THE QUANTUM BLACK HOLE [a]

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ABSTRACT

A review is presented of the problem of black hole quantization. It is argued that quantum mechanics should not only be valid for all micro-physical processes at or near the horizon, but also for the black hole entire. This assumption implies the existence of a scattering matrix for all physical processes, and it has far-reaching consequences not only for the behavior of black holes themselves but also for the formulation of physical laws in a flat background space-time.

1. Introduction. The classical black hole.

The Planck scale is defined to be the regime of physics where quantum effects, relativistic effects and gravitational effects are all of essential importance. One chooses units such that the three constants of nature, $1/2\pi$ times Planck's constant, $\hbar = 1.0546\times10^{-34}$ kg m^2 sec^{-1}, the velocity of light $c = 3\times10^8$ m/sec, and Newton's constant $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} sec^{-2}, are all equal to one. Consequently the length unit is

$$L_{Planck} = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-33} \text{ cm}, \tag{1.1}$$

the mass unit

$$M_{Planck} = \sqrt{\frac{\hbar c}{G}} = 21.8 \ \mu\text{g}, \tag{1.2}$$

and the unit of energy

$$E_{Planck} = M_{Planck}c^2 = \sqrt{\frac{\hbar c^5}{G}} = 1.22 \times 10^{28} \text{ eV}. \tag{1.3}$$

Reconciling the known laws of the gravitational force with those of Quantum Mechanics is one of the most tantalizing problems that are still wide open in Theoretical Physics. Not only do we have difficulties in describing the movements of

[a] Lecture given at the Pacific Winter School, Sogcho, Korea, January 1995.

elementary particles under the influences of their mutual gravitational interactions, we even fail to understand what the fundamental degrees of freedom are, how to do the book keeping. It is instructive however to analyse the Planck regime by approaching it from all possible directions. At energies well below the Planck energy (1.3) we have standard quantum field theory. At energies much *beyond* the Planck regime the gravitational equations become classical differential equations. There we have general relativity theory. In this theory the most compact form of 'matter' appears as black holes, the ultimitely collapsed state. Let us briefly describe such a black hole [1].

Consider a freely falling laboratory where only distances $\mathrm{d}s$ can be measured that are small compared to the inhomogenuities of the gravitational field one might be in. Then one can choose locally a Cartesian coordinate grid there such that the relativistically covariant distance is

$$\mathrm{d}s^2 \;=\; \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu\;;\quad x^0 = t\,, \tag{1.4}$$

where

$$g_{\mu\nu} \;=\; \begin{pmatrix} -1 & & & \oslash \\ & 1 & & \\ & & 1 & \\ \oslash & & & 1 \end{pmatrix}. \tag{1.5}$$

In general relativity space and time are curved, such that rectangular coordinate grids, for which (1.5) would hold everywhere, do not exist. Eq. (1.4) will only be valid for infinitesimal displacements $\mathrm{d}x^\mu$ in the coordinates x^μ and with a space-time dependent metric function $g_{\mu\nu}(x)$. Still, at any isolated space-time point x, local coordinates $\mathrm{d}x^\mu$ exist such that (1.5) is valid there, but no single coordinate frame exists such that (1.5) holds everywhere.

Einstein's gravitational field equations can be solved for the spherically symmetric case, and yield a field $g_{\mu\nu}(\mathbf{x},t)$ given by

$$\mathrm{d}s^2 \;=\; -\left(1-\frac{2GM}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-2GM/r} + r^2\,\mathrm{d}\Omega^2\,, \tag{1.6}$$

where $x^\mu = (t, r, \theta, \varphi)$, and $\mathrm{d}\Omega^2$ stands short for $\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2$. Here $2GM$ is a free constant of the solution, which is written such a way that M corresponds to the total mass that would be responsible for this gravitational field. G is Newton's constant which of course will be put equal to one. The derivation of this solution can be found in many text books on general relativity. It is called the Schwarzschild solution [1].

The coefficient in front of dt^2 can be identified as (twice) the gravitational potential. It is also directly responsible for the gravitational red shift, or time dilation. Clearly, at a distance $r = 2GM$ from the origin the red shift becomes infinite. This singularity there however is only an artefact due to our choice of coordinates. A different set of coordinates exist such that the singularity goes away: the Kruskal coordinates x, y can be defined by [2]

$$\begin{aligned} (r - 2GM)e^{r/2GM} &= xy \; ; \\ e^{t/2GM} &= x/y \; . \end{aligned} \tag{1.7}$$

Now replacing the Schwarzschild coordinates (t, r, θ, φ) by the Kruskal coordinates $(x.y.\theta, \varphi)$ one finds

$$\mathrm{d}s^2 = 2A(x,y)\,\mathrm{d}x\mathrm{d}y \; + \; r^2(x,y)\,\mathrm{d}\Omega^2 \, , \tag{1.8}$$

where

$$A(x,y) = \frac{8G^2M^2}{r}\, e^{-r/2GM} \, , \tag{1.9}$$

and indeed no singularity is seen at $r = 2GM$. The space-time picture in Kruskal coordinates is sketched in Fig. 1.

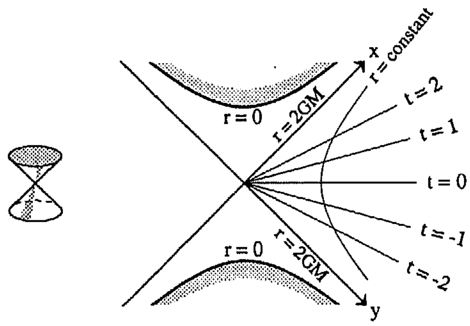

Fig. 1. The Schwarzschild solution in Kruskal coordinates.

Notice that according to Eq. (1.8) all displacements $\mathrm{d}x^\mu$ obeying either $\mathrm{d}x = 0$ or $\mathrm{d}y = 0$ are lightlike vectors. In Fig. 1. therefore the lightcone is everywhere oriented as shown in the Figure. From Eq. (1.7) we see furthermore that a time boost $t \to t + \delta t$ in the Schwarzschild coordinates corresponds to a Lorentz boost in the Kruskal coordinates x and y, if these are seen as light cone coordinates:

$$\begin{aligned} x &\to x \, e^{\delta t/4GM} \; ; \\ y &\to y \, e^{-\delta t/4GM} \; . \end{aligned} \tag{1.10}$$

The horizon is the origin of x - y space. If we limit ourselves to the physics close to this horizon we can ignore the curvature there. Replacing r by a coordinate $\rho = 2r(1-2GM/r)^{1/2}$ and writing

$$\begin{aligned} z =& \rho\cosh(t/4GM)\,, \\ \tau =& \rho\sinh(t/4GM)\,, \end{aligned} \tag{1.11}$$

we see that in terms of z and τ the metric near the horizon is almost flat and we recognise more directly the relation between time boosts in t and Lorentz boosts for the z, τ coordinates. The space of ρ, τ coordinates in case z and τ span a flat space is called *Rindler space*. Most essential features of the horizon can be studied in Rindler space.

Transformations of the form

$$x \to x'(x)\,, \quad y \to y'(y)\,, \tag{1.12}$$

will not affect the orientation of the lightcones of Fig. 1. They allow us to represent all of space-time in a compact domain, whose boundary represents the points at infinity. The useful diagrams obtained this way are called Penrose diagrams[2]. See Fig. 2. The functions x' and y' here have been chosen such that the singularity at $r = 0$ forms a straight line. We see that Schwarzschild space-time consists of four distinct regions, labled $I, \ldots, IV$. Region I is the physically accessible part of the universe. Region III is where one enters while falling through the horizon. This region is connected to region II, a carbon copy of the physical universe I. Finally, region IV is the time reverse of III.

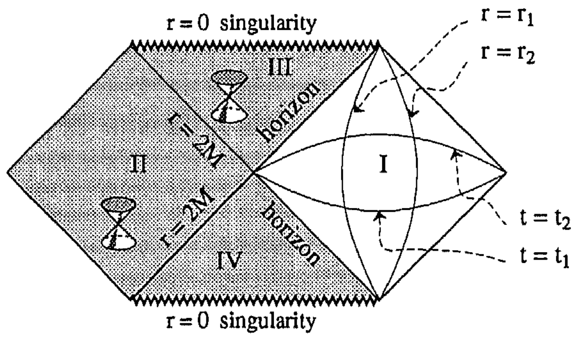

Fig. 2. The Penrose diagram for the Schwarzschild solution. Region I represents space-time outside the black hole.

Regions II and IV can only be reached by going backwards in time, beyond the line $t = -\infty$ in Fig. 1. But a black hole that has been formed by the collapse of matter only takes the Schwarzschild form after a certain time, and so such a black hole does not have regions II and IV. Such an object is represented by the Penrose diagram of Fig. 3b. In a more regular coordinate frame the same structure looks like Fig. 3a.

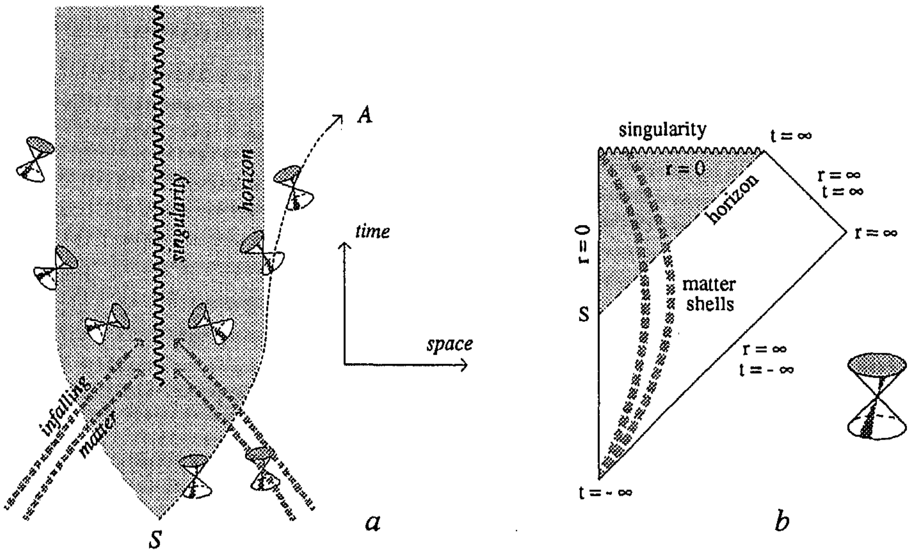

Fig. 3. a) Black hole formed by imploding matter, in approximately Cartesian coordinates. b) Corresponding Penrose diagram

At early times (below in the Figure) space is still regular and material particles are still far apart. Apparently nothing out of the ordinary happens the moment these enter into the shaded region. The particles could be dust particles, or paper clips, or television sets, and still behave as such during this moment. But the shaded region is defined be the behavior of particles and light rays afterwards: no escape to infinity is possible from there. The horizon is the dividing line between this region and the region from which escape is still possible. The horizon itself is a light like surface in space-time. The possible routes of light rays and matter particles can easily be read off from the light cones drawn at various points. These force any particle inside the shaded region to move towards the center ($r = 0$). At this center a true singularity develops. The emergence of such a singularity is in no way in conflict with common sense. Long before this singularity is reached signals are prohibited to reach the outside world, so for the outside observer the presence or absence of such a singularity is of no more than academic interest.

The possibility of black hole formation in astronomical settings is a simple and logical consequence of well-known laws of physics, laws that have been verified convincingly, e.g. by analysis of data from binary pulsars. An observer looking at the event from some distance will see a surface that darkens quickly, and after a short amount of time it will look complately black.

2. Quantum Mechanics; Hawking Radiation.

Now let us switch on quantum mechanics. Of course quantum mechanics should be applied to large black holes as well as small ones, but in the case of large black holes these effects will turn out to be truly minute, so that it makes sense to believe that they will merely give rise to very tiny corrections to the classical picture. At first sight there seems to be nothing wrong with such an approach. Indeed, calculation of the quantum corrections to black hole formation gives something very interesting: black holes aren't black. As was discovered by Hawking [3], the laws of quantum field theory in the environment of a black hole dictate that the hole continuously emits particles with a thermal spectrum:

$$kT = \frac{1}{8\pi GM}\,, \tag{2.1}$$

where G is Newton's constant, k is Boltzmann's constant and M is the black hole mass.

The derivation of this important result can be found in the literature [3]. Let me here sketch a simplified version. Although the argument can be formulated for any kind of quantum fields near the horizon, let us take a simple scalar field $\phi(\mathbf{x},t)$, and let us furthermore neglect the curvature near the horizon by replacing space-time there by Rindler space. In terms of the Cartesian coordinates z,τ of Eq. (1.11) the Hamilton density is

$$\mathcal{H}_M(\mathbf{x},\tau) = \tfrac{1}{2}(\partial_\tau\phi)^2 + \tfrac{1}{2}(\vec{\partial}\phi)^2 + \tfrac{1}{2}m^2\phi^2\,, \tag{2.2}$$

where $\mathbf{x} = (r\varphi\cos\theta, r\theta, \rho)$, and the subscript M stands for Minkowski space. A time boost in τ is generated by $\mathbf{H}_M = \int \mathcal{H}_M \mathrm{d}^3\mathbf{x}$ which can be written as

$$\mathbf{H}_M = \sum_{\vec{k}} a^\dagger(\vec{k})a(\vec{k})\,k_0\,, \tag{2.3}$$

where a and $a^\dagger$ are annihilation and creation operators for the scalar particles.

But for the distant observer not τ but t is the time parameter, and a boost in t corresponds to a Lorentz transformation in terms of the local frame. The generator of such a transformation is the operator

$$\mathbf{H}_R = \int \mathcal{H}_M(\mathbf{x},0)\,z\,\mathrm{d}^3\mathbf{x}\,, \tag{2.4}$$

(where R stands for Rindler), which we write as

$$\begin{aligned}
\mathbf{H}_R &= \mathbf{H}_I - \mathbf{H}_{II}\,;\\
\mathbf{H}_I &= \int_{z>0} \mathcal{H}_M\, z\, \mathrm{d}^3\mathbf{x} = \sum_\omega \omega\, b_I^\dagger(\tilde{k},\omega) b_I(\ k,\omega)\,,\\
\mathbf{H}_{II} &= \int_{z<0} \mathcal{H}_M\, |z|\, \mathrm{d}^3\mathbf{x} = \sum_\omega \omega\, b_{II}^\dagger(\tilde{k},\omega) b_{II}(\tilde{k},\omega)\,,
\end{aligned} \tag{2.5}$$

where $\tilde{k}$ stands for the transverse momentum after Fourier transformation in the transverse coordinates $(x^1,\, x^2)$. This splitting has been performed in such a way that $\mathbf{H}_I$ describes the evolution in region I while commuting with all observables in region II and $\mathcal{H}_{II}$ *vice versa.* The relations between the a, $a^\dagger$ operators on the one hand and the operators b_I, b_{II}, $b_I^\dagger$, $b_{II}^\dagger$ on the other can be readily calculated. One then finds [5]:

$$\begin{aligned}
b_I(\tilde{k},\omega)\sqrt{1-e^{-2\pi\omega}} &= a'(\tilde{k},\omega) + e^{-\pi\omega} a'^\dagger(-\tilde{k},-\omega)\,;\\
b_{II}(\tilde{k},\omega)\sqrt{1-e^{-2\pi\omega}} &= a'(\tilde{k},-\omega) + e^{-\pi\omega} a'^\dagger(-\tilde{k},\omega)\,,
\end{aligned} \tag{2.6}$$

where $a'(\tilde{k},\omega)$ is a linear function of the creation operators $a(\mathbf{k})$ of Eq. (2.3), normalized in such a way that a' and $a'^\dagger$ obey standard commutation rules. Eqs. (2.6) which mix creation and annihilation operators, are called Bogolyubov transformations.

Now consider a freely falling observer. One may convincingly argue that for such an observer the exact location of the horizon appears to be irrelevant. For him only the Cartesian coordinates make sense. If the number of particles he observes close to the horizon does not diverge to infinity he may be said to live essentially in a vacuum state $|\Omega\rangle$, defined by

$$a(\mathbf{k})|\Omega\rangle = a'(\tilde{k},\omega)|\Omega\rangle = 0 \tag{2.7}$$

(the particles this observer *does* see are of no relevance to what follows). Inverting (2.6) we find that this implies

$$b_I(\tilde{k},\omega)|\Omega\rangle = e^{-\pi\omega} b_I^\dagger(-\tilde{k},-\omega)|\Omega\rangle\,, \tag{2.8}$$

and similarly for b_{II}, which means that in terms of the b_I annihilation operators this state is *not* empty.

Integrating the equation (2.8) we find that

$$|\Omega\rangle = \prod_{\omega,\tilde{k}} \sqrt{1-e^{-2\pi\omega}} \sum_{n=0}^{\infty} e^{-n\pi\omega}\, |n_I(\tilde{k},\omega) = n;\ n_{II}(\tilde{k},\omega) = n\rangle\,. \tag{2.9}$$

It should not be surprising that the number of excitations in region I always stays equal to the number of excitations in region II; since the vacuum $|\Omega\rangle$ is Lorentz invariant it has a vanishing Eigenvalue for the generator $\mathbf{H}_R = \mathbf{H}_I - \mathbf{H}_{II}$.

The probabilty W to detect n particles with Rindler energy ω (either in I or in II) is seen to be proportional to $e^{-2\pi n\omega}$. Apparently we are dealing with the density matrix of a thermal state with temperature $kT = 1/\beta = 1/2\pi$. If we wish to normalize the unit of time to the Schwarzschild time parameter we have to insert a factor $4GM$, and hence for the distant observer the temperature turns out to be given by Eq. (2.1).

3. Density of Quantum States.

From a physical point of view this result seems to be totally acceptable. It simply means that a black hole is radio-active, slowly decaying into lighter configurations and eventually disappearing altogether. Comparing the black hole with other evaporating objects is actually quite instructive. The probability to emit a photon (or any other object) into a volume V can be estimated to be

$$W \approx \pi R^2 V^{-1} \rho_{\text{out}} e^{-\beta_H E_{\text{out}}}\,, \tag{3.1}$$

where R is the radius of the black hole, ρ_{out} is the density of photon states inside the volume V, and $\beta_H = 8\pi GM$ is the inverse Hawking temperature. Conversely, the capture cross section is approximately

$$\sigma \approx \pi R^2 = 4\pi M^2\,. \tag{3.2}$$

Thus, if δE is the energy of a nearby photon we can estimate the two-way interaction $|\delta E\rangle_{\text{out}}|E\rangle_{\text{BH}} \Leftrightarrow |E+\delta E\rangle_{\text{BH}}$, where E stands for the energy of a black hole BH. Suppose we had a quantum mechanical amplitude $\mathcal{T}$ for these interactions. Then one would write

$$\begin{aligned} \sigma &= |\langle E+\delta E|\,\mathcal{T}\,|E\rangle|\delta E\rangle|^2 \rho_{E+\delta E}\,; \\ W &= |\langle E|\langle\delta E|\,\mathcal{T}\,|E+\delta E\rangle|^2 \rho_E \rho_{\text{out}} V^{-1}\,. \end{aligned} \tag{3.3}$$

Here ρ_E is the density of quantum levels inside a black hole with energy E. This phase space factor always emerges in expressions for total probabilities.

The two amplitudes in (3.3) need not be identical. One is the time-reverse of the other. But since neither parity P nor charge conjugation C enter into our arguments we can also compare the capture process with its CPT counterpart. All ordinary quantum field theories are CPT invariant. Although it is conceivable that nevertheless such a symmetry does not hold for black holes, it is instructive to assume PCT and to learn its consequences. The absolute values of the amplitudes are then identical in (3.3), so we can divide the two expressions to get

$$\frac{\sigma}{WV}\rho_{\rm out} = \frac{\rho_{E+\delta E}}{\rho_E} \approx e^{\beta_H \delta E}\,, \tag{3.4}$$

and writing $\rho_E \equiv \exp S(E)$ we find the differential equation

$$S(E+\delta E) - S(E) = 8\pi E\delta E \quad \text{or} \quad \mathrm{d}S/\mathrm{d}E = 8\pi E\,. \tag{3.5}$$

Thus, the density of states is given by

$$S(E) = 4\pi E^2 + C\,; \quad \rho_E = C'\, e^{4\pi E^2}\,, \tag{3.6}$$

where C and C' are unknown integration constants. It is tempting to assume these to be of order one but one has to be careful with this assumption; large numbers such as 10^{40} may emerge naturally from quantum gravity. We do conclude that black holes may well exhibit a spectrum of discrete states whose density rapidly increases, exponentially with the square of the mass. Could it be that the lightest of these merge naturally with the ordinary particle states below the Planck mass? This seems to be a reasonable thing to expect; it also seems reasonable to expect that more accurate calculational schemes should exist that tell us how to compute the integration constant C in (3.6).

4. The Quantum Information Problem.

But then a deep and difficult problem emerges. It appears to be impossible to define these quantum states directly. The thermal nature of the outgoing radiation does not seem to result from any statistical treatment of the initial states but an inherent property of the black hole itself. While some particles escape, the quantum modes of other particles falling into the hole become invisible to the external observer, and it is when one averages over all possible invisible modes that a thermal density matrix for the outcoming particles is obtained. So the heat bath is not outside but inside the black hole.

Consider Eq. (2.9). It shows that the state the black hole appears to get into is to be written as

$$|\Omega\rangle = \sum_i |\psi_i\rangle_I\, |\psi_i\rangle_{II}\,. \tag{4.1}$$

Any operator $\mathcal{O}$ employed by a late observer outside the black hole can only act on the features in region I. Hence its expectation value will be expressed as

$$\begin{aligned}\langle\mathcal{O}\rangle &= \sum_{ij} {}_I\langle\psi_i|\,{}_{II}\langle\psi_i|\mathcal{O}|\psi_j\rangle_{II}\,|\psi_j\rangle_I \\ &= \sum_{ij} \rho_{ij}\ {}_I\langle\psi_i|\mathcal{O}|\psi_j\rangle_I \\ &= \mathrm{Tr}\,\rho\mathcal{O}\,,\end{aligned} \tag{4.2}$$

with

$$\rho_{ij} = {}_{II}\langle\psi_i|\psi_j\rangle_{II}\,, \tag{4.3}$$

acting as a density matrix for a thermal mixture of quantum mechanical states.

A fundamental question is now: could one start with collapsing matter in a single quantum state, and end up with a black hole which, while emitting Hawking particles, can only be described as a mixture of states? Does such a transition from pure into mixed states require a revision of the Schrödinger equation for black holes?

One may also formulate this problem differently. Ordinary quantum systems are always described by a unitary evolution operator. This implies that if one compares two mutually orthogonal states in the far past, these states will evolve into mutually orthogonal states in the future. But the final state of a black hole does not seem to depend on what was thrown in; this is the *no-hair theorem.* Since all operators after a while show identical expectation values, the two states cannot continue to be orthogonal to each other – a breakdown of unitarity.

Three different approaches have been advanced in the literature:

- *Indeed the Schrödinger equation for the black hole as a single physical system no longer holds.*

The Schrödinger equation is said to hold only if one takes the entire new universe II into account [4]; it disappears if we try to limit ourselves to "our part of the universe". Equations are proposed that describe transitions from pure into mixed states [6]. However quantum field theory is a delicate construction that may easily fall apart completely if one tries such a drastic alteration in its primary principles. Suppose we have an arbitrary linear equation for density matrices:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{ij} = -i\mathbb{H}_{ij|k\ell}\rho_{k\ell}\,, \tag{4.4}$$

with

$$\mathbb{H}_{ij|k\ell} = \delta_{ik}H_{j\ell} - H_{ik}\delta_{j\ell} + \mathbb{H}^{\mathrm{int}}_{ij|k\ell}\,. \tag{4.5}$$

Pure quantum mechanics would correspond to $\mathbb{H}^{\mathrm{int}} = 0$. Notice then that we have here two universes I and II which in the purely quantum mechanical case would not interact with each other. As soon as interactions would be switched on, *via* $\mathbb{H}^{\mathrm{int}}$, one would expect energy exchange between these two universes. But since the Hamiltonian in one universe is bouded from below and in the other bounded from above (*cf* the minus sign in Eq. (2.5)) such an energy exchange would probably lead to a catastrophic instability [7]. A second idea is more difficult to refute:

- *There is a Schrödinger equation, but the spectrum of black hole states is continuous.*

This is what one gets if one simply keeps the internal states as mutually orthogonal states. Ingoing particles accumulate near the horizon, which is why one tends to get an infinite degeneracy of black holes. Another way to see how this may happen is to consider field theory in the Schwarzschild background. In terms of the Schwarzschild time coordinate t one has an infinite class of stationary wave solutions; these behave as plane waves[8] provided one rewrites $r - 2GM \rightarrow \exp \sigma$, with the horizon at $\sigma \rightarrow -\infty$. There is an infinite universe of mutually orthogonal particle modes *at our side of the horizon!* The arguments of the previous section do imply one important consequence for such black holes: there must exist a lowest state in this continuum, and phase space factors then prohibit its decay. Thus, this proposal requires the existence of arbitrarily long-lived *remnant states.* These remnants, being infinitely degenerate, would not obey Fermi-Dirac or Bose-Einstein statistics, but Boltzmann statistics, and thus defy the usual laws of thermodynamics: they can be created but not destroyed. It may be possible to explain why in an inflatory early universe such remnants were not copiously produced; the main objection against this proposal is its being less than esthetic. Remnants would to some extent have unpredictable behaviour. If we don't want this, the only remaining possibility is:

- *Quantum information returns - encrypted - in the Hawking radiation.*

One might after all argue[9] that ingoing particles do interact with the outgoing ones, if not by ordinary particle exchanges then certainly gravitationally. However, return of information is not what one gets in calculations starting with the usual first principles. As will be shown, one has to reformulate the laws of physics to make this picture consistent.

5. The Scattering Matrix Ansatz; gravitational interactions among light particles crossing the horizon.

None of the presently existing models for quantum gravity possess a built-in safety device that would guarantee physically reasonable behaviour of black holes. Black holes are fundamentally non-perturbative phenomena whereas our models are only understood within some perturbative scheme, and therefore black holes may well exhibit all diseases of a "run-away solution". Notice that in this respect they are essentially different from non-perturbative features such as vortices and instantons in quantum field theories such as QCD. In these theories the ultraviolet behaviour is so well under control that all of our topologically non-trivial field configurations are ultraviolet soft, so that they do not require any new physics for their understanding, just more advanced mathematics. The black hole *does* require new physics in the ultra-violet that seems to fall well outside the usual analysis of divergences of the present theories. Notably this includes string theories, since even they can only be formulated within a scheme of world sheet diagrams with successive increases of topological complexity, *i.e.* a perturbative expansion.

This is why one should not rely on 'model calculations' to 'deduce' the behaviour of black holes. Such calculations tend to lead towards remnants [10], with uncalculable behaviour. This could be interpreted as indicating run-away behaviour. To my opinion this infinite density of quantum states inside a small volume of space is a feature that is physically improbable. We should search for models that avoid this from happening. These models must be very special. One might suspect that a search for a model stable under gravitational collapse will lead to new physical principles, indispensible for a consistent quantum gravity.

Indeed, the requirement that there be a denumerable set of internal quantum states immediately implies an important constraint on models that can be consistent with this: there cannot possibly exist a globally additively conserved quantum number such as baryon number or lepton number or any combination thereof, *unless* it is coupled to a gauge theory which turns the symmetry into a local one [11]. It is easy to see that an exactly conserved global quantum number would enable one to augment this quantum number for a black hole while waiting for it to decay back to its original mass, and this way one would be able to create an unlimited number of quantum states at any given mass. We must conclude that quantum gravity models with such a globally conserved quantum number are examples of self-destructing theories.

Our proposal is now simply to *postulate* unitarity of the black hole evolution matrix, in terms of discrete energy levels [12]. This postulate is very restrictive. Combining it with other requirements such as general relativity in the low-energy domain may actally enable us to *compute* the scattering matrix when virtual black holes are present. We refer to our postulate as the S-matrix Ansatz.

We assume here that Hawking's derivation is correct for large black holes, if these are treated as thermodynamically mixed. So the statistical nature of this result is embraced. However the 'heat bath', which in Hawking's picture is produced by all particle states *inside* the hole, is now attributed to particles that entered earlier, or will leave somewhat later. In contrast with the older picture, we are forced to assume that these early ingoing and late outgoing particles are *not independent* of objects living in region II. How this new picture can be made consistent can be understood only if one takes interactions between ingoing and outgoing matter into account.

Most significant of all mutual interactions that will have to be taken into account is the gravitaional one. Its effects can also be computed accurately, and the result will help us understand how to construct our model. Since we only need to concentrate on the immediate vicinity of the horizon it suffices to consider Rindler space, although no conceptual difficulties will arise if later we replace it by Kruskal space. Thus we first study gravitational interactions between particles moving in a locally flat space-time.

Gravitation is coupled to mass, hence energy. The force becomes strong if the energies approach the Planck energy. What is the gravitational field of an energetic particle? This is simply obtained by first describing accurately the curvature of space-time surrounding a particle at rest, and then boosting the particle to high velocity. One finds [13] that the limit $m \to 0\,,\ v \to c\,,$ $\mathbf{p}$ fixed is particularly simple: space-time in front of the particle is flat, and so is the region trailing behind the particle. But the particle carries a 'shock wave' not unlike Čerenkov radiation. The shock wave causes a displacement of all particles crossing it. If the source particle moves with momentum $p_+ = p^-$ in the $-z$ direction all test particles crossing the wave are displaced by an amount $\delta x^- = 8Gp_+ \log(1/|\tilde{r}|)$ where $\tilde{r}$ is the transverse distance between source and test particle. The displacement is in the lightcone minus direction, parallel to the source particle momentum.

A summary of this calculation is as follows [13]:
Since the mass m of the source particle is negligible we can rewrite the surrounding metric (1.6) in the rest frame (putting $G = 1$) as

$$\mathrm{d}s^2 = \mathrm{d}x^2 - \frac{2m}{r}\mathrm{d}t^2 + \frac{2m}{r}\mathrm{d}r^2\,, \tag{5.1}$$

where $\mathrm{d}x^2$ stands for the flat space-time metric. Taking its 4-velovity in this frame to be $u^\mu = (0,0,0,i)$ we see that we can make the covariant replacements

$$\mathrm{d}t^2 = -(u\cdot\mathrm{d}x)^2\,,\quad r^2 = x^2 + (u\cdot x)^2\,. \tag{5.2}$$

Next we boost the particle such that

$$\begin{aligned} &u^\mu \to (0,0,\gamma,i\gamma)\,,\quad \gamma\to\infty\,;\\ &r^2 \to (u\cdot x)^2\,,\quad \mathrm{d}r \to (u\cdot\mathrm{d}x)\,;\\ &\mathrm{d}s^2 \to \mathrm{d}x^2 + \frac{4m}{r}(u\cdot\mathrm{d}x)^2\,. \end{aligned} \tag{5.3}$$

That this is flat space both in front and behind the source particle can be seen by performing a coordinate transformation:

$$y^\mu_\pm = x^\mu \pm 2mu^\mu \log r\,. \tag{5.4}$$

One has

$$\begin{aligned} \mathrm{d}y^2_\pm \to \mathrm{d}x^2 \pm \frac{4m}{r^2}(x\cdot u)(u\cdot\mathrm{d}x)^2 &= \mathrm{d}x^2 \pm \sigma\,\frac{4m}{r}(u\cdot\mathrm{d}x)^2\,,\\ \text{with}\quad \sigma = \frac{(u\cdot x)}{r} &\to \pm 1\,. \end{aligned} \tag{5.5}$$

The sign σ is $+1$ in front of the particle and -1 behind it. So fitting this with Eq. (5.3) gives us that these two flat spaces are glued together at the plane $(u \cdot x) = 0$ by the rule

$$y_+^\mu - y_-^\mu = 4mu^\mu \log |\tilde{r}| \,. \tag{5.6}$$

Note however that the above is only a whimsical sketch of a true derivation, since the infinite boost limit is rather delicate.

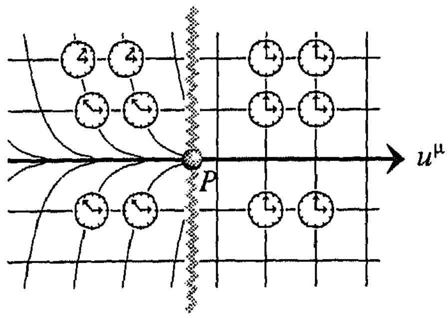

Fig. 4. Snapshot of energetic massless particle P, surrounded by deformed space-time clocks on rectangular grid (right) will emerge displaced in space and time (left). Both in front of and behind the particle space-time is flat.

The physical interpretation of this result is sketched in Fig. 4. The 'shock wave' (gray zigzag line) is the only intrinsic curvature. Clocks placed on a rectangular lattice will emerge at the other side displaced. The displacement is a logarithmic function of the transverse distance. However the local curvature, or the force with which the shock wave may break a brittle object, is proportional to the second derivative of this displacement, or $1/r^2$.

It is important to observe that the displacement function in Eq. (5.6) can be written as $-8\pi Gmu^\mu f(\tilde{r})$, where $f(\tilde{r}) = -(2\pi)^{-1}\log|\tilde{r}|$ obeys a Laplace equation in two dimensions:

$$\partial^2 f(\tilde{r}) = -\delta^2(\tilde{r}) \,, \tag{5.7}$$

6. Calculation of the S-matrix near a Horizon.

Now that we know how fast moving particles affect each other gravitationally, let us return to the S-matrix Ansatz. We postulate that any physical event that takes place in a region of space that is surrounded by a vacuum, is characterized by an asymptotic *in*-state,

$$|p^1, \sigma^1, \ldots, p^N, \sigma^N\rangle_{\rm in} \,,$$

and an asymptotic *out*-state,

$$|p^1,\sigma^1,\ldots,p^K,\sigma^K\rangle_{\rm out}\,,$$

where the first describes the complete set of particles in the limit $t \to -\infty$, and the latter the complete set of particles in the limit $t \to +\infty$. Here p^i are the particle momenta and σ^i their other quantum numbers. N is the number of ingoing particles and K the number of outgoing ones. The mutual inner products of these states,

$$S = {}_{\rm out}\langle p',\sigma|p,\sigma\rangle_{\rm in}\,, \tag{6.1}$$

are well-defined and form a unitary scattering matrix[a], regardless whether a black hole was formed or not.

Our strategy is as follows: *Assume* that a black hole was formed from some initial state to be referred to as $|1\rangle$, and that it leads to a final state $|\chi\rangle$, consisting of (superpositions of) particles running away at $t \to +\infty$. Now consider a tiny change among the ingoing particles, for instance by adding a light extra particle entering with momentum p through the horizon at angular position θ,φ. The momentum p is to be defined with respect to Kruskal coordinates

This way the complete S-matrix can be derived from known laws of nature. In principle, because in practice one will be forced to make approximations, and in particular at energy ranges approaching the Planck mass the laws of nature are unknown, and so we cannot probe all elements of S. But the properties one can derive are restrictive enough to draw important conclusions. Since our main difficulty consists of describing the properties of particles when they just enter or leave the horizon it makes sense to concentrate on the immediate neighborhood of the horizon. There it is permitted to assume space-time to be approximately flat, that is, replace Kruskal space by Rindler space.

Light cone coordinates are then defined[b] as

$$x^+ = z + t\,, \quad x^- = t - z\,, \quad p^0 = p^+ + p^-\,, \quad p_z = p^+ - p^-\,. \tag{6.2}$$

An ougoing particle ($p^- \approx 0$) has a wave function

$$\psi_{\rm out} = e^{-ip^+_{\rm out}x^-}\,. \tag{6.3}$$

[a] In the infinite time limit one may have to take infra-red divergences into account regarding ultra-soft photons and gravitons; which implies that the particle numbers may tend to infinity in this limit. This however is a well-understood aspect of quantum field theory that will not basically affect the details of our arguments.

[b] Not too much attention is paid to factors of 2 arising from this normalization.

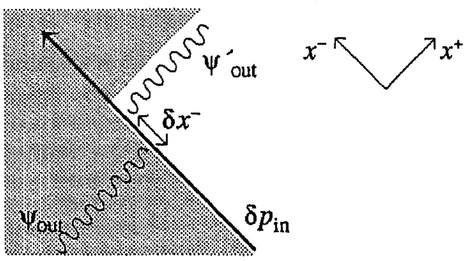

Fig. 5. Effect of ingoing particle on the wave function $\psi_{\rm out}$ of an outgoing particle. The gray region is the inaccessible region of the black hole.

The effect of the gravitational field produced by an extra ingoing particle with momentum $\delta p^-_{\rm in}$ entering the hole at transverse coordinates $\tilde{x}'$ is in first approximation a tiny shift δx^-, depending on the transverse cordinate $\tilde{x}$:

$$\delta x^- = \kappa f(\tilde{x} - \tilde{x}')\delta p^-_{\rm in}\,, \tag{6.4}$$

where $f(\tilde{x})$ is a Green function obeying Eq. (5.7) and $\kappa = 16\pi G$ where G is Newton's constant. We will normalize κ to one[c].

The effect on the outgoing wave functions is:

$$\begin{aligned}\psi_{\rm out} \to \psi'_{\rm out} &= e^{-ip^+_{\rm out}(x^- - \delta x^-)} \\ &= e^{i\hat{P}^+_{\rm out}\delta x^-}\,\psi_{\rm out} = e^{i\int \hat{P}^+_{\rm out}(\tilde{x})f(\tilde{x}-\tilde{x}_1)\delta p^-_{\rm in}(\tilde{x}_1)\,{\rm d}^2\tilde{x}\,{\rm d}^2\tilde{x}_1}\,\psi_{\rm out}\,,\end{aligned} \tag{6.5}$$

where the function $p^-_{\rm in}(\tilde{x}_1)$ is defined as $p^-_{\rm in}\delta(\tilde{x}_1 - \tilde{x}')$. This now one may repeat many times, so as to include the effects of many particles entering at different points $\tilde{x}'$, and since their effects inside the exponential all commute (in this approximation) the outcome is simply a cumulative exponential:

$$\psi_{\rm out} = e^{i\int \hat{P}^+_{\rm out}(\tilde{x})f(\tilde{x}-\tilde{x}_1)p^{-\,{\rm total}}_{\rm in}(\tilde{x}_1)\,{\rm d}^2\tilde{x}\,{\rm d}^2\tilde{x}_1}\,|\chi\rangle\,. \tag{6.6}$$

Here $|\chi\rangle$ is the fixed set of wave packets emitted by a single black hole state $|1\rangle$. By adding or removing as many other particles with momenta $\delta p^-_{\rm in}$ as we please in the in-state we can obtain any other black hole state[d]. If now we characterise the

[c] Note that this then is a normalization different from G=1 used elsewhere in quantum gravity. The difference is a factor 16π. It is a curious aspect of quantum gravity that one will often wish to switch between these two normalizations.

[d] A limitation is that all particles added to the in-state must have negligible energies as seen by the distant observer, and hence we are not yet in a position that we can go from a black hole $|1\rangle$ to an other one with different mass M.

initial black hole by giving the total momentum distribution $p_{\rm in}^-(\tilde{x}')$ of all ingoing particles added to the "reference black hole" $|1\rangle$, and the outgoing states by the momentum distribution $p_{\rm out}^+$ of all outgoing particles added together, we find that all amplitudes can be expressed as

$$\langle p_{\rm out}^+(\tilde{x})|p_{\rm in}^-(\tilde{x}')\rangle = \mathcal{N} e^{i\int p_{\rm out}^+(\tilde{x}) f(\tilde{x}-\tilde{x}_1) p_{\rm in}^-(\tilde{x}_1)\, {\rm d}^2\tilde{x}\, {\rm d}^2\tilde{x}_1} . \tag{6.7}$$

Here $\mathcal{N}$ is one common unknown factor referring to our scattering matrix Ansatz. It is now simple to demand it to be normalized such that the matrix (6.7) be unitary. Its overall phase is of course less relevant being physically unobservable.

It may be argued that the scattering amplitudes thus obtained at the horizon are related (by a mapping) to flat space scattering amplitudes that contained one "infinitely energetic" concentrated beam of particles $|1\rangle$ entering at the left and one such beam $|1'\rangle$ leaving at the left, while the black hole in- and out-states correspond to particles entering and leaving at the right. In this mapping, the states $|1\rangle$ and $|1'\rangle$ are always kept fixed. Thus (see Fig. 6):

$$\langle p_{\rm out}^+|p_{\rm in}^-\rangle_{\rm Black\ Hole} = \langle p_{\rm out}^+; 1'|p_{\rm in}^-; 1\rangle_{\rm Flat\ spacetime} \tag{6.8}$$

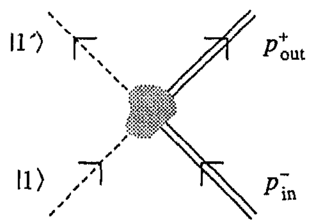

Fig. 6.
Eq. (6.8)

Remembering that according to Eq. (5.7), $f(\tilde{x})$ is the Green function of the transverse Laplacian, we can rewrite eq. (6.7) as a functional integral,

$$\langle p_{\rm out}^+(\tilde{x})|p_{\rm in}^-(\tilde{x}')\rangle = \mathcal{N}' \int \mathcal{D}u^+\mathcal{D}u^- e^{i\int {\rm d}^2\tilde{x}\left[p^+(\tilde{x})u^-(\tilde{x}) + p^-(\tilde{x})u^+(\tilde{x}) - \tilde{\partial}u^+(\tilde{x})\tilde{\partial}u^-(\tilde{x})\right]} , \tag{6.9}$$

where $\mathcal{N}'$ is a new normalization factor[a].

[a] Note that Eq. (5.7) receives corrections due to the curvature of the horizon when the black hole is not infinitely big. These amount to a 'mass term' next to the 'kinetic term' in Eq. (6.9).

Now up to this point we have been treating the in- and out-states in a representation where the longitudinal momenta $p^{\pm}$ and the transverse positions $\tilde{x}$ were fixed. Every particle in such a state corresponds to a Dirac delta contribution to the momentum functions $p^{\pm}(\tilde{x})$. If instead we wish to describe these states entirely in the momentum representation we need to Fourier transform with respect to the transverse dimensions:

$$\begin{aligned}|p_{\rm in}^1,\dots,p_{\rm in}^N\rangle \equiv \mathcal{N}_1 \int \mathrm{d}^2\tilde{x}^1 \dots \int \mathrm{d}^2\tilde{x}^N e^{i\tilde{p}_{\rm in}^1\tilde{x}_{\rm in}^1 + \dots + i\tilde{p}_{\rm in}^N\tilde{x}_{\rm in}^N} \\ \big|p_{\rm in}^-(\tilde{x}) = p_{\rm in}^1\delta^2(\tilde{x}-\tilde{x}^1) + \dots + p_{\rm in}^N\delta^2(\tilde{x}-\tilde{x}^N)\big\rangle\,,\end{aligned} \tag{6.10}$$

where $\mathcal{N}_1$ is again a normalization factor, and for the out-state containing M outgoing particles we write a similar expression. Consequently we have for such states the scattering matrix

$$\begin{aligned}\langle\{p_{\rm out}^i\}|\{p_{\rm in}^j\}\rangle = \int \mathrm{d}^2\tilde{x}_{\rm in}^1 \dots \mathrm{d}^2\tilde{x}_{\rm out}^K\, \mathcal{N}'' \int \mathcal{D}u^+\mathcal{D}u^- \\ \exp\int \mathrm{d}^2\tilde{x}\big[-i\tilde{\partial}u^+\tilde{\partial}u^- + i\sum_{i=1}^{K} p_\mu^{(i)}x^{(i)\mu}\big]\,,\end{aligned} \tag{6.11}$$

where $K = N + M$ stands for the total number of ingoing and outgoing particles, and the four vectors $x^{(i)\mu}$ are defined as $(\tilde{x}, u^+, u^-)$ at the points where particle number (i) enters or leaves the horizon. Again $\mathcal{N}''$ is a normalization factor.

It is here that one notes the striking resemblance with string theory amplitudes. The functional integral coincides with the one for strings that show infinitesimal oscillations around a straight configuration, and the integrations over the variables $\tilde{x}$ are exactly the Koba-Nielsen integrations over moduli space. The only important difference with a closed string amplitude is the fact that here the string tension constant is purely imaginary. In strong-interaction particle theories such an imaginary part arises if the string is unstable against decay due to pair creation of quarks.

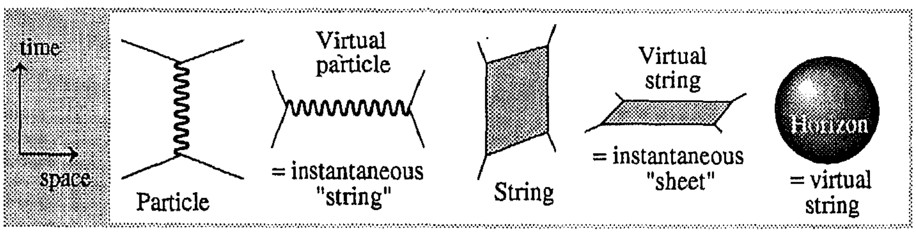

Fig. 7. Identification of horizon as an instantaneous virtual string.

That a black hole horizon may be identified with a string may sound surprising since a black hole is obviously a three-dimensional object whereas a string is only one-dimensional in space. The situation is clarified in Fig. 7. The horizon can behave as a virtual string because time stands still there. Both the ingoing and the outgoing particles act as closed string loops on the mass shell in the amplitude (6.11). They exchange a virtual string that wraps exactly once around the black hole, as also sketched in Fig. 8.

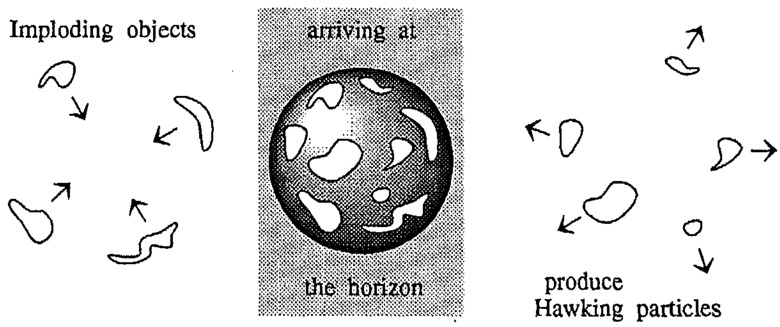

Fig. 8. Black hole formation and evaporation in s string picture.

7. The new Hilbert Space; an Operator Algebra [14]

Even though from a "string" point of view the amplitudes look familiar, they appear to be very odd in terms of more conventional physical descriptions. Usually one has a Fock space. The states are described as

$$\left|\mathbf{p}^{(1)},\sigma^{(1)},\mathbf{p}^{(2)},\sigma^{(2)},\ldots,\mathbf{p}^{(N)},\sigma^{(N)}\right\rangle, \tag{7.1}$$

where the number N of particles is well-defined and one enumerates the particle momenta $\mathbf{p}^{(i)}$ and their spins and possible other quantum numbers $\sigma^{(i)}$.

In stead of this we now have to deal with a Hilbert space of states where the *longitudinal part* of the *total particle momentum distribution* (in Kruskal space)

$$|\{p^-(\tilde{x})\}\rangle \quad \text{or} \quad |\{p^+(\tilde{x})\}\rangle \tag{7.2}$$

is given. Now in Eq. (6.10) we defined an element of the basis (7.1) in terms of states (7.2) to obtain a string amplitude (6.11) defined in a (somewhat) more conventional Fock space, that of string theory. However, the scattering matrix (6.9) will only be unitary in the Hilbert space (7.2), not evidently in (7.1).

It is now convenient to introduce another basis from the basis of states (7.2), by means of a functional Fourier transformation (using as before the variable notation $p_{\rm in} = p^- = p^-_{\rm in}$ and $p_{\rm out} = p^+ = p^+_{\rm out}$):

$$|\{u^{\rm in}(\tilde x)\}\rangle \equiv \mathcal{N}_0 \int \mathcal{D}p_{\rm in}(\tilde x)\, e^{-i\int {\rm d}^2\tilde x\, p_{\rm in}(\tilde x)u^{\rm in}(\tilde x)}|\{p_{\rm in}(\tilde x)\}\rangle\,, \tag{7.3}$$

where $\mathcal{N}_0$ provides for an appropriately normalized measure. One then may define an operator $u^{\rm in}(\tilde x)$ that turns out to obey the commutation rule

$$[p_{\rm in}(\tilde x), u^{\rm in}(\tilde x')] = -i\delta^2(\tilde x - \tilde x') \tag{7.4}$$

and similarly we have

$$[p_{\rm in}(\tilde x), p_{\rm in}(\tilde x')] = 0\,, \tag{7.5}$$

$$[u^{\rm in}(\tilde x), u^{\rm in}(\tilde x')] = 0\,; \tag{7.6}$$

$$[p_{\rm out}(\tilde x), u^{\rm out}(\tilde x')] = -i\delta^2(\tilde x - \tilde x')\,, \tag{7.7}$$

$$[p_{\rm out}(\tilde x), p_{\rm out}(\tilde x')] = 0\,, \tag{7.8}$$

$$[u^{\rm out}(\tilde x), u^{\rm out}(\tilde x')] = 0\,. \tag{7.9}$$

The relation (6.7) now implies

$$u^{\rm out}(\tilde x) = -\int {\rm d}^2\tilde x'\, f(\tilde x - \tilde x')p_{\rm in}(\tilde x') \tag{7.10}$$

and since $\tilde\partial^2 f(\tilde x - \tilde x') = -\delta^2(\tilde x - \tilde x')$,

$$\tilde\partial^2 u^{\rm out}(\tilde x) = p_{\rm in}(\tilde x)\,, \tag{7.11}$$

and similarly

$$\tilde\partial^2 u^{\rm in}(\tilde x) = -p_{\rm out}(\tilde x)\,, \tag{7.12}$$

so that

$$\begin{aligned} [p_{\rm out}(\tilde x), p_{\rm in}(\tilde x')] &= -i\tilde\partial^2\delta^2(\tilde x - \tilde x')\,,\\ [u^{\rm out}(\tilde x), u^{\rm in}(\tilde x')] &= if(\tilde x - \tilde x')\,. \end{aligned} \tag{7.13}$$

In this algebra the operator $u^{\rm out}$ can be interpreted as the precise location of the observed outgoing particles with respect to the horizon, or equivalently, the horizon with respect to these particles. Similarly, one could interpret $u^{\rm in}$ as an operator fixing the position of the past event horizon. We now observe that $u^{\rm out}$ and $u^{\rm in}$ do not commute. If now we define a *white hole* as an object with well-defined event horizon in the past, so that it is the time-reverse of a black hole, then we observe that white holes may be regarded as *quantum mechanical superpositions* of the class of all black holes, and vice versa. Thus white holes, long speculated about in general relativity, now have a natural place in our quantum mechanical theory of black holes. The in-Hilbert space has the ingoing particles p_{in} well-defined, so that consequently the future event horizon $u^{\text{out}}(\tilde{x})$ is precisely localizable. In the appropriate basis elements for the out-Hilbert space we have the outgoing particles and their momenta p_{out} precisely listed, and that is why they generate a well-localizable past horizon. According to the S-matrix the mapping relating the in-Hilbert space (in its appropriate basis) to the out-Hilbert space (in the basis with the outgoing particles) produces quantum mechanical superpositions, and this is why a white hole consists of superpositions of black holes.

8. The transverse Gravitational Force [15].

What has not yet been achieved is an understanding of the discrete nature of Hilbert space near the horizon. To achieve that most likely the *transverse* components of the gravitational force have to be taken into account. So-far, the gravitational shifts, as described in Sect. 6, were assumed to be entirely in the longitudinal (x^+ or x_-) direction. Neglecting the sideways components is tantamount to ignoring the transverse components of the ingoing momenta. Assuming then the shifts to be less than the Planck length implies that also the outgoing momenta are kept such that these shifts cannot be resolved, *i.e.* the outgoing momenta were kept below the Planck values also. Can one improve the procedure?

The transverse components of the momenta are not independent degrees of freedom. They are the generators of translations in the transverse directions. Their commutator relations with the other operators are well-defined. Let $i,\ j = 1$ or 2 be transverse indices. Then

$$\begin{aligned} [\tilde{p}^i(\tilde{x}),\, u(\tilde{x}')] &= -i\delta^2(\tilde{x}-\tilde{x}')\tilde{\partial}_i u(\tilde{x})\,, \\ [\tilde{p}^i(\tilde{x}), p(\tilde{x}')] &= ip(\tilde{x})\tilde{\partial}_i\delta^2(\tilde{x}-\tilde{x}')\,, \\ [\tilde{p}^i(\tilde{x}),\tilde{p}^j(\tilde{x}')] &= i\tilde{p}^j(\tilde{x})\tilde{\partial}_i\delta(\tilde{x}-\tilde{x}') - i\tilde{p}^i(\tilde{x}')\tilde{\partial}_j\delta^2(\tilde{x}-\tilde{x}')\,, \end{aligned} \tag{8.1}$$

which is required to hold for the in-Hilbert space as well as the out-Hilbert space.

These operators can be constructed explicitly. One gets

$$\begin{aligned} \tilde{p}^{\text{in}}(\tilde{x}) &= p_{\text{in}}(\tilde{x})\tilde{\partial}u^{\text{in}}(\tilde{x})\,, \\ \tilde{p}_{\text{out}}(\tilde{x}) &= p_{\text{out}}(\tilde{x})\tilde{\partial}u^{\text{out}}(\tilde{x})\,. \end{aligned} \tag{8.2}$$

Now the transverse gravitational shifts would have been correctly reproduced if these operators would obey commutation rules similar to the ones of the longitudinal momenta, eq. (7.13):

$$[\tilde{p}^i_{\text{out}}(\tilde{x}),\tilde{p}^j_{\text{in}}(\tilde{x}')] \stackrel{?}{=} -i\delta^{ij}\tilde{\partial}^2\delta^2(\tilde{x}-\tilde{x}')\,, \tag{8.3}$$

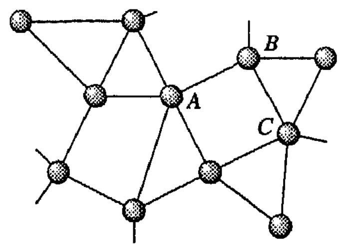

Fig. 9. Lattice at the horizon.

but this does not at all follow from (8.2). Indeed, (8.3) in conjunction with (8.2) violates the Jacobi identities. This should have been expected. The continuous operators $p^{\pm}(\tilde{x})$ cannot be the ones we are looking for as soon as we try to deal with a Planckian resolution.

It is natural to suspect that $p^{\pm}(\tilde{x})$ must be replaced by discrete operators. A proposal could be to introduce a lattice on the horizon. The momenta are now defined only on the lattice sites A, B, $C, \ldots$, and in stead of the operators $p^{\pm}(\tilde{x})$ we now define the momenta only on these sites, $p_A^{\pm}$, $p_B^{\pm}, \ldots$. In addition the transverse coordinates $\tilde{x}_A$, $\tilde{x}_B, \ldots$ are now also operators, to be defined separately for the in- and for the out-Hilbert space. Their conjugated operators are $\tilde{p}_A$, $\tilde{p}_B, \ldots$. Thus, so-far, the deviation from the continuum picture is rather small. The commutation relations are now simply

$$\begin{aligned}
[x_A^{i,\mathrm{in}}, x_B^{j,\mathrm{in}}] &= 0\,, \\
[p^i_{\mathrm{in},A}, p^j_{\mathrm{in},B}] &= 0\,, \\
[x_A^{i,\mathrm{in}}, p^j_{\mathrm{in},B}] &= i\delta^{ij}\delta_{AB}\,,
\end{aligned} \tag{8.4}$$

where now the indices take the three values 1,2,3. In the out-Hilbert space we have similar commutation rules.

To describe the scattering matrix however we now need a relation between the in- and the out-operators which should be a natural extension of (7.10)–(7.12). This requires a discrete version of the Laplacian. To obtain something that in a continuum limit would turn into a transverse Laplacian we need to introduce the notion of "nearest neighbors" on these points. For reasons to be explained shortly, we take this to be a random lattice, as depicted in Fig. 9. Every site A now has a collection of $N(A)$ neighbors. An expression that will approximately produce a Laplacian in the continuum limit is

$$\tilde{\partial}^2 u(\tilde{x}) \to -u_A + \langle u \rangle_{\text{linked to } A}\,, \tag{8.5}$$

where the average is taken of the u values on all points B that are neighbors of A.

Temporarily we multiply this with $N(A)$, which is nothing but a redefinition of the norm, so in the end we will have to correct by factors $\langle N\rangle$, the average of the number of neibors. We then find that (7.13) is replaced by

$$[p^i_{\mathrm{out},A}, p^j_{\mathrm{in},B}] = i\delta^{ij} C_{AB}\,, \tag{8.6}$$

with

$$\begin{aligned} &C_{AB} = 1 \quad \text{iff} \quad A \text{ and } B \text{ are neighbors,} \\ &C_{AA} = -N(A)\,. \end{aligned} \tag{8.7}$$

What is still not quite right in this model is that "neighbors" may still have widely separated values of $\tilde{x}$ so there are regions in Hilbert space where things go wrong. However we may have captured a more interesting picture than the continuum model. Suppose that we keep the total momenta bounded, both for the in-states and the out-states. This would imply bounds both on $\sum_A p_A^2$ and on $\sum_A x_A^2$. But since the x 's and the p 's are each other's conjugates this implies a limitation on the total number of allowed states, as we know from the harmonic oscillator. So we did achieve discreteness of the Hilbert space. Also note that the uncertainty relations prohibit any "continuum limit" beyond the Planck length. There is a natural lattice length.

9. Non-gravitational Interactions [16]

It is quite instructive to try to elaborate the S-matrix Ansatz as accurately as one can. For example, there are other than gravitational interactions when ingoing and outgoing objects meet each other at not too high energies.

The simplest other interaction that one can take into account is electromagnetism. The procedure is as in Sect. 6, but now we take also electric charge into account. A charged ingoing particle produces an electric current

$$j_\mu(x^+, x^-, \tilde{x}) \;=\; e'\delta^+_\mu \delta(x^+)\delta^2(\tilde{x} - \tilde{x}')\,, \tag{9.1}$$

where e' is the electric charge and $\tilde{x}'$ the point where the charged particle enters the horizon. It produces a vector potential

$$A_\mu \;=\; e\delta^+_\mu \delta(x^+) A(\tilde{x}), \quad \text{with} \quad \tilde{\Delta} A(\tilde{x}) \;=\; -\delta^2(\tilde{x} - \tilde{x}')\,. \tag{9.2}$$

The effect of this vector potential is that the wave of another particle with charge e traversing the shock front will undergo a phase shift:

$$\Psi_{\mathrm{out}} \to \Psi_{\mathrm{out}} e^{i\delta\Lambda(\tilde{x})}\,, \quad \text{with} \quad \delta\Lambda \;=\; eA(\tilde{x})\,. \tag{9.3}$$

Table 1.

Your favorite 4 dimensional "Standard Model"	**Induced operator theory in 2 dimensions**
• Spin 2: $g_{\mu\nu}(\mathbf{x},t)$ gauge generator: $u^\mu(\mathbf{x},t)$	Spin 1 "string" variable $x^\mu(\tilde{x})$
• Spin 1: $A_\mu(\mathbf{x},t)$ gauge generator $\Lambda(\mathbf{x},t)$ mod $2\pi/e$	Scalar (spin 0) variable $\phi(\tilde{x})$ mod $2\pi/e$
• Spin 0: $\varphi(\mathbf{x},t)$	c-number, $C(\tilde{x})$
∘ Spin $\frac{1}{2}$: $\psi(\mathbf{x},t)$	? (non-propagating)
• Spin $\frac{3}{2}$ gravitino gauge generators: $\theta(\mathbf{x},t)$, $\overline{\theta}(\mathbf{x},t)$	spin $\frac{1}{2}$ fermion ψ, $\overline{\psi}(\tilde{x})$
• Magnetic monopole $M_m(\mathbf{x},t)$	Disorder operator $\sigma\phi = \phi\sigma e^{i\theta(\tilde{x})}$
• Mass generation through Higgs mechanism: $(\partial^2 - M^2)A_\mu = J_\mu$	Mass term for ϕ: $(\tilde{\partial}^2 - M^2)\phi = \rho$
• Confinement: $\langle M_m(\mathbf{x},t)\rangle \neq 0$	Mass term for $\sigma(\tilde{x})$: $(\tilde{\partial}^2 - M^2)\sigma = \rho^{\text{dual}}$
• Non-Abelian gauge field, gauge generator $\Omega(\mathbf{x},t)$	Spin 0 non-linear sigma model: $\mathcal{L} = \int \tilde{\partial} g^{-1}(\tilde{x})\, \tilde{\partial} g(\tilde{x})\, \mathrm{d}^2\tilde{x}$

Combining this phase factor with the contribution from the gravitational shift (6.6-7) we get

$$\begin{aligned}\langle p^+_{\text{out}}(\tilde{x})|p^-_{\text{in}}(\tilde{x}')\rangle = \mathcal{N} e^{i\int p^+_{\text{out}}(\tilde{x})f(\tilde{x}-\tilde{x}_1)p^-_{\text{in}}(\tilde{x}_1)\,\mathrm{d}^2\tilde{x}\,\mathrm{d}^2\tilde{x}_1} \\ \times\, e^{i\int \rho_{\text{out}}(\tilde{x})\rho_{\text{in}}(\tilde{x}')f_1(\tilde{x}-\tilde{x}')\mathrm{d}^2\tilde{x}\mathrm{d}^2\tilde{x}'}\,,\end{aligned} \tag{9.4}$$

where $\rho_{\text{in,out}}$ are the charge densities of the in- and outgoing matter. f_1 is a Green function obeying the same Laplace equation as f except for finite size black holes where there is a slight difference due to curvature of the horizon.

We can rewrite the new term as

$$\int \mathcal{D}\phi e^{i\int \mathrm{d}^2\tilde{x}\left((\rho_{\text{out}}-\rho_{\text{in}})\phi(\tilde{x}) - \frac{1}{2}(\tilde{\partial}\phi)^2\right)}\,, \tag{9.5}$$

which differes from the original expression only by two universal phase factors for the in- and out-state separately; only the cross term involving the product $\rho_{\text{in}}\rho_{\text{out}}$

corresponds to an observable contribution to the scattering matrix. Now we observe that in all respects the charge density $\rho_{\rm out}(\tilde{x}) - \rho_{\rm in}(\tilde{x})$ acts as a fifth component of the momentum $p(\tilde{x})$, exactly as in a Kaluza-Klein theory. Indeed, since the total electric charge is quantized, $\int d^2\tilde{x}\,\rho(\tilde{x}) = Ne$, the functional integrand will be periodic in ϕ with period $2\pi/e$. Here e is of course the quantum of electric charge.

The effects of other types of forces are more difficult to evaluate. In Table 1 we use the observation that the induced operator variables on the two-dimensional surface of the horizon have spins that are one below the spin values of the dynamical fields in 4 dimensions. The effect of a scalar field in 4 dimensions is that its value on the horizon does not Lorentz transform and hence acts as a c-number in the effective interactions there. In particular, if a scalar fields causes a Higgs mechanism in 4 dimensions then the two-dimensional theory also generates a mass term (since the photon field then becomes short range); this mass term then breaks the corresponding symmetry explicitly. If in 4 dimensions confinement takes place the effect in two dimensions is that the *disorder operator* $\sigma(\tilde{x})$ looses its invariance under translations, and consequently the corresponding effect on the *order operator* $\phi(\tilde{x})$ can no longer be well defined; the order operator itself is then ill-defined.

How exactly to formulate the effect of a 4-dimensional spin $\frac{1}{2}$ field $\psi(\mathbf{x},t)$ on the two-dimensional operators is not yet quite understood.

Further elaborations on the theme of the Scattering Matrix Ansatz are described in Refs [17].

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