INT. SYMP. ON ELECTRON AND PHOTON INTERACTIONS AT HIGH ENERGIES 1973 – NORTH-HOLLAND

GAUGE FIELD THEORIES

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Es gibt viele Theorien
Die sich jedem Check entziehen
Diese aber kann man checken
Elend wird Sie dann verrecken

1 Introduction

It is not bon ton in a scientific exposé to show excitement, and I will accordingly try to avoid this. Yet it is at this moment very difficult to do so, because it seems more and more that we are on the right track. I am probably very much in the same position as B. Lee one year ago; he was one of the first to note and emphasize the importance of the present development, and his excitement filters through in the title of his talk: "Perspectives on theory of weak interactions" [44].

In preparing this talk I have tried as much as possible to avoid the use of complicated mathematical or graphical arguments. This type of considerations cannot be expected to be palatable to a general audience. In a way, this is a pity, because there is great beauty in the intricate patterns of gauge field theory. A way to communicate something in this respect is to give a historical review of the subject; this may at the same time serve as a bibliography, and to credit the often excellent work of several modest authors that has been crucial and instrumental, and that for no rational reason remains unrecognized by the general public. Such a review is presented in the appendix.

In this talk I will furthermore completely avoid speculating on the precise type of model that could be relevant in the description of nature. Such questions will be tackled in the subsequent talk by Llewellyn-Smith. To us it is sufficient to know that the general idea of a weak gauge symmetry acquires credibility through the discovery of neutral hadron currents in neutrino experiments.

Many arguments may be presented to emphasize the attractiveness of gauge field theories: renormalizability, unification of interactions, high-energy behaviour, etc. A great deal of sense and nonsense has been said on these matters over the past two years, and I am not going to add any "deep" comments. At the very minimum we can say that a new chapter has been added to field theory; and that there is a way to extrapolate the verified methods of quantum-electrodynamics to weak interactions. What we need now is some hard fact, some irrefutable argument that proves the relevance of gauge field theory (apart from electromagnetism and gravitation) to nature. The discovery of neutral currents cannot yet serve as such; while encouraging it is conceivable that this has nothing to do with

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a gauge symmetry. The only, but very important thing that can be said now is that we have evidence of couplings not contained in the Cabibbo theory. The actual discovery of weak vector mesons with the appropriate masses would however go a long way in proving the applicability of gauge considerations. But that is certainly not the only way that progress can be made, and sections 2 and 3 are devoted to other lines of thought.

Recently, an interesting argument in favour of a gauge field theory of strong interactions has attracted attention. In sect. 2 we will give a simple minded account of this development. In sect. 3 we discuss the important question of higher order effects that has been the source of some arguments in the recent literature. It may well be of value in connection with actual model building.

Of course, an important part of this talk is to report on the status of the proof or renormalizability of gauge field theory. As usual most of the information transmits in the form of folklore, rather then by actual reading, and consequently a great amount of misunderstanding is circulating. Many people seem to think that the proofs rely on path integral methods and tend therefore to dismiss any claims of rigour. This however is simply incorrect. Path integral techniques have been instrumental to guess the correct form of the Feynman rules for gauge theories. Even so, the fundamental work of 't Hooft [22, 27] relied subsequently on combinatorial methods to prove correctness of these rules. There can be no doubt to these rules for the unrenormalized theory. The subsequent work of 't Hooft et al. [45] and B. Lee and Zinn-Justin [46] and also Ross and Taylor [47] concerns the actual renormalization and the generalization of the combinatorial methods. It should be emphasized that most of the functional manipulations employed by B. Lee and Zinn-Justin have a direct diagrammatic parallel and are therefore not to be put on one line with the shadier path integral techniques of yesteryear. As to our own methods, I think that it can be said safely that the proof of renormalizability is on the same level as that of quantum electrodynamics. In saying this, I have in mind also the continuous dimension regularization method [23, 48] and the recent work on renormalization theory of Epstein and Glaser [49]. The latter in contrast to the method of BPHZ, that has not yet been adapted completely to the problems of gauge field theory. In particular, it seems that unrealistic infrared problems occur if one tries to carry through the Zimmermann method. Nevertheless, Stora and co-authors [50] have achieved some progress in treating gauge theories along these lines. Without doubt we may assume that the last sceptics will be satisfied in the near future.

In sect. 4 we outline the renormalization program and the associated problems. In sect. 5 we discuss the definition of physical particles à la Kibble [38], and finally, in sect. 6 we consider the question of the validity of PCAC and low-energy theorems. We do this to emphasize that gauge theories explain this: the various consequences of the algebra of charges of Gell–Mann. Anyone dismissing gauge theories must keep this in mind.

2 Asymptotic freedom

A subject that has aroused considerable interest in recent months is the renormalization group [51] and the question of asymptotic freedom.

Within the example of the φ^4 theory, Symanzik [52] has been investigating the behaviour of perturbation theory in the infrared ultraviolet region. The essential tool is the Callan–Symanzik [53, 54] equation, which can be derived by very simple dimensional considerations. In first approximation for the φ^4 theory the situation may be put as follows. Consider any amplitude. It will depend on the external momenta, on the φ-mass and on the coupling constant. One may ask: is there any relation between this amplitude and the same amplitude where all external momenta are scaled upwards or downwards with a certain common factor? It turns out that the change in the amplitude under this scaling of the external momenta can be compensated by a suitable change in the coupling constant and the φ-mass. Furthermore, the *sign* of the change in these quantities can be determined by the study of one loop diagrams, even if the result is supposedly true for the whole summed up perturbation series. That is one of the beautiful aspects of this work: the one loop diagrams determine the behaviour of the whole perturbation series. Let us stress that these one loop diagrams are not really the dominating diagrams. The point is that the behaviour of the multiloop diagrams is determined, in a known way, by the one loop diagrams.

Everything now centers on the question of the sign of the change of the coupling constant. It has been noted some time ago for the φ^4 theory that increasing the external momenta is equivalent to increasing the magnitude of the coupling constant [52]. This provided the sign of the coupling constant is taken such that in the corresponding classical theory the energy is bounded from below. The opposite happens if the sign of the coupling constant is reversed; Symanzik has argued that the possibilities of negative energies in the classical theory not necessarily implies the same for the quantum theory.* If that is true then the φ^4 theory with the "wrong" sign for the coupling constant would be an example of a theory that becomes asymptotically free for very high external momenta. Moreover, Symanzik demonstrated that in the limit of large momenta amplitudes can be computed with arbitrary accuracy, and he obtained equations showing logarithmic behaviour of these amplitudes. Parisi [56] pointed out that this asymptotic behaviour implies Bjorken scaling (note: in the φ^4 theory there are less logarithms). Recently Callan and Gross [56] also investigated this question.

At the 1972 Marseille conference, in a remark following Symanzik's lecture [52] on that subject, 't Hooft indicated that for Yang–Mills theories the sign of the change in the coupling constant is such as to give an asymptotically free theory for large momenta. The importance of this fact in connection with scaling in strong interactions was not emphasized by anyone at that time; for this reason this result was not publicized until recently [55].

Recently Gross and Wilczek [57] and Politzer [58] also dicovered the fact that Yang–Mills theories are asymptotically free. And Coleman and Gross [59] established that any renormalizable field theory that is asymptotically free (for large momenta) must contain non-abelian gauge fields. Let us note however the following fact: as soon as spontaneous symmetry breaking is attempted some kind of φ^4 theory has to be introduced. And we then have the dilemma pointed out above, namely how to get an asymptotically free theory without negative energies. But, as Symanzik emphasized to me, nobody knows the behaviour with respect to energy except in the simple case of the φ^4 theory. These arguments concerning the Higgs fields are therefore of limited value, if any.

Thus, if we believe that ultimately the strong interactions have an underlying renormalizable field theory, then we are led to believe, because of scaling, that this underlying theory is a nonabelian gauge theory. Quite succesful models of strong interaction gauge theories have already been proposed, in particular we mention the work of Bars et al. [60]. Moreover, such a type of model, invented inde-

* Symanzik, private communication, tends to agree with the reasoning of Coleman and Weinberg [73] that negative energies seem to be unavoidable in φ^4 theories with the wrong sign.

pendently by De Wit [61] (note: not DeWitt) has been shown to give rise to a Cabibbo angle when combined with some gauge theory of weak and e.m. interactions. All this is very encouraging. There is an interesting point of a rather philosophical nature to these models which we will discuss now.

3 Higher order effects

Quite soon after the discovery that spontaneously broken gauge theories are renormalizable, i.e. that one can compute things, it was discovered that the higher order weak corrections to weak processes are of order $\alpha = \frac{1}{137}$ [62]. Naturally the suspicion arises that the weak corrections to strong interactions are also of order α [63], which would be in disagreement with experiment. Indeed, order α parity violations are experimentally excluded.

Let us first classify theories according to what can happen. A given theory of strong interactions will contain a number of parameters that must be chosen to fit the experimental data. Concentrating for definitness on parity we may distinguish the following possibilities:

(i) The strong interaction lagrangian posesses a symmetry that guarantees parity conservation. Moreover, the higher order weak parity violating corrections remain weak.
(ii) Same, but the weak corrections are of order α.
(iii) The strong interaction lagrangian possesses no particular symmetry with respect to parity, but by chosing parameters parity violation is initially set zero. The weak corrections remain weak.
(iv) Same, but weak corrections are of order α, and can be made smaller again by readjusting the parameters.
(v) Same, but cannot be made smaller by parameter readjustment.

Experimentally, classes (ii) and (v) are ruled out. No examples of class (v) theories are known to me. Esthetically, we could perhaps prefer a theory of the type (i). The question is to what extent our principle that there shall be no order α parity violations would help us in selecting models. The answer is that this seems to be of little help. As was already stressed by De Wit [61], one may prefer models of the type (i) over those of type (iii) (and further (iii) over (iv)), but it is not clear to me that one can say anything beyond that. Let us consider the problem in some more detail.

Any piece of field theory (the Wilson expansion, see ref. [63], or Bogoliubov's causality condition in some suitable form) assures us that if a diagram containing one or more vector mesons with large mass M is finite, then it behaves as $1/M^2$. This guarantees that for such diagrams the lowest order correction is $G \approx \alpha/M^2$, the next $G\alpha$, etc.*. So, to find effects of order α without factors $1/M^2$, we must look to diagrams that are infinite, i.e. need renormalization. But generally spreaking in any renormalizable theory in a renormalizable gauge this means that there is a parameter that we can adjust, and we would have a theory of the type (iv). However, there is an exception to this rule, which is that individual diagrams are divergent, so that the above reasoning does not apply, while at the same time there is a symmetry that assures that the infinities always cancel out (although not certain finite parts). In this way finite mass differences can arise in gauge theories, see for example ref. [27], eq. (8.4), which gives a mass difference behaving like ln (M^2) for large M. In the case at hand that can happen if there

* In higher order Higgs particle couplings proportional to M may contribute.

exists a symmetry guaranteeing parity conservation in strong interactions. So in first instance one can imagine a combined gauge symmetry of strong and weak interactions, where the strong gauge symmetry is such as to force parity conservation. The question is if the requirement of absence of order α parity violations helps us in chosing the weak and strong gauge groups. Now, there must be Ward identities arising from this strong interaction symmetry that rule out parity violations in renormalizable type vertices. As long as the weak interactions leave those Ward identities unchanged, this result will remain true even if the weak interactions are switched on. That is, if the generators of the weak symmetry commute with those of the strong symmetry there will be no order α parity violations. This important result is contained in Weinberg's paper, ref. [63]. However, for gauge theories the weak and strong generators must commute, or else one must introduce further vector mesons. We therefore conclude that the constraints resulting from these considerations are redundant, renormalizability has put these same constraints already. Conversely stated, in a gauge model of the type (i) the parity violations are automatically weak.

There is a further point*. Non-abelian spontaneously broken theories posess scalar particles, i.e. mesons. At first sight it seems very difficult to find a principle that rules out parity violating couplings to these mesons. In that case these theories would be of the less desirable (from this point of view) type (iii).

All in all, it seems doubtful that these arguments will lead us anywhere. We are faced with an old problem in a new setting: how come that the world splits up so neatly into various kinds of interactions with very different magnitudes and symmetries. Up to now, gauge theories have not given a satisfactory answer to this problem. We refer to the talk by Llewellyn–Smith for a review of models unifying weak, e.m. and strong interactions.

4 Renormalization of gauge field theories

As indicated in the appendix, 't Hooft [22, 27] concluded a development in gauge field theory with the precise description of the Feynman rules in arbitrary gauges for the unrenormalized lagrangian. A large class of models containing charged vector mesons have gauges where the corresponding Feynman rules are of the renormalizable type. The next step consists of showing that after renormalization a sensible theory results. This has meanwhile been done [45–47, 65].

In short, the following complications occur. First, in order to have any idea about the properties of the renormalized theory a suitable regularization procedure must be found. Let us stress that wholly independent of the question as to what subtraction procedure one uses (including sophisticated formulations where everything is finite), a gauge invariant regulator method is needed to demonstrate that all subtractions can be defined in a consistent way, without violation of the gauge symmetry. Moreover, anyone having done calculations, even at the one loop level, knows by experience that without such a procedure a hopeless confusion is inevitable. Two procedures have been suggested:

(i) Slavnov [66] and B. Lee and Zinn-Justin [46] have suggested the introduction of terms of the form $(D_\alpha G^a_{\mu\nu} D_\alpha G^a_{\mu\nu})$ in the lagrangian. These terms do not break the gauge invariance, and lead to propagators behaving like k^{-4}. A disadvantage is the occurrence of many new vertices, while further-

* I am indebted to Ch. Llewellyn Smith for this observation.

more one-loop infinities remain and must be regulated separately. There may be further difficulties with respect to unitarity etc., but since most authors prefer now the other scheme, we will not go into that.

(ii) The continuous dimension method. Various authors [23, 48] have proposed regularization by allowing non-integer values for n, the dimensionality of space–time. Some of these publications center on one loop diagrams only, and define the continuation by considering the result after all the loop integrations have been done. The use of Feynman parameters is essential to this prescription. However, this cannot be extended easily to multiloop diagrams, because then some of the divergencies of the theory transfer from the loop integrations to the Feynman parameter integrations. Another way out is to define the continuation to non-integer n before loop integrations are performed.

Many theorists are confused about the differences between the dimensional regularization method and analytic renormalization [67, 68]. Let us very briefly indicate the salient features. Integrals over closed loops are of the form

$$\int \mathrm{d}_4 p \frac{f(p)}{(p^2+m^2)^\alpha}.$$

Gauge invariance implies a tight interplay between the numerator functions $f(p)$ (involving γ-matrices, Kronecker delta's, etc.) and the denominators. Changing either of the two will generally destroy gauge invariance; the analytical renormalization scheme requires change of the exponent α, and is therefore not well suited for gauge theories. The dimensional regularization method changes the only object left over, namely $\mathrm{d}_4 p \to \mathrm{d}_n p$. Obviously this change must be such that the algebra of this symbol remains unchanged, for instance

$$\mathrm{d}_n(\mu p) = \mu^n \mathrm{d}_n p.$$

Finally, in the analytic regularization scheme as implemented by Speer [68], the denominators in different loops get different exponents. That kind of change is not permissible in gauge theories, because gauge invariance ties up loops with one another so that all of them must have the same properties. We refer to ref. [23] for further details.

Given a proper regulator method that respects unitarity and causality, the method of SPBEG [49] (Stueckelberg–Petermann–Bogoliubov–Epstein–Glaser) can be applied straight away. Alternatively, one can try to work within the BPH scheme. These renormalization procedures lead to a renormalized lagrangian, with finite Green's functions. After that however, a number of problems specific to gauge theories remains.

The starting point in any gauge theory is a gauge invariant Lagrangian. It will depend on the vector gauge fields and other fields. The vector fields will be denoted by W_μ^a, the index a takes as many values as there are generators in the underlying gauge group. This number will be denoted by N. The other fields may include fermions, and will be denoted by A^i. They will belong to some representation of the group.

Thus we have a lagrangian

$$\mathscr{L}_{\mathrm{inv}}(W, A)$$

that is invariant for the infinitesimal transformation

$$W_\mu^a \to W_\mu^a + g c_{bc}^a W_\mu^b \Lambda^c - \partial_\mu \Lambda^a, \quad 1 \leqq a \leqq N; \qquad A^i \to A^i + g s_{ja}^i A^j \Lambda^a.$$

The c are the structure constants of the group, and the s are the generators in the representation of the A^i. Further g is the coupling constant.

After spontaneous symmetry breaking some of the fields A acquire a vacuum expectation value, and we write

$$A = m + B \qquad \langle B \rangle_0 = 0.$$

Sustituting this into the transformation laws leads to new laws where some of the B's will now have field independent parts in their transformation law. The trick is to diagonalize with respect to these field independent parts. This diagonalization is equivalent to a separation in physical and unphysical fields, and will be considered in the following section. The important point is that the lagrangian remains invariant under the transformation laws, even in the presence of a spontaneous breakdown.

The prescription to obtain Feynman rules goes in several steps.

(i) Chose a set of functions C^a, $1 \leqq a \leqq N$, of the fields. The C^a must not be gauge invariant, they must be so that fixing the C^a implies fixing a gauge, even in zero'th order in g. Example of a good C:

$$C^a = \partial_\mu W_\mu^a.$$

(ii) Apply an infinitesimal gauge transformation to the fields in C^a. Then the C^a will change also:

$$C^a \to C^a + \mathcal{M}_b^a \Lambda^b,$$

where the $\mathcal{M}_b^a$ may contain fields.

(iii) Amend the invariant lagrangian as follows

$$\mathcal{L} = \mathcal{L}_{\text{inv}} - \tfrac{1}{2}C^2 + \varphi_a^* \mathcal{M}_b^a \varphi_b.$$

The complex field φ is the Faddeev–Popov ghost. This lagrangian defines a set of Feynman rules to which one has the additional prescription that every closed loop of ϕ-particles must be given a minus sign.

The so defined lagrangian contains physical and unphysical fields. It has been shown in all rigour that the S-matrix elements between physical states only are independent of the choice of C (and the corresponding Faddeev–Popov ghost lagrangian).

Let now the C^a contain only terms linear in the fields. It has been shown that all Green's function's of the theory can be made finite by the addition of a counter lagrangian $\Delta\mathcal{L}$, that is well defined within the context of the dimensional regularization scheme. And it has further been shown that $\mathcal{L} + \Delta\mathcal{L}$ is of the form:

$$\mathcal{L} + \Delta\mathcal{L} = \mathcal{L}'_{\text{inv}} - \tfrac{1}{2}C^2 + \varphi_a^* \mathcal{M}_b'^a \varphi_b,$$

where now $\mathcal{L}'_{\text{inv}}$ is invariant under infinitesimal transformations as above, but with different coefficients c' and s' that are power series in the coupling constant. And $\mathcal{M}'$ arises by applying these "renormalized" gauge transformations to the C^a. That is, the renormalized lagrangian has the same structure as the unrenormalized lagrangian, but the symmetry is different. Finally it has been shown that the new coefficients c' satisfy the Jacobi identity (the group property of ref. [45]). Similarly for the s'. For any group that by an infinitesimal change in the structure constants cannot go over into another group we have the result that the renormalized group equals the unrenormalized group. This implies that the renormalized lagrangian can be obtained from the unrenormalized lagrangian by parameter substitution and field renormalization, i.e. multiplicative renormalization. Here we will not dwell on pathological cases not covered by this argument, although these can be handled also. The above summarizes the treatment of ref. [45].

B. Lee and Zinn–Justin have followed a somewhat different strategy. Mostly working with the group SU2 they chose a gauge function C that is such that $-\frac{1}{2}C^2$ is invariant for transformations with space-time independent Λ. Then the renormalized theory is invariant also for these global transformations, and the question of the symmetry of the renormalized lagrangian becomes much simpler. Nevertheless, also the group property must be proven if one goes beyond SU2 to demonstrate for instance within SU3 the absence of d coefficients in the renormalized transormation laws. Apart from this their treatment, employing mostly functional techniques, parallels the combinatorial treatment of ref. [45].

Except for the difficulties associated with finite mass differences these works firmly establish renormalizability of gauge theories.

All the above mentioned proofs rely heavily on the Slavnov–Taylor identities. These identities do not directly relate irreducible vertices and self-energy parts, which makes the procedure quite complicated. Recently new identities have been proposed by B. Lee [69] that relate directly irreducible graphs. The Lee identities seem to us better suited for the renormalization procedure, and further progress may be expected.

Some complications arise in the case of non-simple C, i.e. if C contains terms non-linear in the fields. We think we know what happens, and will describe that at some convenient occasion; here we only report that as far as we can see no special difficulties occur.

For practical purposes we may summarize:

(i) Make sure that the unrenormalized lagrangian contains all possible interactions compatible with the symmetry. Write down the transformation laws of the fields.
(ii) Chose a simple C (linear in the fields). Find the ghost lagrangian.
(iii) Use the dimensional regularization method and introduce counterterms in the lagrangian that make all Green's functions finite. This can be done by suitable wave function renormalization and parameter renormalization.

Presto: a good theory results. This prescription is described in detail in ref. [70].

5 Physical particles

The physical particles are defined by their transformation laws. We follow the method essentially due to Kibble [38]. For a definition up to all orders in the coupling constant see ref. [45].

After spontaneous breakdown the transformation laws will be of the form

$$W^a_\mu \to W^a_\mu + gc^a_{bc} W^b_\mu \Lambda^c - \partial_\mu \Lambda^a, \qquad 1 \leqq a \leqq N \; 1 \leqq a \leqq N; \qquad B^i \to B^i + gs^i_{ja} B^j \Lambda^a + t^i_a \Lambda^a,$$

with $t^i_a = gs^i_{ja} m^j$ where the m^j are the vacuum expectation values described before.

We now assume that the quantities t^i_a are diagonal:

$$t^i_a = M(i)\delta^i_a.$$

If this is not the case then by a suitable rearrangement of the fields B and W one can easily bring t in this form.

The fields B separate in κ fields U (with $\kappa \leqq N$) that have a non-zero M and fields P with zero M:

$$U^i \to U^i + g s^i_{ja} B^j \Lambda^a + M(i)\Lambda^i, \qquad 0 \leqq i \leqq \kappa \leqq N; \qquad P^i \to P^i + g s^i_{ja} B^j \Lambda^a,$$

$$B^i = \begin{cases} U^i & 1 \leqq i \leqq \kappa \leqq N, \\ P^i & \text{else.} \end{cases}$$

Moreover for $a \leqq \kappa$ we may define fields $\bar{W}$ so that the $\bar{W}$ have no constant part in their transformation law:

$$W^a_\mu = \bar{W}^a_\mu - \frac{1}{M(a)} \partial_\mu U^a, \quad 1 \leqq a \leqq \kappa; \qquad W^a_\mu = \bar{W}^a_\mu, \quad a > \kappa;$$

$$\bar{W}^a_\mu \to \bar{W}^a_\mu + g c^a_{bc} W^b_\mu \Lambda^c + \frac{g}{M(a)} \partial_\mu (s^a_{jb} B^j \Lambda^b), \ 1 \leqq a \leqq \kappa; \ \bar{W}^a_\mu \to \bar{W}^a_\mu + g c^a_{bc} W^b_\mu \Lambda^c - \partial_\mu \Lambda^a, \quad a > \kappa.$$

We now have:

$\bar{W}^a_\mu$	$1 \leqq a \leqq \kappa$	massive vector particles,
$\bar{W}^a_\mu$	$a > \kappa$	massless vector particles,
U^i	$i \leqq \kappa$	unphysical Higgs–Kibble ghosts,
P^i	$i > \kappa$	physical particles.

The Faddeev–Popov ghost is always unphysical.

The lagrangian is invariant for these transformations, also in zero'th order of the coupling constant g. It is easy to see that the fields U cannot have a quadratic part in this lagrangian, which shows their unphysical nature.

6 PCAC and divergence equations

Both PCAC and divergence equations [71, 72] are supposed to be experimentally verified through the Goldberger–Treiman relation and various low-energy theorems, for instance the Adler–Weisberger equation. We are therefore interested in these relations in case of a gauge theory of weak interactions, for example the Weinberg model [28], when extended to include interactions with baryons.

First of all, the currents employed in these equations are the physical currents as encountered in weak interactions. Thus they are the currents to which the physical vector mesons of the previous section are coupled. And we are interested in matrix-elements of the divergence of these currents between physical states. For these purposes it is best to work in the physical (or so-called unitary) gauge:

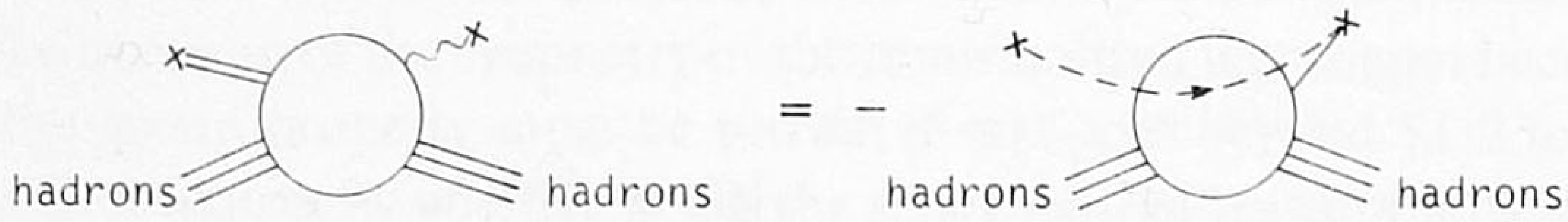
$$\begin{aligned} C^a &= bU^a, \quad b \to \infty, \quad a \leqq \kappa, \\ &= \partial_\mu W^a_\mu \qquad\qquad\quad a > \kappa. \end{aligned}$$

For simplicity we will assume $\kappa = N$, i.e. no massless vector mesons. The term $-\frac{1}{2}C^2$ to be added to the lagrangian is the only term quadratic in the fields U. The propagator for these fields becomes simply $1/b^2$, that is they behave as particles with very large mass. The Faddeev–Popov ghost lagrangian is:

$$\mathscr{L}_{FP} = b\{M(a)\varphi^*_a \varphi_a + g\varphi^*_a s^a_{jb} \varphi_b B^j\}.$$

The F–P propagators are simply constants, $1/bM(a)$, and within the dimensional regularization scheme the integrals arising from F–P closed loops are zero.

Consider now a source that emits the unphysical U and a source emitting (or absorbing) a physically polarized vector meson not necessarily on mass shell, and furthermore many strongly interacting particles on mass-shell. The Slavnov–Taylor identities read:



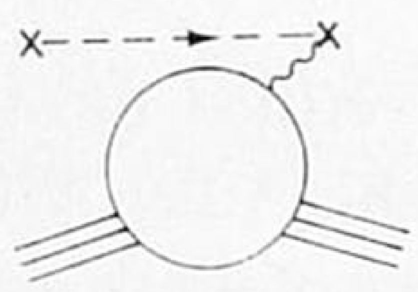
In here:

$\sim\sim$ stands for the $\bar{W}$,
—— stands for the $W = \bar{W}-(1/M)\partial U$,
$=\!=$ stands for $bU = C$,
- - - stands for the F–P ghost.

The leading terms for $b \to \infty$ are of the order $1/b$; the F–P ghost has the propagator $1/bM$. To first order in the weak interaction coupling constant we may ignore the right hand side (there is a g in the source vertex, and a g where the W couples). The coupling of the U to the other particles separates in two pieces: (i) The coupling generated by the replacement $W = \bar{W}-(1/M)\partial U$. (ii) The rest.

The first type of coupling is precisely $\partial_\mu j_\mu$, where j_μ is the current to which the $\bar{W}$ is coupled. The second type gives the extent to which the divergence of the current is non-zero, and in a suitable model that would be the pion field. In a renormalizable theory we would in any case expect it to be a smooth object. This gives PCAC as usual.

To second order in the weak interactions we have as extra contribution:



We have ignored the U-part in the W-propagator because it gives zero in the limit $b \to \infty$. Now the coupling of the F–P ghost to the W-source is defined by the gauge transformation properties of the W-field. We find precisely the same result as obtained with the divergence equations of ref. [71]:

$$\partial_\mu j^a_\mu = c\pi^a + g c^a_{bc} j^b_\mu W^c_\mu .$$

In fact, our derivation is largely identical with the derivation given by Bell, ref. [72]. It may be noted though that our identification of the pion field depends on the strong iso-spin assignment of the weak currents. Within 10–20% this is certainly true.

We see that a gauge theory of weak interactions when suitably coupled to strongly interacting particles automatically implies PCAC and divergence equations, i.e. for instance the Adler–Weisberger relation expressing g_A in the π–N scattering length.

This brings us automatically to the question of PCAC and divergence equations for the neutral currents. As more experimental data concerning for instance N* production comes available these very interesting questions may be investigated and resolved.

Acknowledgments

I am particularly indebted to Drs T. Appelquist, Ch. Llewellyn-Smith and K. Symanzik for very informative discussions.

Appendix. Historical review and bibliography

There are several different lines of approach to gauge field theory. Here we will distinguish the following:

a) Study of massless Yang–Mills fields, as a prelude for the study of gauge theories of weak and strong interactions, or of gravitation.
b) It has been known that quantum-electrodynamics with massive photons is a renormalizable theory, despite the appearant breakdown of gauge invariance. One may try to understand the mechanism and then extend it to theories containing also charged vector mesons (non-abelian gauge theories).
c) In studying weak interaction processes one may try to invent couplings such that the amplitudes behave decently if the energy becomes very large.

Approaches b) and c) have been tried by many authors, with partial success. The mechanism is too complicated to be found in this way. Of course, knowing now that it works one has understood also how to carry through these programs, and they furnish better insight in the structure of gauge field theories.

The progress along line of approach a) has been slow and painful. Almost everybody made a mistake of one kind or another, or landed up in a blind alley. We will try to outline the historical development, indicating briefly the advances and shortcomings of the various treatments.

Ever since the invention of Yang–Mills gauge theory [1] the impact on physics has been considerable, even if up to now no explicit realization in nature has been demonstrated. One of the earliest papers known to me where symmetry considerations were applied to the system of weak and e.m. interactions is the work of Schwinger [2]. This was even before the V–A theory of weak interactions became established, and under these circumstances it is very hard to write a complete theory. After the V–A theory and especially the CVC hypothesis became popular, the first work relevant in this context is due to Bludman [3], who investigated the possibility of global and also local (i.e. of the Yang–Mills type including the associated vector bosons) gauge invariance of weak interactions. In this context he introduced chiral SU2 for leptons as well as the charges associated with the various currents. Glashow [4], in 1961, advocated the idea of partial symmetry, that is that apart from certain terms (presumably the mass terms) the lagrangian would be invariant under some symmetry. This is very close indeed to spontaneous symmetry breaking. He also extended Bludman's work to include electromagnetism. Gell-Mann [5] emphasized these ideas, and continuing this line of thought he proposed his well known algebra of currents and charges.

A further example of the fruitfulness of the gauge idea is the discovery of SU3 by Gell-Mann [6] and Ne'eman [7] which owes much of its existence to the works of Sakurai [8] and also Salam and Ward [9]. These authors elaborated the idea that the vector mesons of strong interactions are gauge vector bosons.

Investigating gravitation Feynman [10], following a suggestion of Gell–Mann made the first step in the study of massless Yang–Mills fields. His approach did contain already many of the ideas and techniques used by subsequent workers. It may be noted though that due to the fact that the published paper is a transcript of a tape recording one has to be something of a Feynman to understand it. What I mean is that one recognizes these elements now that we know them. Anyway, from this paper it is clear that he used path integral techniques (including manipulations that are close to the later work of Faddeev and Popov) to get a heuristic derivation of the precise diagrammatic rules; and combinatorial techniques were then employed to verify the unitarity of these rules. A fictitious particle, almost but not quite what we now call the Faddeev–Popov ghost, was introduced (the nearest description is a mixture of the Faddeev–Popov and Higgs–Kibble ghosts). In doing all this work Feynman made a mistake by assuming that the massless Yang–Mills fields can be obtained in the limit of zero mass of the massive theory, which turns out not to be the case. There is a very good reason to approach the theory in this way, which is the following. A massless vector particle has two states of polarization, whereas a massive vector particle has three such states. If one goes in a covariant way off mass-shell the massless vector particle obtains a third state of polarization. That is, it is impossible to write a manifestly covariant propagator for a massless vector particle that has only two states of polarization. In other words, for massless vector particles there are no manifestly covariant and unitary Feynman rules. Since unitarity is the thing to be investigated one obviously is in an unfortunate position. For massive particles there is no such problem.

Essentially due to this, and some deficiences in his combinatorial methods Feynman was not able to go beyond one closed loop. DeWitt [11] in his 1964 Letter and his subsequent monumental work derived most of the things that we know of now. That is, he considered the question of a choice of gauge and the associated ghost particle. Indeed, he writes the ghost contribution in the form of a local lagrangian containing a complex scalar field obeying Fermi statistics. Somewhat illogically this ghost is now often called the Faddeev–Popov ghost. His method however was not very transparant, and unitarity (and causality) remained a ticklish problem. Faddeev and Popov [12] using path integral techniques derived the rules in the Landau gauge. In particular the way in which the ghost came out was very direct and practicable. Their technique was not sufficiently flexible to discuss a general gauge in a simple way, in particular it was hard to understand the previously established rules in the Feynman gauge. To be precise, the ghost introduced by Faddeev and Popov had no orientation (no arrow), and this goes with a simple vertex structure only in the Landau gauge. However, their approach gave a nice, simple and workable definition of the *S*-matrix in case of a lagrangian that obeys some local symmetry, and to some extent they re-established the direct connection between lagrangian and Feynman rules that one is used to. To some extent, because neither the choice of gauge nor the ghost rules were expressed in terms of local fields in the lagrangian; yet an important step in that direction had been taken. For this reason this work has been of great importance in the subsequent developments. Further progress was made by Fradkin and Tyutin [13], who introduced the possibility of a general choice of gauge through the introduction of a term in the lagrangian.

Using an entirely gauge invariant formulation of field theory Mandelstam [14] derived the rules also. His formulation is rather far removed from the usual formulation, and not very well adapted to practical purposes. Hence this approach has not become very popular. Yet both Mandelstam's and DeWitt's papers contain very interesting techniques that may be worthwhile to exploit further.

The combinatorial work on diagrams was for reasons mentioned above mainly concentrated at massive Yang–Mills fields. Even if this theory is ultimately not the correct one to study, the generalized Ward identities and the combinatorial techniques developed can be taken over without difficulty to complete gauge theories.

That for massive theories [15] there were difficulties if one goes beyond one closed loop was soon recognized. Also the path integral treatment of Boulware [16] indicated the presence of non-renormalizable vertices occurring at the level of two or more closed loops. Nobody knew whether the infinities caused by these vertices would cancel out, and in fact, because the massless theory was known not to have such vertices it seemed reasonable to assume that they would. However, subsequently it was shown explicitly [17] that massless Feynman rules, modified by the addition of a mass in the denominator did violate unitarity for two closed loop diagrams. Next Slavnov and Faddeev [18] and Van Dam et al. [19] established that the limit of zero mass of the massive theory is different by a finite amount from the zero mass theory already for one closed loop. However, this did not yet establish that massive Yang–Mills fields are not renormalizable. For that one needs at least one actual calculation showing that the remaining divergencies do not cancel out. But an important psychological barrier had been taken.

In the development up to this point very little and somewhat random use had been made of Ward–Takahashi (WT) identities. Such identities may be used to show unitarity of a theory containing ghost loops; they also play a critical role in carrying through the actual renormalization procedure, for also the renormalized theory must be shown to be unitary.

One may distinguish three types of WT identities:

(i) Relations between individual bare vertices and propagators;
(ii) Relations for slightly modified S-matrix elements;
(iii) Relations between Green's functions. These are sometimes called generalized Ward identities.

In quantum-electrodynamics the identities (i) and (iii) take the well-known form

$$k_\mu \Gamma_\mu(q, p) = S_F^{-1}(q) - S_F^{-1}(p), \qquad k = q-p$$

for bare and dressed vertices and propagators respectively. The identities (ii), in the usual gauges of quantum-electrodynamics say that one gets zero if of one or more external lines, whether on or off mass-shell, the polarization vector $e_\mu(k)$ is replaced by k_μ while all the other lines are kept on mass-shell and provided with physical polarization vectors. Note that the above identity involves only the vertices and propagators of the particles coupled to the photon (i.e. the electron); there are obviously no identities involving only photon quantities.

In case of non-abelian gauge field theories the situation is much more complicated. First, in addition to identities of the same type as in quantum-electrodynamics one has a whole set of identities involving only the Yang–Mills fields and the associated ghosts. Furthermore, the identties (iii) are no more the dressed version of the identities (i). And the identities (ii) are in general more complicated and have less direct physical meaning than the above mentioned rule of quantum-electrodynamics.

To analyze the renormalizability of massive Yang–Mills fields it is first of all mandatory to develop combinatorial techniques so that remaining difficulties cannot be blamed to the less than rigorous method of path integrals. With the help of generalized Ward identities of the type (iii) containing indeed the full symmetry content of the theory, the massive theory was reduced to a minimally divergent form [20]. These identities were derived by means of the so-called Bell–Treiman transformation [15], that is a canonical transformation that is in form a gauge transformation. By explicit calculation with some brute cut-off method it was verified that some of the remaining non-renormalizable divergencies did not cancel. This more or less ruined the last hopes that massive Yang–Mills theories were renormalizable. The only possibility left was that the cut-off method used was not good enough in view of the symmetries of the problem. The problem of the cut-off method was also relevant to the massless case, because even if the Feynman rules indicate a renormalizable theory, the Bell–Jackiw–Adler [21] anomaly has taught us that power counting is not to be trusted in the presence of symmetries.

'tHooft, tackling this problem in the massless Yang–Mills theory [22] made several important advances. First, using combinatorial methods, he established WT identities of the type (i), and exploited them to derive identities of type (ii) as well as to establish unitarity. Secondly, he devised a cut-off method for one loop diagrams that later was generalized to a complete procedure [23]. Thirdly, he extended the method of Faddeev and Popov [12] and Fradkin and Tyutin [13] so that it became very easy to write Feynman rules containing oriented ghosts in a very large class of gauges. And these rules were all given in terms of a local lagrangian, which opened up the way for a complete renormalization program. It must be emphasized that the close understanding of the theory in terms of diagrams side by side with formal expressions such as path integrals was a necessary ingredient in this and further developments.

This is perhaps the right place to mention the work of Mohapatra [24]. Working within the usual canonical formalism, which indeed can be carried through completely in the so-called axial gauge ($W_3^a = 0$), invented by Arnowitt and Fickler [25], he derived the Feynman rules for massless Yang–Mills fields. They agree with the results found by other authors.

The work of B. Lee and Gervais, and B. Lee, and Symanzik [26] on the σ-model demonstrated that the renormalizability of a theory was not spoiled in case of a spontaneous breakdown of the symmetry. This dispelled the fear that in the process infrared divergencies would change over into ultraviolet divergencies. Inspired by this work, 'tHooft [27] developed several models of gauge theories with massive vector mesons. Among them is a model very similar to the Weinberg [28] model which could have relevance to strong (and weak) interactions, and he furthermore demonstrated that certain electromagnetic mass differences are finite in this model.

Meanwhile Slavnov [29] and Taylor [30] deduced the generalized Ward identities of the type (iii) for massless Yang–Mills theories. They did this using the important observation of Fradkin and Tyutin [13] that one can exploit certain non-local Bell–Treiman transformations to investigate the properties of the theory. Both Slavnov and Taylor worked within the framework of the path integral approach: in addition Taylor verified the combinatorics of the diagrams. These generalized Ward identities, which we will call Slavnov–Taylor identities from now on, are very similar to the generalized Ward identities of the massive theory, and can be written down also for gauge theories with spontaneous breakdown. The Slavnov–Taylor identities contain the full combinatorial content of the theory. This follows, because from them one can rederive the symmetry of the lagrangian. As such they form one of the essential tools in the renormalization program.

The net result of all this work was that one knew precisely the Feynman rules for gauge theories for any choice of gauge, as well as the associated generalized Ward identities. To complete the renormalization program a gauge invariant regulator method was of vital importance; subsequently the subtraction procedure had to be carried through. This work has now been largely completed, but we will refrain in this appendix from further discussion.

To come back now to approach b) mentioned above, the historical development can be outlined as follows. First, Englert and Brout [31] demonstrated that gauge bosons, abelian or non-abelian, acquire a mass in case of a spontaneous breakdown of the symmetry. Whilst they observed that the zero mass scalar meson played the rôle of maintaining gauge invariance in the vector meson propagator, they did not make explicit that this was the only rôle of the Goldstone bosons, and hence that the latter were in fact unobservable in this type of theory. This aspect of the theory was the focal point of Higg's work [32], and he arrived at the known results. His very nice and now well-known treatment brought to conclusion an argument entertained by Klein and B. Lee [33], and Gilbert [34] that started with an important observation by Anderson [35] concerning Goldstone's [36] and Nambu and Jona–Lasinio's [37] fundamental work on superconductivity and spontaneous breakdown of symmetries. In subsequent papers various authors elaborated on this subject [38].

As early as 1966 Englert, Brout and Thiry [39] published a paper in which they obtained a substantial portion of the Feynman rules in case of a spontaneously broken symmetry. They had the correct vector meson propagator in the Landau gauge as well as the corresponding ghost. However, they had not discovered the necessity for the Faddeev–Popov ghost, which is also very difficult to find in their approach. On the basis of the form of the propagator Englert et al. suggested that the theory be renormalizable. Unfortunately this paper went largely unnoticed by the scientific community.

Meanwhile progress was made concerning model building. As indicated before, Bludman [3] published in 1958 a model of weak interactions in which the known leptons were placed in chiral SU2 multiplets. Glashow [4] extended this to include electromagnetism. He arrived at a lagrangian for the interaction of leptons and vector bosons that is in fact nothing but what is also contained in the Weinberg lagrangian. There are three vector mesons, W^+, W^- and W^0, and the photon, and a mixing angle which is nowadays called the Weinberg angle. What Glashow did not have, evidently, is the relation fixing the neutral vector meson mass as a function of this angle. In 1964 Salam and Ward [40] proposed $SU2 \times U1$ as underlying group structure, thereby also introducing the photon-neutral vector boson mixing angle. Their result coincided with that of Glashow; like Glashow they did not introduce spontaneous breakdown as a mass generating mechanism. Finally, in 1967, Weinberg [28] added spontaneous breakdown of the symmetry as a mechanism to give mass to the vector mesons and leptons. In that formalism the mass of the neutral vector meson becomes a function of the mixing angle, and consequently several reactions involving neutral currents contain this angle as the only free parameter. This particular mass spectrum is therefore to be considered as typical for a gauge theory with spontaneous breakdown. Weinberg furthermore aired his suspicion that the theory might be renormalizable, but no indication of the Feynman rules was given. Salam [41] in his Nobel symposium lectures demonstrated a remarkable intuitive insight, but also did not come to grips with the Feynman rules. For this reason the arguments were unconvincing, and this line of research did not bear further fruits until recently. In fact, these works have been of great value as guideline for recent more sophisticated attempts of model building. With regard to the field theoretical aspects people have tried to extend the reasoning by trying to work consistently in the so-called unitary gauge. This is in my opinion about as rational as studying function theory while refusing to go in the complex plane.

With respect to approach c), from the high-energy behaviour of weak interactions, there are several attempts scattered in the literature. Recently Llewellyn-Smith [42] and Cornwall et al. [43] have succeeded in deriving the complete structure of a spontaneously broken symmetry from such principles. This is important, since this argument has a more direct physical appeal.

Discussion

J. Iliopoulos (Orsay)

Could you explain the difference between theories of class (ii) and (v)?

Veltman

Class (ii): One starts with a theory in which parity conservation is a consequence of the strong interaction symmetry. Then the weak interactions produce order α violations of parity. This is not allowed.

Class (v): One has 25 knobs with which one can adjust the strong interactions to have conservation of parity. Then the weak interactions are switched on and one finds that one lacks a knob (parameter) to tune parity violation to zero. This is to be excluded also.

J. C. Pati (Maryland)

First I like to make a comment and then ask a question: Regarding your comment that models of class (i), i.e. non-abelian gauge theories of strong interactions, in which strong gauge mesons get their masses through Higgs–Kibble mechanism) could be in trouble with regard to parity because of Yukawa coupling of the corresponding Higgs mesons to the Fermions, this statement is not true in general. Consider, for example, the model of Salam and myself, in which we introduce strong interactions by an octet of color gauge gluons. The Higgs mesons giving masses to the gauge bosons are taken to belong to (2+2, 1, 3*) or three triplets each transforming as (1, 1, 3*) under the gauge group $SU(2)_L \times Z(1) \times SU(3'')_{L+R}$. Here, these Higgs mesons can not have Yukawa coupling to quarks (which transform as **3*** under SU(3'')) due to gauge invariance. But they do give mass to the strong gauge bosons. (The quarks get mass through the Higgs mesons transforming as (2, 1, 1), whose Yukawa coupling constant is small compared to e if (m_q/m_w) is small).

My question is related to your initial remark that a list of things follows as a consequence of gauge principle. I understand it in so far as current algebra is concerned; but I do not understand it in so far as CVC or PCAC etc. are concerned. It seems to me that after spontaneous symmetry breaking, this is so far no guaranteed mechanism for these to hold. Can you comment on this?

Veltman

Imagine a vector meson coupled to a current of strongly interacting particles. The $k_\mu k_\nu/M^2$ term in the propagator, when multiplied by the currents must become harmless for large k. That means that k_ν multiplied into the current must behave as a constant for large k. In this sense we expect the divergence of the current to be "soft" or smoothly behaved.

H. Stern-Kluberg (Saclay)

I would like to comment about your suspicion: "class (i) is empty". Do you think that the existence of Higgs mesons is fundamental, or more likely a technical tool? Moreover it was shown that spontaneous breaking occurs through radiative corrections, and an example of a 2-dimensional field they shows such behaviour and contains only fermions and a photon.

Veltman

Please interpret my statement that I suspect that class (i) is empty just the way I say it. If the class is not empty it is a very interesting class of theories.

C. N. Yang (Stony Brook)

The recent activities are very exciting in this field. By combining the concept of gauge fields and the concept of broken symmetry, it was proved – and you say that is a closed chapter – that there is renormalizability.

I would like to ask you, in your opinion, when the dust settles down, what essential parts of the recent developments are likely to remain, and what new ingredients, in a general way, are still needed for a successful theory?

Veltman

There are several things that seem to be of deeper significance. First, electromagnetism and gravitation are gauge theories, and now there opens the real possibility that other interactions have also some intricate gauge symmetry. It appears that the gauge idea, the Yang–Mills structure, is a fundamental property. But note that these theories owe their interest to the fact that they are renormalizable. I do not know how fundamental this notion of renormalizability is, but it is really miraculous that these theories are renormalizable. This, of course, barring gravitation of which we do know very little. It seems to me that this can be no accident.

One of the things that appeals less to me is for instance the question of the anomalies. It is strange to think that such far fetched technicalities would determine the structure of the world. There must be something that we have not understood about parity. Again and again it strikes me as something that fits not naturally in the scheme.

Another such thing is the Higgs mechanism. Despite its real beauty in various circumstances it hurts that one must artificially introduce new particles. Perhaps there are ways out, and one can imagine that the Higgs system is a way to describe an effective lagrangian at low energies. In the literature there are already attempts in that direction, viz. Coleman and E. Weinberg [73].

Perhaps a deep understanding of strong interactions is necessary before we can make further progress. Perhaps also more can be learned by studying gravitation, which after all exists too. Who knows!

R. Gatto (Roma)

I want to add to your statement that an asymptotic free theory produces scaling. It is interesting that under general hypotheses also the reverse statement is true, as was pointed out some time ago by Ferrara, Grillo, Parisi, and myself [74]. In fact it could be shown that Bjorken scaling implied an infinite number of local conservation laws $\partial^{\alpha_1} O_{\alpha_1 \dots \alpha_n} = 0$ a situation that essentially requires a free asymptotic theory.

Iliopoulos

Asymptotic freedom occurs only when the gauge group does not contain an abelian subgroup. This should hold for both strong and weak gauge groups since the photon has a tendency to mix with the strong neutral gauge bosons. Therefore, if asymptotic freedom is a criterion for choosing your model, you should exclude all those containing an abelian subgroup.

L. Stodolsky (Max-Planck-Inst., München)

Can we forget such theories if no new particles – W mesons, Higgs bosons, or the like – turn up?

Veltman

It seems that I can best refer you to the talk of Llewellyn Smith, who reports on model building.

Förster (Aachen)

I would like to ask if anybody is prepared to comment on a recent paper of Nielsen and Olesen where it is shown that there are vortex solutions in the Higgs model. That appears to be related to gauge theories. The reason why I am asking is that there is a speculation that there is a connection with the work of people like Rebbi or the original idea of Nambu that the hadron spectrum has something to do with a string-like spectrum and the string is related to a gauge theory now.

Veltman

No comment.

H. Terazawa (Rockefeller Univ.)

I wonder if the speaker has any comment on the work by Umezawa and Takahashi who pointed out that the Higgs mechanism has some trouble. If I remember correctly, they said that in some cases, the mechanism would break either the Ward-Takahashi identity or the important criterion that any Heisenberg operator can be expressed as a functional of asymptotic field operators.

Veltman

No comment.

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