MASSIVE YANG-MILLS FIELDS

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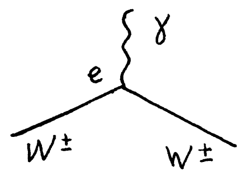
Here we will discuss :

A - why massive Yang-Mills fields may be important in physics ;

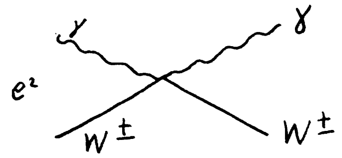
B - the state of affairs in perturbation theory of Yang-Mills fields.

A - Yang-Mills fields may play a rôle in the theory of strong interactions. In particular the field-current identity model [1)] is very suggestive of a Yang-Mills theory.

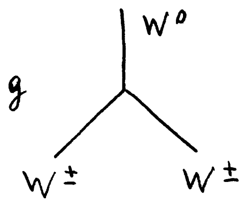
Furthermore, the intermediate vector bosons of weak interactions may be of the Yang-Mills type. This may be seen in a naive way as follows. Let us carry the idea of the C.V.C. hypothesis to an extreme, and let us assume that there exists a neutral vector-boson, differing from the photon by having a non-zero mass, and by coupling weakly to hadrons and leptons. In addition we have of course the charged bosons of weak interactions. Because they are charged, they are coupled to photons :

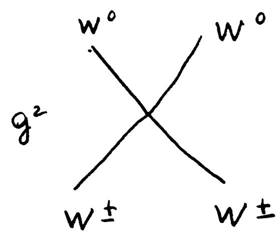


In fact, there is also the two-photon coupling :

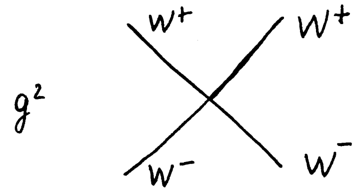


In the spirit of the C.V.C. hypothesis, one must then also introduce a coupling between the neutral and the charged bosons :





If one introduces also a charged boson vertex



one has exactly a massive Yang-Mills theory. Here we must point out that there is no indication that neutral bosons exist, so if the situation is as sketched here the neutral boson must be heavier than the charged one.

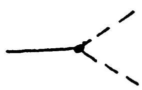
Before one can carry through a realistic program of studying weak interactions with Yang-Mills type bosons, one must first consider the problem of bosons as yet equally heavy coupled to each other in the way indicated above but not yet to leptons or hadrons. Even that problem has not been settled, and we will now indicate the state of affairs.

B - Perturbation theory of massless Yang-Mills fields has been studied by Feynman and others [2]. They have shown that the following rules must be used for computing the S matrix (we take Landau gauge) :

Vector-boson propagator : $\dfrac{\delta_{\mu\nu} - \dfrac{k_\mu k_\nu}{k^2}}{k^2 - i\varepsilon}$

"ghosts" scalar particle propagator, only to be used in closed loops : $\dfrac{1}{k^2 - i\varepsilon}$

Vector-boson vertices :  and 

"ghost" vertex : 

- sign for every closed loop of ghost particles.

Thus one encounters the following self-energy diagrams in fourth order :

 etc.

The S matrix computed in this manner is unitary and renormalizable. However, there are terrible infra-red problems.

For massive Yang-Mills fields the Feynman rules are as usual and well-known :

Vector-boson propagator : $\dfrac{\delta_{\mu\nu} - \dfrac{k_\mu k_\nu}{M^2}}{k^2 + M^2 - i\varepsilon}$

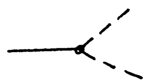
Vector-boson vertices : 

It is very hard to estimate the divergences of the theory since there are very many cancellations between the various diagrams of a given order. For the purpose of studying the divergencies, one performs the Bell-Treiman [3] transformation which amounts to a reshuffling of diagrams in such a way that the S matrix is not affected. It can be shown that for diagrams having no or one closed loop the following set of rules gives the same values for the S matrix elements as the above ones :

Vector-boson propagator : $\dfrac{\delta_{\mu\nu} - \dfrac{k_\mu k_\nu}{k^2}}{k^2 + M^2 - i\varepsilon}$

"Ghost" scalar particle propagator, only to be used in closed loops : $\dfrac{1}{k^2 - i\varepsilon}$

Vector-boson vertices : 

"Ghost" vertex : 

- sign for every closed loop of ghost particles.

These rules are as for the massless case apart from the mass appearing in the vector-boson propagator. From the point of view of divergences, the one loop diagrams behave like in a renormalizable theory.

No complete treatment for diagrams with more than one closed loop has yet been given. The situation is likely to be more complicated than in the massless case since one can show that the boson self-energy diagram with two closed loops formed according to the last mentioned rules do not satisfy the requirements of unitarity.

REFERENCES

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2) R.P. Feynman - Acta Physica Polonica 24, 69 (1963) ;
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3) M. Veltman - Nuclear Phys. B7, 637 (1968).