C VIOLATION IN STRONG INTERACTIONS

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Once more physicists are facing the breakdown of a principle, namely CP invariance, which was supposed to be generally valid. As is the hallmark of a general principle, the consequences are simple to understand on the one hand, but far reaching on the other. It is just these features which make the subject so attractive for study both theoretically and experimentally; we are therefore happy to dedicate this account to Professor V. F. Weisskopf, whose deep interest in such matters, and whose warm personality and high standards in science and scientific life have made such an impact on CERN.

In this discussion we will limit ourselves to the question of possible C violation in strong interactions and where it may manifest itself in an observable manner. As yet it is still an open question whether the CP violation observed in $K_L \rightarrow 2\pi$ decay [1] is due to a perturbation by a rather strong interaction [2]; moreover, even if we assume that such is the case, we are still in doubt about number of properties of this interaction. The first question that arises is: does there exist a class of interactions to which we can attribute this CP violation? To this purpose we must first establish what we understand under a "class of interactions". Three notions are important in this respect, namely symmetry properties, the involved particles, and strength, and interaction classes are distinguished from each other through behaviour with respect to one or more of these qualifications. Thus, a class of interactions may distinguish itself from another class through different behaviour with respect to some symmetry; a good example is the so-called medium strong or SU_3 breaking interaction whose existence became significant only after the discovery of SU_3 symmetry. These

SU_3 breaking interactions are not yet sharply defined through a strength of coupling constant; this is in contrast to the weak interactions that always have been characterized by their small strength, whether they are leptonic, non-leptonic, strangeness violating or strangeness conserving. Thus, when parity violation was used to explain the famous $\theta-\tau$ puzzle, without any further ado this parity violation was generalized to all weak interactions. The electromagnetic interactions are characterized through the photon being involved, and also by their strength and behaviour with respect to isospin, which is used as identification if the photon is only virtually present. Indeed, it would be very difficult to distinguish an I spin breaking interaction among strongly interacting particles of a strength of about 10^{-2}, and unless some further observable difference is detected it remains a question of semantics.

Let us now investigate what properties we reasonably can attribute to the C violating interaction. First we must discuss somewhat in detail the K meson system. One knows that the $\Delta I = \frac{1}{2}$ selection rule is broken in $K^+ - K^0 \rightarrow 2\pi$ decay, a 5 % $\Delta I = \frac{3}{2}$ or $\frac{5}{2}$ amplitude admixture being observed. This is a somewhat stronger breaking than expected from electromagnetism, but there are some arguments that explain this discrepancy [3]. The possibility that these $\Delta I = \frac{3}{2}$ or $\frac{5}{2}$ amplitudes arise from a C violating, I spin breaking perturbing interaction [4] is not very plausible, because one would expect in such a case a 5 % effect in $K_L \rightarrow 2\pi$ instead of the observed 0.25 % [5]. Thus, we will continue to assume the $\Delta I = \frac{3}{2}$ or $\frac{5}{2}$ amplitudes as resulting from e.m. perturbations, that conserve C. If the C violating interaction breaks I spin also it must thus be of strength 10^{-2}–10^{-3}, and give rise to a $\Delta I = \frac{3}{2}$ or $\frac{5}{2}$ amplitude small with respect to the 5 % electromagnetic amplitudes. Although this is not experimentally excluded it is clearly more attractive to assume that the C violating interactions conserve I spin. This lifts also somewhat the restriction on the magnitude of the coupling constant [6].

Thus we assume the following selection rules:

1) $\Delta I = 0$;

2) parity conservation. Parity non-conserving effects seem to appear only at the level of weak interactions. An example of a test is the absence of an electric dipole moment for the neutron [7];

3) $\Delta S = 0$. A glance in any table on properties of elementary particles shows that strangeness is broken only at the level of weak interactions;

4) obviously, to be able to act in the K^0 system the interaction must involve strongly interacting particles.

Altogether we have an interaction between strongly interacting particles with strength $\gtrapprox 10^{-2}$, $\Delta I = 0$, $\Delta S = 0$, and P conservation. Within the present possibilities for distinguishing classes of interactions we arrive at the conclusion that we are dealing with strong interactions that may or may not break SU_3.

From some general considerations we may further arrive at certain limitations. On the basis of a simple theorem due to Soloviev, Pais and others one may find it plausible that no C violation occurs in the SU_3 conserving interactions. Recently this point has been analyzed anew by Cabibbo [8], who has been able to state a number of theorems of this nature for matrix elements rather than for interaction Lagrangians, and it appears reasonable that in many cases C violation even in strong interactions only shows up at the level of SU_3 breaking interactions. For the time being the SU_3 behaviour of a C violating interaction is quite academic, but ultimately (if indeed the C violation is to be found in the strong interactions) this question must be settled.

In the following we will concentrate our attention on the detection of a $\Delta I = 0$, C violating interaction, occasionally mentioning tests for $\Delta I \neq 0$, C violating interactions. Let us discuss some interesting reactions. The ideal systems for direct observation are those systems that are eigenstates of C, and we will limit ourselves here to such systems, excluding discussion of possible C violating effects, for example, in collision processes.

From the table of elementary particles and resonances [9] we find as candidates (with the exclusion of K_1^0 or K_2^0 and some doubtful cases):

$$\pi^0, \eta, X^0 (= \eta 2\pi), \rho^0, \omega, \phi.$$

Further we have the (by far the most interesting) proton-antiproton system.

π^0 *decay*. The only particles lighter than the pion are leptons or photons. The C violating decays are:

$$\pi^0 \to (\gamma) \to e^+ + e^-$$

$$\pi^0 \to 3\gamma, \text{ etc.}$$

The first process is forbidden in lowest order of electromagnetic interactions because of parity, and also gauge invariance. Of course, this transition may proceed in higher order, see the figure. The process $\pi^0 \to 3\gamma$ has not yet been looked for with an interesting accuracy. An estimate of the rate on the basis of simple phenomenological considerations has been made by Berends [10], and the conclusion is that this process is probably very rare [11].

η decay. The η decay into $\pi^+\pi^-\pi^0$ offers the extremely interesting possibility of an interference between an electromagnetically induced and a C violating transition. As has been noted however [12] angular momentum barrier effects play a very important role here, and it is not easy to estimate possible effects. The decay modes in question and their estimated strength are ($e^2 = 1/137$ = e.m. coupling constant):

Mode	I spin viol.	Strength	C behaviour	Wave function
0	$\Delta I = 0$	gk^3 or $g'e^2k^3$	C viol.	$\partial^3_{\mu\nu\alpha}\eta\varepsilon^{ijk}\partial_{\mu\nu}\pi^i\partial_\alpha\pi^j\pi^k$
1	$\Delta I = 1$	e^2 [13]	C cons.	$\eta(\pi^i\pi^i)\pi^j$
2	$\Delta I = 2$	ge^2k or $g'k$	C viol.	$\partial_\mu\eta\partial_\mu\pi^i\pi^j\pi^l\varepsilon^{ijk}$

Only the wave functions with minimum of derivatives for a given I spin mode are considered. The uninteresting $\Delta I = 3$ mode has been left out. ∂_μ stands for $\partial/\partial x_\mu$.

In here g is the coupling constant of the C violating isospin conserving interactions; for completeness we added also the case that the C violating interaction breaks I spin also and called that coupling constant g'. Latin indices indicate I spin components. k represents angular momentum barrier effects:

$$k = \frac{m_\eta Q}{M^2}$$

where Q is the average kinetic energy of the pions, about 50 MeV, and M is some unknown reference mass, certainly larger than the mass of the pion. m_η is the η mass.

As has been pointed out [14] interference between a C conserving and a C violating mode may result in that the ratio

$$\frac{\text{Number of events with } \pi^+ \text{ energy} > \pi^- \text{ energy}}{\text{Number of events with } \pi^+ \text{ energy} < \pi^- \text{ energy}}$$

is different from 1. Study of the Dalitz plot may eventually reveal whether mode 0 or 2 is the interfering one. It may be noted that the known dominant S wave structure of the Dalitz plot implies the dominance of mode 1.

Let us write down the energy dependence of the matrix element for the different modes. Denoting the energy (including rest mass) of π^+, π^- and π^0 by E_+, E_- and E_0 we have:

Mode	Energy dependence of matrix element
0	$E_\eta^3\{E_0^2(E_- - E_+) + E_+^2(E_0 - E_-) + E_-^2(E_+ - E_0)\}$ $= \frac{1}{4}x(x^2 - 3y^2)\cdot E_\eta^3$
1	Const.
2	$E_\eta(E_+ - E_-) = E_\eta x$

where $x = E_+ - E_-$, $y = E_0 - \frac{1}{3}m_\eta$. The mode 0 matrix element is antisymmetrical between the three pions. Neither mode 0 nor mode 2 give rise to the decay $\eta \to 3\pi^0$ [15].

Another η decay mode is $\eta \to \pi^+\pi^-\gamma$. In this decay C violating interactions can interfere, but they would suffer quite strong angular momentum barrier effects. Moreover, the C violating modes are suppressed by an extra factor g as compared to the main electromagnetic mode.

X^0 *decay*. As the X^0 has the same quantum numbers as the η, everything said above is applicable to X^0 decay also. Thus the 3π mode (not observed yet) is of particular interest, especially because barrier effects should be less important. Here we have the drawback of a competing strong process, namely $X^0 \to \eta\pi\pi$. The decay $X^0 \to \eta\pi\pi$ is G parity conserving and any interfering C violation must break isospin also. Thus this decay is suitable for detection of C violating interactions with $\Delta I = 1$.

ρ *decay* [16]. $\rho^0 \to \eta\pi^0$ is forbidden if C is conserved. $\rho^\pm \to \eta\pi^\pm$ may proceed through C violation or (electromagnetically) I spin violation. The branching ratio expected for the *C* violating case could be at most $g^2 \lessapprox 10^{-2}$ with respect to the main mode $\rho \to 2\pi$, the e.m. decay should be down by a factor $e^4 \sim 10^{-4}$. It is interesting to note that $\rho^0 \to \eta\pi^0$ could simulate a resonance of *I* spin 0 in the ρ region.

ω and ϕ decay [16]. The decays $\omega \to \eta\pi$, $\omega \to 3\pi$ and $\phi \to \eta\pi$, $\phi \to 3\pi$ conserve *G* parity and can therefore be used only to detect *C* and *I* spin violating interactions. Note that the normal $\phi \to 3\pi$ is strongly suppressed (by SU_6) so that any irregular decay could show up stronger. Barrier effects are very important here, too.

Very interesting are $\omega \to \pi\pi\gamma$ and $\phi \to \pi\pi\gamma$. Depending on the pion configuration *C* is violated or conserved (as $\Delta I = 0$ or 1 there is no limitation from isospin). Thus interference may show up as asymmetries between the π^+ and π^- distributions. A favourable circumstance is the possible enhancement of the *C* violating mode through the ρ meson: $\omega \to \rho\gamma$ ($\phi \to \rho\gamma$) is forbidden by *C*. As has been noted elsewhere [17] this decay may be used for completely different purposes, namely the detection of S wave $\pi\pi$ resonances.

The $p\bar{p}$ *system* [18]. The above-mentioned decay modes may all be used to detect the existence of *C* violations, and eventually we could get information on *I* spin behaviour and also on the strength of the *C* violating interaction. But as no strange particles are involved it is not easy to see how information with respect to SU_3 could be obtained. For these purposes the $p\bar{p}$ system is well suited: K and K* mesons are quite copiously produced and if *C* violation shows up here a systematic study could reveal properties with respect to SU_3. In this context also tests of the kind as proposed in Ref. [19] ($p\bar{p} \to \Lambda\bar{\Lambda}$) may provide very useful information.

In the following we will not try to give a general discussion. We merely note the following interesting fact: if *C* is conserved the energy spectra and total numbers of K^+ and K^- in the reactions

$$p + \bar{p} \to K^{\pm} + \text{anything}$$

must be identical (in the $p\bar{p}$ centre-of-mass system). If *C* is not conserved this need not be the case, which we will demonstrate on a simple example, namely the channel:

$$p + \bar{p} \to K\bar{K}\pi^{\pm}$$

with $p\bar{p}$ annihilation at rest. There are two reasons why we take this channel: first, it has been demonstrated [20] that with certain assumptions $\tilde{U}(12)$, one of the relativistic generalizations of SU_6 forbids this transition, which we interpret that it could be that the SU_3 invariant transitions are somewhat suppressed so that other effects may show up more easily; and second, because the isotopic spin structure is very simple, which saves us writing. For a proton and an antiproton at rest in a state of zero angular momentum the only non-zero spinor combinations are

$$\bar{u}_{\bar{p}}\gamma^{\mu}u_{p} \text{ and } \bar{u}_{\bar{p}}\gamma^{5}u_{p}$$

i.e., the ${}^3S_1$ and ${}^1S_0$ state, respectively. Both have the parity minus, the ${}^3S_1$ state has $C = -1$ (like e.m. current), the ${}^1S_0$ state has $C = +1$. We will limit ourselves to the ${}^1S_0$ state.

With respect to isotopic spin the $p\bar{p}$ system is an equal mixture of isotopic spin 0 and 1

$$(p\bar{p}) = \tfrac{1}{2}\{(\bar{N}N)+(\bar{N}\tau_3 N)\}.$$

Thus the ${}^1S_0$ state contains an equal mixture of states with I spin 0 and 1, both with parity $-$ and C $+$, i.e., of η and π^0 like states. Thus the ${}^1S_0$ state is a mixture of two states with different G parity. Clearly then any final state with a definite G parity may be reached by both the G conserving and the G violating ($= C$ violating if I spin is conserved) interactions, and interference effects may show up.

Another possibility is that two final states, with different G parity, are reached from the same initial state by C violating and C conserving interactions and interfere in an observable way. As a first example we consider the case where the K mesons are in an S wave with respect to each other. As we consider only the $\bar{K}^0K^+$ and K^0K^- combinations only the I spin 1 combination of the kaons is important, and this combination has the G parity minus (being an isovector with $C = +1$). Together with the pion we have a system with $G = +1$, and the C conserving (violating) transition proceeds from the I spin 0 (1) state of $p\bar{p}$. The general matrix element is

$$M_1 = a\{(\bar{N}N)\pi^i(\bar{K}\tau^i K)\} + b\{\bar{N}\tau^i N)\pi^j(\bar{K}\tau^k K)\varepsilon^{ijk}\}$$

where we indicated only the isospin structure. If b is non-zero C is violated. In the absence of final state interactions a and b have the same phase (are real), but the final state interactions destroy this property. The ratio of $p\bar{p} \to K^+\bar{K}^0\pi^-$ to $p\bar{p} \to K^-\bar{K}^0\pi^+$ is given by

$$\frac{\text{rate }(K^+\bar{K}^0\pi^-)}{\text{rate }(K^-K^0\pi^+)} = \left|\frac{a+ib}{a-ib}\right|^2 = \frac{|a|^2+|b|^2-2\mathrm{Im}(ab^*)}{|a|^2+|b|^2+2\mathrm{Im}(ab^*)}$$

which is not necessarily 1.

To demonstrate the other class of interference phenomena in this particular channel we may consider interference between systems where the K mesons are in an S or P wave, respectively. The latter combination has the G parity plus, and the general matrix element is the sum of the S wave matrix element M_1 above and the P wave matrix element M_2

$$M = M_1 + M_2$$

$$M_2 = a'\{(\bar{N}N)\partial_\mu \pi^i(\bar{K}\overleftrightarrow{\partial_\mu}\tau^i K)\} + b'\{(\bar{N}\tau^i N)\partial_\mu \pi^j(\bar{K}\overleftrightarrow{\partial_\mu}\tau^k K)\varepsilon^{ijk}\}$$

where

$$(\bar{K}\overleftrightarrow{\partial_\mu}\tau^i K) = (\partial_\mu \bar{K})\tau^i K - \bar{K}\tau^i(\partial_\mu K).$$

In M_2 we have C violation if $a' = 0$. Interference between a' and a may give rise to different energy spectra for K^+ and K^-, but after integration over all energies these interferences drop out and total numbers of K^+ and K^- are not influenced by this effect. This type of interferences may be more easily accessible to detection if for some reason the C violating transition is enhanced or the C conserving one depressed (angular momentum barriers, resonant states, etc.).

Thus we arrive at the conclusion: if the energy spectra and total numbers of K^+ and K^- are different for any one channel C is violated.

Obviously similar statements can be made with respect to pions or resonant states instead of kaons.

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REFERENCES

1) J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Phys. Rev. Letters **13** (1964) 138.
2) J. Prentki and M. Veltman, Physics Letters **15** (1965) 88;
T. D. Lee and L. Wolfenstein, Columbia University preprint;
B. Okun, preprint;
In the days of parity violation *T* violation in strong interactions was discussed by B. Jacobsohn and E. Henley, Phys. Rev. **113** (1959) 225, 234;
A number of experiments of the type

$$\alpha + C \rightleftarrows d + N$$

has been performed, and any violation of detailed balance there is below about 3 %. As these reactions are governed by SU_3 conserving forces and moreover do not involve strange particles we do not expect big effects. See also Ref. [8];
D. Bodansky et al., Phys. Rev. Letters **2** (1959) 101.
3) N. Cabibbo, Phys. Rev. Letters **12** (1964) 62;
M. Gell-Mann, Phys. Rev. Letters **12** (1964) 83.
4) Recently Salzman and Salzman have proposed *T* violation in electromagnetism, Physics Letters **15** (1965) 91;
See also S. Barshay, Rutgers, the State University, preprint.
5) T. T. Wu and C. N. Yang, Phys. Rev. Letters **13** (1964) 380;
J. N. Truong, Phys. Rev. Letters **13** (1964) 358;
As the mass matrix governing the definition of K_L and K_S will only be affected by a few %, there will be no compensation between a phase in the mass matrix and the $\frac{3}{2}$ or $\frac{5}{2}$ amplitude phase (which can in these circumstances be anything between 0° and 360°). In other words, K_L will, up to a few percent, still be eigenstate of *CP*, but we can say nothing of the $\frac{3}{2}$ or $\frac{5}{2}$ amplitudes. See also Ref. [6].
6) S. Weinberg, Phys. Rev. **110** (1958) 782;
In this case the $\Delta I = \frac{3}{2}$ and $\frac{5}{2}$ transitions suffer only small perturbations, of a few percent, from the *C* violating interaction, and their phase with respect to the mass matrix will be close to zero. The main contribution to $K_L \to 2\pi$ should come from the $\Delta I = \frac{1}{2}$ transition, being out of phase with the mass matrix by a small amount. If the $\Delta I = \frac{1}{2}$ mode is the main constituent of the mass matrix, the phase of the latter may be very close to the $\Delta I = \frac{1}{2}$ amplitude phase.
7) For an analysis of the neutron electric dipole question, see R. Jengo and R. Odorice, Physics Letters **16** (1965) 168 and D. Boulware, Harvard University, preprint.
8) N. Cabibbo, Rockefeller Institute preprint.
Here further references to the subject are listed.
9) A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz and M. Roos, Rev. Mod. Phys. **36** (1964) 977.
10) F. Berends, Physics Letters **16** (1965) 178.

11) The authors are indebted to Dr. V. Soergel for illuminating discussions on the experimental possibilities for detection of $\pi^0 \to 3\gamma$.

12) Read again Ref. [2], first paper.

13) See, S. Okubo and B. Sakita, Phys. Rev. Letters **11** (1963) 50;
From SU_3 one may obtain an estimate of the total rate $\eta \to 2\gamma$ using the rate $\pi^0 \to 2\gamma$ as input (result: $\tau^{-1} \sim 140$ eV). As $\eta \to 3\pi$ is about just as abundant as $\eta \to 2\gamma$ one can then calculate the coupling constant f involved for an S wave decay (mode 1). The result is larger than expected from electromagnetism: $f^2/4\pi = 2.5 \cdot 10^{-3}$ instead of $e^4 \sim 10^{-4}$. It seems that this enhancement is a common feature of virtual e.m. processes, as well as weak processes (the non-leptonic are generally a factor 10 stronger than leptonic ones).

14) R. Friedberg, T. D. Lee and M. Schwartz, private communication to T. D. Lee and L. Wolfenstein, Ref. [2], second paper.

15) Recently the $\eta \to 3\pi$ decay has been discussed by M. Nauenberg, Stanford University preprint, and also, before *C* violation was suspected, by M. Foster et al., University of Wisconsin preprint (submitted to Phys. Rev. Letters).

16) Y. Fujii and G. Marx, Stanford University preprint.

17) J. Prentki and M. Veltman, private communication. See also CERN preprint TH. 565.

18) Discussions with Profs. R. Armenteros, C. Peyrou, J. Steinberger and Dr. L. Montanet on this system have been most valuable.

19) G. Cohen-Tannoudji and A. Messiah, Physics Letters **15** (1965) 191.

20) R. Delbourgo, Y. Leung, M. Rashid and J. Strathdee, International Centre for Theoretical Physics (Trieste) preprint.