

The Holographic Principle

Opening Lecture

Gerard 't Hooft

Institute for Theoretical Physics
University of Utrecht, Princetonplein 5
3584 CC Utrecht, the Netherlands

and

Spinoza Institute
Postbox 80.195
3508 TD Utrecht, the Netherlands

e-mail: g.thoof@phys.uu.nl
internet: <http://www.phys.uu.nl/~thoof/>

Abstract

After a pedagogical overview of the present status of High-Energy Physics, some problems concerning physics at the Planck scale are formulated, and an introduction is given to a notion that became known as “the holographic principle” in Planck scale physics, which is arrived at by studying quantum mechanical features of black holes.

1. Introduction.

To open an International School at which many important issues of modern elementary particle physics will be discussed, it seems appropriate to start with a bird's eye view of the recent developments in the field. Another motivation to do so is that fundamental physics today appears to have reached a new stage, at which some reflection is needed over the past, in order to explain our present standpoints and views, and to justify the kinds of questions that we think we have to ask today, in order to enable us to proceed in our field.

Before the '70s, we had the following picture of the fundamental forces. First, there was *Quantum Electrodynamics* (QED), a very successful scheme to describe (nearly) all electric and magnetic features of our beloved particles. It was understood how to perform impressively accurate calculations by using perturbation expansions with respect to $\alpha = e^2/4\pi\hbar c \approx 1/137$, a small parameter¹. Although it was understood how to *renormalise* the apparent divergences of the theory, it was still something of a mystery why this procedure worked at all, and indeed, sometimes (for instance when electromagnetic mass differences were to be calculated) it did not appear to work.

As for the *weak forces*, we only had an 'effective' expression for the interaction², that however was not renormalisable, which made it impossible to calculate any of the higher order radiative corrections.

The *strong force* was in an even worse state. It was usually treated as a 'black box', for which only the symmetry pattern was well established. Several simplistic but quite instructive models could be written down (the Gell-Mann-Lévy model³, the dual resonance model⁴), but they were mutually incompatible, and since the coupling strength was large, perturbation expansions, even if you could renormalise, appeared to be meaningless.

Then, the revolution of the '70s came. The discovery that non-Abelian gauge theories are renormalisable⁵ enabled us to make a very important step. We could now ask the question: "What is the *most general* perturbatively renormalisable quantum field theory?"⁶

The answer turned out to be that our theory must consist of three kinds of basic particles, to be represented by fundamental fields. They are distinguished by the value S of the intrinsic spin:

$S = 1$: These particles cannot be described unless you have a (Abelian or non-Abelian) gauge theory. The gauge group may be any local, compact Lie group G , for instance $G = SU(3) \otimes SU(2) \otimes U(1) \otimes \dots$

$S = \frac{1}{2}$: These particles must be described by Dirac fields, ψ^L and ψ^R , of which ψ^L transforms as a $\mathbf{2} \times \mathbf{1}$ representation of the algebra $SU(2)^{\text{Left}} \otimes SU(2)^{\text{Right}}$ of the Lorentz group (in Euclidean notation) and ψ^R transforms as a $\mathbf{1} \times \mathbf{2}$. Each of these fields may be in any kind of finite-dimensional representation of the gauge group G , but there is a very important restriction: an *anomaly* may arise in the contributions of triangle diagrams to the matrix elements of axial currents. It is not allowed to have currents with anomalies in them, coupled to gauge fields. If ψ^L and ψ^R are in different representations of the gauge group G , one must require that their contributions to the axial anomalies nevertheless cancel out⁷. In the current version of the Standard Model, this is indeed the case.

$S = 0$: Any set of scalar fields ϕ may be present, in any finite representation of G . Its self-interactions must be polynomial of degree four, and its interaction with the fermions must be through gauge-invariant Yukawa terms of the form $\bar{\psi}^L \phi \psi^R + \text{h.c.}$ Via the Higgs mechanism⁸, these fields may produce masses for vector particles and Dirac particles.⁹

The present Standard Model obeys all of these requirements — and more: the renormalization group equations tell us that the coupling strengths vary as the energies increase, in such a way that perturbation expansions can be applied up to extremely high energies*. Yet many questions are still not answered:

- What determined Nature’s choice for the gauge group G , the fermionic and scalar representations of G , the number of leptonic and quark generations, and the values of the coupling strengths, in particular the details of the Kobayashi-Maskawa matrix? ¹¹
- How do we explain the large hierarchy of scales in Nature? The disparity between the Planck scale, the electro-weak scale, and the scale(s) of the neutrino masses is just one of several questions of this sort.
- What is the role of supersymmetry, and how is supersymmetry broken? ¹²
- How do we couple the gravitational force, and why is the cosmological constant as small as it is, or if it vanishes altogether, again, why? There is no known symmetry that ‘protects’ the cosmological constant against renormalisation effects.

Different answers to some or all of these questions are presently being investigated. Judging from past experiences, it must be of extreme importance to ask the right questions.

Are there any further useful results to be expected from experiments? Three classes of experimental avenues have not yet been completed, and may give us great improvements in our understanding, although all of these are becoming more and more difficult, demanding increasing skills of the experimenters:

a) At increasing, higher energies, the following is to be expected, and I think will be done:

- The Higgs is there to be discovered.
- Supersymmetry partners of all presently known particles may be detected, hopefully some time soon. At first sight, the fact that supersymmetric patterns were discovered in nuclear physics ¹³ has little to do with the question of supersymmetry among elementary particles, but it may indicate that, as the spectrum of particles is getting more and more complex, some supersymmetric patterns might easily arise, even if there is no ‘fundamental’ reason for their existence.
- Other new structures may also be found at higher energies. The most pleasant surprises will be the unexpected ones, which may open up new fields. It is generally believed that the present model will break down beyond a TeV or so, and this energy level will be within reach in a decade or so ¹⁴.

These high energy experiments address the unknown physics in a direct manner, and they are therefore most important.

b) On rare occasions, new results may also be expected from precision experiments at lower energies. There have been a number of interesting examples in the recent past:

- Atomic parity violations could be measured with better than one percent precision,

* An uncertain factor here is the Higgs self-coupling, since the Higgs mass is still unknown. ¹⁰

in spite of the fact that these are minute effects¹⁵, yielding independent confirmation of the effects due to W and Z exchanges within atomic nuclei.

- The K_L/K_S system is a beautiful laboratory. Precision measurements can be made of the parameter ε'/ε , which may reveal features from deeply inside, or possibly beyond, the Standard Model, as we will learn at this School¹⁶.
- Other known fundamental principles of Theoretical Physics can be put to a test, such as CPT invariance, relativity tests, the ratios Q/M can be compared between particles and antiparticles¹⁷, the Quantum Mechanics of gravitating systems can be investigated, etc.
- New ideas were launched suggesting that Newton's law of the gravitational force might change at scales below a mm. This can be experimentally tested.¹⁸
- Tiny mass terms that produce mixing between various neutrino species can be detected in dedicated experiments.¹⁹
- And there are doubtlessly many more subtle effects that may be discovered and that will alter our views concerning the fundamental interactions.

c) A third source of information is cosmology. It used to be well within the domain of Science-Fiction, but nowadays cosmological models are becoming more mature. They yield precise predictions that can be verified by astrophysical observations. Models of the inflationary universe probe deeply into regions at extremely high energy, and so the information they deliver is unique:

- Structures in the spectrum of the cosmic background radiation are predicted and more detailed observations are to be expected.
- The distribution of galaxies is speculated to be due to quantum fluctuations in a very early universe. They will be calculated and compared to what is observed.
- The search for dark matter continues. The outcome will deeply affect our thinking about the fundamental interactions.
- Statistical analysis of distant galaxies may finally also reveal the presence of a cosmological constant term in the Einstein-Hilbert action.
- Other tests of the models, for instance the baryon-antibaryon asymmetry and CP violation.

In spite of this long list, there are reasons to worry about the increasingly difficult barriers from behind which we are trying to understand the small-distance structure of our world. Which purely theoretical approaches will help us find the answers? We have to concentrate on *fundamental inconsistencies* in our present picture. There are many of these:

- As it was already mentioned, the hierarchies seen in the distance scales are not properly explained by what is presently known.
- The apparent absence of a cosmological constant is at odds with what we understand about Quantum Gravity.
- Indeed, quantizing gravity is still a deep problem. Superstring theory is vigorously

trying to bring the gravitational force under control, but it surely is a wild animal. From superstrings came D -branes, from D -branes came “ M -theory”, but it has as yet not been possible to even come close to an accurate formulation of the laws. These ideas are of extreme importance, but new avenues must still be found.

What is known for sure is that Quantum Mechanics works, that the gravitational force exists, and that General Relativity works. The approach advocated by me during the last decades is to consider in a direct way the problems that arise when one tries to combine these theories, in particular the problem of gravitational instability. These considerations have now led to what is called “the Holographic Principle”, and it in turn led to the more speculative idea of deterministic quantum gravity. This theory, and the effects of *dissipation of information*, will be discussed in a separate lecture. Our central issue is: *What is Nature’s bookkeeping system at the Planck scale?*

2. The inevitable existence of black holes

In sufficiently large amounts of matter, gravitational collapse is inevitable.²⁰ There are various ways to derive this fact. First, one may consider a stationary, spherically symmetric configuration of matter, held together by gravity. Near the surface, we assume that there is a region $r = r_1$ where the temperature is low enough so that the density ρ_1 there is sufficiently high, say that of water. Assume that inside the sphere $r = r_1$, there is a certain amount of mass M_1 . The pressure p at this surface is still negligible. It is now easy to argue that M_1 must be subject to an upper limit. In any case, the pressure p rises if we look at smaller distances r from the centre. If the density were constant, and the general relativistic effects negligible, then one could readily compute the pressure at the centre. However, the density is likely to increase if we go down. In fact, if we assume our material to be *non-exotic*, then a finite compressibility follows. Matter is defined to be non-exotic if the speed of sound v_s is less than the speed of light, c . This means that the gradient of the gravitational field will rise, and hence the gradient in the pressure will become steeper, and this will cause an instability. If M_1 was chosen large enough, there will be a point $r = r_2 < r_1$ where the pressure becomes infinite. Even without any other relativistic arguments, this gives us as a limit: $\frac{2}{3}\pi G_N \rho_1 r_1^2 < 1$. Adding general relativistic effects correctly will give a more stringent limit, as I will briefly explain later.

But first: does matter have to be non-exotic? Suppose the speed of sound exceeds the speed of light. Would there be an immediate contradiction? Special relativity would normally demand that no signal can go faster than light. However, the reason why we demand this is causality: no signal should be able to propagate *backwards* in time. This is then combined with demanding Lorentz-invariance. However, matter in equilibrium represents a preferred Lorentz frame, so we could drop the latter demand. Still, there must be restrictions. Consider two regions in which matter has different local velocities. Imagine two adjacent pipes in which matter streams in opposite directions, and in both, sound goes faster than light, also in opposite directions. Due to the time shifts when Lorentz transforming, an outside observer may see both signals move backwards in time. This, in principle, could then generate a closed loop of information transport in space-time, which is an undesirable situation. This we must forbid.

Even exotic matter, however, will not be able to stop black holes from being

formed. This is seen if we insert the complete general relativistic equations instead of our above pseudo-relativistic argument. These are the so-called Tolman-Oppenheimer-Volkoff equations²¹ These equations tell us how density and pressure increase when followed inwards, given some equation of state. It is an elementary exercise to solve these equations for constant density. Even then, one finds that, due to space-time curvature, the pressure diverges to infinity at a finite radius, if we start with too much mass and a too high density at a too small radius on the surface.

We can however also produce a black hole from ordinary matter at zero pressure. Consider a spherically symmetric arrangement of matter in the form of a shell, with some finite thickness. We allow the shell to contract due to its own gravitational field. Inside the shell, there is no gravitational field at all, something that one can understand using the same arguments that tell us that inside a conducting metal sphere there is no electric or magnetic field. If the original amount of material was big enough, the contraction will proceed, and, in the limit of zero pressure and purely radial, spherically symmetric motion, the equations can easily be solved exactly. We obtain flat space-time inside, and a pure Schwarzschild metric outside. As the ball contracts, a moment will arrive when the Schwarzschild horizon appears. From that moment on, an outside observer will no longer detect any radiation from the shell, but a black hole instead.

3. Hawking radiation and quantum states

The standard generally relativistic black hole solution has as a special feature that shortly after its formation, no signals will be seen coming out. It should be truly black. As is well-known, this picture changed when Hawking²² discovered an elementary consequence of quantum field theory when applied to fields living in the black hole metric. The rearrangement of creation and annihilation operators is such that the states near the horizon are not truly vacuum, but they contain a precisely computable density of particles, which are emitted as black body radiation at a temperature given by²²

$$kT_H = \frac{\hbar c^3}{8\pi G M_{\text{BH}}} . \quad (3.1)$$

This result allows us to compute the *density of quantum states* of a black hole. The easiest way to do this is by using thermodynamics. However, one could object that a black hole is not truly in thermodynamic equilibrium; if energy is added to a black hole, its mass and its size will increase, and consequently its temperature will drop.

We can avoid thermodynamics by deriving the spectral density of a black hole directly from its Hawking temperature. All one needs is some form of time reversal invariance²³. We have at our disposal both the *emission rate* (the Hawking radiation intensity), and the *capture probability*, or the effective cross section of the black hole for infalling matter.

In units at which $G = \hbar = c = 1$, the cross section σ is approximately:

$$\sigma \approx 2\pi R^2 = 8\pi M^2 , \quad (3.2)$$

and slightly more for objects moving in slowly. The emission probability Wdt for a given particle type, in a given quantum state, in a large volume $V = L^3$ is:

$$Wdt = \frac{\sigma(\mathbf{k})v}{V} e^{-E/kT} dt, \quad (3.3)$$

where \mathbf{k} is the wave number characterizing the quantum state, v is the particle velocity, and E is its momentum.

Now we *assume* that the process is also governed by a Schrödinger equation. This means that there exist quantum mechanical transition amplitudes,

$$\begin{aligned} \mathcal{T}_{\text{in}} &= {}_{\text{BH}}\langle M + E | \mathcal{T} | M \rangle_{\text{BH}} | E \rangle_{\text{in}}, \\ \text{and } \mathcal{T}_{\text{out}} &= {}_{\text{BH}}\langle M | {}_{\text{out}}\langle E | \mathcal{T} | M + E \rangle_{\text{BH}}, \end{aligned} \quad (3.4)$$

where the states $|M\rangle_{\text{BH}}$ represent black hole states with mass M , and the other states are energy eigenstates of particles in the volume V . In terms of these amplitudes, using the so-called Fermi Golden Rule, the cross section and the emission probabilities can be written as

$$\sigma = |\mathcal{T}_{\text{in}}|^2 \varrho(M + E) / v, \quad (3.5)$$

$$W = |\mathcal{T}_{\text{out}}|^2 \varrho(M) \frac{1}{V}. \quad (3.6)$$

where $\varrho(M)$ stands for the level density of a black hole with mass M . The factor v^{-1} in Eq. (3.5) is a kinematical factor, and the factor V^{-1} in W arises from the normalization of the wave function.

Now, time reversal invariance relates \mathcal{T}_{in} to \mathcal{T}_{out} . To be precise, all one needs is *PCT* invariance, since the parity transformation P and charge conjugation C have no effect on our calculation of σ . Dividing the expressions (3.5) and (3.6), and using (3.3), one finds:

$$\frac{\varrho(M + E)}{\varrho(M)} = e^{E/kT} = e^{8\pi M E}. \quad (3.7)$$

This is easy to integrate:

$$\varrho(M) = e^{4\pi M^2 + C} = e^S. \quad (3.8)$$

We can rewrite this as

$$\varrho(M) = 2^{A/A_0}, \quad (3.9)$$

where A is the horizon area and A_0 is a fundamental unit of area,

$$A_0 = 4 \ln 2 L_{\text{Planck}}^2. \quad (3.10)$$

This suggests a spin-like degree of freedom on all surface elements of size A_0 , see Fig. 1.

The importance of this derivation is the fact that the expressions used as starting points are the *actual* Hawking emission rate and the *actual* black hole absorption cross section. This implies that, if in more detailed considerations divergences are found near the horizon, these divergences should not be used as arguments to adjust the relation between entropy and level density by large renormalization factors.

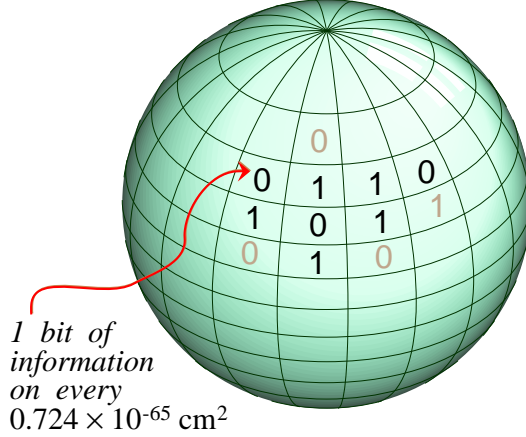


Figure 1. Information on a black hole horizon.

4. The quantum information problem.

It is tempting to conclude from the arguments presented above that the ‘black hole states’ form a natural extension of the spectrum of elementary particles. The lightest particles are known and have been identified as photons, neutrinos, electrons, muons, mesons, baryons, and onwards to the heavy leptons, the Higgs and so forth. The series could continue with as yet unknown particles in the ‘desert’ between 1 TeV and 10^{19} GeV, and beyond that region the first superstring recurrences could exist. The ‘most pointlike objects’ beyond the Planck mass must undoubtedly be black holes, simply because any sufficiently compact object with sufficiently high mass must carry a gravitational field and a horizon associated with that. Apparently, we now know the spectrum of the objects in this range, apart from the unknown multiplicative constant e^C in Eq. (3.8).

It should be possible to handle these objects just as all quantum objects when we consider quantum mechanical amplitudes at high energies: they are represented as propagators describing intermediate states. Theoretical Physics should give us the computational rules, comparable to Feynman rules, for computing these amplitudes. What have we got?

The behaviour of quantum fields near the horizon of a black hole follows from the expression for ds , the infinitesimal invariant distance element according to General Relativity:

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{1 - 2M/r} + r^2 d\Omega^2, \quad (4.1)$$

where $d\Omega^2$ stands for $d\theta^2 + \sin^2 \theta d\phi^2$. Writing

$$r - 2M = e^\sigma, \quad dr = (r - 2M) d\sigma, \quad (4.2)$$

we see that

$$ds^2 = \left(1 - \frac{2M}{r} \right) \left(-dt^2 + r^2 d\sigma^2 + \frac{r^2}{1 - 2M/r} d\Omega^2 \right). \quad (4.3)$$

At high energies, the conformal factor has little effect on the wave equations for light particle species. Consider a wave equation close to the horizon, such as

$$\partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \phi - m^2 \sqrt{-g} \phi = 0. \quad (4.4)$$

Since it contains the inverse of the metric tensor $g_{\mu\nu}$, the contribution from the angular part in Eq. (4.3) becomes insignificant, and likewise the mass term. Thus, the wave equations become 2-dimensional, and they generate plane waves in the σ - t direction as if σ - t space were flat. This means that one gets an unlimited number of oscillations near the horizon, as $\sigma \rightarrow -\infty$. Quite unlike the case for systems such as the hydrogen atom, we see that the boundary condition at the horizon is ill-defined. The quantum states can generate an unlimited number of modes here. From this exercise, one would conclude that the density of quantum states for a black hole is not at all finite.

The physical origin of this divergence is not difficult to identify. Particles may move into a black hole, but, as long as we stick to linear field equations, the particles emerging from the black hole cannot at all be related to the ingoing ones. There cannot be any reflection against the horizon, since there should be an infinite time delay. Here we see that the situation with ingoing and outgoing spinors and vector particles will be equally hopeless.

Two other conundrums are closely related to the problem just signalled. First, we have the *quantum decoherence* problem. This problem becomes apparent when the Hawking effect is calculated explicitly. The initial state of elementary particles before the formation of a black hole is described in terms of various Fourier modes of their fields. All of these modes then are associated to observable operators. In a Heisenberg picture of the quantum states, these operators become time dependent. Part of the Fourier modes of the initial fields now enter into the black hole, and only the operators associated to the modes that emerge out of the black hole correspond to observables at a later time. The expectation values of these late-time observables turn out to be described by a thermal density matrix. In terms of the basis of states generated by the late-time observables alone, this density matrix turns out to have eigenvalues less than one, which is characteristic for a not fully coherent quantum state. This situation is similar to what one gets in a condensed matter system if one allows observable particles to escape, and subsequently omits the quantum states that they represent.

In the case of a black hole, the missing particles are the absorbed ones. If we were forced to keep these particles in our quantum description, an even worse infinity of quantum states would result.

A description of the ‘information problem’ that is easier to understand is the following. Choose a coordinate frame in which the formation of a black hole looks more or less regular. Ingoing particles are then seen to enter at rather late times. If now the returning particles were assumed to be not totally independent of the ingoing ones, one would have to accept the observation that, somehow, the information contained in the ingoing particles has been transferred to the outgoing ones. The outgoing ones, however, all belong to the Fourier modes that arose as quantum oscillations at the point where the black hole was formed, way back in the past. How could the required information have been imprinted on these particles, if they have already been there for such a long time?

A mathematically impeccable observation was made by Hawking²⁴: the black hole space-time describes *two* universes, not one, and these two are connected by a ‘wormhole’. After a black hole is formed, the quantum wave functions of elementary particles spread over these two universes, and they become intertwined. Cutting off the information concerning the contents of the ‘hidden’ universe will leave the other universe in a quantum mechanically decoherent state.

From a physical point of view, however, this argument is unsatisfactory. It implies that black holes are fundamentally different from all other forms of matter in the sense that they appear to produce decoherence. In all respects this result is equivalent to saying that the scattering matrix elements involving black holes *are not fixed by our theory*, but carry an uncertainty, distributed in some well-defined way. So, what we really have here is an ‘uncertain theory’. Our theory is incomplete. We should not be satisfied with that. Perhaps new physics can remove this uncertainty.

5. The Scattering Matrix Ansatz.

How do we ‘improve’ our theory? Naturally, one may think of including more interactions; obviously, the procedures applied thus far assumed ingoing and outgoing fields not to interact — Eq. (4.4) is after all linear in ϕ . At first sight it seems that including interactions will resolve the paradox. As σ approaches $-\infty$, the plane ϕ waves enter conformally into ever smaller regions of space and time. Effectively, the gravitational couplings increase rapidly, and as e^σ approaches the Planck length, this effective coupling becomes super strong. An alternative way to verify this is by switching towards a coordinate frame that is locally regular near the horizon. Such a frame is given, e.g., by the Kruskal coordinates $\{x, y\}$:

$$xy = \left(\frac{r}{2M} - 1\right) e^{r/2M} ; \quad x/y = e^{t/2M} . \quad (5.1)$$

Writing

$$x^0 = x - y ; \quad x^1 = x + y , \quad (5.2)$$

one finds the metric to be regular near $x \approx y \approx 0$. Particles sent in in the far past will align close to the axis $x = 0$, and particles going out in the distant future align close to $y = 0$. A boost in the Schwarzschild time parameter t corresponds to a Lorentz transformation in (x, y) space, where the scale is set by the mass parameter M . If we consider time lapses long compared to M , the Lorentz boosts separating ingoing and outgoing particles become horrendous. Thus we see that ingoing and outgoing particles meet each other near $x = y = 0$ at tremendously large c.m. energies. Even if we could neglect Standard Model interactions at these energies, the gravitational interactions, which grow with the energy squared, can no longer be ignored from some point onwards.

This observation however does not resolve the decoherence problem. Even with the interactions in place, one may still argue that information is drained by the black hole, and a theory for pure states interacting with pure states without decoherence does not follow. A more powerful approach is wanted.

It is strongly advocated now to start from the other end: we must *assume* that there exists a quantum mechanically fully coherent scattering matrix S . The assumption is

somewhat dogmatic; we cannot prove it from first principles, other than demanding the *existence* of a theory. Even if standard techniques at best only provide us with some ‘distribution’ for the physical scattering matrix elements, we assume that the ‘true’ scattering matrix elements are exactly defined. Even if no theory would exist to derive them, they could in principle be derived from experiment.

Demanding consistency with existing theory however gives us important constraints. Indeed, the scattering matrix can now almost be derived from the information we already have. The calculations have been presented at length elsewhere, so here we give a summary.

The dominant interaction is assumed to be the gravitational one, simply because the c.m. energies tend to infinity. Other interactions, such as in particular the electro-magnetic one, can be corrected for later (the effects from electro-magnetism are important, but they do not affect the main structure that will be obtained). The procedure then is as follows.²⁵

First, assume a black hole with some well-specified initial history of *ingoing* particles, for instance we specify the way in which a star imploded to give this black hole, and afterwards more objects may have fallen in at later times. We assume that all this leads to a *pure* quantum state, to be referred to as the state $|1\rangle$. It evolves and decays in some prescribed way. It leads to some superposition of many possible states for the outgoing particles, including states describing the final explosion.

Now, we consider the same state $|1\rangle$, but we either add or remove one ingoing particle, and we call this state $|1, \delta p\rangle$, where δp stands for the momentum (and possible other details) of the extra ingoing object. What can be done now is a calculation, as detailed as possible, of the effects this extra ingoing particle has on the outgoing objects. Surely there is interaction. The gravitational one is most interesting. It leads to a *shift* of the outgoing wave functions. This shift is simply the Shapiro delay due to the gravitational field of the ingoing object. The calculation is in principle entirely straightforward, but has to be done with some care since the ingoing object goes essentially with the speed of light. The shift of the Kruskal y coordinate is found to be

$$\delta y = p_{\text{in}} \cdot G(|\tilde{x}_{\text{in}} - \tilde{x}_{\text{out}}|), \quad (5.3)$$

where G is a simple calculable function of the coordinates \tilde{x} on the horizon. In the limit where the black hole is large and the separations on the horizon small, we can approximately view \tilde{x} as flat coordinates, and in that limit, the function G is proportional to $\log|\tilde{x}_{\text{in}} - \tilde{x}_{\text{out}}|$. We then find that this shift obeys a Laplace equation:

$$\tilde{\partial}^2 \delta y = -C \cdot p_{\text{in}} \cdot \delta^2(\tilde{x}_{\text{in}} - \tilde{x}_{\text{out}}). \quad (5.4)$$

G is therefore a Green function. Because of this shift, the outgoing state $|\psi_{\text{out}}\rangle$ turns into

$$\exp\left(i \int d^2 \tilde{x} P^+_{\text{out}}(\tilde{x}) \delta y(\tilde{x})\right) |\psi_{\text{out}}\rangle. \quad (5.5)$$

Writing $\delta y(\tilde{x}) = \int d^2 \tilde{x}' G(\tilde{x} - \tilde{x}') \delta p_{\text{in}}(\tilde{x}')$, the new state is

$$|\psi'_{\text{out}}\rangle = \left(\exp i \int \int d^2 \tilde{x} d^2 \tilde{x}' P^+_{\text{out}}(\tilde{x}) G(\tilde{x} - \tilde{x}') \delta p_{\text{in}}(\tilde{x}') \right) |\psi_{\text{out}}\rangle. \quad (5.6)$$

And now we can repeat this many times. Let $P^-(\tilde{x}')$ stand for the total momentum (in Kruskal coordinates) for all particles added, or subtracted using a minus sign, from the state $|1\rangle$. Then we have

$$|\psi_{\text{out}}\rangle = \left(\exp i \int \int d^2\tilde{x} d^2\tilde{x}' P^+_{\text{out}}(\tilde{x}) G(\tilde{x} - \tilde{x}') P^-_{\text{in}}(\tilde{x}') \right) |1\rangle. \quad (5.6)$$

This way, the state $|1\rangle$ can be used as a universal reference state. The true state is then specified by giving $P^-_{\text{in}}(\tilde{x})$.

After some simple manipulations, we find that both the initial and the final state could be described by giving the transverse coordinates $\tilde{x}^{(i)}$ and the radial momenta $p^{(i)}$ for all particles. We get

$$\begin{aligned} & {}_{\text{out}}\langle q^1, \tilde{y}^1, q^2, \tilde{y}^2, \dots || p^1, \tilde{x}^1, p^2, \tilde{x}^2, \dots \rangle_{\text{in}} = \\ & = \int \mathcal{D}X^+(\tilde{x}) \mathcal{D}X^-(\tilde{x}) \exp \left(\int d^2\tilde{x} \left[-i\tilde{\partial}X^+ \tilde{\partial}X^- + \right. \right. \\ & \left. \left. + \sum_i i\delta^2(\tilde{x} - \tilde{x}^i) p^i X^+(\tilde{x}) - \sum_i i\delta^2(\tilde{x} - \tilde{y}^i) q^i X^-(\tilde{x}) \right] \right), \end{aligned} \quad (5.7)$$

and if the in- and outgoing particles are described by transverse wave functions $e^{i\tilde{p}^i \tilde{x}^i}$ and $e^{i\tilde{q}^i \tilde{y}^i}$, then another set of integrations has to be performed, over the transverse coordinates \tilde{x}^i and \tilde{y}^i . All of this yields an amplitude that is very much reminiscent of a string amplitude, with the exception of the i in front of the ‘kinetic’ term $(\partial X)^2$ in Eq. (5.7). In Eq. (5.7), Newton’s constant G_N has been normalized according to

$$8\pi G_N = 1. \quad (5.8)$$

6. Fock space.

The result of the previous section appears to be beautiful. We managed to construct the S -matrix using only known facts about the gravitational interaction between fast moving objects. In addition, it appears not to be too difficult to impose unitarity for this scattering matrix. Unitarity just fixes the *measure* of the functional integration in (5.7). Only the phase then remains undetermined, but it was arbitrary anyway since the amplitudes in question violate many of the conventional conservation laws such as all combinations of baryon and lepton number.

There are, however, two problems, both having to do with the Hilbert space in terms of which this scattering matrix appears to be defined.

Problem # 1: the space of all momenta, $\{P^\pm(\tilde{x})\}$, is infinite dimensional, even for small black holes, whereas we expected a finite total number of states (the entropy was supposed to be finite). So, this Hilbert space is far too large.

Problem # 2: the space of all momentum distributions, $\{P^\pm(\tilde{x})\}$, is far too *small* to accommodate for all possible particle configurations. If two or more particles enter at

the same transverse point \tilde{x} , then, in our expressions, only the total momentum counts. States for which the total momentum distributions are identical will be indistinguishable, and since we want our scattering matrix to be unitary, these states must be *identical*. This is not Fock space for elementary particles as we are used to.

It is important to note, on the other hand, that string amplitudes, which are like Eq. (5.7) but without the i in the kinetic term, share the same feature: states with two or more particles entering the string world sheet at one point, are indistinguishable from states with just a single particle entering at that point. Distinctions only come after the \tilde{x} integrations, at which the particle number becomes unambiguous.

7. The holographic principle.²⁶

We have reached a point where, for a proper description of the particle states in the vicinity of a black hole, a *two*-dimensional function is required: the momentum distribution over a two-dimensional coordinate on the horizon. In addition, this function must be further reduced, since it must effectively contain not more than one $\mathbb{Z}(2)$ variable per surface element A_0 (see Eq. (3.10)). A comparison with a holographic photograph is quickly made. In a holographic set-up, a laser beam shines onto some three-dimensional object, and the reflected light interferes with an unperturbed laser beam. The interference pattern is registered on a photographic plate. In turn, after having developed the plate, we can shine a laser beam on it. An image of the three-dimensional object re-emerges. This appears to be a way to register three-dimensional objects on a two-dimensional photographic plate.

Now imagine that the photographic plate has a limited resolution, and that its colouring can only be black-and-white, no gray tones. In that case, the image we see of the original object will be blurred somewhat, since information went astray. This must actually be the situation in our description of particles entering a black hole: the momentum distribution cannot represent as many details as a fully three-dimensional description: our image of the universe is blurred. Of course, since it is the Planck scale where this limit is attained, in practice we perceive our universe very sharply.

Although this holographic nature of our description of the particles appears to apply only for particles entering a black hole, one may argue that it must have a much more universal validity. According to general relativity, there should exist a direct mapping that relates physical phenomena in one setting (with a gravitational field present) to another one (freely falling coordinates). Normally, the mapping goes both ways. It is indeed unlikely that freely falling particles can be described in more detail than the limits set by the holographic principle: one bit of information per surface element of size A_0 . It can be computed that the energy needed to detect more details would be so large that gravitational collapse would be inevitable; the entire scene would be absorbed by a black hole – and indeed be impossible to observe at all!

This is what we found out about Nature's book keeping system: the data can be written onto a surface, and the pen with which the data are written has a finite size.

References

1. see for instance B.E. Lautrup et al, *Phys. Reports* **3C** (1972) 193.
2. R.E. Marshak et al, *Theory of Weak Interactions in Particle Physics*, Wiley Intersc., 1969, SBN 471 57290 X; E.C.G. Sudarshan and R.E. Marshak, Proc. Padua-Venice Conf. on “Mesons and Recently discovered Particles”, 1957, p. V - 14, reprinted in: P.K. Kabir, “Development of Weak Interaction Theory”, Gordon and Breach, 1963, p. 118; E.C.G. Sudarshan and R.E. Marshak, *Phys. Rev.* **109** (1958) 1860; R.P. Feynman and M. Gell-Mann, *Phys. Rev.* **109** (1958) 193
3. M. Gell-Mann and M. Lévy, *Nuovo Cim.* **16** (1960) 705; B.W. Lee, *Nucl. Phys.* **B9** (1969) 649; J.-L. Gervais and B.W. Lee, *Nucl. Phys.* **B12** (1969) 627; K. Symanzik, *Lett. Nuovo Cim.* **2** (1969) 10, *Commun. Math. Phys.* **16** (1970) 48. B.W. Lee, “Chiral Dynamics”, Gordon and Breach, New York, 1972, ISBN 0 677 01380 9 (cloth); 0 677 01385 X (paper)
4. S. Mandelstam, *Phys. Reports* **13c** (1974) 259; G. Veneziano, *Nuovo Cimento* **57A** (1968) 190; J.H. Schwarz, *Phys. Reports* **8C** (1973) 269; V. Alessandrini et al, *Phys. Reports* **1C** (1971) 269.
5. G. 't Hooft, *Nucl. Phys.* **B33** (1971) 173; *id*, *Nucl. Phys.* **B35** (1971) 167 .
6. J. Cornwall, D. Levin and G. Tiktopoulos, *Phys. Rev. Lett.*, **30** (1973) 1268, **31** (1973) 572; C.H. Llewellyn Smith, *Phys. Lett.* **B46** (1973) 233.
7. S.L. Adler and W.A. Bardeen, *Phys. Rev.* **182** (1969) 1517; W.A. Bardeen, *Phys. Rev.* **184** (1969) 1848.
8. P.W. Higgs, *Phys. Lett* **12** (1964) 132; *Phys. Rev. Lett.* **13** (1964) 508; *Phys. Rev.* **145** (1966) 1156.
9. F. Englert and R. Brout, *Phys. Rev. Lett.* **13** (1964) 321.
10. See for instance: E. Accomando et al, *Phys. Reports* **299** (1998) 1, and P.M. Zerwas, *Physics with an e^+e^- Linear Collider at High Luminosity*, Cargèse lectures 1999, preprint DESY 99-178.
11. M. Kobayashi and T. Maskawa, *Progr. Theor. Phys.* **49** (1973) 652.
12. P. Fayet and S. Ferrara, *Phys. Reports* **32C** (1977) 249; S. Ferrara, *Supersymmetry*, North Holland, World Scientific, 1987, Vols 1 and 2. Supersymmetry has a vast literature. See for instance the collection of papers in: S. Ferrara, *Supersymmetry*, Vol. 1, North Holland, Amsterdam, etc., 1987.
13. F. Iachello and A. Arima, *The Interacting Boson Model*, Cambridge Univ. Press, 1987.
14. A. Zichichi, *Subnuclear Physics, The First Fifty Years. Highlights from Erice to ELN*. Eds. Barnabei, P. Pupillo, F. Roversi Monaco. Published by the University of Bologna and its Academy of Sciences, Bologna, 1998, Galvani celebrations.
15. M.A. Bouchiat and C. Bouchiat, *J. Phys.* (Paris) **35** (1974) 899, *ibid.* **36**(1975)

- 493;
 C.S. Wood, *et al*, *Science* **257** (1997) 1759;
 S.C. Bennet and C.E. Wieman, *Measurement of the $6S \rightarrow 7S$ transition polarizability in atomic cesium and an improved test of the standard model*, arXiv:hep-ex/9903022.
16. M. Jamin, *Theoretical status of ϵ'/ϵ* , hep-ph/9911390.
 17. A. Zichichi, *Antimatter*, in Proceedings of the 8th International Workshop on “Neutrino Telescopes”, 23-26 February 1999, Venice, Italy.
 18. N. Arkani-Hamed *et al*, *A Small Cosmological Constant from a Large Extra Dimension*, hep-th/0001197.
 19. Y. Suzuki and Y. Totsuka, *Nucl. Phys. Proc. Suppl* **77** (1999) 546 P; L. Maiani, *Cold Dark Matter and Massive Neutrinos, in the Universe and in the Lab*. Talk presented at the Symposium “Future of Universe, Future of Earth, Future of Civilization”, Budapest - Debrecen, Hungary, 2-6 July 1999 (CERN preprint).
 20. C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation*, Freeman, San Francisco, 1973;
 S. Chandrasekhar, *The Mathematical Theory of Black Holes*, Clarendon Press, Oxford University Press;
 K.S. Thorne, *Black Holes: the Membrane Paradigm*, Yale Univ. Press, New Haven, 1986.
 21. R.C. Tolman, *Relativity, Thermodynamics and Cosmology* (Clarendon, Oxford, 1934), pp. 242-243; R. Oppenheimer and G. Volkoff, *Phys. Rev.* **55** (1939) 374.
 22. S.W. Hawking, *Commun. Math. Phys.* **43** (1975) 199.
 23. G. 't Hooft, *Nucl. Phys.* **B256** (1985) 727.
 24. S.W. Hawking, *Phys. Rev.* **D14** (1976) 2460.
 25. G. 't Hooft, *Phys. Scripta* **T15** (1987) 143; *id.*, *J. Mod. Phys.* **A11** (1996) 4623 (gr-qc/9607022).
 26. G. 't Hooft, “Dimensional Reduction in Quantum Gravity”, Essay dedicated to Abdus Salam, Utrecht preprint THU-93/26 (gr-qc/9310026); *id.*, “Black holes and the dimensionality of space-time”, in *Proceedings of the Symposium “The Oskar Klein Centenary”*, 19-21 Sept. 1994, Stockholm, Sweden. Ed. U. Lindström, World Scientific 1995, p. 122;
 L. Susskind, L. Thorlacius and J. Uglum, *Phys. Rev.* **D48** (1993) 3743 (hep-th 9306069).