INTERMEDIATE BOSON PRODUCTION BY NEUTRINOS

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We consider the production, by neutrinos incident on nuclei, of the hypothetical intermediate bosons W of weak interactions. The treatment of Lee et al. [1] has been modified in two ways. For the coherent process we adopt more realistic nuclear form factors, and in the incoherent process the exclusion principle is allowed for. As a result both contributions to the cross section are substantially reduced. Always $\varkappa$ will be taken to be zero (no anomalous magnetic moment).

The coherent process

The coherent cross section σ_z(coh) depends on the assumed nuclear charge distribution. Lee et al. used the exponential form

$$\rho(r) \propto e^{-\sqrt{12}\, r/a} \tag{1}$$

with root mean square radius

$$a = \sqrt{\tfrac{3}{5}}\ 1.3 \times 10^{-13}\ A^{\frac{1}{3}}\mathrm{cm}\ .$$

To test the computer programme we repeated their calculations for Fe and obtained their results. Calculations with this distribution were then made for $Cu^{63.5}$ and Al^{27}, which are of particular interest for the CERN experiment. Finally we adopted instead the fermi charge distribution [2] *

$$\rho(r) \propto \{1 + e^{-(r-R)/b}\}^{-1}$$

with (2)

$$R = 1.07\ A^{\frac{1}{3}}\ 10^{-13}\mathrm{cm}, \qquad b = 0.568 \times 10^{-13}\mathrm{cm}$$

The old and revised form factors, which are the fourier transforms of the charge distribution, are compared in fig. 1. The fermi form factors fall off more rapidly with momentum transfer. The cross sections are decreased accordingly, as shown in fig. 2 and fig. 3 for boson masses m of 0.6 and 1.0 proton masses.

The incoherent process

For the total non mesonic production cross section Lee et al. add to the coherent part σ_z(coh) an incoherent contribution

* In the calculations we took the fermi form factor to be zero for $Q > 500$ MeV/c.

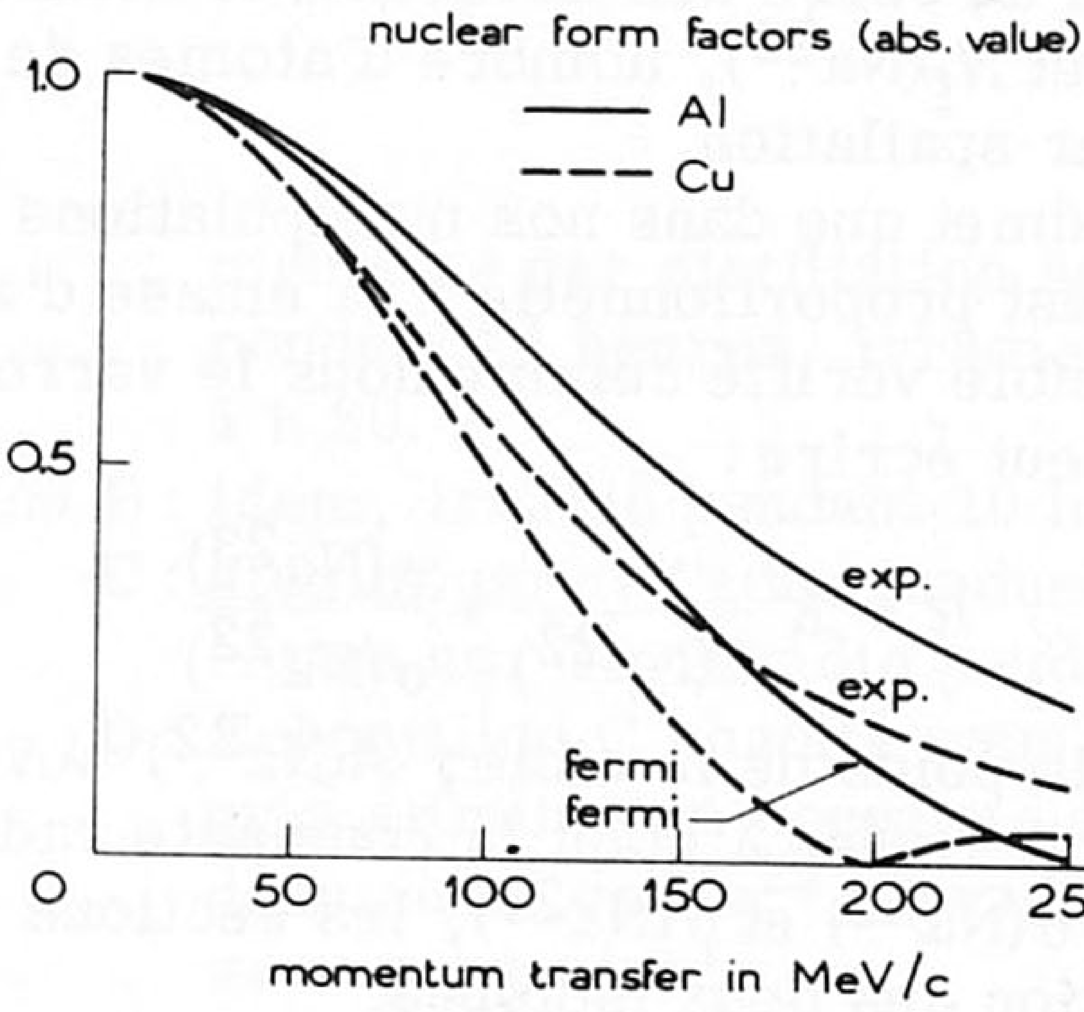

Fig. 1. Exponential and fermi form factors for Al^{27} and $Cu^{63.5}$.

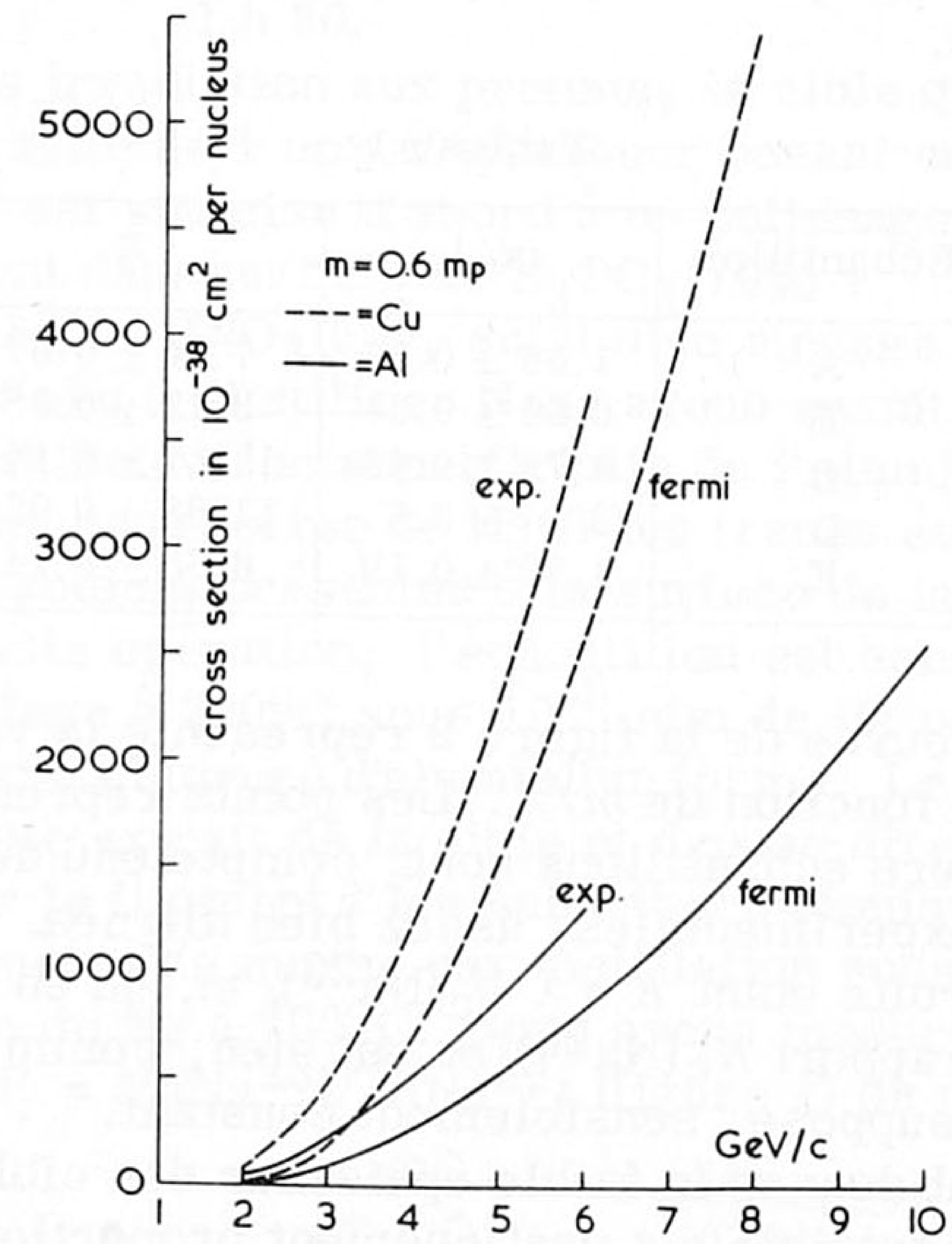

Fig. 2. Coherent cross sections σ_z(coh) for Al^{27} and $Cu^{63.5}$ with boson mass $m_W = 0.6\ m_p$.

$$Z\{\sigma_p - Z^{-2}\sigma_z(\mathrm{coh})\}\ , \tag{3}$$

where σ_p is the cross section for a single proton. This formula can be derived on the assumption that the nucleons in the nucleus do not move and have uncorrulated positions. Here we still ignore the fermi

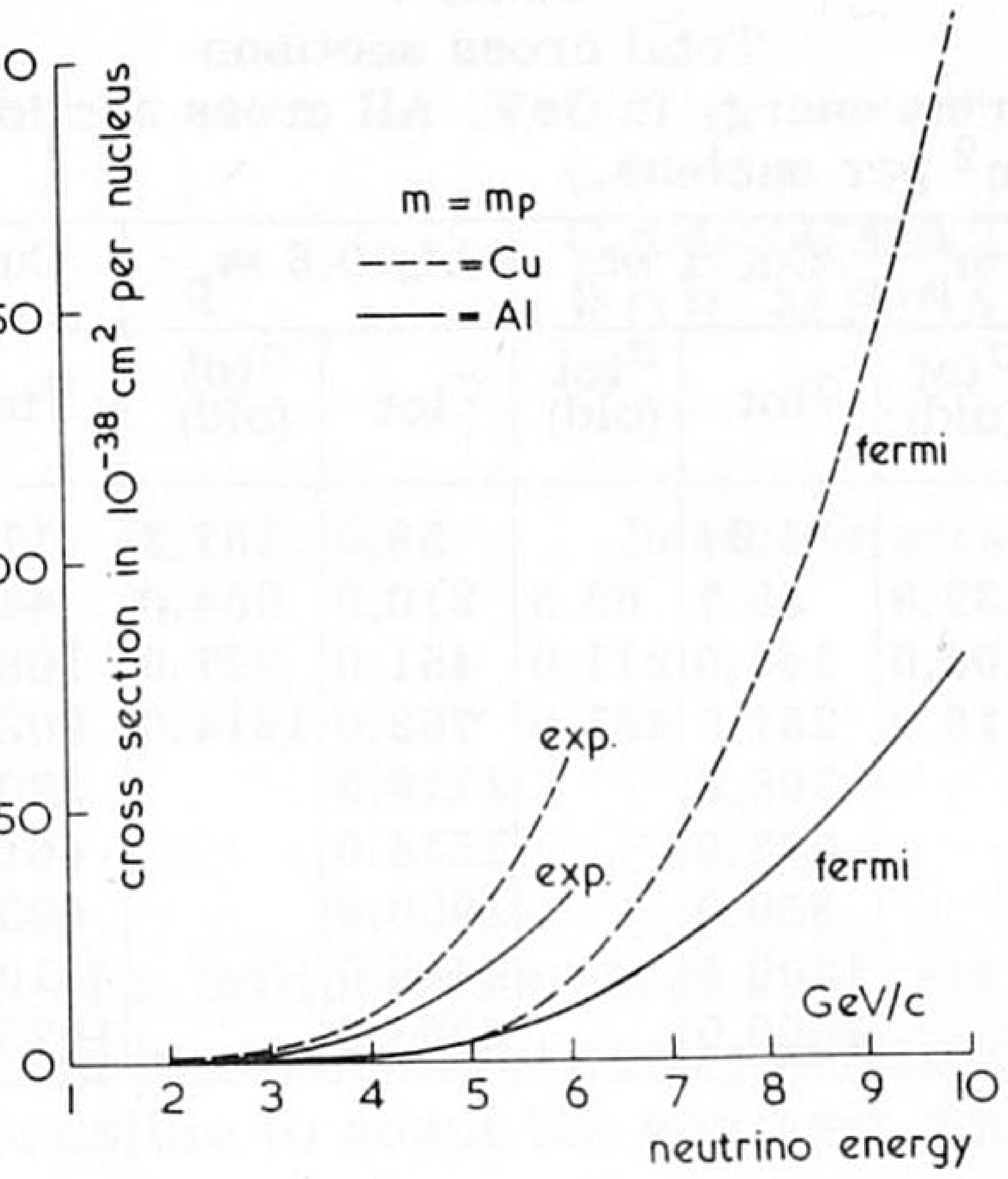

Fig. 3. Coherent cross sections σ_z(coh) for Al^{27} and $Cu^{63.5}$ with boson mass $m_W = 1.0\, m_p$.

motion of nucleons but allow for the correlation implied by the exclusion principle.

When the nucleus is treated as an ideal fermi gas, rather than an ideal classical gas, the formula (3) is replaced by

$$Z\sigma_p' , \qquad (4)$$

where σ_p' is obtained from σ_p by averaging over the fermi motion and excluding final states in which the recoiling proton momentum is within the occupied sphere. If the initial nucleon velocities are ignored the effect of the exclusion is just to introduce in the differential cross section a factor

$$\langle \Theta(K_f - |\boldsymbol{K} - \boldsymbol{Q}|)\rangle_{av} ,$$

where Θ is the step function, $\boldsymbol{Q}$ is the 3-momentum transfer, K_f the fermi momentum, and averaging is over all initial proton momenta $\boldsymbol{K}$. This gives the well known reduction factor

$$R(|\boldsymbol{Q}|^2) = \tfrac{3}{2}\frac{|\boldsymbol{Q}|}{2K_f} - \tfrac{1}{2}\left(\frac{|\boldsymbol{Q}|}{2K_f}\right)^2 \quad \text{for } |\boldsymbol{Q}| < 2K_f , \qquad (5)$$
$$= 1 \quad \text{for } |\boldsymbol{Q}| \geq 2K_f .$$

Now we have for nucleons initially at rest

$$|\boldsymbol{Q}|^2 = Q^2 + (Q^2/2m_p)^2 , \qquad (6)$$

where m_p is proton mass and Q^2 is the squared 4-momentum transfer. The second term is quite small, but has in fact been retained in the calculations. To allow for the exclusion principle then we merely modify as follows the proton form factors *

* Again, to check our programme, we first did the calculation with the same form factors as used in ref. [1]), and obtained their results. Next, we used more recent data (Proc. Int. Conf. on High Energy Physics, CERN 1962, p. 753) but no substantial difference resulted, presumably due to the fact that the changes below Q = 600 MeV/c are small.

already occurring in the formulae:

$$F(Q^2) \rightarrow F(Q^2)\,\{R(Q^2 + (Q^2/2m_p)^2)\}^{\frac{1}{2}} \qquad (7)$$

Results are presented in fig. 4, with K_f = 0.284 m_p. The effect of exclusion is substantial, the fractional reduction increasing with energy. Indeed there can be no effect at all until the kinematically permitted minimum momentum transfer is less than $2K_f$. For $E_\nu \gg m_W + m_\mu$

$$Q_{min} \approx (m_W + m_\mu)^2/2E_\nu .$$

Thus the exclusion principle can enter when

$$\frac{E}{m_W + m_\mu} \gtrsim \frac{m_W + m_\mu}{4K_f} \qquad (8)$$

That the effect is so considerable when the energy is high shows the importance of small transfers (fig. 4).

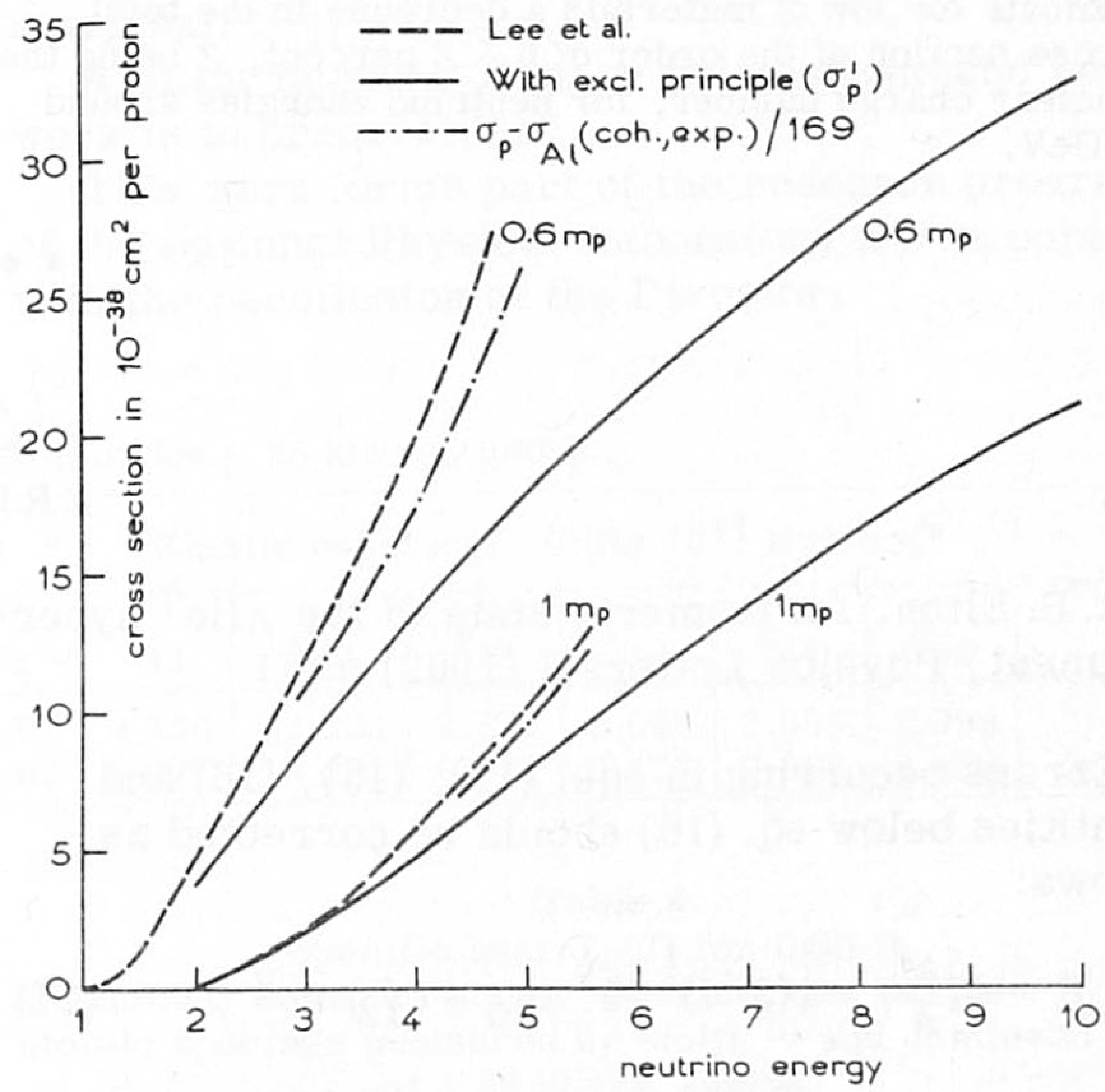

Fig. 4. Incoherent cross section per proton σ_p' for two boson masses, compared with the Lee et al. expression $\sigma_p - Z^{-2}\sigma_z$(coh) for Al($Z$ = 13, exponential form factor) and with the cross section for a single proton σ_p.

We have ignored nucleon motion; this may not be very serious since in the gas model $|\boldsymbol{K}|/m_p \lesssim 0.3$. However, eventually calculations must be made allowing for it. In the ideal gas model this would be in principle straight-forward. We have not carried it out because the extra integration would strain the capacity of the available computer (IBM 709). One effect to be expected is a lowering of the threshold energy. In the region affected by exclusion we would guess that the cross section be further reduced. The momentum transfer to the nucleon has necessarily a forward component; the nucleons moving initially towards the neutrino, which would otherwise be the most effective, are then those most affected by exclusion. However the adequacy of the free gas model is questionable. It appears from our results that transitions to states of low excitation energy are important, and then a more detailed account of nuclear structure may be essential. For heavy nuclei we should at least introduce an effective mass, and perhaps allow for Coulomb effects *. For these reasons we think it may be difficult to obtain quantitatively reliable results for the incoherent process.

Total cross section

Finally in the table we compare total (non mesonic) cross sections computed as described above and with the original prescriptions of Lee et al.

* For the coherent process some preliminary numerical integration of Coulomb effects based on the analysis given in a previous paper [3] has been made. The results indicate for low Z materials a decrease in the total cross section of the order of 0.5 Z percent, Z being the nuclear charge number, for neutrino energies around 6 GeV.

Table 1
Total cross sections
(E_ν = neutrino energy in GeV. All cross sections are in $10^{-38} cm^2$ per nucleus.)

	Al, 1 m_p		Cu, 1 m_p		Al, 0.6 m_p		Cu, 0.6 m_p	
E_ν	σ_{tot}	σ_{tot} (old)	σ_{tot}	σ_{tot} (old)	σ_{tot}	σ_{tot} (old)	σ_{tot}	σ_{tot} (old)
2	1.95		4.34		58.0	107.3	118.4	211.0
3	26.2	32.8	58.7	65.5	210.0	364.0	442.0	795.0
4	66.5	106.0	144.0	211.0	451.0	727.0	1083.0	1759.0
5	121.0	210.0	251.0	421.0	762.0	1214.0	2027.0	3138.0
6	194.0		396.0		1118.0		3203.0	
7	285.0		603.0		1514.0		4610.0	
8	399.0		880.0		1950.0		6233.0	
9	529.0		1230.0		2445.0		8100.0	
10	680.0		1660.0		2984.0		10250.0	

1) T.D. Lee, P. Markstein and C.N. Yang, Phys. Rev. Letters 7 (1961) 429.
2) E.g.: R. Herman and R. Hofstadter, High Energy Electron Scattering Tables (Stanford University Press, 1960).
3) M. Veltman, Physica 29 (1963) 161.

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ERRATA

L. R. B. Elton, An isomeric state of the ${}_\Lambda He^7$ hyperfragment, Physics Letters 2 (1962) p. 41

Errors occurring in eqs. (11), (15), (16) and quantities below eq. (16) should be corrected as follows:

$$\langle Q_2^M \rangle = \tfrac{1}{4}(3/\pi)^{\frac{1}{2}} (5^{\frac{1}{2}} c_{io} + c_{f2})\, ea^2 \,, \qquad (11)$$

$$c_{io} = -\,0.130 \,, \qquad c_{f2} = -\,0.290 \,,$$

$$\langle Q_2^M \rangle = -\,0.141\, ea^2 \,. \qquad (15)$$

$$T(E2) = 1.1 \times 10^{10}\ sec^{-1} \,, \qquad (16)$$

$$\tau_\gamma = 0.88 \times 10^{-10}\ sec \,, \qquad qe = 0.158\, e \,.$$

It will be noticed that now $\tau_\gamma < \tau_\Lambda$, but it must be remembered that the free two-body interaction was employed for V_{12}, in order to obtain a lower limit for τ_γ. A more realistic estimate is to take about 0.6 of the free two-body interaction, which gives the right energy splitting between the ground state and first excited state of He^6. Then $\tau_\gamma = 3.2 \times 10^{-10}$ sec, as against $\tau_\Lambda = 2.5 \times 10^{-10}$ sec. Thus the fact that experimentally $\tau_\gamma > \tau_\lambda$ may now be taken as evidence that the residual two-body interaction V_{12} is substantially smaller than the free two-body interaction.

Finally, it should be stressed that the centre of mass motion does not give any contribution either in lowest configuration or in the configuration mixture employed in this calculation.

W. F. Druyvesteyn and D. J. Van Ooijen, Influence of neutron-irradiation at 78°K on the critical field of superconducting lead, Physics Letters 4 (1963) p. 170

First paragraph, line 9, should read:
"vacancies cannot be determined independently ..."
First paragraph, line 13, should read:
"latter can now be made low by soft-annealing ..."
Figures 1 and 2 on p. 171 have been interchanged erroneously.

E. Marquit, On the experimental verification of the single Regge-Pomeranchuk pole approximation for high-energy elastic scattering, Physics Letters 4 (1963) p. 101

Second paragraph, first line, should read:
One consequence of eq. (1) is that the initial slope of ln X plotted against t should increase with increasing s.