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LIMIT ON MASS DIFFERENCES IN THE WEINBERG MODEL

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Within the Weinberg model mass differences between members of a multiplet generate further mass differences between the neutral and charged vector bosons. The experimental situation on the Weinberg model leads to an upper limit of about 800 GeV on mass differences within a multiplet. No limit on the average mass can be deduced.

1. Introduction

The recent experimental results on e^+e^- annihilation suggest that there may be more elementary particles than contained in the so-called standard model [1]. That is, there may be a heavy lepton of about 2 GeV, possibly with its own neutrino, and also there may be further quarks beyond the usual four. One cannot help but wonder what will happen when PETRA will be switched on: will we see another jump in the ratio R, will further leptons be discovered? If so, is there a natural point where this will stop? These are very interesting questions *, and in this paper we will investigate some aspects of this problem. This we can only do within the framework of some model of weak and e.m. interactions, for which we take the Weinberg model. In particular we will employ the usual simple Higgs system, containing just one isodoublet, which leads to the mass relation $M_0^2 = M^2/c^2$ between the masses of the neutral and the charged vector boson. Here $c = \cos\theta_w$. According to the data we have $c^2 \approx \frac{2}{3}$, while furthermore this mass relation seems to be well satisfied. We will refer to this situation as the Higgs $\Delta I = \frac{1}{2}$ rule.

Let us now suppose that more and more particles turn up in e^+e^- annihilation as the energy increases. Since the photon is not an SU(2) X U(1) singlet we then have automatically coupling of the neutral vector boson to these particles. Depending on the multiplet structure as well as on the mass differences of the new particles there will be quantum corrections to the above mass relation. In view of the fact that this relation seems to hold within 15% we can draw some conclusion from the observed smallness of these quantum corrections. The general suggestion is that the vector boson mass seems to be a natural measure for the maximally possible mass differences.

* This and other points have been raised before. See for instance ref. [2].

Mass differences beyond 13 times this vector boson mass appear to be excluded. Thus, for instance a heavy lepton-neutrino doublet where the neutrino is massless and the heavy lepton is of the order of 850 GeV is excluded, within the above mentioned scheme. And also a doublet of very, very high mass, but with a mass difference of 750 GeV is excluded. Above 1000 GeV symmetry breaking should thus become relatively smaller and smaller, at least for particles that are weakly coupled.

In the following we will compute the quantum corrections due to the addition of one new heavy-lepton-neutrino system, with massless neutrino. Beyond the electric charge we will use μ-decay and ν_μ-e scattering data to fix the free parameters of the Weinberg model, i.e. the weak mixing angle and the vector boson mass. Next the quantum corrections due to these new leptons to low energy ν_μ-e scattering and e^+e^- annihilation at high energy are established. These finite corrections include mass differences as well as wave function corrections, and there is also a finite mixing between the neutral vector boson and the photon. Even if parts of this calculation are already present in the literature [3] we will nevertheless reproduce the whole argument, as it can be kept simple and transparant for the purpose of this article.

2. The model

Within the Weinberg model there is a triplet of vector bosons, B^i_μ, i = 1,2,3, and a singlet B^0_μ. The physical vector boson fields $W^\pm_\mu, W^0_\mu$ and the photon field A_μ are related to the B_μ:

$$W^\pm_\mu = \sqrt{\tfrac{1}{2}}\,(B^1_\mu \mp iB^2_\mu) = B^\pm_\mu\,,$$

$$B^3_\mu = cW^0_\mu + sA_\mu\,, \qquad B^0_\mu = -sW^0_\mu + cA_\mu\,,$$

where c and s denote the cos and sin of the weak mixing angle.

The interaction Lagrangian between a new lepton system and the B_μ ields is taken to be

$$\mathcal{L}_{\mathrm{L,B}} = \tfrac{1}{4} ig \begin{pmatrix} \bar{\nu} \\ \bar{\ell} \end{pmatrix} \gamma^\mu \begin{pmatrix} B^3_\mu + g_1 B^0_\mu & B^1_\mu - iB^2_\mu \\ B^1_\mu + iB^2_\mu & -B^3_\mu + g_1 B^0_\mu \end{pmatrix} (1+\gamma^5) \begin{pmatrix} \nu \\ \ell \end{pmatrix}$$

$$+ \tfrac{1}{4} igg_2\, (\bar{\ell}\gamma^\mu(1-\gamma^5)\,\ell) B^0_\mu - M^2 B^+_\mu B^-_\mu - \tfrac{1}{2} M^2 (B^3_\mu + g_1 B^0_\mu)^2 - m(\bar{\ell}\ell)\,. \tag{2.2}$$

Also vector boson and lepton mass terms are indicated. Here

$$g_1 = -s/c\,, \qquad g_2 = -2s/c\,. \tag{2.3}$$

Note that $B^3_\mu + g_1 B^0_\mu = W^0_\mu/c$.

In terms of the physical fields W and A this Lagrangian becomes:

$$\mathcal{L}_{\mathrm{L,B}} = \tfrac{1}{2}\sqrt{\tfrac{1}{2}}ig\, W_\mu^+(\overline{\nu}\gamma^\mu(1+\gamma^5)\ell) + \tfrac{1}{2}\sqrt{\tfrac{1}{2}}ig\, W_\mu^-(\overline{\ell}\gamma^\mu(1+\gamma^5)\nu)$$
$$+ \tfrac{1}{4}ig\frac{1}{c}W_\mu^0(\overline{\nu}\gamma^\mu(1+\gamma^5)\nu) - igsA_\mu(\overline{\ell}\gamma^\mu\ell)$$
$$+ \tfrac{1}{4}ig\frac{1}{c}W^0(\overline{\ell}\{4s^2-1-\gamma^5\}\ell) - M^2 W_\mu^+ - \frac{1}{2}\frac{M^2}{c^2}W_\mu^0 W_\mu^0 - m\overline{\ell}\ell\,. \qquad (2.4)$$

The only radiative corrections due to this new interaction to p. decay, ν_μ-e, $\overline{\nu}_\mu$-e scattering and e^+e^- annihilation in $\mu^+\mu^-$ arise from self-energy insertions in the vector boson propagators. Only one type of diagram needs to be calculated, see appendix. The result depends on the two masses m_1 and m_2 of the fermions as well as the A/V ratio's λ and λ', and will be denoted by $F(m_1, m_2, \lambda, \lambda')$. Since F contains infinities we must first carry through renormalization.

3. Renormalization

If we neglect the lepton mass term then left- and right-handed leptons do not mix. Consequently there will be no $B_\mu^0 - B_\mu^3$ mixing. The mass independent infinities are proportional to $k_\mu k_\nu - k^2\delta_{\mu\nu}$, and can be cancelled by counterterms of the form

$$(\partial_\mu B_\nu^i - \partial_\nu B_\mu^i)^2\,, \qquad i = 0, 1, 2, 3\,.$$

The mass dependent infinities do contain $B_\mu^3 B_\mu^0$, mixing terms, but as can be easily concluded from the Lagrangian (2.4) and the form of the function F, there are no A_μ^2 and $W_\mu^0 A_\mu$ mixing infinities. Therefore the mass dependent infinities apply only to the field combination $B_\mu^3 + g_1 B_\mu^0 = W_\mu^0/c$. They can thus, according to eq. (2.2), be absorbed into the vector boson mass M.

To be precise, one has according to the Lagrangian (2.2) the following results for the various transitions:

$$B_\mu^+ B_\mu^-: -\tfrac{1}{8}g^2\, F(m, 0, 1, 1)\,,$$
$$B_\mu^3 B_\mu^3: -\tfrac{1}{16}g^2\, F(0,0,1,1) - \tfrac{1}{16}g^2\, F(m, m, 1, 1)\,,$$
$$B_\mu^0 B_\mu^0: -\tfrac{1}{16}g^2 g_1^2\, F(0,0,1,1) - \tfrac{1}{16}g^2(g_1+g_2)^2\, F\left(m, m, \frac{g_1-g_2}{g_1+g_2}, \frac{g_1-g_2}{g_1+g_2}\right),$$
$$B_\mu^3 B_\mu^0: -\tfrac{1}{16}g^2 g_1\, F(0,0,1,1) + \tfrac{1}{16}g^2(g_1+g_2)\, F\left(m, m, 1, \frac{g_1-g_2}{g_1+g_2}\right). \qquad (3.1)$$

The non-logarithmic parts, containing the infinities are:

$$
\begin{aligned}
&B^+_\mu B^-_\mu:\ \tfrac{1}{2}g^2 i\left(-\frac{2\pi^2}{n-4}+C\right)\{\tfrac{2}{3}(k_\mu k_\nu - k^2\delta_{\mu\nu}) - m^2\delta_{\mu\nu}\}\,,\\
&B^3_\mu B^3_\mu:\ \tfrac{1}{2}g^2 i\left(-\frac{2\pi^2}{n-4}+C\right)\{\tfrac{2}{3}(k_\mu k_\nu - k^2\delta_{\mu\nu}) - m^2\delta_{\mu\nu}\}\,,\\
&B^0_\mu B^0_\mu:\ \tfrac{1}{2}g^2 i\left(-\frac{2\pi^2}{n-4}+C\right)\{2g_1^2(k_\mu k_\nu - k^2\delta_{\mu\nu}) - g_1^2 m^2\delta_{\mu\nu}\}\,,\\
&B^3_\mu B^0_\mu:\ \tfrac{1}{2}g^2 i\left(-\frac{2\pi^2}{n-4}+C\right)\{-g_1 m^2\delta_{\mu\nu}\}\,.
\end{aligned}
\tag{3.2}
$$

The $B^0_\mu B^3_\mu$ term equals the $B^3_\mu B^0_\mu$ expression. There is no momentum dependent infinity in $B^3_\mu B^0_\mu$. Furthermore it is seen that the mass dependent infinities appears only in $B^+_\mu B^-_\mu$ and the combination $(B^3_\mu + g_1 B^0_\mu)^2$, and moreover these two are equal. Thus all the above terms can be renormalized away, in particular the mass dependent terms can be absorbed into M.

4. The finite parts

The infinities have been taken care of, and the whole renormalization procedure amounts to simply throwing them away. We are left with the finite parts which will be written down for the physical fields $W^\pm_\mu$, W^0_μ and A_μ. The $k_\mu k_\nu$ terms can be ignored for our purpose; a proper treatment would also involve calculation of Higgs ghost selfenergy diagrams, but in the processes that we will consider they would only give contributions proportional to the electron mass. We find for the various amplitudes:

$$
\begin{aligned}
&W^+_\mu W^-_\mu:\ f_+ = \tfrac{1}{2}ig^2\pi^2\int \mathrm{d}x\ \{4x(1-x)k^2 + 2xm^2\}\ln(m^2x + k^2x(1-x))\,,\\
&W^0_\mu W^0_\mu:\ f_{11} = \frac{ig^2\pi^2}{4c^2}\left[\int \mathrm{d}x\ \{4x(1-x)k^2(6s^4+2c^4-1) + 2m^2\}\ln(m^2\right.\\
&\qquad \left. + k^2x(1-x)) + \int \mathrm{d}x\, 4x(1-x)k^2 \ln k^2x(1-x)\right]\,,\\
&W^0_\mu A_\mu:\ f_{12} = \frac{ig^2\pi^2 s}{2c}\int \mathrm{d}x\, 4x(1-x)k^2(1-4s^2)\ln(m^2 + k^2x(1-x))\,,\\
&A_\mu A_\mu:\ f_{22} = 2ig^2\pi^2 s^2\int \mathrm{d}x\, 4x(1-x)k^2\ln(m^2 + k^2x(1-x))\,.
\end{aligned}
$$

By means of the usual methods the corrected propagators can be found. The charged

W-propagator **is:**

$$\Delta^{c}_{\mu\nu} = \frac{1}{(2\pi)^4 i}\,\frac{\delta_{\mu\nu}}{k^2 + M^2 - f_+/(2\pi)^4 i}\ .$$

The W^0, A_μ and W^0–A_μ mixing propagators follow by inverting the matrix $\delta_{ij} - F_{ij}\Delta^i$, where Δ^1 and Δ^2 are the bare W^0 and A_μ propagators:

$$\delta_{ij} - F_{ij}\Delta^j = \begin{pmatrix} 1 - f_{11}\Delta^1 & -f_{12}\Delta^2 \\ -f_{12}\Delta^1 & 1 - f_{22}\Delta^2 \end{pmatrix}.$$

The inverse is, up to terms .g^4 in the determinant:

$$\frac{1}{(1 - f_{11}\Delta^1)(1 - f_{22}\Delta^2)} \begin{pmatrix} 1 - f_{22}\Delta^2 & f_{12}\Delta^2 \\ f_{12}\Delta^1 & 1 - f_{11}\Delta^1 \end{pmatrix}.$$

This leads to the corrected propagators:

$$\Delta^{11}_{\mu\nu} = \frac{1}{(2\pi)^4 i}\,\frac{\delta_{\mu\nu}}{k^2 + M^2/c^2 - f_{11}/(2\pi)^4 i}\,,$$

$$\Delta^{12}_{\mu\nu} = \frac{1}{(2\pi)^4 i}\,\frac{\delta_{\mu\nu} f_{12}/(2\pi)^4 i k^2}{(k^2 + M^2/c^2 - f_{11}/(2\pi)^4 i)(1 - f_{22}/(2\pi)^4 i k^2)}$$

$$\Delta^{21} = \Delta^{12}\,,$$

$$\Delta^{22} = \frac{1}{(2\pi)^4 i}\,\frac{\delta_{\mu\nu}}{k^2 - f_{22}/(2\pi)^4 i}\ .$$

Note that f_{12} and f_{22} are proportional to k^2, **so** that no difficulties appear in the limit $k^2 = 0$. The factor containing f_{22} in Δ^{12} can therefore be omitted.

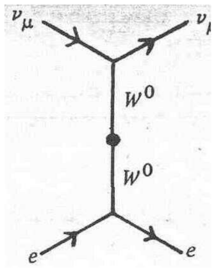
5. Fitting parameters

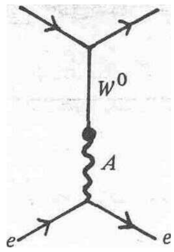
There are three free parameters, namely g, s and M. They can be fixed by comparison with three experimental quantities, for which we take the electric charge e, the fermi coupling constant G from μ-decay, and the neutral current cross-section ratio $\sigma(\overline{\nu}_\mu e)/\sigma(\nu_\mu e)$,

$$e^2 = \left(\frac{g^2 s^2}{1 - f_{22}/(2\pi)^4 i k^2}\right)_{k^2=0}\,, \qquad \frac{e^2}{4\pi} = \alpha = \tfrac{1}{137}\,, \tag{5.1}$$

$$G = \frac{1}{8} g^2 \left(\frac{1}{M^2 - f_+/(2\pi)^4 i}\right)_{k^2=0} = \frac{1.02 \times 10^{-5}}{\sqrt{2} m_p^2}\,, \qquad m_\nu = \text{proton mass} \tag{5.2}$$

For ν_μ-e scattering we have two diagrams





This leads to the amplitude:

$$\frac{g^2}{16c^2}\left(\frac{1}{M^2/c - f_{11}/(2\pi)^4 i}\right)_{k^2=0} \times (\overline{\nu}_\mu \gamma^\alpha (1+\gamma^5)\nu_\mu)(\overline{e}\gamma^\alpha(a+b\gamma^5)e)_{k^2=0}\,, \tag{5.3}$$

with

$$a = 4s^2 - 1 - 4sc\, f_{12}/(2\pi)^4 ik^2\,, \qquad b = -1\,. \tag{5.4}$$

The total cross section for ν_μ-e scattering is:

$$\sigma_{\text{tot}}^{\nu e} = \frac{g^4}{64\pi c^2 (M^2/c^2 - f_{11}/(2\pi)^4 i)^2}\, m_e E\, [(a+b)^2 + \tfrac{1}{3}(a-b)^2]\,.$$

The total cross section for $\overline{\nu}$-e scattering follows by replacing b by $-b$. The ratio of these cross sections is:

$$\frac{\sigma^{\overline{\nu}e}}{\sigma^{\nu e}} = \frac{\xi^2 - \xi + 1}{\xi^2 + \xi + 1}\,, \qquad \xi = \frac{a}{b}\,. \tag{5.6}$$

Thus from this ratio the parameter ξ at zero momentum transfer can be determined. Actually, since both $\sigma^{\overline{\nu}e}$ and $\sigma^{\nu e}$ are not well-known it is better to use hadron neutral current reactions. The result is that $\xi(0) \approx -\frac{1}{3}$, from the zeroth order relation $\xi = 1 - 4s^2$ and the experimental result $s^2 \approx \frac{1}{3}$.

Solving for s^2, g^2 and M^2 we find:

$$1 - 4s^2 = \frac{\xi}{1 - (e^2/12\pi^2)\ln m^2}\,, \tag{5.7}$$

$$g^2 = \frac{4e^2}{1 - \xi - (e^2/12\pi^2)\ln m^2}\,, \tag{5.8}$$

$$M^2 = \frac{e^2}{2G(1-\xi)}\left(1 + \frac{e^2}{12\pi^2(1-\xi)}\ln m^2\right) + \frac{e^2}{32\pi^2(1-\xi)}(m^2 \ln m^2 - \tfrac{1}{2}m^2). \tag{5.9}$$

In these equations ξ in the point $k^2 = 0$ is understood.

6. Quantum corrections to $\sigma^{\nu e}$

The amplitude for this process is given by eqs. (5.3) and (5.4). Using the relation

$$f_{11}\,|_{k^2=0} = \frac{1}{c^2}\,(f_+|_{k^2=0} + \tfrac{1}{4}ig^2\pi^2\, m^2)\,,$$

we have

$$\frac{g^2}{M^2 - c^2 f_{11}/(2\pi)^4 i} = \frac{8G}{1-(m^2/8\pi^2)G}.$$

Thus the amplitude for this process requires a quantum correction $1 + m^2 G/8\pi^2$, and the cross section a correction

$$1 + \frac{m^2}{4\pi^2}\; G = 1 + 1.83 \times 10^{-7}\,\frac{m^2}{m_p^2}\,.$$

Assume now that there are N lepton-neutrino doublets, with lepton masses m_i, $i = 1, ..., N$. The corrections add up, and from the requirement that this correction should be less than 15% we get:

$$1.83 \times 10^{-7} \sum_{i=1}^{N} \frac{m_i^2}{m_p^2} < 0.15\,.$$

The presently known leptons do not contribute significantly to this relation. It would however be saturated by one heavy lepton with a mass of 905 $m_p \approx 850$ GeV.

The calculation can be repeated easily for the case that both leptons are massive, with masses m_1 and m_2. The quantum correction becomes

$$1 + \frac{G}{4\pi^2}\left\{m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_2^2 - m_1^2}\ln\frac{m_2^2}{m_1^2}\right\}.$$

If $m_2 = m_1 + \epsilon$ we find, neglecting terms ϵ/m_1:

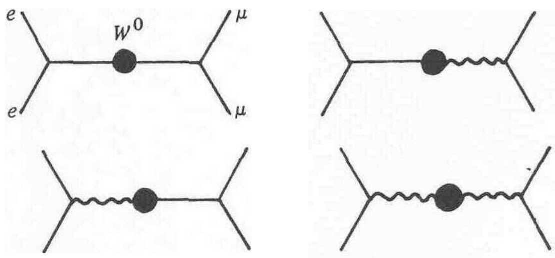
$$1 + \frac{G}{3\pi^2}\,\epsilon^2\,.$$

Thus mass differences should be less than 750 GeV.

7. **Quantum corrections** to $e^+e^- \to \mu^+\mu^-$

The e^+e^- annihilation processes are distinguished from the previously discussed processes by the fact that k^2 is substantially different from zero. In the near future, at PETRA, we may even expect values of the order of 0.15 for the ratio $-k^2c^2/M^2$. The interference effects between weak and e.m. amplitudes may be observable.

The process to be considered here is e^+e^- annihilation into a muon pair. There are four contributing diagrams



The first three diagrams together give the amplitude

$$\frac{1}{16}g^2 \frac{1}{k^2 + M^2/c^2 - f_{11}/(2\pi)^4 i} \left(\bar{e}\gamma^\alpha\left(\frac{a}{c}+\frac{b}{c}\gamma^5\right)e\right)\left(\bar{\mu}\gamma^\alpha\left(\frac{a}{c}+\frac{b}{c}\gamma^5\right)\mu\right), \tag{7.1}$$

with a and b as given in eq. (5.4). This may be rewritten as:

$$\frac{1}{2}G \frac{1}{1-(G/8\pi^2)m^2 + 8G(c^2/g^2)k^2 + 8G(c^2/(2\pi)^4 ig^2)(f_{11}(k^2) - f_{11}(0))}$$
$$\times (\bar{e}\gamma^\alpha\{\xi(0) + \xi(k^2) - \xi(0) + \gamma^5\}e)(\bar{\mu}\{\xi(0) + \xi(k^2) - \xi(0) + \gamma^5\}\mu)\,. \tag{7.2}$$

In this expression we must still express c^2/g^2 in terms of the various experimental quantities e, G and $\xi(0)$. One has:

$$\frac{c^2}{g^2} = \frac{(3+\xi)(1-\xi)}{16e^2} - \frac{3+\xi^2}{16 \times 12\pi^2}\ln m^2\,, \qquad \xi = \xi(0)\,. \tag{7.3}$$

This seems to generate a term of the form k^2 In m^2, which would suggest a dependence on the units that m is expressed in. However, it turns out that this term is precisely needed to cancel a similar dependence in $f_{11}(k^2) - f_{11}(0)$, so that in the resulting logarithms only ratios such as k^2/m^2 appear. In fact, one finds for the denominator expression in (7.2) the result:

$$1 - \frac{G}{8\pi^2}m^2 + \frac{G}{2e^2}(3+\xi)(1-\xi)k^2$$
$$+\frac{G}{2\pi^2}\int \mathrm{d}x \left\{x(1-x)\frac{1+\xi^2}{2}k^2 + \frac{1}{2}m^2\right\} \ln\left\{1 + \frac{k^2}{m^2}x(1-x)\right\}$$

$$+\frac{G}{2\pi^2}\int \mathrm{d}x x(1-x)k^2 \ln\left\{\frac{k^2}{m^2}x(1-x)\right\}, \qquad \xi = \xi(0). \tag{7.4}$$

The quantity $\xi + \xi(k^2) - \xi$ can be worked out:

$$\xi + \xi(k^2) - \xi = \xi\left[1 + \frac{e^2}{2\pi^2}\int \mathrm{d}x x(1-x)\ln\left\{1 + \frac{k^2}{m^2}x(1-x\right\}\right]. \tag{7.5}$$

Most of these corrections are rather small as k^2/m^2 becomes small, and are thus insensitive to the existence of heavy leptons. In the limit of small k^2/m^2 expression (7.5) becomes equal to ξ, while (7.4) reduces to:

$$1 - \frac{G}{8\pi^2}m^2 + \frac{G}{2e^2}(3+\xi)(1-\xi)k^2$$
$$+\frac{G}{24\pi^2}k^2 + \frac{G}{12\pi^2}k^2\ln\frac{k^2}{m^2} - \frac{5G}{36\pi^2}k^2 + i\,\frac{G}{12\pi}k^2\theta(-k^2). \tag{7.6}$$

The last term is the imaginary part of the last logarithm in (7.4). It reflects the fact that the neutrino is taken massless, so that the decay width of W^0 into a neutrino pair enters in the propagator.

For PETRA energies the above corrections become essentially of order $\alpha = \frac{1}{137}$. Unless there exists a fantastically large number of multiplets (of the order of a thousand) it is very unlikely that any of these will be observed in the near future.

8. Conclusions

From the observed smallness of quantum corrections to the neutral vector boson mass a limit of about 800 GeV on mass differences within multiplets can be obtained. This however rests on the assumption of the validity of the Higgs $\Delta I = \frac{1}{2}$ rule, i.e. the assumption that the spontaneous symmetry breaking involves only a Higgs doublet. In zeroth order this leads to the result that the parameter β, defined by [4]

$$\beta = M^2/c^2M_0^2$$

equals one. Here M_0 is the neutral vector boson mass, and in particular in the Weinberg model $\beta = 1$. Experimentally this parameter β is measured by measuring both the ν and $\bar{\nu}$ neutral current cross sections; the ratio of the two determines essentially the weak mixing angle, while the ratio of one of these cross sections to the prediction of the Weinberg model equals β^2. To the extend that the experimental results seem to give $\beta \approx 1$, we are encouraged to believe the limit mentioned above.

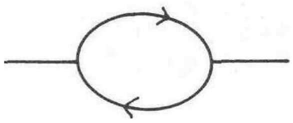
If however the Higgs $\Delta I = \frac{1}{2}$ rule is incorrect then we must use the k^2 dependent corrections given in (7.6). They may become observable once we get near to the pole of the propagator, i.e. if we get to energies where the neutral vector boson can actually be created. This is supposedly around 80 GeV. As can be seen from eq. (7.6), also the accuracy with which ξ is measured is of relevance here.

Finally we would like to note that if in some future machine the neutral vector boson is created then its decay width will also get contributions from all kinds of neutrinos. In this sense the W^0 will become a probe for the existence of all kinds of neutral particles, much like the photon which probes all charged particles. In the W^0 case we have the advantage that at the resonance the decay width will give us immediately the equivalent of the famous ratio R in lower energy annihilation experiments. At PETRA energies the weak production of neutral particles is suppressed by a factor of about 50 relative to photon produced charged particles, and it seems unlikely that we will be able to obtain information on neutral particles from future measurements at PETRA. All this emphasizes the need for an e^+e^- machine going to much higher energies [2].

The author is indebted to Drs. B. de Wit and G. Passarino for helpful discussions.

Appendix

The only diagram to be computed is a one fermion loop diagram.



Assuming masses m_1 and m_2 for the two fermion propagators, and the vertices $\gamma^\mu(1+\lambda\gamma^5)$ and $\gamma^\nu(1+\lambda'\gamma^5)$ the diagram can be worked out. In this we suppose anticommuting γ^5. The result is:

$$\Gamma(m_1, m_2, \lambda, \lambda')$$

$$= -\int \mathrm{d}_n q \, \frac{\mathrm{Tr}\{\gamma^\mu(1+\lambda\gamma^5)(-i\gamma q + m_1)\gamma^\nu(1+\lambda'\gamma^5)(-i\gamma(k+q)+m_2)\}}{((k+q)^2+m_2^2)(q^2+m_1^2)}$$

$$= -4i\pi^{n/2}\Gamma(2-\tfrac{1}{2}n)\int \mathrm{d}x\, (m_2^2 x + m_1^2(1-x) + k^2 x(1-x))^{(n-4)/2}$$

$$\times\ \{-(1+\lambda\lambda')(-2k_\mu k_\nu x(1-x) + 2\delta_{\mu\nu}k^2 x(1-x) + m_2^2 x\delta_{\mu\nu} + m_1^2(1-x)\delta_{\mu\nu})$$

$$+ (1-\lambda\lambda')m_1 m_2 \delta_{\mu\nu}\}\,.$$

Using

$$\pi^{n/2}\Gamma(2-\tfrac{1}{2}n) = -\frac{2\pi^2}{n-4} + C + \mathrm{O}(n-4)\,,$$

where C is a constant, we find:

$$F(m_1, m_2, \lambda, \lambda') = -4i\left(-\frac{2\pi^2}{n-4} + C\right) [\tfrac{1}{3}(1 + \lambda'\lambda)(k_\mu k_\nu - k^2\delta_{\mu\nu})$$

$$- \tfrac{1}{2}(1 + \lambda'\lambda)(m_1^2 + m_2^2)\delta_{\mu\nu} + (1 - \lambda'\lambda)m_1 m_2 \delta_{\mu\nu}]$$
$$+ 4i\pi^2 \int \mathrm{d}x \ln(m_2^2 x + m_1^2(1-x) + k^2 x(1-x))$$
$$\times [2(1 + \lambda'\lambda)x(1-x)(k_\mu k_\nu - k^2\delta_{\mu\nu}) - (1 + \lambda'\lambda)(m_2^2 x + m_1^2(1-x))\delta_{\mu\nu}$$
$$+ (1 - \lambda'\lambda)m_1 m_2 \delta_{\mu\nu}] .$$

References

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