Baryon-Resonance Production by Neutrinos.

S. M. BERMAN

Stanford Linear Accelerator Centre - Stanford, Cal.

M. VELTMAN

CERN - Geneva

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Summary. — The production of the $\frac{3}{2}$, $\frac{3}{2}$ resonance by neutrinos incident on nucleons is considered in the framework of the Rarita-Schwinger formalism for spin-$\frac{3}{2}$ particles. With the help of the conserved vector theory, and the Goldberger-Treiman relation, the value of certain coefficients is estimated. Wherever information is lacking, the static theory is used. The interaction determined in this way is then used to obtain total cross-sections. The result is that the total cross-section on complex nuclei as function of neutrino energy behaves similarly to, but is in absolute value larger than, the elastic process cross-section. Properties of the spin-$\frac{3}{2}$ density matrix under the combined assumptions of time-reversal invariance and Born approximation are derived and the general expression for the angular distribution of the decay pion in the $\mathcal{N}^*$ rest frame is given.

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1. – Introduction.

In the recent neutrino experiments (1) a considerable amount of one-pion events have been observed. It is tempting to attribute some of these to the

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(1) G. BERNARDINI, G. VON DARDEL, P. EGLI, H. FAISSNER, F. FERRERO, C. FRANZINETTI, S. FUKUI, J.-M. GAILLARD, H. J. GERBER, B. HAHN, R. R. HILLIER, V. KAFTANOV, F. KRIENEN, M. REINHARZ and R. A. SALMERON: *International Conference on Elementary Particles* (Sienna, 1963).

production of the $\mathcal{N}^*$ ($I=\frac{3}{2}$, $J=\frac{3}{2}$ resonance) through the reaction

$$\nu + \mathcal{N} \to \mu^- + \mathcal{N}^* \qquad \mathcal{N}^* \to \mathcal{N} + \pi$$

This idea is supported by the predominant production of π^+ with respect to π^0 (roughly 3 to 1); the pure $\mathcal{N}^*$ production would give a π^+/π^0 ratio of 5 to 1, which is then modified through charge-exchange effects to a somewhat lower value.

As a further motivation for an investigation of the $\mathcal{N}^*$ production process, we stress the fact that for incident neutrinos the process has a cross-section on protons which is 3 times the cross-section on neutrons. Therefore, this process is well suitable for experiments on hydrogen. Moreover, all final particles are charged in that case, which facilitates experimental investigation.

In this paper, we will treat the $\mathcal{N}^*$ in the Rarita-Schwinger scheme (2), analogous to the isobar model (3) used in photoproduction of $\mathcal{N}^*$. In Sect. **2** we consider the $\mathcal{N}\mathcal{N}^*$ current and write down its most general form. At zero momentum transfer, we will obtain the coefficients of the vector parts from photoproduction (4), while the values of the axial vector current are estimated with the help of the Goldberger-Treiman method (5). To go further, especially to find the relative sign between vector and axial vector current, we compare, in Sect. **3**, with the static theory as developed earlier (6). For non-zero momentum transfer we use the result of DOMBEY and also DENNERY (7), which states that both the vector and axial-vector form factors are essentially those of the ordinary nucleon-nucleon current which enter in the « elastic » process

$$\nu + \mathrm{n} \to \mu^- + \mathrm{p}\,.$$

In Sect. **4** we give the pion angular distribution and the properties of the $\mathcal{N}^*$ density matrix which follow from the combined assumptions of time-reversal invariance and Born approximation for the weak interactions.

Finally, in Sect. **5**, we quote some numerical results, assuming a certain axial form factor. The computations for the antineutrino production are made with the assumption that the nucleon currents coupled to neutrino and antineutrino, respectively, are in the same isospin multiplet.

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(2) W. RARITA and J. SCHWINGER: *Phys. Rev.*, **60**, 61 (1941).

(3) M. GOURDIN and PH. SALIN: *Nuovo Cimento*, **27**, 193, 309 (1963).

(4) Similar methods are used in determining the Ω^- leptonic decay rate by S. L. GLASHOW and R. H. SOCOLOW: *Phys. Lett.*, **10**, 142 (1964).

(5) M. L. GOLDBERGER and S. B. TREIMAN: *Phys. Rev.*, **110**, 354, 1178 (1958).

(6) J. S. BELL and S. M. BERMAN: *Nuovo Cimento*, **25**, 404 (1962).

(7) N. DOMBEY: *Phys. Rev.*, **127**, 653 (1962); PH. DENNERY: *Phys. Rev.*, **127**, 664 (1962).

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2. – The nucleonic current.

The matrix element for the process (Fig. 1) is assumed to be of the form

$$j_\alpha J_\alpha \,,$$

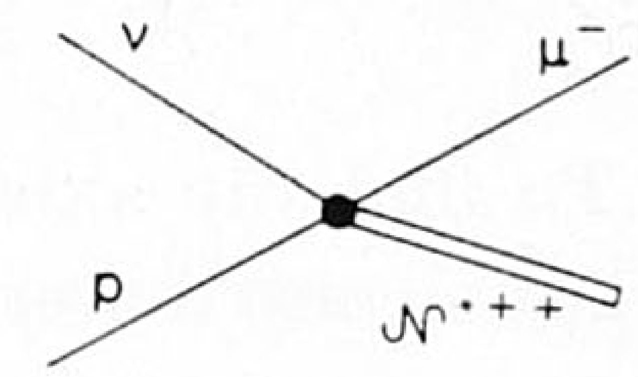

Fig. 1. – Diagram showing the inelastic process $\nu + p \to \mathcal{N}^{*++} + \mu^-$.

where j_α is the usual lepton current:

$$j_\alpha = [\bar{\mu}\gamma^\alpha(1+\gamma^5)\nu]$$

and J_α is the nucleonic current. The $\mathcal{N}^*$ is described by four spinor quantities N^*_α ($\alpha = 1, ..., 4$) satisfying the subsidiary conditions

$$\gamma^\alpha N^*_\alpha = 0\,, \qquad k_\alpha N^*_\alpha = 0\,,$$

where k is the $\mathcal{N}^*$ four-momentum. Using these conditions the most general form for J_α is

$$(1)\qquad \left\{\begin{aligned} J_\alpha &= J^V_\alpha + J^A_\alpha\,,\\ J^V_\alpha &= \left[\bar{N}^*_\beta(k)\left\{a_1\delta_{\beta\alpha} + \frac{Q_\beta}{\Delta}\left(ia_2\gamma^\alpha + \frac{ia_3}{\Delta}\sigma_{\alpha\lambda}Q_\lambda + \frac{a_4}{\Delta}Q_\alpha\right)\right\}\gamma^5 N(p)\right]\,,\\ J^A_\alpha &= \left[\bar{N}^*_\beta(k)\left\{b_1\delta_{\beta\alpha} + \frac{Q_\beta}{\Delta}\left(ib_2\gamma^\alpha + \frac{b_3}{\Delta}(p+k)_\alpha + \frac{b_4}{\Delta}Q_\alpha\right)\right\} N(p)\right]\,,\end{aligned}\right.$$

where $\Delta = M + m$.

To appearance of γ^5 in J^V_α is because the parity of $\mathcal{N}^*$ is positive. q, q', k, p are ν, μ, $\mathcal{N}^*$, $\mathcal{N}$ four-momenta, $Q = p - k =$ four-momentum transfer. $a_1 - a_4$, $b_1 - b_4$ are functions of Q^2, m^2 and M^2 (we use the metric $p^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2 = -m^2$, $p_4 = ip_0$, and the further notation $m =$ nucleon mass, $M = \mathcal{N}^*$ mass). In the following, we will neglect the dependence of a_i and b_i on m^2 and M^2.

To determine the coefficients $a_1 - a_4$ for $Q^2 = 0$, we turn to $\mathcal{N}^*$ photoproduction. Writing down the most general form as above (and using gauge invariance in addition), GOURDIN and SALIN ([3]) deduced the values for $a_1 - a_4$, by comparison with experiment. Using their results in the manner given by the conserved vector current hypothesis ([8]) we find for $\mathcal{N}^{*+}$ production on

([8]) R. P. FEYNMAN and M. GELL-MANN: *Phys. Rev.*, **109**, 193 (1958).

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neutrons

$$J^V_\alpha = g_V\{\bar{N}^{*+}_\beta(\delta_{\beta\alpha} - i/\Delta Q_\beta\gamma^\alpha)\gamma^5 N\} \tag{2}$$

with

$$g_V = 6G = a_1 \qquad \text{and} \qquad G = (1.02/\sqrt{2})\cdot 10^{-5} m^{-2}\,.$$

To find the axial current coefficients we note that the one-pion-exchange diagram contributes to the b_4 term. Taking the divergence of the axial current gives

$$Q_\alpha J^A_\alpha = [N^*_\beta Q_\beta D_0 N]\,,$$

where

$$D_0 = b_1 + b_2\frac{(M-m)}{\Delta} + b_3\frac{(M^2-m^2)}{\Delta^2} + b_4\frac{Q^2}{\Delta^2}\,.$$

If b_2 and b_3 are of the same order as b_1 and not singular for $m = M$, we may neglect them in D. Assuming then that D_0 is for low Q^2 dominated by the one-pion pole, we find

$$\frac{b_4}{\Delta^2} = g_1 g_2/[m_\pi^2\sqrt{3}(Q^2+m_\pi^2)]\,, \qquad b_1 = -g_1 g_2/\sqrt{3}\,m_\pi^2 = -2.2G/\sqrt{3}\,, \tag{3}$$

where g_1/m_π and g_2/m_π are the coupling constants for the vertices $\pi^+ \to \mu^+ + \nu$, $p+\pi^+ \to N^{*++}$, $g_1 = 1.48\cdot 10^{-7}$, $g_2 = 2.32$, and m_π is the pion mass. The sign in b_4 is arbitrary, we chose it plus for convenience. As far as b_2 and b_4 are concerned, we remark that the static theory gives the result that they are zero to lowest order in Q/m.

3. – Comparison with the static theory.

In order to determine the relative sign between J^A_α and J^V_α, as well as to check the rel·tive magnitude of the coefficients, we compute the quantity

$$J_{\alpha\beta} = \sum J_\alpha J^*_\beta$$

in the rest system of N^*. Summation over final polarization states, and averaging over initial spin states is indicated by $\sum$.

For J_α we write

$$J_\alpha = [\bar{N}^*_\nu\{\delta_{\nu\alpha}(\gamma^5 - \lambda) - i/\Delta Q_\nu\gamma^\alpha\gamma^5\}N]\,. \tag{4}$$

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From this we find, using $k^2 = -M^2$,

$$(5)\qquad J_{\alpha\beta} = (\tfrac{1}{3}p_0 k_0)\Big[\lambda^2\{\delta_{\alpha\beta} + (k_\alpha k_\beta/M^2)\}\{\Delta M - (kQ)\} +$$
$$+ (k_\alpha Q_\beta + Q_\alpha k_\beta)\left\{\frac{(Qk)^2 + Q^2M^2}{M^2\Delta^2} + \frac{m-M}{\Delta}\right\} - \frac{M}{\Delta}Q_\alpha Q_\beta +$$
$$+ \delta_{\alpha\beta}\left\{\frac{(M\Delta - (Qk))(Q^2M^2 + (Qk)^2)}{M^2\Delta^2} - \frac{Q^2M^2 + (Qk)^2}{M\Delta} - (Qk) - Mm + M^2\right\} +$$
$$+ k_\alpha k_\beta\left\{\frac{2(Qk)^2 + 2Q^2M^2}{M^2\Delta^2} + \frac{2(Qk)(m-M) - Q^2M^2}{M^2\Delta} - \frac{(Qk)}{M^2} - \frac{2m}{M} + 1\right\} -$$
$$- \lambda\varepsilon_{\alpha\beta\sigma\tau}k_\sigma Q_\tau\left\{\frac{(Qk) - M\Delta}{M\Delta} - 1\right\}\Big].$$

If both m and M are large with respect to Q, we have in the N^* rest system ($k_4 = iM$, $p_4 = i\sqrt{\boldsymbol{Q}^2 + m^2}$) to first order in $\boldsymbol{Q}^2$

$$Q_0 = M - m + \boldsymbol{Q}^2/2m\,.$$

and therefore

$$(Qk) = M(M-m) - M\boldsymbol{Q}^2/2m\,.$$

Moreover, we note that for $\beta = \alpha = 4$ only the terms proportional to Δ^{-2} survive, in accordance with $N_4^* = 0$ in the N^* rest system. Together, they give rise to a term of order $\boldsymbol{Q}^3/M^3$. Also, for $\alpha = 4$, $\beta \neq 4$ only the $k_\alpha Q_\beta$ and $Q_\alpha Q_\beta$ terms contribute, but are small due to the value of Q_0. Neglecting then terms of relative order Q/M, we have

$$(6)\qquad \begin{cases} J_{\alpha\beta} = (\tfrac{1}{3}p_0k_0)\Big[2mM\lambda^2\delta_{\alpha\beta} - \dfrac{M}{\Delta}Q_\alpha Q_\beta + \\ \qquad + \dfrac{M}{2m}\boldsymbol{Q}^2\delta_{\alpha\beta} + i\lambda\varepsilon_{\alpha\beta\sigma 4}Q_\sigma M\left\{\dfrac{2m}{\Delta} + 1\right\}\Big]\,, & \alpha, \beta \neq 4\,, \\ J_{\alpha\beta} = 0\,, & \alpha, \beta = 4: \end{cases}$$

For $m = M$, this formula is identical with the formula obtained from the static theory ([6]). This shows that we used the correct relative sign between J^A and J^V. For the ratio between axial and vector coupling constants, the static theory gives $\lambda/(1+\mu) = 1.15/4.71 = 0.24$, whereas we found earlier $\lambda/(1+\mu) = 2.2/(6\sqrt{3}) = 0.21$.

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4. – Density matrix and angular distributions (⁹).

We begin this Section with some general remarks about $\mathcal{N}^*$ production. Since there are four possible magnetic substates of a spin-$\frac{3}{2}$ particle corresponding to the values $m=\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ the density matrix for the produced particle will be a 4×4 matrix. Hermiticity of the density matrix allows the possibility of, in general, 16 independent elements. If there are no other strongly interacting particles in the final state, except the produced $\mathcal{N}^*$, then application of time-reversal invariance, coupled with the fact that first order in the weak coupling constant [10] is presumed sufficiently accurate, will reduce the number of independent components of the density matrix from 16 to 10. This may be seen as follows.

In the rest frame of $\mathcal{N}^*$, a useful orthogonal system describing the reaction consists of a polar axis along $\hat{N}$ (a unit vector along the normal to the production plane, *i.e.*, along $\boldsymbol{q}\times\boldsymbol{q}'$) the beam direction $\hat{q}$ (in the $\mathcal{N}^*$ rest system) and the orthogonal direction $\hat{L}=\hat{N}\times\hat{q}$.

The density matrix ϱ_{st} in the $\mathcal{N}^*$ rest frame can be expressed in the form

$$\varrho_{st}=\psi_i^{*(s)}T_{ij}\psi_j^{(t)}\,,\qquad i,j=1,2,3\,,\qquad s,t=\tfrac{3}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{3}{2}, \tag{7}$$

where s and t label the magnetic quantum numbers of the spin-$\frac{3}{2}$ particle referred to an axis of quantization which is taken here as $\hat{N}$. The indices i and j refer to the Cartesian co-ordinate system of q, L, N. The most general form for T_{ji} which satisfies hermiticity ($T_{ji}=T^*_{ij}$) and the combined conditions of time reversal and Born approximation (*i.e.*, $q\to -q$, $L\to -L$, $N\to N$, $\sigma\to-\sigma$ and complex conjugation) can be written as

$$\begin{aligned}T_{ij}=\;&A_1N_iN_j+A_2q_iq_j+A_3L_iL_j+A_4(q_iL_j+L_iq_j)+\\&+[A_5N_iN_j+A_6q_iq_j+A_7L_iL_j+A_8(q_iL_j+L_iq_j)](\sigma\cdot\hat{q})+\\&+[A_9N_iN_j+A_{10}q_iq_j+A_{11}L_iL_j+A_{12}(q_iL_j+L_iq_j)](\sigma\cdot\hat{L})+\\&+iA_{13}(q_iL_j-L_iq_j)(\sigma\cdot\hat{N})+iA_{14}(q_iN_j-N_iq_j)+\\&+iA_{15}(q_iN_j-N_iq_j)(\sigma\cdot\hat{q})+iA_{16}(q_iN_j-N_iq_j)(\sigma\cdot\hat{L})+\\&+iA_{17}(L_iL_j-L_jN_i)+iA_{18}(L_iN_j-N_iL_j)(\sigma\cdot\hat{q})+iA_{19}(L_iN_j-N_iL_j)(\sigma\cdot\hat{L}).\end{aligned} \tag{8}$$

(⁹) Many of the results given in this Section have been developed earlier by Prof. M. M. BLOCK. We wish to thank Prof. BLOCK for communicating his results prior to publication and further wish to thank him for his interest and advice.

(¹⁰) We may apply time reversal to the complete final system ($\mu\pi\mathcal{N}$) since the final ($\pi\mathcal{N}$) system is in a definite state of angular momentum and parity, *i.e.*, $J^P=\frac{3}{2}^+$.

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The coefficients $A_1, \ldots, A_{19}$, are all real as required by Hermiticity, and are functions of the neutrino energy and muon angle. We note that there can be no terms proportional to σ_i since the condition $\sigma_i \psi_i = 0$ is required to guarantee the absence of spin $\frac{1}{2}$ components in ψ_i.

Although T_{ij} contains 19 arbitrary coefficients, the reduction from Cartesian form (which contains spin-$\frac{1}{2}$ projections) to the form given by eq. (7) will give only ten independent components to ϱ_{st}. There are six conditions which result from the form of T_{ij} given in eq. (8), and which yield the relations

$$(9a) \qquad \varrho_{\frac{3}{2},\frac{3}{2}} = \varrho_{-\frac{3}{2},-\frac{3}{2}} ,$$

$$(9b) \qquad \varrho_{\frac{1}{2},\frac{1}{2}} = \varrho_{-\frac{1}{2},-\frac{1}{2}} ,$$

$$(9c) \qquad \varrho_{\frac{3}{2},\frac{1}{2}} = -\varrho_{-\frac{1}{2},-\frac{3}{2}} ,$$

$$(9d) \qquad \varrho_{\frac{3}{2},-\frac{1}{2}} = -\varrho_{\frac{1}{2},-\frac{3}{2}} .$$

(Note that eqs. (9c) and (9d) represent two conditions each, since the off-diagonal elements ϱ_{st} can be complex.)

Equations (9a) and (9b) imply that the produced $\mathcal{N}^*$ is unpolarized, *i.e.*,

$$\sum_m m\,\varrho_{m,m} = 0 .$$

However, since $\varrho_{\frac{3}{2},\frac{3}{2}}$ is not necessarily equal to $\varrho_{\frac{1}{2},\frac{1}{2}}$, there is the possibility of an alignment. Symmetry about the perpendicular to the diagonal of ϱ follows from eqs. (9c) and (9d).

Since parity is conserved in the $\mathcal{N}^*$ decay, it is readily seen that the angular distribution $D(\theta, \varphi)$ of the decay proton (or pion) in the $\mathcal{N}^*$ rest frame contains at the most, four arbitrary constants, and is of the form

$$(10) \quad D(\theta, \varphi) = B_1 \cos^2\theta + B_2 \sin^2\theta \cos^2\varphi + B_3 \sin^2\theta \sin^2\varphi + B_4 \sin^2\theta \sin\varphi \cos\varphi ,$$

where θ is the angle between the direction of one of the decay particles and $\hat{N}$. The azimuthal angle φ is measured with respect to the plane of $\hat{q}$ and $\hat{N}$. Parity-violating terms, as well as parity-conserving terms, contribute to the decay angular distribution, but the term B_4 is purely a vector-axial vector interference term. Any polarization of the decay proton is due to parity violation in the production process, *i.e.*, if parity were conserved, in addition to time reversal and Born approximation, the decay proton would be unpolarized.

If time reversal and Born approximation hold, then there are six additional independent correlations involving the decay nucleon polarization $\boldsymbol{P}$

which are of the form

$$\text{i)}\quad (\boldsymbol{P}\cdot\hat{\mathcal{N}})\cos\theta\sin\theta\cos\varphi\,,\qquad \text{ii)}\quad (\boldsymbol{P}\cdot\hat{\mathcal{N}})\cos\theta\sin\theta\sin\varphi\,,$$

$$\text{iii)}\quad (\boldsymbol{P}\cdot\hat{q})\sin^2\theta\cos\varphi\sin\varphi\,,\qquad \text{iv)}\quad (\boldsymbol{P}\cdot\hat{L})\sin^2\theta\cos\varphi\sin\varphi\,,$$

$$\text{v)}\quad (\boldsymbol{P}\cdot\hat{q})\sin^2\theta\cos^2\varphi\,,\qquad \text{vi)}\quad (\boldsymbol{P}\cdot\hat{q})\sin^2\theta\sin^2\varphi\,,$$

which should be nonvanishing. In addition there are the six correlations

$$\sin\theta\cos\theta\sin\varphi\,,\quad \sin\theta\cos\theta\cos\varphi\,,\quad (\boldsymbol{P}\cdot\hat{\mathcal{N}})\,,\quad (\boldsymbol{P}\cdot\hat{\mathcal{N}})\sin^2\theta\sin^2\varphi\,,$$

$$(\boldsymbol{P}\cdot\hat{\mathcal{N}})\sin^2\theta\cos^2\varphi\,,\qquad (\boldsymbol{P}\cdot\hat{\mathcal{N}})\sin^2\theta\cos\varphi\sin\varphi\,,$$

which whould vanish if time reversal and Born approximation are valid. The strength of the ten nonvanishing correlations are then linearly related to the ten components of the $\mathcal{N}^*$ density matrix.

The complete form of the pion angular distribution for arbitrary energy and momentum transfer is somewhat lengthy, and is given below. In order to take advantage of a simplification which results when the angle between leptons is small we introduce a polar co-ordinate system in the $\mathcal{N}^*$ rest system, which is slightly rotated from the co-ordinate system used above in discussing the density matrix. In this case the polar angle θ' is measured with respect to an axis along the line which bisects the two lepton momenta and the azimuthal angle φ' is measured with respect to the plane containing the two lepton

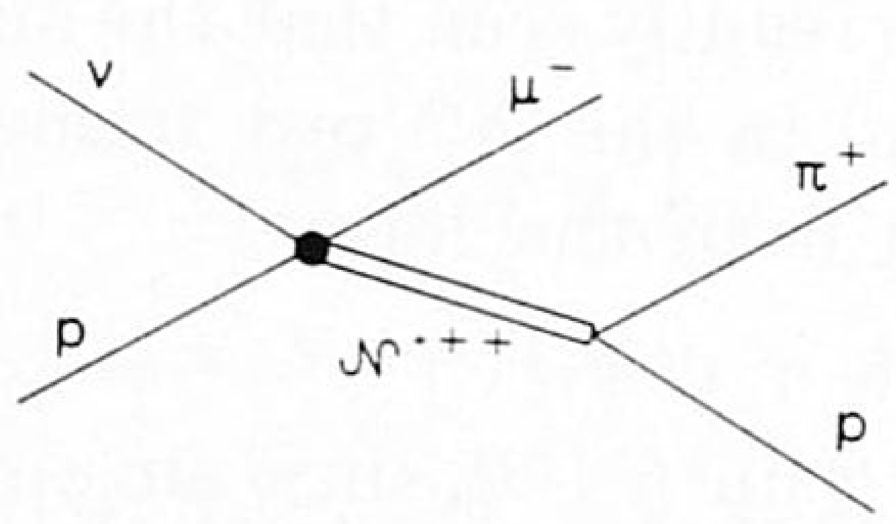

Fig. 2. – Diagram showing the inelastic process $\nu+p\to p+\pi^++\pi^-$ via the $\mathcal{N}^*$.

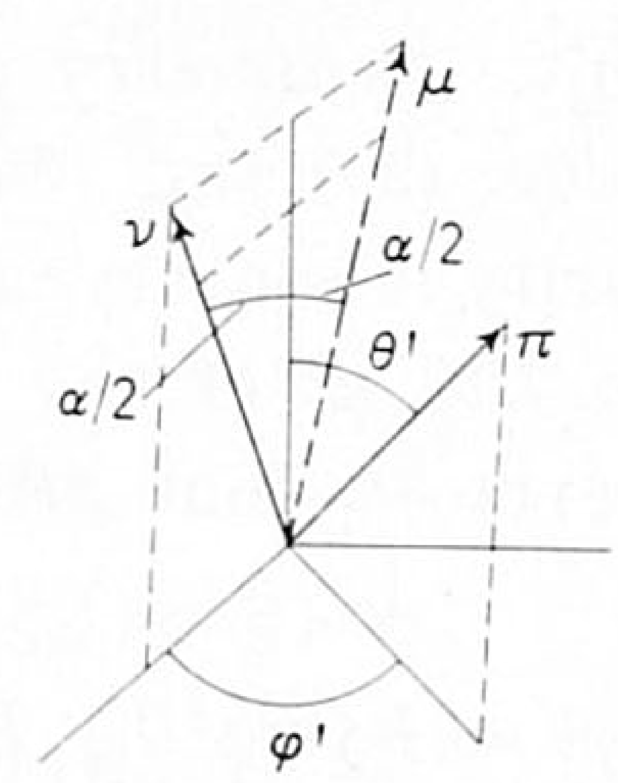

Fig. 3. – Co-ordinate system used in describing pion angular distribution.

momenta (Fig. 3). For small angles α, the angle between neutrino and muon, the decay distribution takes the simple form

$$D(\theta',\varphi') = 1 + 3\cos^2\theta' + O(\sin^2\alpha/2) + O(\delta E\sin\alpha/2) + O(\delta m\sin\alpha/2)\,, \tag{11}$$

where $\delta E = E - E'$ and $\delta m = M - m$. E is the neutrino energy, E' the muon energy, both in the $\mathcal{N}^*$ rest system.

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For an interaction of the form

$$\bar{N}^*_\mu[A_1\delta_{\mu\nu}\gamma^5 + iA_2 Q_\mu\gamma^\nu\gamma^5 + B_1\delta_{\mu\nu}]Nj_\nu\,, \tag{12}$$

the general form of the angular distribution can be expressed as follows (note that eq. (12) is of the same form as eq. (4) with $A_1 = 1$, $B_1 = -\lambda$, $A_2 = -1/\Delta$):

$$\begin{aligned}
f_0 = {} & A_1^2 a + B_1^2 b + 2A_2^2 M\,\delta E^2 + 2A_1A_2\,\delta m\,\delta E + \\
& + \sin^2\frac{\alpha}{2}\{A_1^2 a + B_1^2 b + 4A_1B_1D + 6A_2^2 MD^2 + 8A_2^2 MEE' + \\
& + 2A_2^2 m\,\delta E^2 - 2A_2^2\,\delta E^3 + 12A_1A_2EE' - 2A_1A_2\,\delta m\,\delta E + \\
& + 2A_1A_2(E^2 + E'^2) + 6A_2B_1(m + M)D + 2A_2B_1bD\} + \\
& + \sin^4\frac{\alpha}{2}\{8A_2^2 mEE' - 14A_2^2EE'\,\delta E + 6A_2^2 mD^2 - 6A_2^2(E^3 - E'^3) + \\
& + 6A_1A_2\,\delta E^2 + 16A_1A_2EE' - 6A_2B_1D\,\delta E\} + \\
& + \cos^2\theta'\Big[3A_1^2 a + 3B_1^2 b + 6A_2^2 M\,\delta E^2 + 6A_1A_2\,\delta m\,\delta E + \\
& + \sin^2\frac{\alpha}{2}\{-12A_1B_1D - 6A_2^2 MD^2 - 6A_2^2\,\delta m\,\delta E^2 - 6A_2^2\,\delta E^3 - 24A_1A_2EE' - \\
& - 6A_1A_2\,\delta m\,\delta E + 6A_1A_2D^2 - 6A_2B_1D(m + M + \delta E)\} + \\
& + \sin^4\frac{\alpha}{2}\{-6A_2^2 m\,\delta E^2 - 6A_2^2 mD^2 + 6A_2^2\,\delta E^3 + 6A_2^2EE'\,\delta E + \\
& + 6A_2^2(E^3 - E'^3) - 6A_1A_2(D^2 + \delta E^2) + 12A_2B_1D\,\delta E\}\Big] + \\
& + \cos\varphi'\sin\theta'\cos\theta'\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}\Big[12A_1B_1\,\delta E + \\
& + 12A_2^2 MD\,\delta E + 6A_1A_2D\,\delta m + 6A_2B_1(M + m)\,\delta E + \\
& + \sin^2\frac{\alpha}{2}\{12A_2^2 D\,\delta E(m - \delta E) + 12A_1A_2D\,\delta E - 12A_2B_1(E^2 + E'^2)\}\Big] - \\
& - \sin^2\varphi'\sin^2\theta'\sin^2\frac{\alpha}{2}\Big[12A_1B_1D - 3A_1^2 a - 3B_1^2 b + 6A_2^2 MD^2 + \\
& + 24A_1A_2EE' + 6A_2B_1(M + m)D + \\
& + \sin^2\frac{\alpha}{2}\{6A_2^2 mD^2 - 6A_2^2EE'\,\delta E - 6A_2^2(E^3 - E'^3) + 6A_1A_2\,\delta E^2 - 6A_2B_1D\,\delta E\}\Big]\,,
\end{aligned} \tag{13}$$

where $a = M - m - (E - E')$, $b = M + m - (E - E')$, $D = E + E'$.

For completeness, the total cross-section in the laboratory system takes the form (for definition of g, Γ and M_0 see following Section)

$$\sigma_{tot}=\frac{g^2}{2^6(2\pi)^5m_\pi^2 m q_0}\int\frac{d^3q'}{q_0'}\frac{d^3l}{l_0}\frac{d^3P}{P_0}\frac{\delta_4(l+P+q'-q-p)}{(k^2+M_0^2)^2-\Gamma^2k^2}F_0\,,$$

where $k=P+l=q+p-q'$, and P and l are final proton and pion momenta. The Lorentz-invariant integral is considered in the $\mathcal{N}^*$ rest frame given above, and F_0 is then related to f_0 by

$$F_0=\frac{128M_0^2l^2EE'(m+p_0)}{3}f_0\,,$$

where, in computing F_0, the relations $k^2=-M_0^2$ and $\Gamma=0$ are used.

5. – Conclusions.

On the basis of the above, we now try to calculate total cross-sections. The expression to be evaluated corresponds to the diagram of Fig. 2, where the $\mathcal{N}^*$ propagator $D_{\mu\nu}(M)$ is given by

$$D_{\mu\nu}(M)=\left(\delta_{\mu\nu}-\tfrac{1}{3}\gamma^\mu\gamma^\nu-\frac{i}{3M}(\gamma^\mu k_\nu-k_\mu\gamma^\nu)+\frac{2}{3M^2}k_\mu k_\nu\right)(-i\gamma k+M)(k^2+M^2)^{-1}$$

with

$$M=M_0-(i/2)\Gamma(\sqrt{-k^2})\sqrt{-k^2}/M_0\,,\qquad M_0=1238\text{ MeV},$$

and

$$\Gamma(x)=\frac{g^2}{6\pi m_\pi^2}[(x+m)^2-m_\pi^2][(x^2-m_\pi^2-m^2)^2-4m^2m_\pi^2]^{\frac{3}{2}}(2x)^{-5}\,,$$

$g=$ renormalized $\pi\mathcal{N}\mathcal{N}^*$ coupling constant $=2.32$.

Obviously, due to the form of the denominator in the $\mathcal{N}^*$ propagator, we get contributions to the total cross-section from a region

$$M_0-\Gamma<\sqrt{-k^2}<M_0+\Gamma\,.$$

If $\Gamma\ll M_0$, we may simplify the expression belonging to the diagran of Fig. 2 by working to lowest order in Γ/M_0 everywhere except in the denominator of the $\mathcal{N}^*$ propagator. In this case one finds in the laboratory system ($\boldsymbol{P}=0$)

$$\sigma_{tot}=\frac{g^2}{32\pi^2q_0m}\int_{m_\mu}^{(E_\nu-m_\pi)}|\boldsymbol{q}'|\,dq_0'\int_{-1}^{1}\frac{dy\,\Gamma(k^2)M_0}{(k^2+M^2)^2-\Gamma^2k^2}f_{\mu\nu}H_{\mu\nu}\,,\tag{14}$$

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where y is the cosine of the angle between ν and μ directions, and

$$(15)\quad \begin{cases} f_{\mu\nu} = \mathrm{Tr}\,\{(\gamma q')\gamma^{\mu}(1+\gamma^5)(\gamma q)\gamma^{\nu}(1+\gamma^5)\}\,, \\ H_{\mu\nu} = \mathrm{Tr}\,\{(k^2+M_0^2)D_{\alpha\beta}(M)F_{\beta\mu}(-i(\gamma p)+m)\overline{F}_{\alpha\nu}\}\,, \end{cases}$$

where

$$F_{\mu\nu} = \{\delta_{\mu\nu} - (1/(m+M_0))\,Q_{\mu}\gamma^{\nu}\}\gamma^5 F_V(Q^2) - \lambda F_A(Q^2)\delta_{\mu\nu}\,,$$

$$\overline{F}_{\mu\nu} = \gamma^4 F^*_{\mu\nu}\gamma^4\,, \qquad g = (6.12/\sqrt{2})\cdot 10^{-5}m^{-2}\,, \qquad \lambda = 2.2/(6\sqrt{3})\,.$$

The above expression reduces to the expression obtained for a stable $\mathcal{N}^*$ if one takes the limit $\Gamma \to 0$, using the formula

$$\lim_{a\to 0}\int_0^{\infty} \mathrm{d}k_0 \frac{a}{(k_0^2 - b^2)^2 + a^2} = \pi/2b\,.$$

Using the above, together with the form factors [11]

$$F_A = F_V = \frac{1}{1+Q^2/M_x^2}\,, \qquad M_x = 570\ \mathrm{MeV}\ \text{and}\ M_x = 420\ \mathrm{MeV},$$

we find production cross-sections as given in Table I. The numbers between brackets refer to $M_x = 420$ MeV.

TABLE I.

GeV	σ_{tot} in 10^{-38} cm²	
E_ν	$\nu + n \to \mu^- + \mathcal{N}^{*+}$	$\bar{\nu} + p \to \mu^+ + \mathcal{N}^{*0}$
1	0.255 (0.13)	0.05 (0.03)
1.25	0.33 (0.16)	0.082 (0.05)
1.5	0.39 (0.18)	0.117 (0.066)
2	0.47 (0.21)	0.19 (0.1)
2.5	0.54 (0.23)	0.26 (0.14)
3	0.60 (0.25)	0.32 (0.15)
4	0.70 (0.28)	0.44 (0.19)
5	0.79 (0.31)	0.54 (0.22)
10	1.17 (0.42)	0.97 (0.37)

(11) We use equal vector and axial form factors, and take the vector form factor as given by C. DE VRIES, R. HOFSTADTER and H. JOHANSSON: *Proceedings of Stanford University Conference on Nucleon Structure* (1963).

Cross-sections for $\nu+p\to\mu^-+\mathcal{N}^{*++}$ and $\bar{\nu}+n\to\mu^++\mathcal{N}^{*-}$ are obtained by multiplying the listed cross-sections for ν and $\bar{\nu}$, respectively, by 3.

For computation of the $\bar{\nu}$ cross-sections, we make the assumption that the nucleon currents multiplying the lepton current for neutrino, respectively antineutrino, are in the same isospin multiplet ([12]). It can be shown that this gives the same result as the usual assumption that the baryon current transforms as a current of the first kind ([13]) under G-parity.

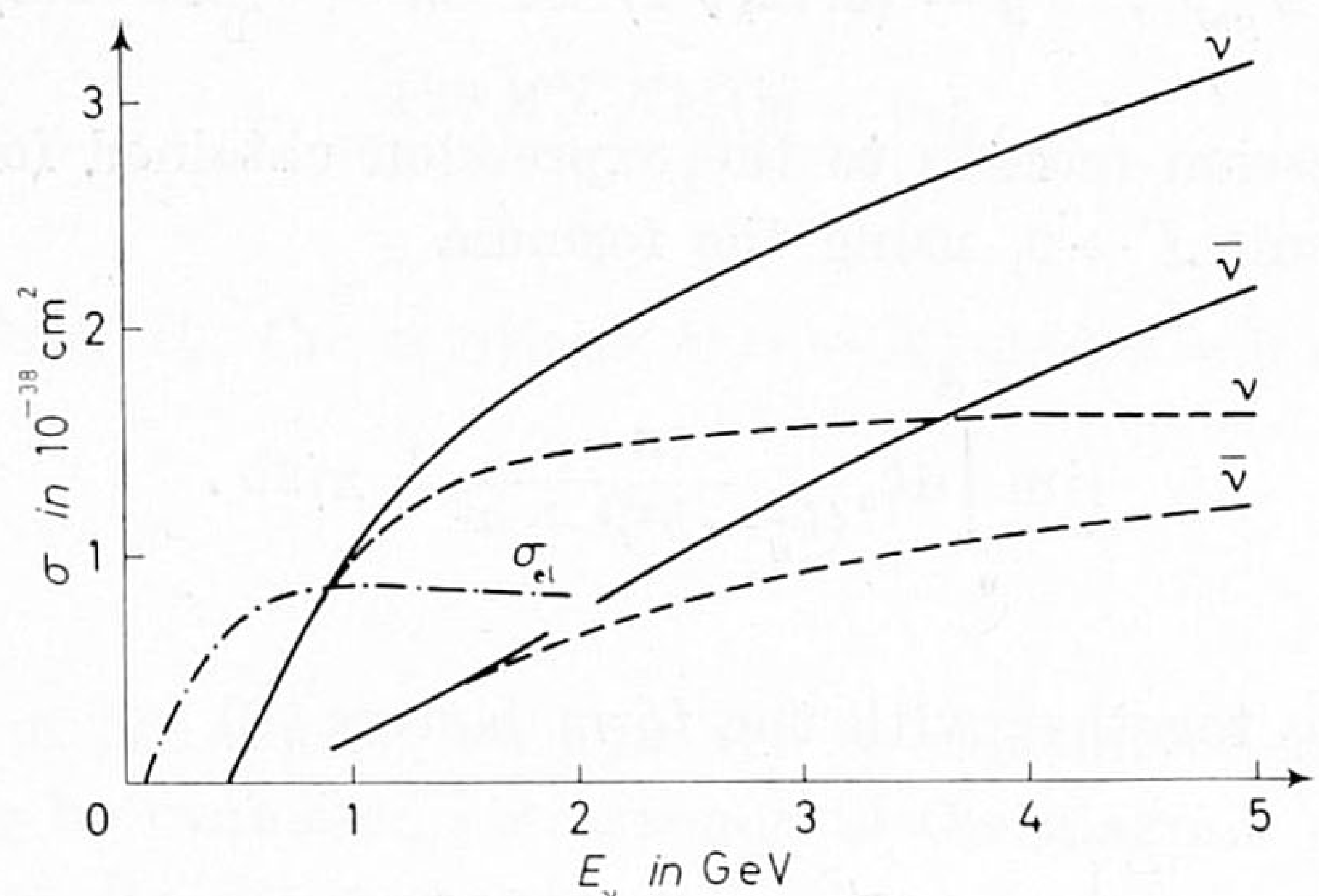

Fig. 4. – Total cross-sections for $\mathcal{N}^*$ production by neutrinos and antineutrinos impinging on a system of one proton and one neutron, as a function of laboratory system energy. The form of the interaction is taken to be

$$g\left[\bar{N}^*_\alpha\left\{(6\sqrt{3}\,\gamma^5-2.2)\,\delta_{\alpha\beta}-\frac{6i\sqrt{3}}{\Delta}\,Q_\alpha\gamma^\beta\gamma^5\right\}N\right][\bar{\mu}(1+\gamma^5)\nu]\,.$$

Cross-sections are given for two choices of form factors as indicated, with Q^2 in units of $(\text{MeV})^2$:

$$\text{——}\quad F_A=F_V=\frac{1}{1+Q^2/(570)^2}\,;\qquad ---\quad F_A=F_V=\frac{1}{(1+Q^2/(900)^2)^2}\,;$$

$$-\cdot-\cdot-\quad F_A=F_V=\frac{1}{(1+Q^2/(840)^2)^2}\,.$$

The graph of σ_{el} refers to the elastic reaction $\nu+n\to p+\mu$ as calculated by YAMAGUCHI ([14]).

In Fig. 4 we have plotted the $\mathcal{N}^*$ production cross-section as a function of neutrino energy, with two choices of form factors. As is seen from Fig. 4, the cross-section differs by a factor of two at $E_\nu=5$ GeV for the two choices

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([12]) T. D. LEE and C. N. YANG: *Phys. Rev.*, **119**, 1410 (1960).
([13]) S. WEINBERG: *Phys. Rev.*, **112**, 1375 (1958).
([14]) Y. YAMAGUCHI: CERN report 61-2 (1961).

of form factors indicating that large values of Q^2 are contributing to the cross-section. For low energies the cross-sections are essentially equal for either choice of form factor since these form factors have approximately the same radii.

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RIASSUNTO (*)

Si studia, nel quadro del formalismo di Rarita-Schwinger per le particelle aventi spin $\frac{3}{2}$, la produzione della risonanza $\frac{3}{2}$, $\frac{3}{2}$ da parte di neutrini incidenti su nucleoni. Con l'aiuto della teoria del vettore conservato e della relazione di Goldberger-Treiman, si stima il valore di alcuni coefficienti. Dove mancano le informazioni si usa la teoria statica. Si usa poi l'interazione determinata in tal modo per ottenere la sezione d'urto totale. Risulta che la sezione d'urto totale su nuclei complessi in funzione dell'energia del neutrino si comporta in modo simile alla sezione d'urto del processo elastico, ma in valore assoluto ne è maggiore. Si deducono le proprietà della matrice di densità dello spin $\frac{3}{2}$ nell'ipotesi combinate della invarianza all'inversione del tempo e della approssimazione di Born e si dà l'espressione generale della distribuzione angolare del pione di decadimento nel sistema in quiete del $\mathcal{N}^*$.

(*) *Traduzione a cura della Redazione.*

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