# TOWARDS A THEORY FOR THE QUANTUM MECHANICS OF GRAVITATIONAL COLLAPSE

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## ABSTRACT

The difficulty in reconciling the principles of general relativity with those of quantum mechanics shows particularly clearly when black holes are compared with elementary particles. In a viable theory these notions should be placed on an equal footing. In this lecture we show that indeed the mathematical structure of modern string theories for elementary particles suggests an interpretation in terms of dynamical properties of the black hole horizon.

## 1. INTRODUCTION

In its standard form, the theory of general relativity contains equations of motion that correspond to an extremum principle. There is a lagrangean,

$$\mathcal{L} = \sqrt{-g}\, R \ , \tag{1.1}$$

whose integral can be required to be stationary under infinitesimal variations:

$$\delta \int \mathcal{L}\, d^4x = 0 \ . \tag{1.2}$$

As is well-known, such a system can, in principle, be "quantized" by replacing Poisson brackets by commutators. At first sight, the resulting "quantum theory of gravity" seems to be quite reasonable: the gravitational force is transmitted by a graviton, a particle with spin 2, which, due to the correct sign of the gravitational constant G, has positive energy and production probabilities.

Nevertheless, there are problems. The statement that quantum gravity is not renormalizable is equivalent to saying that at very short distances the interactions run out of control. When particle physicists talk about "renormalizable theories of gravity", what they really mean is that in a perturbative expansion with respect to G the individual terms do not get infinite contributions from the small-distance limit; but stil, at distance scales

$$\ell = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}} = 1.6.10^{-33} \text{ cm} , \tag{1.3}$$

or smaller, the interactions are so strong that their cumulative effects defy any description. Certainly, the perturbation expansion itself starts to diverge badly.

A key ingredient in quantum field theory, as much as in general relativity, is locality: of any pair of points in space-time, regardless how close together, one can tell which is earlier and which is later. Precisely this locality must be given up in a true theory of quantum gravity: two points can be separated only by using particles at an energy scale inversely proportional to their distance:

$$mc^2 = E \gtrsim \hbar c/\ell , \tag{1.4}$$

but if we try to describe the gravitational field of such particles we get a Schwarzschild horizon at a distance

$$\ell = 2Gm/c^2 = (2G\hbar/c^3\ell) \tag{1.5}$$

so that indeed the distance (1.3) (the Planck scale) is never surpassed.

Paradoxically, the most impressive results in understanding physics at the Planck length scale were obtained by ignoring the problem. String theory originated in a different branch of particle physics and its proponents claim that it should produce the resolution of our problems of locality, among all others. In the next section we give a very brief outline of string theory. Much more extensive reviews can be found elsewhere [1] . What we really need is the "classical" string equation (2.21).

Next we turn to an apparently quite different subject: black holes. It is also quite well-known that black holes are expected to emit radiation with a thermal spectrum. In sect. 3 we argue that this radiation is actually due to a very delicate boundary condition in the Schwarzschild coordinate system and is perhaps not as well understood as some authors suggest . In any case it is not so unreasonable to suspect that the tiniest black holes merge into the spectrum of elementary particles, which are also unstable, but do not behave thermally.

If one includes gravitational self-interaction between in- and outgoing particles a quite different picture emerges and in the last sections we show that black holes indeed look very much like strings... .

## 2. STRING THEORY IN A NUT SHELL

Only a few notions from string theory will be needed in the sequel. We here summarize some of its basic features; more extensive and "complete" accounts can be found in the enthusiastic literature [1,2] .

If in a scattering experiment a virtual particle is exchanged, its effect can be seen in the form of a pole in the amplitude. Consider for instance the collision of two particles, 1 and 2, yielding after the collision particles 3 and 4. Let $p^1$, $p^2$, $p^3$,$p^4$ be their 4-momenta. Define

$$k_\mu = p^1_\mu + p^2_\mu = p^3_\mu + p^4_\mu \ ;$$

$$q_\mu = p^3_\mu - p^1_\mu = p^2_\mu - p^4_\mu \ ; \tag{2.1}$$

$$s = - k^2_\mu \ ; \ t = - q^2_\mu \ . \tag{2.2}$$

Here, $s$ is the (energy)$^2$ in the rest frame; $q_\mu$ is the exchanged momentum. $t$ can be seen to be directly related to the scattering angle $\theta$ in the center of mass frame:

$$t = A(m^i,s) + B(m^i,s)\cos\theta \ . \tag{2.3}$$

A Lorentz-invariant amplitude could look like

$$G(s,t) = \sum_i \frac{\lambda^s_i}{k^2+m^2_i-i\varepsilon} + \sum_j \frac{\lambda^t_j}{q^2+M^2_j-i\varepsilon} \ . \tag{2.4}$$

So there are poles of the form

$$\frac{1}{m^2_i-s} \quad \text{and} \quad \frac{1}{M^2_j-t} \ . \tag{2.5}$$

Here, $m_i$ and $M_j$ are masses, and we note that the poles are located at those values of the external momenta for which the particles could be actually produced.

Notice the symmetry $s \leftrightarrow t$ (crossing symmetry). We ignore for simplicity the possibility of producing particles in the u-channel:

$$- u = (p^4_\mu - p^1_\mu)^2 \ , \tag{2.6}$$

which could be suppressed because of unfavorable quantum numbers in that channel.

If the residues of the poles (2.5) are independent of the other parameter $t$ or $s$, respectively, then the exchanged particles are scalars. Otherwise we get

$$\frac{\mathrm{Pol}_i(t)}{m^2_i-s} \quad \text{and} \quad \frac{\mathrm{Pol}_j(s)}{M^2_j-t} \tag{2.7}$$

where the degrees of the polynomials correspond to the spins of the exchanged particles.

A model amplitude having an infinite series of poles was suggested by G. Veneziano :

$$G(s,t) = \frac{\Gamma(-s)\Gamma(-t)}{\Gamma(-s-t)} = B(-s,-t) \tag{2.8}$$

where $\Gamma(x)$ is Euler's gamma function. One easily checks that the pole structure is as in (2.7), where

$$m^2_i = M^2_i = n = 0,1,2,\dots \ , \tag{2.9}$$

and the degrees of the polynomials, hence the spins, increase with increasing n. In fact, because the polynomials are not pure Legendre polynomials we have superpositions of various spins. The model found its first applications in strong interaction theory where indeed bound states have a mass spectrum resembling (2.9).

The physical interpretation of Veneziano's formula (2.8) is quite interesting. Let us assume that the external particles are massless:

$$(p^i)^2 = 0 \qquad \text{for each } i \ . \tag{2.10}$$

Then (2.8) can be rewritten as

$$\begin{aligned} B(-s,-t) &= \int_0^1 x^{-s-1}(1-x)^{-t-1}dx \\ &= \int_0^1 \frac{dx}{x(1-x)} \exp[2(p^1.p^2)\log x - 2(p^1.p^3)\log(1-x)] \ . \end{aligned} \tag{2.11}$$

The logarithms can be regarded as Green functions in a two-dimensional space, which can either be taken to be the entire complex plane, or a compact, simply connected subspace, such as the unit circle. Take the latter case and transform it conformally into the upper half plane (Fig. 1).

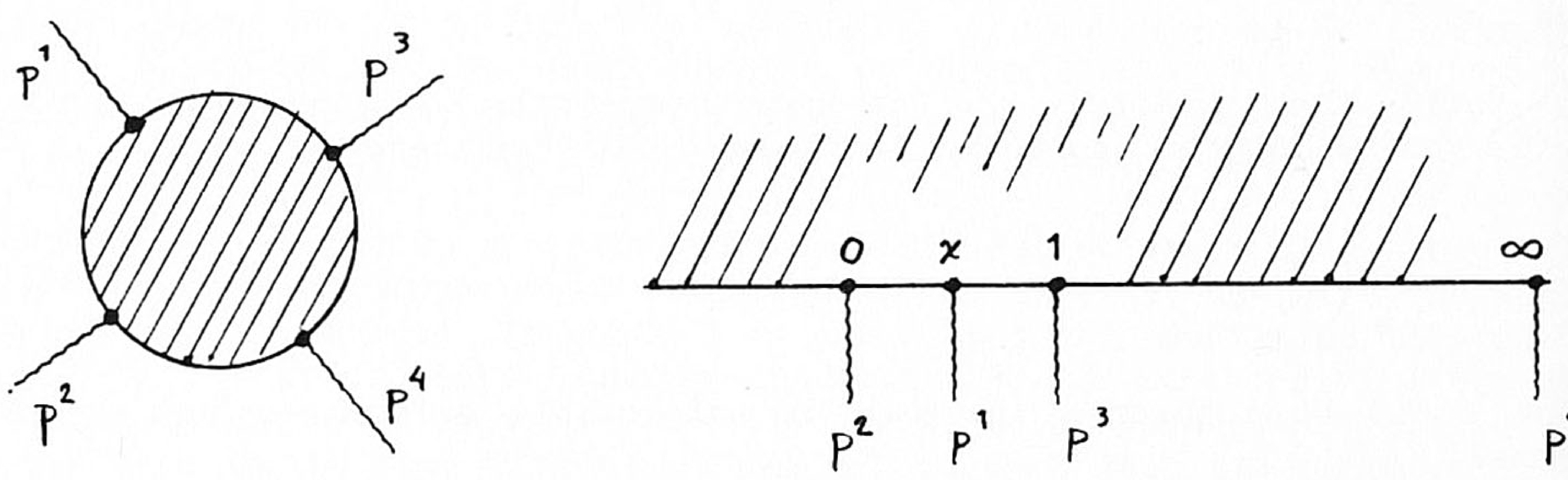


Fig. 1. Electronic simulation with conducting sheet, reproducing the exponent in eq. (2.11).

The exponent in (2.11) can be seen as the dissipated power in an electronic circuit where four different kinds of currents $p_\mu$ ($\mu = 1,\ldots,4$) are sent into or out from a uniformly conducting sheet, at the points, 0, x, 1 and $\infty$. The voltage due to the currents 2 and 3 will be minus the real part of

$$f_\mu(z) = 2p_\mu^2 \log z - 2p_\mu^2 \log(1-z) \ , \tag{2.12}$$

which satisfies

$$\partial_z^2 f_\mu(z) = 2\pi\rho_\mu(z) \ ;$$

$$\rho_\mu(z) = p_\mu^2\,\delta(z) - p_\mu^3\,\delta(z-1) \ . \tag{2.13}$$

(If the source were not at, but inside the border, the $2\pi$ would have to be replaced by $4\pi$.)
We now notice that the amplitude (2.8) can be rewritten as

$$G(s,t) = \int_0^1 \frac{dx}{x(1-x)} \int D\eta_\mu(z) \exp \int d^2z \left(\frac{1}{4\pi} (\vec{\partial}_z \eta_\mu(z))^2 + \rho_\mu(z)\eta_\mu(z)\right) . \tag{2.14}$$

The integral over $\eta_\mu$ is a functional integral over its imaginary values. We will not go into the problem of defining the measure of this functional integral (requiring this measure to be truly invariant under conformal transformations leads us into the complexities of superstring theory). Notice that, using the restriction (2.10), we can replace (2.13) by

$$\rho_\mu(z) = \sum_i \pm \rho_\mu^i \, \delta(z-x^i) , \tag{2.15}$$

with $x^i = (x,0,1,\infty)$, and the sign depending on whether the particle was in- or outgoing.

Identifying $e^{\int \rho_\mu \eta_\mu(z) dz}$ in (2.14) with a particle wave function $e^{ipx}$, we see that $\eta_\mu$ must be considered to be i times a coordinate $x_\mu$. Apparently, we have a functional integral over all configurations

$$- i\eta^\mu(\sigma+i\tau) = x^\mu(\sigma,\tau) . \tag{2.16}$$

The corresponding classical equation is

$$\vec{\partial}_z^2 x^\mu = - 2\pi i \, \rho_\mu(\sigma,\tau) . \tag{2.17}$$

The quantity (2.16) describes a string-like object moving around in 4-space. The reason why Veneziano's model works well in theories such as QCD is that indeed in QCD one has quarks held together by field configurations in the form of vortices, and we may view the strings as idealizations of the vortices.

Eq. (2.14) can be seen to follow from a string action:

$$S = \int d\sigma \, d\tau \sqrt{\left(\frac{\partial x^\mu}{\partial \sigma} \cdot \frac{\partial x^\mu}{\partial \tau}\right)^2 - \left(\frac{\partial x^\mu}{\partial \sigma}\right)^2 \left(\frac{\partial x^\mu}{\partial \tau}\right)^2} , \tag{2.18}$$

if the parametrization z is chosen sufficiently carefully. The energy of a string is

$$\frac{1}{2\pi} \int \frac{d\sigma}{\sqrt{1-v_\perp^2}} , \tag{2.19}$$

where $\sigma$ is a coordinate along the string and $v_\perp$ the velocity perpendicular to the string. In QCD the mass unit for the spectrum (2.9) is roughly 1 GeV. The string tension T, as an energy per unit of length, then turns out to be

$$T \approx \frac{1}{2\pi} (\mathrm{GeV})^2 = 14 \text{ tons} . \tag{2.20}$$

Putting the dimensions right, (2.17) becomes

$$\partial^2 x^\mu = - i\rho_\mu / T . \tag{2.21}$$

In unified superstring theories one takes T to be enormous:

$$T = \mathcal{O}(M_{PL}^2) \ . \tag{2.22}$$

Often, the string will be without end points (closed string theory). In this case, (2.21) still applies, but $\rho_\mu$ then becomes a distribution defined in the interior of the complex z plane.
A consequence of (2.22) is that the Schwarzschild radius R of a superstring must be

$$R \geqslant 2GTL = \mathcal{O}(L) \tag{2.23}$$

where G is the gravitational constant and L the total length of a string. Apparently, in superstring theories, gravitational collapse should not be ignored! Paradoxically, strings are usually treated as if they are immune to gravitational collapse. Could this be because superstrings "are" black holes?

## 3. THE BLACK HOLE

We write

$$M = Gm \ , \tag{3.1}$$

and the black hole metric as

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2 \ . \tag{3.2}$$

The Kruskal coordinates x and y, defined by

$$xy = \left(1 - \frac{r}{2M}\right)e^{r/2M} \ ,$$
$$x/y = -\ e^{t/2M} \tag{3.3}$$

can replace r and t, after which we can extend analytically [5] to the entire space $(x,y \mid xy < 1)$.
That this metric may arise naturally when a large amount of matter is accumulated is being discussed extensively at this School. The physical interpretation of the various regions in x-y space has also been discussed at various places [5,6].

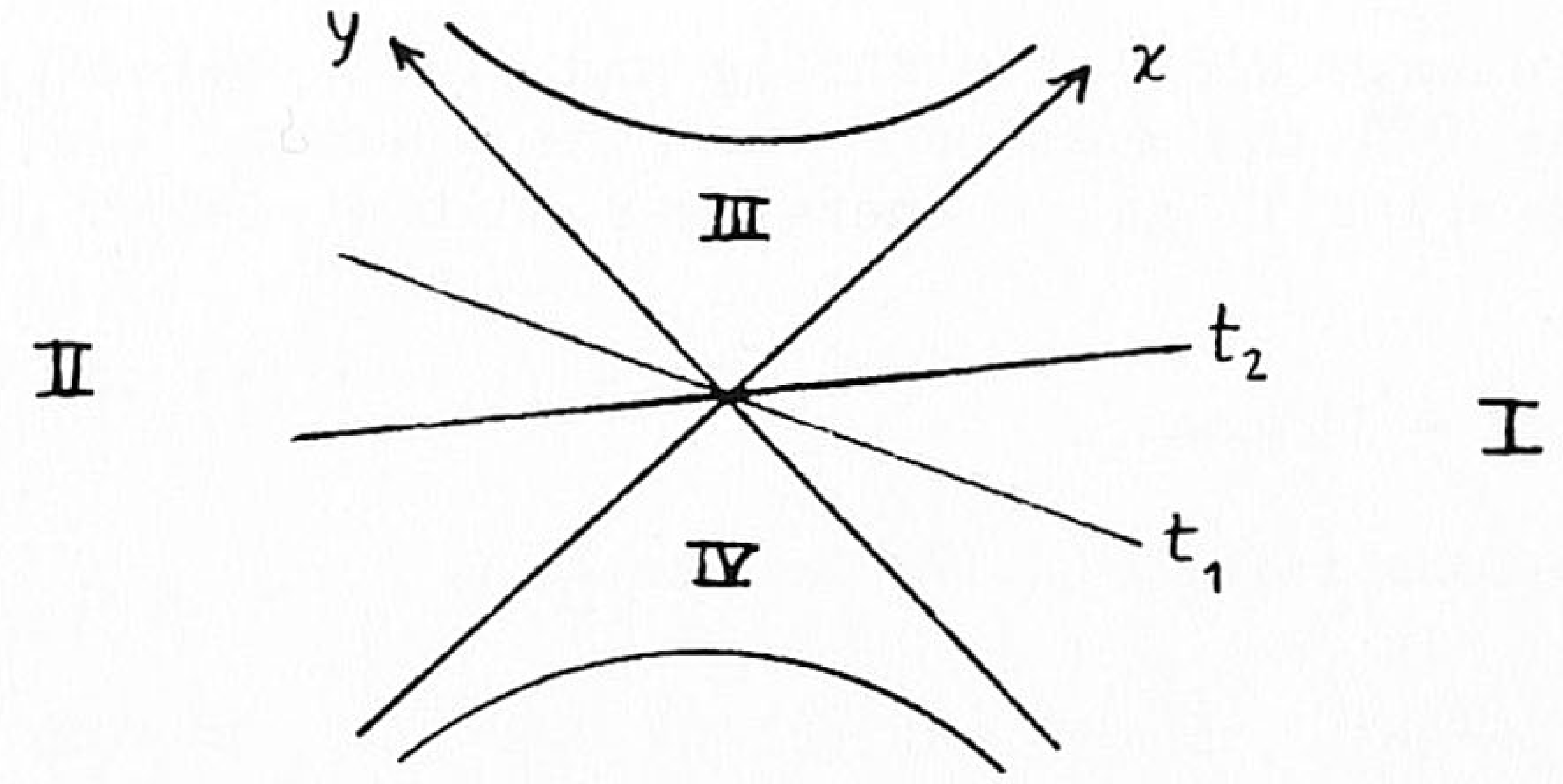


Fig. 2. Black hole in Kruskal coordinates.

Of particular interest is the region close to the origin. Writing

$$\zeta^2 = \left(\frac{r}{2M} - 1\right)e^{r/2M} ,$$
$$\tau = t/4M , \qquad (3.4)$$

we see that

$$\tfrac{1}{2}(x+y) = \zeta \sinh\tau ,$$
$$\tfrac{1}{2}(x-y) = \zeta \cosh\tau . \qquad (3.5)$$

At the origin the metric is regular:

$$ds^2 \to -\frac{16M^2}{e}\, dxdy + 4M^2 d\Omega^2 . \qquad (3.6)$$

Since we shall mainly concentrate on the origin of xy space we could just as well replace (3.6) by a flat Minkowski metric. Consider a flat Minkowski space with coordinates $x_{tr}$, z, t, and

$$ds^2 = dx_{tr}^2 + dz^2 - dt^2 \qquad (3.7)$$

(where $x_{tr}$ is a two component vector replacing the angles $\theta$ and $\varphi$, and where t is not to be confused with the Schwarzschild time t in (3.2)). The Rindler coordinates $\zeta$ and $\tau$ are now defined by [7]

$$z = \zeta \cosh\tau$$
$$t = \zeta \sinh\tau . \qquad (3.8)$$

An observer for whom $\tau$ acts as a time coordinate feels a strong gravitational field, singular at $\zeta = 0$. Since his time-translations correspond to Lorentz transformations in the original Minkowski space, the observer in this new world, called Rindler space [7], experiences laws of nature that are constant in time.

Let the Minkowski Hamiltonian be the integral over a Hamilton density:

$$H_M = \int \mathcal{H}(\vec{x})d^3\vec{x} . \qquad (3.9)$$

Of course, (3.9) is the generator of time-translations. The generator of Lorentz transformations in Minkowski space is the Rindler Hamiltonian:

$$H_R = \int d^3\vec{x}\, \left(\mathcal{H}(\vec{x})z - P_3(\vec{x})t\right) , \qquad (3.10)$$

where $P_3(\vec{x})$ is the momentum density. We can split $H_R$ into two parts (taking t = 0):

$$H_1 = \int_{z>0} \mathcal{H}(\vec{x})z\, d^3\vec{x} ; \qquad (3.11)$$

$$H_2 = \int_{z<0} \mathcal{H}(\vec{x})\, |z|\, d^3\vec{x} ; \qquad (3.12)$$

$$H_R = H_1 - H_2 \,. \tag{3.13}$$

In most field theories one finds easily:

$$[H_1, H_2] = 0 \,. \tag{3.14}$$

The physical interpretation of (3.14) is that no signals can be transmitted between the regions I ($z > |t|$) and II ($z < -|t|$).

Suppose we had a scalar field $\varphi$ in Minkowski space satisfying the Euler-Lagrange equations generated by the Lagrangean

$$\mathcal{L} = -\tfrac{1}{2}(\partial_z\varphi)^2 + \tfrac{1}{2}(\partial_t\varphi)^2 - \tfrac{1}{2}(\partial_{tr}\varphi)^2 - \frac{m^2}{2}\varphi^2 \tag{3.15}$$

If we write

$$\zeta = e^{\sigma} \qquad (-\infty < \sigma < \infty) \tag{3.16}$$

then

$$\mathcal{L}\, dz\, dx_{tr}\, dt = \ell\, d\sigma\, dx_{tr}\, d\tau \,, \tag{3.17}$$

with

$$\ell = -\tfrac{1}{2}(\partial_\sigma\varphi)^2 + \tfrac{1}{2}(\partial_\tau\varphi)^2 + e^{2\sigma}\left(-\tfrac{1}{2}(\partial_{tr}\varphi)^2 - \frac{m^2}{2}\varphi^2\right) \,. \tag{3.18}$$

Close to the horizon ($\sigma \to -\infty$), this Lagrangean generates plane waves:

$$(\partial_\tau^2 - \partial_\sigma^2)\varphi \to 0 \,. \tag{3.19}$$

The left-going waves will never reach the horizon itself, whereas the right-going ones are infinitely old. Therefore at first sight no boundary condition at $\sigma = -\infty$ (or $\zeta = 0$) seems to be necessary. Nevertheless, we expect "Hawking radiation". This is a thermal flux of particles ariving from $\sigma = -\infty$. This follows if one makes one extra assumption: total energy in the original Minkowski space must be finite. Since this inevitably involves linear combinations of different eigenstates of $H_1$ and $H_2$, one ends up with a quantummechanically mixed state in terms of $H_1$ alone, hence the thermal nature of the radiation .
Any "eigenstate" $|E_1,E_2\rangle$ with

$$\begin{aligned} H_1|E_1,E_2\rangle &= E_1|E_1,E_2\rangle \,; \\ H_2|E_1,E_2\rangle &= E_2|E_1,E_2\rangle \,, \end{aligned} \tag{3.20}$$

being different from the Minkowski vacuum yet Lorentz-invariant, must have infinite energy in Minkowski space.

The infinite spectrum of equation (3.19) in the space (3.16), not bounded for small $\sigma$ values, causes a nasty divergence at the horizon. Also, in Rindler space, but also for finite size black holes, no correlation whatsoever is expected between ingoing and outgoing particles. In a free theory all these states are orthogonal to each other, and nothing can affect the truly stochastic nature of the thermal radiation.

As stated in the introduction, this situation seems to be unacceptable if we were to consider black holes as just some sort of elementary particles. What one expects there is a scattering matrix S:

$$|\psi_{out}\rangle = S|\psi_{in}\rangle \ . \tag{3.21}$$

The reason why (3.21) may perhaps not be incompatible with Rindler space dynamics is that the assumption of having only free fields in Minkowski space is clearly wrong. At sufficiently far negative values of the coordinate $\sigma$, sooner or later, gravitational forces (which are extremely non-linear), will dominate. If we would build such a combination of states (3.20) that the total energy in Minkowski space would become large, then gravitational interactions will cause severe curvature of space-time. Could the particles that went in, not be knocked out again by this curved space-time? Perhaps values of $\zeta$ smaller than the Planck length are forbidden. Are particles bounced back somehow?

## 4. THE SHIFTING HORIZON

Shifts in the parameter $\tau$ of transformation (3.8) correspond to Lorentz transformations in Minkowski space. Even the lightest particles become exceedingly energetic after a certain amount of "time" $\tau$. It is therefore particularly important to study the gravitational effect of such energetic particles on space-time [8,9].

The metric of a very light particle at rest can be approximated by

$$g_{\mu\nu} \simeq \delta_{\mu\nu}\left(1 + \frac{2m_o}{r}\right) + \frac{4m_o}{r} u_\mu u_\nu \ , \tag{4.1}$$

with

$$r^2 = x^2 + (x.u)^2 \ , \tag{4.2}$$

$$u^2 = -1 \ . \tag{4.3}$$

$u_\mu$ is the 4-velocity of the particle. Since (4.1) - (4.3) have been written in a Lorentz-invariant way, they remain true also when the particle is boosted to tremendous energies. We let $m_o \to 0$, $u_\mu \to \infty$, with

$$P_\mu = m_o u_\mu \simeq (0,0,p,ip) \tag{4.4}$$

fixed. If $(x.u) \neq 0$ we have

$$r \to |x.u| = |x.p|/m_o \ , \tag{4.5}$$

so in most of space-time, r is large and indeed $|m_o/r| \ll 1$.
Now consider the new coordinates $y^\mu_\pm$ with

$$y^\mu_\pm = x^\mu \pm 2m_o u^\mu \log r \ . \tag{4.6}$$

At

$$(x.u) > 0 : \quad ds^2 \to dy_+^2 \ , \tag{4.7}$$

And at

$$(x.u) < 0 : \quad ds^2 \to dy_-^2 \ . \tag{4.8}$$

So space-time is flat at points $x_\mu$ with $(x.p) \neq 0$. At $(x.p) = 0$ we must make the transition from the $y_+$ to the $y_-$ coordinates: At $(x.p) = 0$:

$$y_+^\mu = y_-^\mu + 2p^\mu \log y_{tr}^2 \ . \tag{4.9}$$

The second derivative of this shift with respect to $y_{tr}$ produces real curvature across the seam. Notice that the shift, $\Delta y^\mu$, obeys the two-dimensional Laplace equation

$$\partial_{tr}^2(\Delta y^\mu) = 8\pi \ p^\mu \ \delta^2(y_{tr}) \ . \tag{4.10}$$

Thus, spacetime can still be represented by flat coordinates $y^\mu$, provided that we draw the geodesics that cross the plane $(y.p) = 0$ in a special way (Fig. 3)

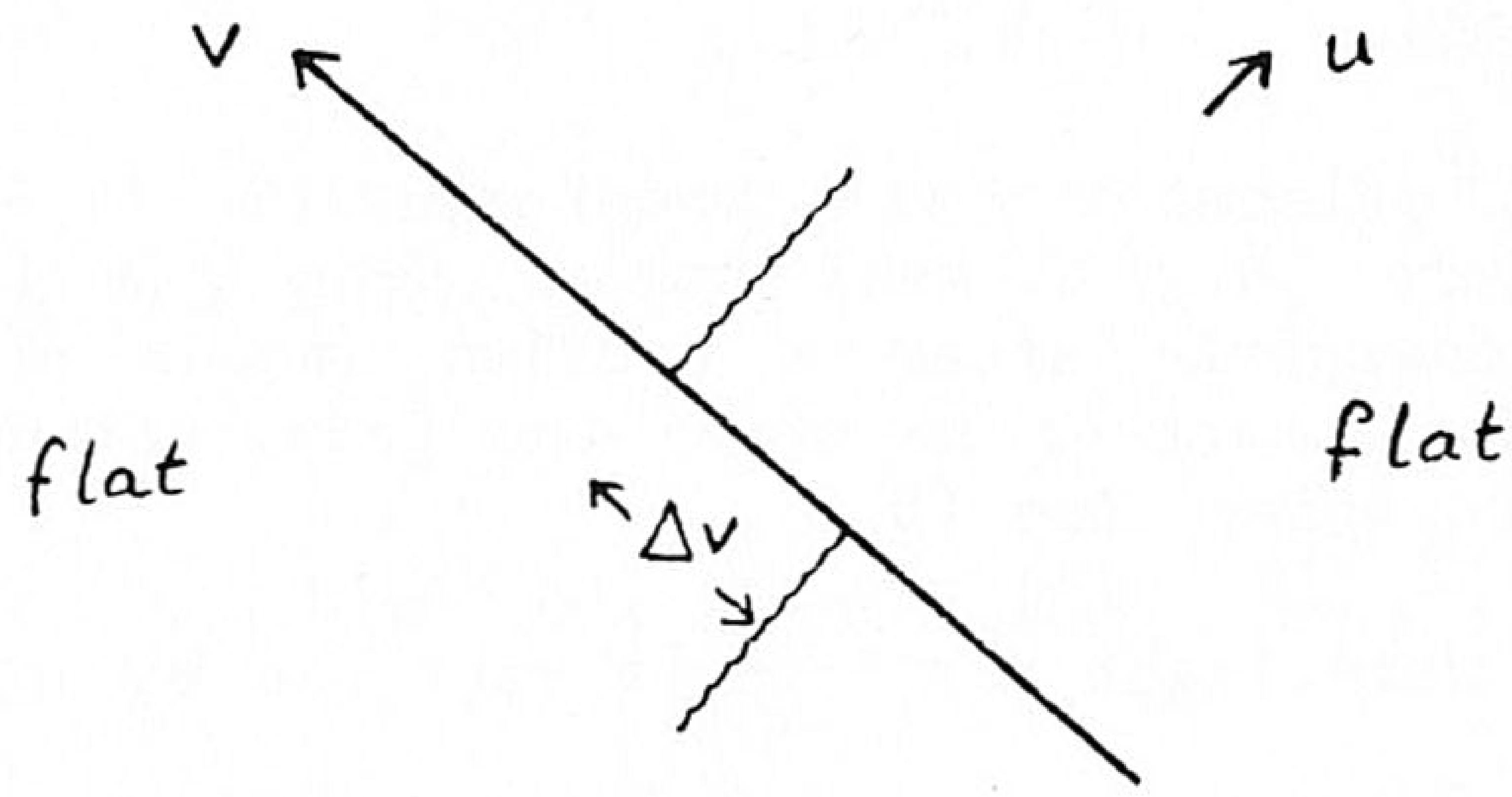


Fig. 3. A fast particle moving in the plane $u = 0$ causes a drag $\Delta v$ in all geodesics that cross the plane $u = 0$, whose value depends on the transverse distance $y_{tr}$ from the particle according to eq. (4.10).

In a finite-size black hole a particle approaching the horizon causes a similar drag. The laplacian $\partial_{tr}^2$ is then replaced by an angular laplacian plus a constant [9], see Fig. 4.

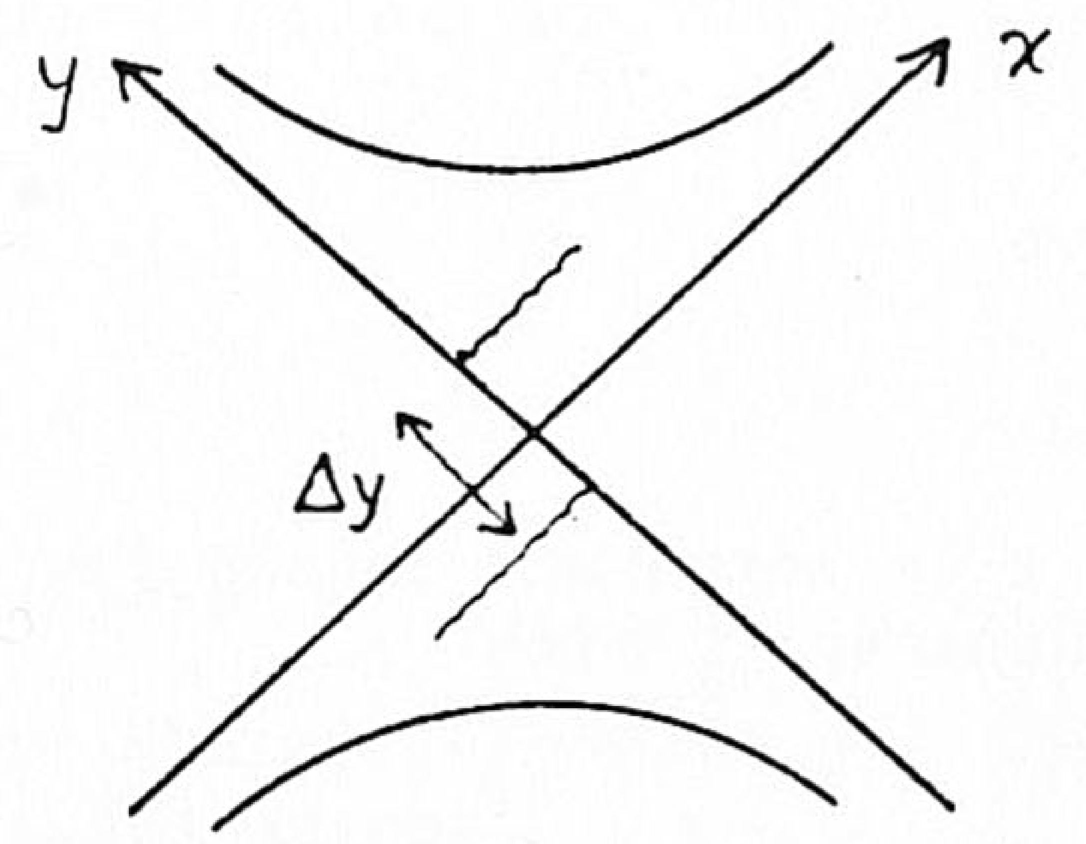


Fig. 4. A particle that moved into a black hole a long time ago causes a shift $\Delta y$ in the trajectories of outgoing particles.

If we have very fast particles both on the u- and on the v-axis, complications arise. Clearly the two sets of particles disturb each other's motion, and the surrounding space-time will no longer be flat. As a first approximation however one may assume that the shifts due to the u-particles and the v-particles can be added linearly.

## 5. A LINK WITH STRING THEORY

Consider now a black hole that has been formed in the distant past. A freely falling observer detects very few particles. It seems to be very well possible to consider the analytic extension of his space-time to the entire Kruskal frame ($xy < 1$). Of course we realize that this analytic extension went beyond the line $t = -\infty$, which is before the black hole was actually made. In reality a large amount of collapsing matter had accumulated on that line and the analytic extension the mathematical model one gets was invalid. Nevertheless the mathematical model one gets is important. For example, if an outgoing particle is actually detected one enters into an element of Hilbert space which contains the past singularity of Kruskal space, a feature further discussed in ref. [10].

The future horizon is defined to be the boundary of that region of space-time from which escape out of the black hole is still possible. The past horizon is the boundary of the region of Kruskal space that can be reached from outside the black hole starting at $t = -\infty$. The analytic extension discussed above is realistic exactly up to the past horizon. The space-like horizon is defined to be the intersection of future and past horizon.

Suppose we take that part of the world that is accessible to us and add to it some ingoing particles with momenta $p^-_{in}$, and some outgoing particles with momenta $p^+_{out}$. How do these particles affect the shape of the space-like horizon?

As long as $p^-_{in}$ and $p^+_{out}$ are not too large we may assume their effects to be additive (as will turn out later, the products of the interesting values for $p^-_{in}$ and $p^+_{out}$ will be negligibly small). From the previous section, eq. (4.10), we deduce for the coordinates $x^{\pm}(\theta.\varphi)$ of the space-like horizon:

$$\partial^2_{tr} x^- = 8\pi G p^-_{in}\ \delta^2(\Omega-\Omega_{in}) \ ; \tag{5.1}$$

$$\partial^2_{tr} x^+ = -\ 8\pi G p^+_{out}\ \delta^2(\Omega-\Omega_{out}) \ , \tag{5.2}$$

where $\Omega_{in}$, $\Omega_{out}$ are the angles $(\theta,\varphi)$ at which the particles crossed the horizon. $\partial^2_{tr}$ is the laplacian with respect to $\theta$ and $\varphi$. The constant term is neglected.
Writing the 4-vector

$$p^{\mu}_{ex} = \left(p^-_{in}, p^+_{out}\right) \qquad ((5.3)$$

and replacing $\theta$ and $\varphi$ by any set $\sigma,\tau$ via conformal transformations, we see that

$$\partial^2_{tr} x^{\mu} = 8\pi G \rho^{\mu}_{ex} \tag{5.4}$$

where $\rho^{\mu}_{ex}$ is the distribution of external momenta on the $\theta,\varphi$ (or $\sigma\tau$) plane.

The analogy with the string theory in section 2 is striking, apart from a factor $-i$ that at first sight seems to be very troublesome, but may find a quite natural explanation: a factor −1 may be needed if, instead of the coordinates $x^{\mu}(\sigma,\tau)$ we consider the image of an ideal surface as seen through the "gravitataional lens" caused by the in- and outgoing particles, and the factor i may be seen as resulting from transformations of the sort

$$x^{\pm} \rightarrow \pm i\, x^{\pm} \,, \tag{5.5}$$

which are Lorentz transformations with complex arguments. Also one must bear in mind that string theory is usually defined in a $\sigma\tau$ space with metric (+,−), in order to obtain timelike surfaces.

A complete formalism that explains string theory from black hole dynamics is still lacking. The above however suggests that these concepts are closely related. If indeed the factor $\pm i$ can be accounted for we expect the string constant T to be

$$T = 1/8\pi G \,, \tag{5.6}$$

simply by comparing (2.21) with (5.4).

## 6. CONCLUSION

Even without understanding all technical details of the suggested link between string theory and black hole dynamics, we can consider its topological aspects. A conventional particle being a point in 3-space traverses a trajectory in 4-space. A virtual particle, giving rise to an instantaneous interaction between two points (such as a photon producing the Coulomb force), is a one-dimensional subspace of 3-space, but instantaneous in time (Fig. 5).

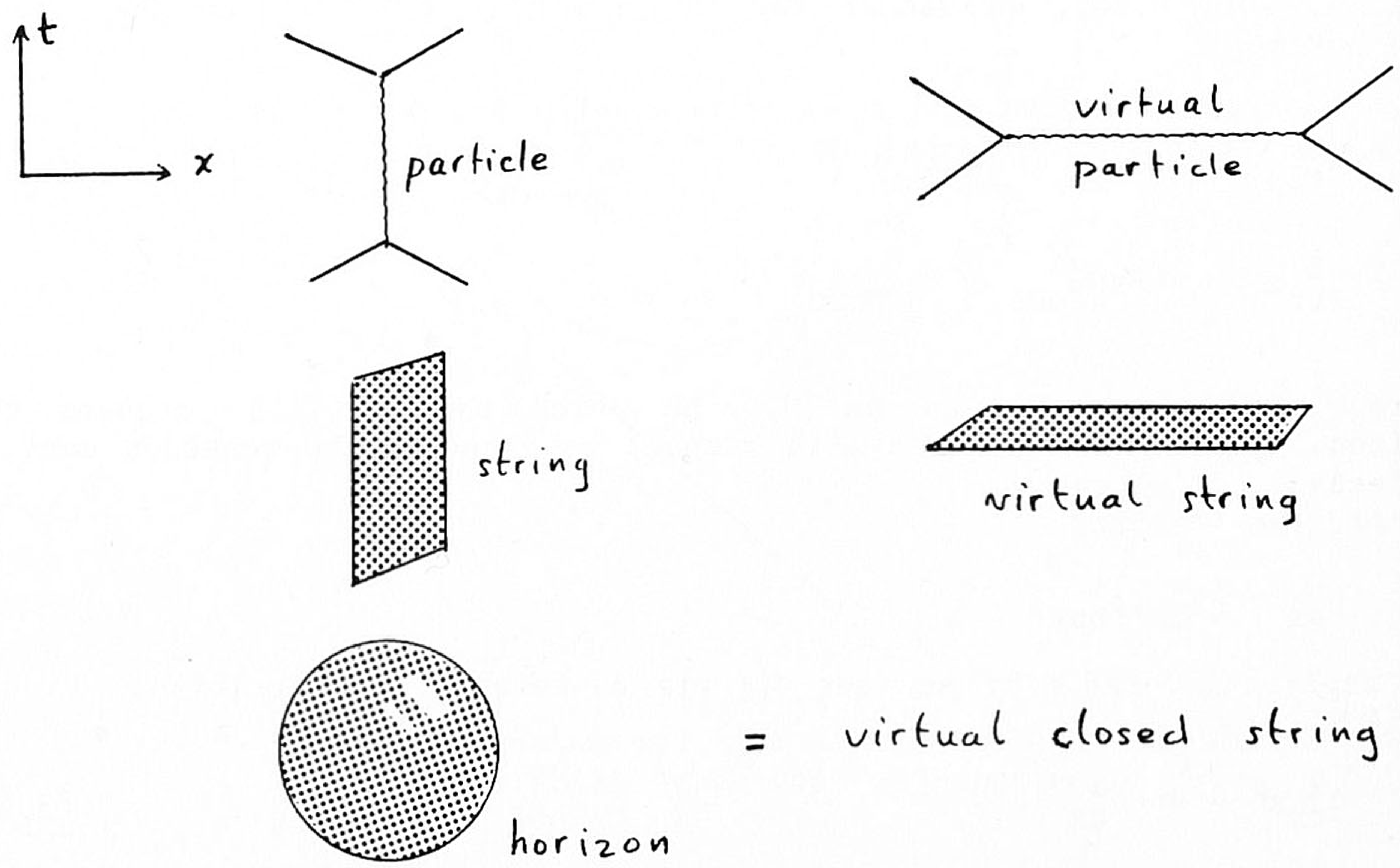


Fig. 5. A virtual string is sheet-like in 3-space and instantaneous in time. The space-like horizon can be a virtual closed string.

A virtual string can therefore be a sheet in 3-space. This is why the space-like horizon, being an $S_2$ sphere, has the same topology as a virtual closed string.

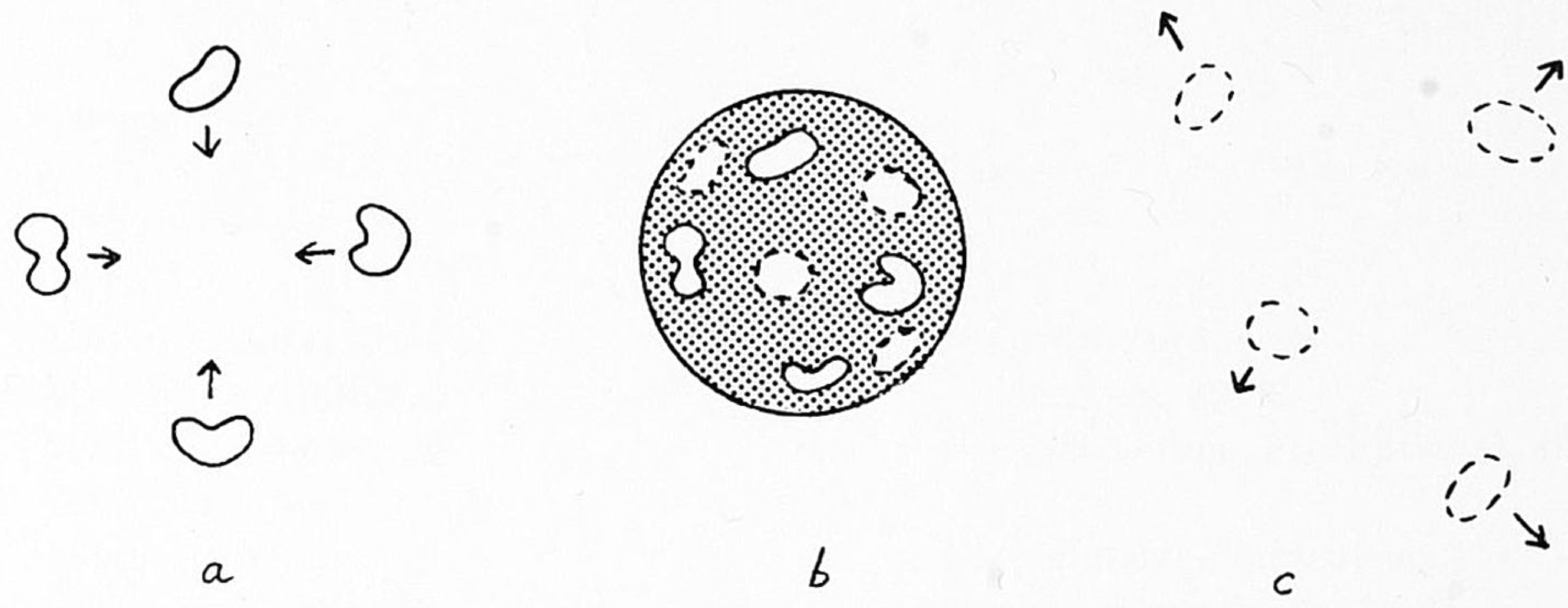

Fig. 6. Imploding particles (a), arrive at the horizon (b) and produce Hawking radiation (c).

In Fig. 6 we show how a black hole can perhaps be seen as a virtual closed string. Particles in the form of little closed stringscome together, exchange a virtual string that leaves holes; these holes connect to the outgoing Hawking particles, againclosed strings. Our aim was to bring black hole physics and string theory together. What we see is that in some sense the functional integral (2.14) for a string may actually describe the oscillations of a black hole horizon.

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