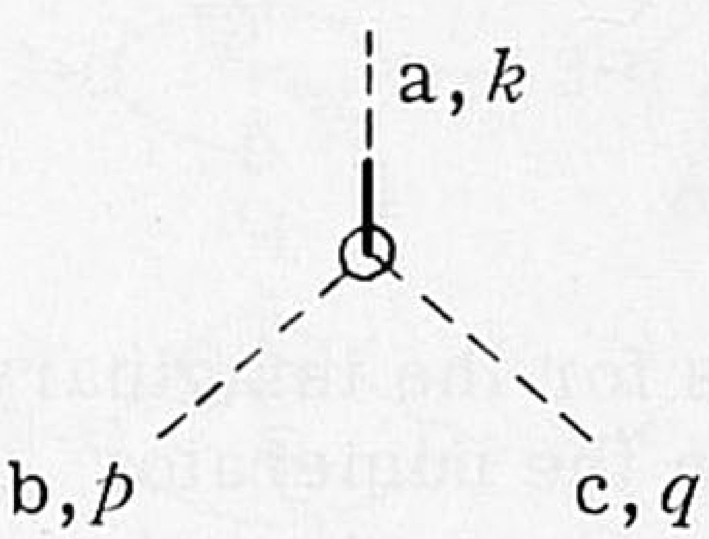
$$= - \frac{ig}{M^2} \epsilon_{abc} [-\tfrac{1}{2}(qk - pk) k_\alpha - \tfrac{1}{2} k^2 (p - q)_\alpha] . \qquad (3.12)$$

Note

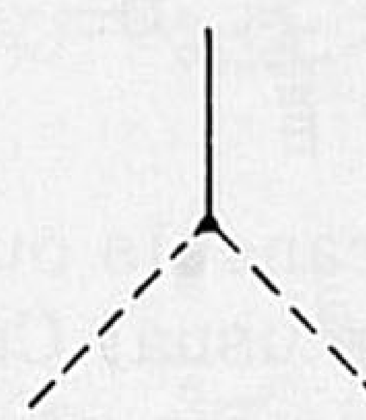


$$= 0 . \qquad (3.13)$$

In case of (3.10) multiplied by k_α we must introduce a special notation to remember the line which was originally the solid line:



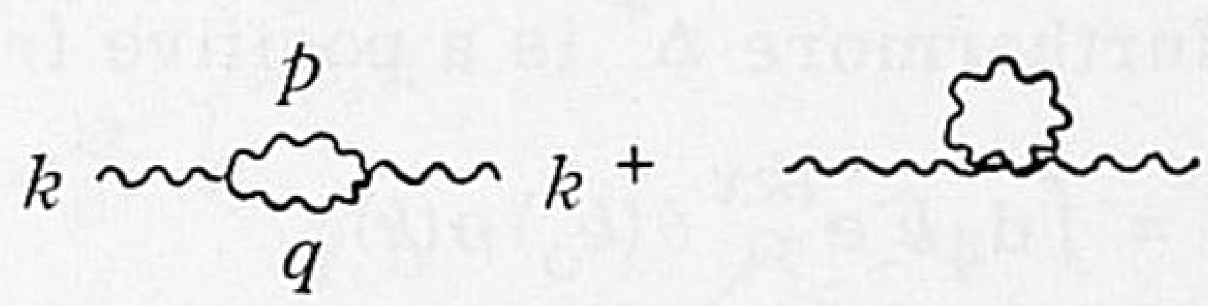
$$= \frac{i}{M} k_\alpha .$$



$$= \frac{1}{2M} g \epsilon_{abc} (p^2 - q^2) . \qquad (3.14)$$

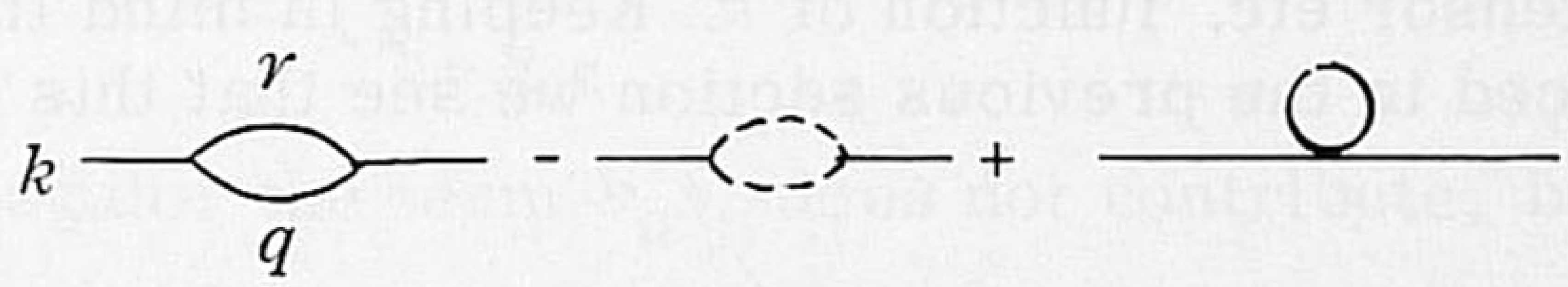
To exhibit the equivalence of set 1 and set 2 for one closed loop we consider the lowest order self energy diagram for a W on the mass-shell. One must also multiply the outgoing lines with the appropriate polarisation vectors $e^{\epsilon}_{\mu}(k)$.

Set 1



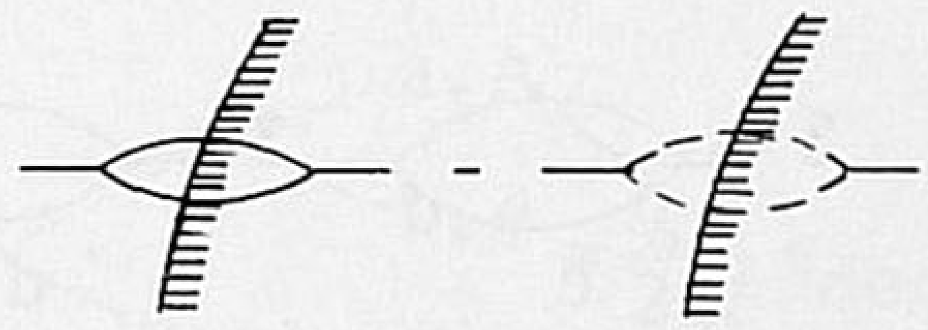
$$k^2 = -M^2 .$$

Set 2



$$k^2 = -M^2 .$$

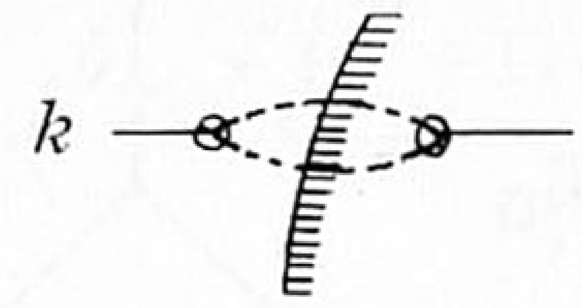
Note the minus sign in front of the second diagram. Let us now see if the imaginary part of this last set of diagrams receives any contribution of the ghost intermediate states. To obtain the imaginary part one may apply the usual Cutkosky [5] cutting technique. One finds:



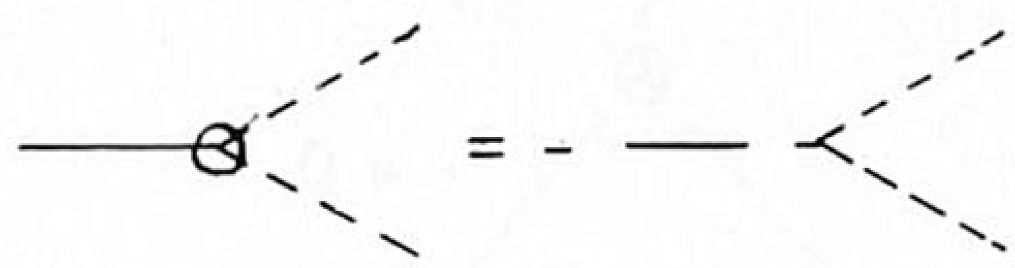
Consider now in particular the two ghost intermediate state. One has

 = $+ \frac{k_\mu k_\nu}{M^2}$

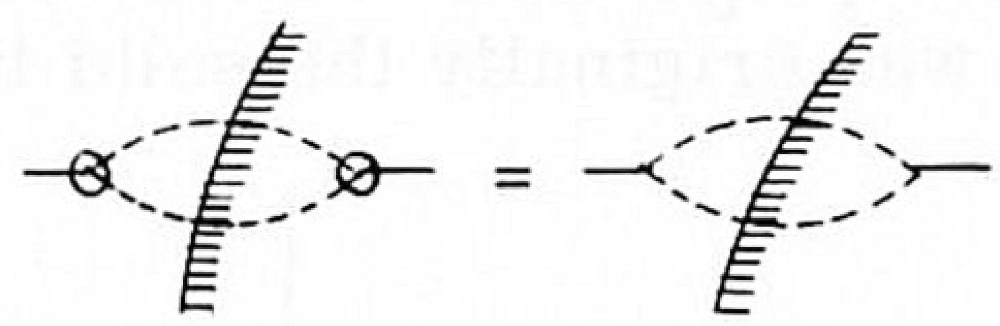
Therefore the two ghost contribution to the first diagram is



Remember that everything occurring on the right-hand side must be complex conjugated. Now, for a W on the mass shell, and when multiplied by a polarisation vector $e_\alpha(k)$ with $e_\alpha(k)\, k_\alpha = 0$, one verifies



Thus



and the 2 ghost contribution cancels out.

In the above we applied the usual Cutkosky cutting rules for the imaginary part. One may wonder whether the presence of momenta in the numerator perhaps spoil this rule. This is in fact not so; as has been demonstrated elsewhere [6] the essential features necessary for application of the cutting rules are that in coordinate space the propagators Δ_F are true time ordered products:

$$\Delta_F(x) = \theta(x_0)\, \Delta^+(x) + \theta(-x_0)\, \Delta^-(x)\,,$$

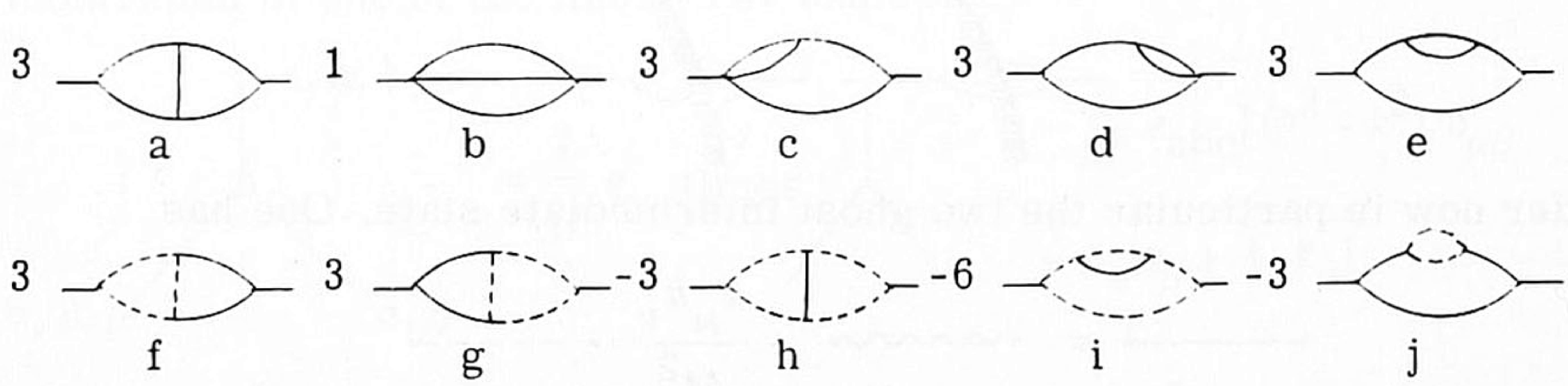
where $\Delta^- = (\Delta^+)^*$ and where furthermore Δ^+ is a positive frequency function

$$\Delta^+(x) = \int d_4k\, e^{ikx}\, \theta(k_0)\, \rho(k)\,.$$

ρ may be a scalar or tensor etc. function of k. Keeping in mind the regulator techniques introduced in the previous section we see that this is the case here.

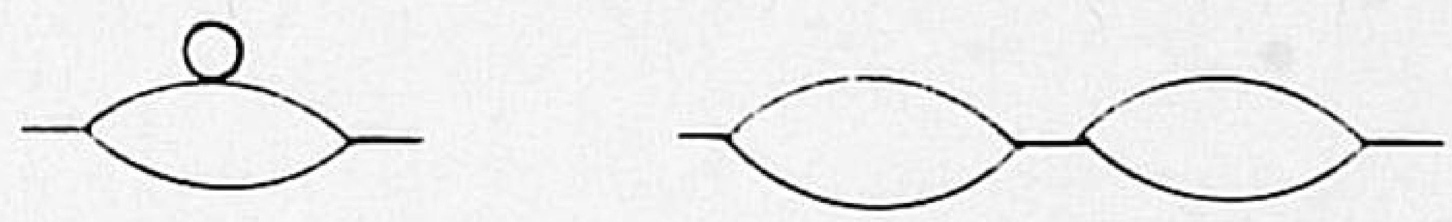
4. DIAGRAMS WITH TWO CLOSED LOOPS

Let us now consider the two closed loop self-energy diagrams of set 2:



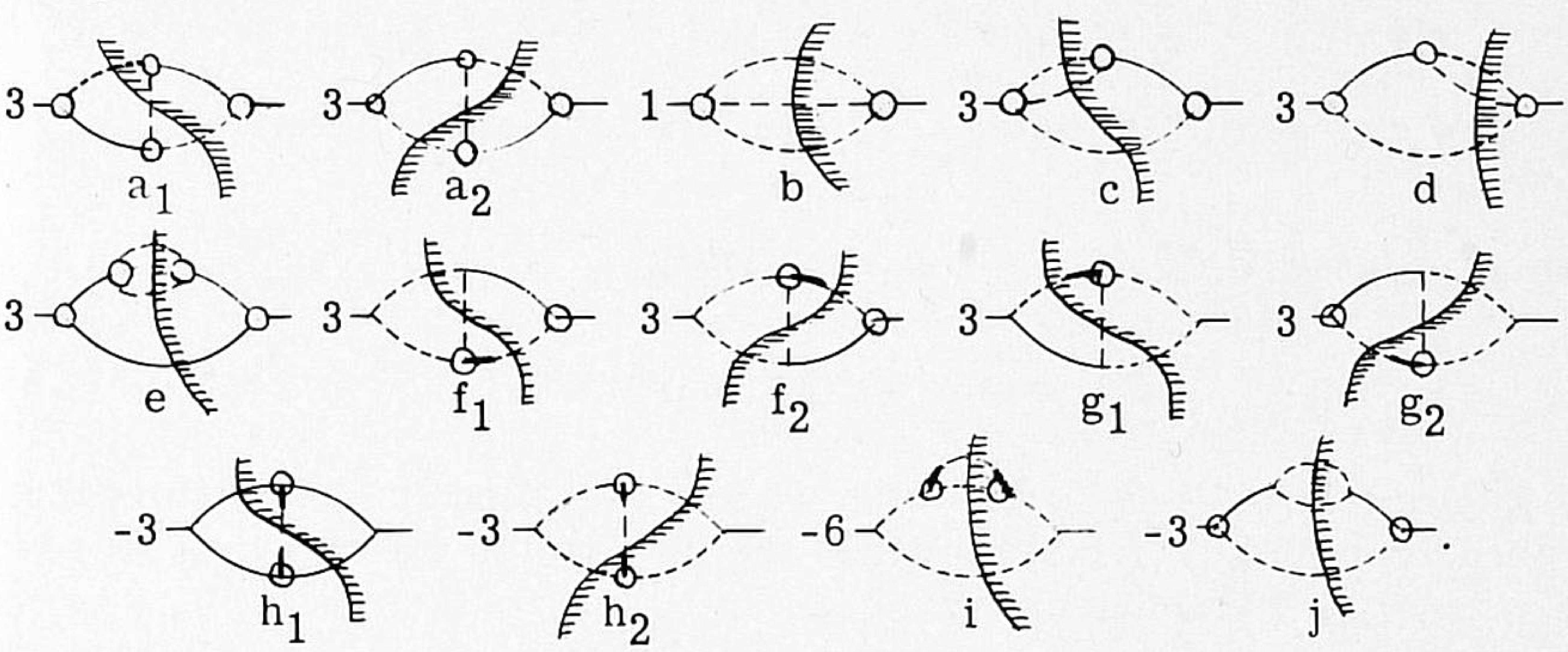
(4.1)

We did not include diagrams that do not have a three-particle cut, like



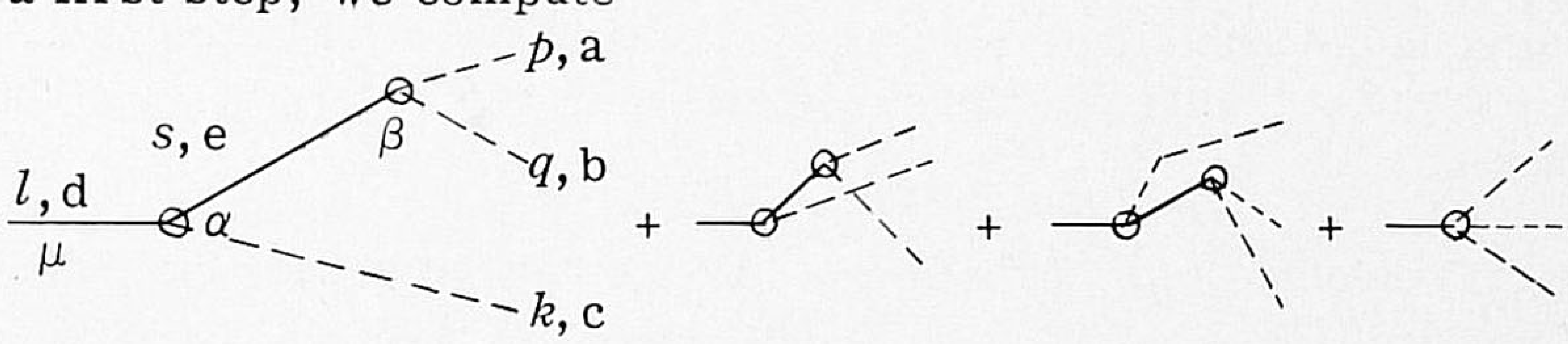
In the above list the numbers indicate how many times a given diagram should be counted. Apart from the sign these numbers follow simply if one applies the Dyson-Wick procedure.

We will compute the contribution of the three-ghost state to the imaginary part of this set of diagrams, and establish a non-zero result. Remembering that the solid line represents the sum of a W and a ghost propagator we must compute:



(4.2)

As a first step, we compute

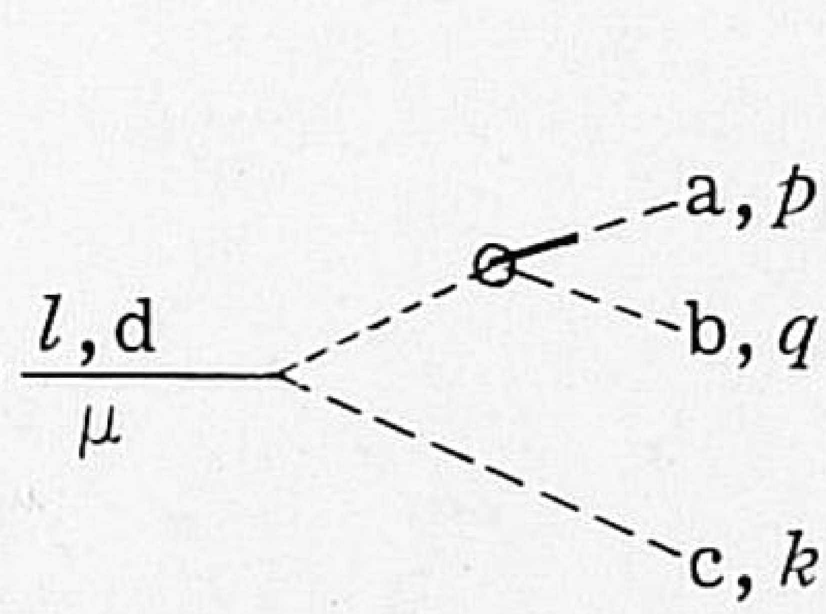
I = 

. (4.3)

In the propagator the term $s_\alpha s_\beta$ does not contribute, because of (3.13). The result is

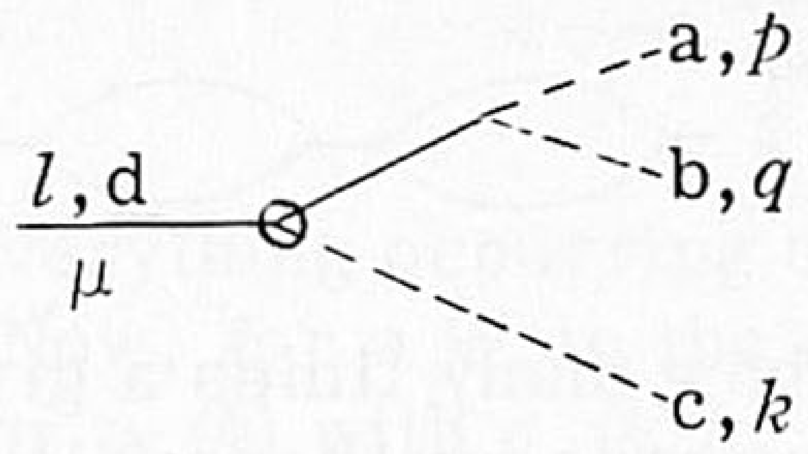
$$I = -\frac{ig^2}{M}\left[\epsilon_{deb}\,\epsilon_{eca}\,p_\mu - \epsilon_{dea}\,\epsilon_{ebc}\,q_\mu\right]. \tag{4.4}$$

Next we need:

II = 

$$= \frac{ig^2}{2M}\,\epsilon_{dec}\,\epsilon_{eba}\,k_\mu\,, \tag{4.5}$$

where we used $q^2 = -\kappa^2 M^2$. Finally

III = 

$= -\frac{ig^2}{2M}\,\epsilon_{dec}\,\epsilon_{eba}(q-p)_\mu\,,$ (4.6)

where we used $q^2 = p^2$ $(= -\kappa^2 M^2)$.

Multiplication of I with itself (the outgoing W being characterized by indices g, ν and momentum l) gives the contributions of diagrams $a_1 \ldots e$. One obtains:

$$a_1 \ldots e = -\frac{2g^4}{M^2}\,\delta_{dg}\,\{2p_\mu p_\nu + 2q_\mu q_\nu + p_\mu q_\nu + q_\mu p_\nu\}\,. \tag{4.7}$$

This must of course still be integrated over the available phase space ($l^2 = -M^2$):

$$\int d_4q \int d_4p \int d_4k\ \delta_4(l-p-q-k)\,\theta(q_0)\,\theta(p_0)\,\theta(k_0) \times \delta(q^2+\kappa^2M^2)\,\delta(p^2+\kappa^2M^2)\,\delta(k^2+\kappa^2M^2)\,. \tag{4.8}$$

This is only non-zero if $|\kappa| < \frac{1}{3}$, which we assume. Because of the fact that p, q, k occur completely symmetrical in the integration we may in (4.7) replace $q_\mu q_\nu$ by $p_\mu p_\nu$; furthermore:

$$p_\mu q_\nu = p_\mu(l-p-k)_\nu\,.$$

The l_ν term does not contribute since $l_\nu e_\nu(l) = 0$ where $e_\nu(l)$ is the polarisation vector for the outgoing W. Moreover, we may replace k_ν by q_ν. Thus

$$p_\mu q_\nu \rightarrow -p_\mu p_\nu - p_\mu q_\nu$$

that is, we may replace $p_\mu q_\nu$ by $-\frac{1}{2}p_\mu p_\nu$. Altogether we get for diagrams $a_1 \ldots e$:

$$a_1 \ldots e = -\frac{6g^4}{M^2}\,\delta_{dg}\,p_\mu p_\nu \tag{4.9}$$

still to be integrated over phase space, expression (4.8).

The rest of the diagrams must be computed one by one. One finds:

$$f_1 = f_2 = \frac{g^4}{2M^2}\,p_\mu(q-p)_\nu\,\delta_{dg} \rightarrow \frac{g^4}{2M^2}\,\delta_{dg}(-\tfrac{3}{2}p_\mu p_\nu)\,, \tag{4.10}$$

$$g_1 = g_2 = \frac{g^4}{2M^2}\,(q-k)_\mu k_\nu\,\delta_{dg} \rightarrow \frac{g^4}{2M^2}\,\delta_{dg}(-\tfrac{3}{2}p_\mu p_\nu)\,, \tag{4.11}$$

$$h_1 = h_2 = \frac{g^4}{2M^2}\,p_\mu k_\nu\,\delta_{dg} \rightarrow \frac{g^4}{2M^2}\,\delta_{dg}(-\tfrac{1}{2}p_\mu p_\nu)\,, \tag{4.12}$$

$$\mathrm{i} = -\frac{g^4}{M^2}\, k_\mu k_\nu \delta_{\mathrm{dg}} \qquad \rightarrow \frac{g^4}{2M^2}\,\delta_{\mathrm{dg}}(-2p_\mu p_\nu)\,, \tag{4.13}$$

$$\mathrm{j} = -\frac{g^4}{M^2}\,(q-p)_\mu (q-p)_\nu \delta_{\mathrm{dg}} \rightarrow \frac{g^4}{2M^2}\,\delta_{\mathrm{dg}}(-6p_\mu p_\nu)\,. \tag{4.14}$$

All together

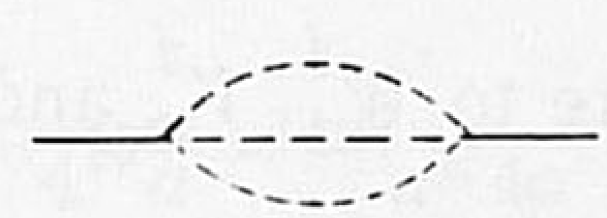
$$\begin{array}{rr} \mathrm{a}_1 \ldots \mathrm{e} & -12 \\ 6(\mathrm{f}_1 + \mathrm{g}_1) & -18 \\ -(6\mathrm{h}_1 + 6\mathrm{i} + 3\mathrm{i}) & \underline{+33} \\ & 3(g^4/2M^2)\,\delta_{\mathrm{dg}}\, p_\mu p_\nu \,. \end{array} \tag{4.15}$$

Integration over phase space does not give zero, for instance for $\mu = \nu = 1$ one has the factor $p_1 p_1$ to be integrated in eq. (4.8).

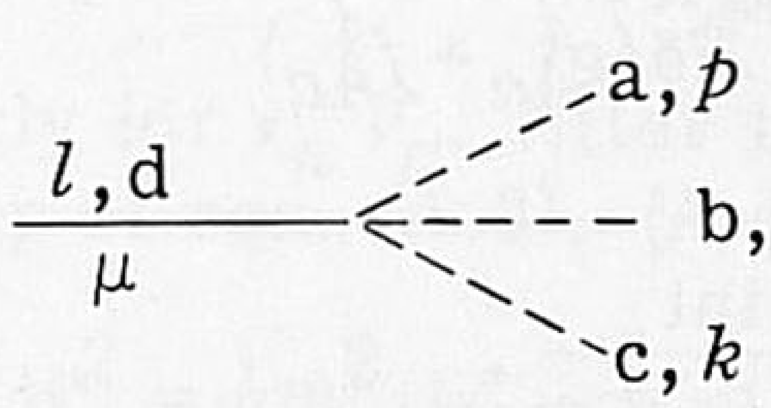
Analogous calculations show vanishing contributions for the 2W, 1 ghost and 1W, 2 ghost intermediate states, quite apart from phase space considerations. These latter calculations are simple if one uses the techniques given in ref. [1]. Calculation of the two ghost intermediate state requires integration over a closed loop, which we have not done.

5. CONCLUSIONS

In our opinion the regulator technique settles in a satisfactory manner the question of the relation between Lagrangian and Feynman rules in case of derivative coupling, or vector meson fields. As to the question of the Yang-Mills fields, we wish to note here only one amusing fact. If one adds to the diagrams (4.1) the diagram



where



$$\frac{ig^2}{2M}\left[\epsilon_{\mathrm{dea}}\,\epsilon_{\mathrm{ebc}}\, q_\mu - \epsilon_{\mathrm{deb}}\,\epsilon_{\mathrm{eca}}\, p_\mu\right]$$

is a vertex introduced in ref. [1], then the three-ghost intermediate state contribution to the imaginary part vanishes.

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APPENDIX A

Interaction Lagrangian with vector-meson fields

Consider a Lagrangian of the following type

$$\mathcal{L} = -\tfrac{1}{4}g^2_{\mu\nu} - \tfrac{1}{2}m^2W^2_{\mu} + \mathcal{L}_{\mathrm{int}}(W_{\mu}, g_{\mu\nu})\,; \qquad \text{(A.1)}$$

W_{μ} stands for n vector-meson fields $W^1_{\mu}(x) \ldots W^n_{\mu}(x)$. Further

$$g^i_{\mu\nu} = \partial_{\mu} W^i_{\nu}(x) - \partial_{\nu} W^i_{\mu}(x)\,.$$

Thus derivatives enter only in the combination $g_{\mu\nu}$. As $\mathcal{L}$ does not depend on the $\partial_4 W^i_4$ the canonical momenta conjugate to W^i_4 vanish. Following Wentzel [3] we will treat the W^i_4 as dependent fields; this dependence is such that regulator fields must be added to the W^i_4. Also we need regulator vector fields in connection with the $g^i_{\mu\nu}$.

We therefore modify the Lagrangian eq. (A.1) to obtain

$$\mathcal{L} = -\tfrac{1}{4}g^2_{\mu\nu} - \tfrac{1}{2}m^2W^2_{\mu} + \tfrac{1}{4}f^2_{\mu\nu} + \tfrac{1}{2}M^2V^2_{\mu} + \tfrac{1}{2}(\partial_{\mu}\varphi)^2 + \tfrac{1}{2}M^2\varphi^2$$

$$+ \mathcal{L}_{\mathrm{int}}(W_{\mu} + \frac{1}{m}\partial_{\mu}\varphi, g_{\mu\nu} + f_{\mu\nu})\,. \qquad \text{(A.2)}$$

Here φ and V_{μ} stand for n scalar fields φ^i and n vector fields V^i_{μ} respectively. Of course they have negative metric. Further

$$f^i_{\mu\nu} = \partial_{\mu} V^i_{\nu}(x) - \partial_{\nu} V^i_{\mu}(x)\,.$$

There exist momenta conjugate to W^i_a, V^i_a and φ^i $(a = 1, 2, 3)$:

$$\Pi^i_a = \frac{\delta\mathcal{L}}{\delta\partial_0 W^i_a} = \frac{\delta\mathcal{L}}{i\delta g^i_{4a}} = ig^i_{4a} + \frac{\delta\mathcal{L}_{\mathrm{int}}}{i\delta(g^i_{4a} + f^i_{4a})}$$

$$\chi^i_a = \frac{\delta\mathcal{L}}{\delta\partial_0 V^i_a} = -if^i_{4a} + \frac{\delta\mathcal{L}_{\mathrm{int}}}{i\delta(g^i_{4a} + f^i_{4a})}\,,$$

$$\Pi^i_{\varphi} = \frac{\delta\mathcal{L}}{\delta\partial_0\varphi^i} = -\dot{\varphi}^i - \frac{i}{m}\frac{\delta\mathcal{L}_{\mathrm{int}}}{\delta(W^i_4 + m^{-1}\partial_4\varphi^i)}\,.$$

The equation of motion for W^i_4 is

$$0 = \partial_\mu \frac{\delta \mathcal{L}}{\delta g^i_{\mu 4}} - \frac{\delta \mathcal{L}}{\delta W^i_4} = -i\partial_a \Pi^i_a + m^2 W^i_4 - \frac{\delta \mathcal{L}_{\text{int}}}{\delta (W^i_4 + m^{-1}\partial_4 \varphi^i)} .$$

This we interpret as a relation giving W^i_4 in terms of the other fields:

$$W^i_4 = \frac{i}{m^2} \partial_a \Pi^i_a + \frac{1}{m^2} \frac{\delta \mathcal{L}_{\text{int}}}{\delta (W^i_4 + m^{-1}\partial_4 \varphi^i)} .$$

Similarly for V^i_4:

$$V^i_4 = -\frac{i}{M^2} \partial_a \chi^i_a . \tag{A.3}$$

The combinations we need are:

$$g^i_{4a} + f^i_{4a} = -i(\Pi^i_a - \chi^i_a) , \tag{A.4}$$

$$W^i_4 + \frac{1}{m} \partial_4 \varphi^i = W^i_4 - \frac{i}{m} \dot{\varphi}^i = \frac{i}{m} \Pi^i_\varphi + \frac{i}{m^2} \partial_a \Pi^i_a . \tag{A.5}$$

These are also the only combinations occurring in $\mathcal{L}_{\text{int}}$. The Hamiltonian becomes:

$$\mathcal{H} = \Pi_a \dot{W}_a + \chi_a \dot{V}_a + \Pi_\varphi \dot{\varphi} + \tfrac{1}{2} g^2_{4a} - \tfrac{1}{2} f^2_{4a} + \tfrac{1}{2}\dot{\varphi}^2 - \tfrac{1}{2}(\partial_a \varphi)^2 + \tfrac{1}{2} m^2 W^2_4$$

$$- \tfrac{1}{2} M^2 \varphi^2 - \tfrac{1}{2} M^2 V^2_4 - \tfrac{1}{4} g^2_{ab} + \tfrac{1}{4} f^2_{ab} + \tfrac{1}{2} m^2 W^2_a - \tfrac{1}{2} M^2 V^2_a - \mathcal{L}_{\text{int}} .$$

Here we need to use the fact that only $\int \mathcal{H}\, d_3 x$ is of interest for the derivation of the Feynman rules, that is, we are allowed to perform partial integration with respect to the spatial coordinates on the various terms in $\mathcal{H}$. We write:

$$\Pi_a \dot{W}_a = i\,\Pi_a \partial_4 W_a = i\,\Pi_a g_{4a} + i\,\Pi_a \partial_a W_4$$

$$\rightarrow i\,\Pi_a g_{4a} - i(\partial_a \Pi_a) W_4$$

and similarly for $\chi_a \dot{V}_a$. After this everything is as in sect. 2. We obtain with the help of eqs. (A.3), (A.4) and (A.5):

$$\tfrac{1}{2}\dot{\varphi}^2 + \tfrac{1}{2} m^2 W^2_4 = \tfrac{1}{2} m^2 (W_4 - \frac{i}{m} \dot{\varphi})(W_4 + \frac{i}{m} \dot{\varphi}) = \tfrac{1}{2} i m (W_4 + \frac{i}{m} \dot{\varphi})(\Pi_\varphi + \frac{1}{m} \partial_a \Pi_a) .$$

Together with $-i(\partial_a \Pi_a) W_4$ and $\Pi_\varphi \dot{\varphi}$ this makes

$$-\tfrac{1}{2}\Pi^2_\varphi + \frac{1}{2m^2} (\partial_a \Pi_a)^2 .$$

Further

$$i\,\Pi_a g_{4a} + i\,\chi_a f_{4a} + \tfrac{1}{2} g^2_{4a} - \tfrac{1}{2} f^2_{4a} = \tfrac{1}{2}\Pi^2_a - \tfrac{1}{2}\chi^2_a .$$

The final result is

$$\mathcal{H} = \tfrac{1}{2}\Pi_a^2 + \frac{1}{2m^2}(\partial_a\Pi_a)^2 - \tfrac{1}{2}\chi_a^2 - \frac{1}{2m^2}(\partial_a\chi_a)^2 - \tfrac{1}{2}\Pi_\varphi^2 - \tfrac{1}{4}g_{ab}^2 + \tfrac{1}{4}f_{ab}^2$$

$$- \tfrac{1}{2}(\partial_a\varphi)^2 + \tfrac{1}{2}m^2W_a^2 - \tfrac{1}{2}M^2V_a^2 - \tfrac{1}{2}M^2\varphi^2 + \mathcal{H}_{\text{int}}\,.$$

$$\mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{int}}(W_a + \frac{1}{m}\partial_a\varphi, \frac{i}{m^2}\partial_a\Pi_a + \frac{i}{m}\Pi_\varphi, g_{ab} + f_{ab}, -i\Pi_a + i\chi_a)\,.$$

To obtain Feynman rules we must replace in $\mathcal{H}_{\text{int}}$ the various fields and momenta by their free field-values:

$$\Pi_\varphi \to -\dot{\varphi}^{\text{o}}\,,$$

$$\partial_a\Pi_a \to -im^2W_4^{\text{o}}\,, \qquad \partial_a\chi_a \to iM^2V_4^{\text{o}}\,,$$

$$\Pi_a \to ig_{4a}^{\text{o}}$$

$$\chi_a \to -if_{4a}^{\text{o}}\,,$$

and the vertices in the Feynman rules are directly given by the structure of $\mathcal{L}$:

$$-\mathcal{L}_{\text{int}}(W_\mu^{\text{o}} + \frac{1}{m}\partial_\mu\varphi^{\text{o}}, g_{\mu\nu}^{\text{o}} + f_{\mu\nu}^{\text{o}})\,.$$

The propagators for the combinations $W_\mu + m^{-1}\partial_\mu\varphi$ and $g_{\mu\nu}^{\text{o}} + f_{\mu\nu}^{\text{o}}$ have no contact terms. Also the expression

$$\langle 0|T(g_{\mu\nu}^{\text{o}}(x), W_\lambda^{\text{o}}(x'))|0\rangle = \langle 0|T(\partial_\mu W_\nu^{\text{o}} - \partial_\nu W_\mu^{\text{o}}, W_\lambda^{\text{o}})|0\rangle$$

$$= \langle 0|T(\partial_\mu(W_\nu^{\text{o}} + m^{-1}\partial_\nu\varphi^{\text{o}}) - \partial_\nu(W_\mu^{\text{o}} + m^{-1}\partial_\mu\varphi^{\text{o}}), W_\lambda^{\text{o}} + m^{-1}\partial_\lambda\varphi^{\text{o}})|0\rangle$$

does not give rise to any contact terms.

APPENDIX B

The use of indefinite metric

Indefinite metric has been discussed and used by many authors par excellence [7]. Only for completeness we give in this appendix a short review of what are for us the essential details connected with the use of indefinite metric.

Consider two spaces H and H' (the "covariant" and "contravariant" spaces) in which suitable bases $|a_i\rangle$ and $|b_i'\rangle$ have been chosen. Here and in the following the prime denotes a vector in H'. To a given state α in nature corresponds a vector $|\alpha\rangle$ in H and a vector $|\alpha'\rangle$ in H'. There is a linear relation between the vectors $|\alpha\rangle$ and $|\alpha'\rangle$ describing a given state:

$$|\alpha'\rangle = \eta|\alpha\rangle.$$

Let us assume that the operator η is diagonal and real on the given bases:

$$\eta|a_i\rangle = \eta_i|b_i'\rangle. \tag{B.1}$$

In here the η_i are real numbers and in all generality one may assume the η_i to be ±1. Note that $|a_i\rangle$ is a vector in H, but $\eta|a_i\rangle$ as well as $|b_i'\rangle$ are vectors in H'.

We now define a scalar product between a vector in H and a vector in H' by:

$$\langle b_i'|a_j\rangle = \langle a_j|b_i'\rangle = \delta_{ij},$$

and if

$$|\alpha\rangle = \lambda_i|a_i\rangle \qquad |\beta'\rangle = \mu_j|b_j'\rangle,$$

then

$$\langle\beta'|\alpha\rangle = (\langle\alpha|\beta'\rangle)^* = \sum_j \mu_j^*\lambda_j.$$

This scalar product is the link between physical reality and the mathematical framework. To the state α in nature corresponds $|\alpha\rangle$ in H and $|\alpha'\rangle$ in H'; the norm of $|\alpha\rangle$ is defined to be $\langle\alpha'|\alpha\rangle$. This norm is not necessarily positive (although real with our restrictions on η). For, let

$$|\alpha\rangle = \lambda_i|a_i\rangle.$$

Then

$$|\alpha'\rangle = \eta_i\lambda_i|b_i'\rangle,$$

and

$$\langle\alpha'|\alpha\rangle = \sum_j \eta_j\lambda_j^*\lambda_j.$$

This may be negative if some of the η_j are negative. Of course, for the usual probability interpretation of quantum-mechanics to make sense, physically meaningful states $|\alpha\rangle$ must have $\langle\alpha'|\alpha\rangle > 0$.

We now must find the form of the basis transformations that leave this scalar product invariant. Let U be a basis transformation in H:

$$\underline{|\alpha\rangle} = U|\alpha\rangle.$$

In H' we have then the transformation:

$$\underline{|\beta'\rangle} = \eta U \eta^{-1}|\beta'\rangle.$$

This is such that $\underline{|\beta'\rangle} = \eta\,\underline{|\alpha\rangle}$ if $|\beta'\rangle = \eta|\alpha\rangle$. The requirement

$$\langle\beta'|\alpha\rangle = \underline{\langle\beta'|}\,\underline{|\alpha\rangle}$$

leads to

$$\langle \beta' | \alpha \rangle = \langle \beta' | (\eta U \eta^{-1})^+ U | \alpha \rangle$$

and we have the requirement

$$\eta U^+ \eta = U^{-1} , \tag{B.2}$$

where we used $\eta^+ = \eta = \eta^{-1}$. Note that $U^+ = \eta U^{-1} \eta$ is a transformation in H'. Thus a legitimate basis-transformation is not necessarily unitary, but satisfies (B.2). In particular the S-matrix which transforms the "in" basis into the "out" basis will satisfy (B.2) rather than unitarity. This guarantees that the amplitude to find a particular configuration $|\beta_{in}\rangle$ in a state $|\alpha_{in}\rangle$ is conserved:

$$\langle \beta'_{in} | \alpha_{in} \rangle = \langle \beta'_{out} | \alpha_{out} \rangle$$

and also norm is conserved

$$|\langle \alpha'_{in} | \alpha_{in} \rangle|^2 = |\langle \alpha'_{out} | \alpha_{out} \rangle|^2 ,$$

but there are difficulties with transition probability. In this scheme one must define as transition probability

$$W_{\alpha\beta} = |\langle \beta' | S | \alpha \rangle|^2 N_\beta^{-1} N_\alpha^{-1} , \tag{B.3}$$

where $N_\alpha = \langle \alpha' | \alpha \rangle$ and $N_\beta = \langle \beta' | \beta \rangle$ are the norms of the states $|\alpha\rangle$ and $|\beta\rangle$. This guarantees a total transition probability of 1 for a state β going over into any state α:

$$\sum_\alpha W_{\alpha\beta} = \sum_\alpha |\langle \beta' | S | \alpha \rangle|^2 N_\beta^{-1} N_\alpha^{-1} = \sum_\alpha \langle \beta' | S | \alpha \rangle \langle \alpha | S^{+'} | \beta' \rangle N_\beta^{-1} N_\alpha^{-1}$$

$$= \sum_\alpha \langle \beta' | S | \alpha \rangle \langle \alpha' | \eta S^+ \eta | \beta \rangle N_\beta^{-1} N_\alpha^{-1} . \tag{B.4}$$

Assuming η to be diagonal on the set $|\alpha\rangle$ we have

$$\sum_\alpha |\alpha\rangle N_\alpha^{-1} \langle \alpha' | = 1 .$$

For instance, if $|\gamma\rangle$ is one out of the set $|\alpha\rangle$:

$$\sum_\alpha |\alpha\rangle N_\alpha^{-1} \langle \alpha' | \gamma \rangle = \sum_\alpha \delta_{\alpha\gamma} |\alpha\rangle N_\alpha^{-1} \langle \alpha' | \alpha \rangle = |\gamma\rangle .$$

Thus

$$\sum_\alpha W_{\alpha\beta} = \langle \beta' | S \eta S^+ \eta | \beta \rangle N_\beta^{-1} = 1 .$$

However, the definition (B.3) does not automatically imply $W_{\alpha\beta} \geqslant 0$. A negative $W_{\alpha\beta}$ makes no sense. Thus, unless the S-matrix has some special properties, the theory is not physically meaningful.

Let now A be some operator in H. In the usual theory without indefinite metric, i.e. $\eta = 1$, one has an operator A^+, and the eigenvalues of the product A^+A are positive. The important property is that the definition $A^+ = \tilde{A}^*$ (~ means transpose, * conjugate) is invariant with respect to the choice of basis, for, if U is a unitary basis transformation than

$$A' = \mathrm{U}^{-1} A\,\mathrm{U}\,,$$

$$A'^+ = (\mathrm{U}^{-1} A\,\mathrm{U})^+ = \mathrm{U}^+ A^+ \mathrm{U}^{-1+} = \mathrm{U}^{-1} A^+ \mathrm{U}.$$

Note that this property is not true for A^* or $\tilde{A}$.

In case of indefinite metric η the corresponding definition independent of the choice of basis is:

$$\hat{A} = \eta A^+ \eta\,.$$

Indeed, if $A' = \mathrm{U}^{-1} A\,\mathrm{U}$, then (B.2) leads to

$$\hat{A}' = \mathrm{U}^{-1} \hat{A}\,\mathrm{U}.$$

Furthermore, with our restrictions on η one has the important property that $\hat{A}A$ has only real eigenvalues:

$$(\langle \alpha' | \hat{A} A | \alpha \rangle)^* = (\langle \alpha' | \eta A^+ \eta A | \alpha \rangle)^*$$

$$= \langle \alpha | A^+ \eta A \eta | \alpha' \rangle = \langle \alpha' | \eta A^+ \eta A | \alpha \rangle = \langle \alpha' | \hat{A} A | \alpha \rangle.$$

However, $\hat{A}A$ may have negative eigenvalues.

If one now wants to employ the canonical formalism one must start with a Lagrangian built up from A and $\hat{A}$. For instance we may take for some field A the free Lagrangian

$$\mathcal{L} = \partial_\mu \hat{A}\, \partial_\mu A + m^2 \hat{A} A\,. \tag{B.5}$$

We took a sign opposite to the usual sign. In this paper we deal mostly with "real" fields, $\hat{A} = A$, but for the moment we consider the general case. It may be emphasized again that A^+ has a meaning only if a basis has been chosen.

The momenta $\hat{\Pi}$ and Π conjugate to $\hat{A}$ and A are

$$\hat{\Pi} = \frac{\delta \mathcal{L}}{\delta \partial_0 \hat{A}} = -\partial_0 A = -\dot{A}\,, \tag{B.6}$$

$$\Pi = -\partial_0 \hat{A} = -\dot{\hat{A}}. \tag{B.7}$$

Note that $\hat{\Pi} = \eta \Pi \eta$. The Π do have the opposite sign with respect to the usual relation between Π and the time derivative of the field. We impose the usual canonical commutation rules:

$$[\hat{\Pi}(x'), \hat{A}(x)]_{x_0 = x'_0} = -i\,\delta_3(\boldsymbol{x}-\boldsymbol{x}'), \tag{B.8}$$

etc. With these commutation rules and the definition of $\mathcal{H}$:

$$\mathcal{H} = \Pi \dot{A} + \hat{\Pi}\dot{\hat{A}} - \mathcal{L} \tag{B.9}$$

we can be assured that the equations of motion following from the chosen $\mathcal{L}$ give a variation of time that is the same as that obtained by commutation with $H = \int d_3x\, \mathcal{H}$:

$$[H, A] = -i\dot{A} \tag{B.10}$$

etc. Then, by the use of the interaction representation the S-matrix may be expressed in terms of H_{int} with momenta and fields replaced by their free field values.

For the free Hamiltonian we find:

$$\begin{aligned}\mathcal{H} &= -\Pi\hat{\Pi} - \hat{\Pi}\Pi - \partial_i\hat{A}\,\partial_i A + \hat{\Pi}\Pi - m^2 A\hat{A}\\ &= -\partial_a\hat{A}\,\partial_a A - \hat{\Pi}\Pi - m^2\hat{A}A.\end{aligned} \tag{B.11}$$

This $\mathcal{H}$ has the opposite sign with respect to the usual expression. The opposite sign in the relations (B.6), (B.7) and the usual relation (B.8) leads to an extra minus sign in the commutation relations between operators a, b, $\hat{a}$ and $\hat{b}$ defined as usual:

$$A(x) = \sum_{\boldsymbol{k}} \frac{1}{\sqrt{2Vk_0}} \{a(\boldsymbol{k})\,\mathrm{e}^{ikx} + \hat{b}(\boldsymbol{k})\,\mathrm{e}^{-ikx}\} \tag{B.12}$$

with $k_0 = +\sqrt{\boldsymbol{k}^2 + m^2}$. Apart from a - sign as in (B.11) the expression for H in terms of these operators is as usual:

$$H = -\sum_{\boldsymbol{k}} k_0 \{\hat{a}(\boldsymbol{k})a(\boldsymbol{k}) + \hat{b}(\boldsymbol{k})b(\boldsymbol{k})\} + \text{const} \tag{B.13}$$

again with $k_0 = +\sqrt{\boldsymbol{k}^2 + m^2}$. For our purposes we want H to have only positive eigenvalues. This may be achieved as follows. First we choose a basis. States may be characterized by the number of particles, say n. We define η by (see eq. (B.1)):

$$\eta|n\rangle = (-1)^n |n'\rangle.$$

One verifies that $\hat{a}a$ has only negative eigenvalues:

$$\begin{aligned}\langle n'|\hat{a}a|n\rangle = \langle n'|\eta a^+ \eta a|n\rangle &= \sqrt{n}\langle n'|\eta a^+\eta|n-1\rangle\\ &= (-1)^{n-1}\sqrt{n}\langle n'|\eta a^+|n-1'\rangle = n(-1)^{n-1}\langle n'|\eta|n'\rangle\\ &= n(-1)^{2n-1}\langle n'|n\rangle = -n.\end{aligned} \tag{B.14}$$

Note that the vectors $|n\rangle$, $|n'\rangle$ are the basis vectors of the two spaces. To the "physical" state with n particles correspond the vectors $|n\rangle$ and $\eta|n\rangle = (-1)^n|n'\rangle$. Furthermore one verifies that with

$$a\,|n\rangle = \sqrt{n}\,|n-1\rangle\,,$$

$$a^{+}\,|n'\rangle = \sqrt{n+1}\,|n+1'\rangle\,,$$

we have indeed the opposite sign for the commutator $[a, \hat{a}]$:

$$[a,\hat{a}]\,|n\rangle = (a\,\hat{a} - \hat{a}\,a)\,|n\rangle = -\,|n\rangle\,. \tag{B.15}$$

Finally we must consider the time ordered product:

$$\langle 0'\,|T(\hat{A}(x), A(x'))\,|0\rangle = \theta(x_0 - x_0')\langle 0'\,|\hat{A}(x)\,A(x')\,|0\rangle + \theta(x_0' - x_0)\langle 0'\,|A(x')\,\hat{A}(x)\,|0\rangle\,.$$

This may be evaluated in the usual manner with the help of (B.15). Note that the one-particle intermediate states contributing here have positive energy. One finds

$$\langle 0'\,|\hat{A}(x)\,A(x')\,|0\rangle = \frac{-1}{(2\pi)^3}\int \mathrm{d}_4 k\; \mathrm{e}^{ikx}\,\theta(k_0)\,\delta(k^2+m^2)\,, \tag{B.16}$$

which differs from the usual expression by a minus sign. The Fourier transform of the resulting propagator is:

$$-\,\frac{1}{k^2+m^2-i\epsilon}\,. \tag{B.17}$$

Had we chosen the usual sign in the Lagrangian $\mathcal{L}$ then the energy eigenvalues would have been negative, in eq. (B.16) we would have $\theta(-k_0)$ instead and in eq. (B.17) the opposite sign for $i\epsilon$, i.e. $k^2+m^2+i\epsilon$ in the denominator would have resulted. It is the propagator (B.17) that we need, and this dictates the metric η and the sign to be taken in $\mathcal{L}$.

A physically acceptable theory may for instance be achieved as follows:

(i) The "unphysical" or "ghost" particles are always created in pairs. Then the S-matrix commutes with η:

$$\eta S \eta = S\,,$$

and unitarity in its usual form results:

$$S^{+}S = \eta S^{+}\eta S = 1\,.$$

Defining physical states to be states with an even number of ghost particles, we see that the S-matrix never gives a transition to a state with an odd number of ghosts, i.e. unphysical states.

(ii) The limit $m \to \infty$ for the mass of the ghost particles is taken (provided this limit makes sense). For finite energy a no-ghost state can never be connected by the S-matrix to a state containing ghosts. This approach is the basis for the regulator procedures introduced by Pauli and Villars, which is also the technique employed in this paper.

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