THE GIANT LEAP TO THE PLANCK LENGTH

Gerard 't Hooft
Institute for Theoretical Physics
University of Utrecht, Princetonplein 5
Postbox 80 006, 3508 TA Utrecht
The Netherlands

1 INTRODUCTION. THE PRESENT THEORY

During the years 1970-1975 a revolution took place in the world of fundamental particle physics. There were major breakthroughs due to joint efforts of experimental physicists and theoreticians. The experimentalists had enabled themselves to look much more deeply into the subatomic particles than ever before, and the theoreticians had all of a sudden arrived at unified schemes that made it possible for the first time to speculate very far beyond the regions from which the data were known.

Before 1970 the internal structure of the subnuclear particles was a mystery to us. It is important to remember what the prejudices were of the theoreticians at that time. It was generally thought that the mathematical basis of quantum field theoretical models was incorrect, and that abstract mathematical schemes based solely upon consistency requirements for the scattering matrix should ultimately explain everything. Quite unexpectedly however quantum field theoretical models revived, and an accurate field theoretical description of all known particles and forces was discovered.

The objections theorists had had previously against field theoretical models turned out to be a large part formalities. The point is that a quantum field theory in which a local Lagrangian is used as a starting point cannot be defined with infinite mathematical precision. Calculations will have to be done in terms of an infinite series of successive corrections, the so- called perturbation expansion. It is well-known that this series will *not converge*. Naturally one then questions whether such theories make any logical sense at all. This concern was aggravated by the fact that in computing the successive perturbative correction terms one often hit upon infinite -hence meaningless – integrals. Only if one limits oneself to the comparison of measurable properties of the fundamental particles these infinite expressions disappear, but the reason why this happens was often not explained very well and so the entire scheme was considered highly suspect.

Now we know that one has to approach this subject with a more pragmatic point of view. If we take the first few terms of a perturbative expansion and then terminate the series as soon as it tends to become divergent one gets a highly accurate, but not *infinitely* accurate model. This implies that our models have a limited region of applicability: there are energy ranges beyond which they become meaningless. Our integrations should be terminated there, and consequently our renormalization constants will be large but not infinite. The need for renormalization of masses, charges and such, is then a natural feature of any model. The formal limitations in accuracy in practice are usually very far beyond the accuracy both of our actual calculations, and of the experiments.

The construction we arrived at became known as "The Standard Model"[1]. The model is fairly complicated, and for the reasons explained above it cannot predict the interactions of particles and fields under all conceivable circumstances, but it seems to be a very good description of what is happening nearly everywhere in our universe. I will now give a rough description of this model and its laws.

The basic entities are "Dirac fermions", elementary particles that show a certain amount of spinning motion, called "spin". When measured in multiples of a fundamental constant of nature, Planck's constant divided by 2π, these particles are said to have "spin $\frac{1}{2}$". We start with particles that can only move with the speed of light. We call them "massless particles". For these particles the *axis of rotation* is always parallel to the velocity vector, and one can derive that this statement is unique and independent of the velocity of the observer. This is why one can distinguish unambiguously particles that spin "to the left" from particles that spin "to the right", with respect to this axis.

The forces these fermions exert onto each other are now described by introducing another set of particles, the *gauge bosons*. These are particles with spin one. They can either be considered as the "energy quanta" of various kinds of electric and magnetic fields, or one can view these particles themselves as the transmitters of these forces. When that picture is used one describes the force as being the consequence of an *exchange* of a gauge boson between two fermions: one fermion emits a boson and the other fermion absorbs it. It the *mass* of the boson is negligible then the efficiency of this exchange process is inversely proportional to the square of the distance: the Coulomb force law. If the boson has a certain amount of mass when at rest, then the force it transmits will range only up to a distance inversely proportional to that mass, and decrease very rapidly beyond that distance.

An important feature of the gauge boson force between two fermions is that before and after the emission of a gauge boson the fermion keeps the *same helicity*. This means that a left rotating particle remains left rotating, and a right rotating particle remains right rotating. This is of special importance for the neutrinos: neutrinos *only* exist as left rotating objects. Right handed neutrinos have never been observed (at least not for sure). In contrast, *anti*-neutrinos only come rotating towards the right, not left. By emitting a gauge boson a neutrino may turn into an electron, but then the electron must rotate towards the left.

We can now make a listing of all fermions and gauge particles. See Fig. 1. Here we see that the fermions are divided into *leptons* and *quarks*. The left rotating objects are indicated by an "L" and the right rotating ones by an "R". For the anti-leptons and

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[1] There are many text books on the Standard Model. See for instance: I.J.R. Aitchison and A.J.G. Hey, "Gauge Theories in Particle Physics", Adam Hilger, 1989. More pedestrian: N. Calder, "The Key to the Universe", BBC, London 1977.

anti-quarks, which are not shown, the L and R are interchanged. Of all fermions only the quarks are sensitive to the forces of the eight gauge bosons in the box called "$SU(3)$". This force is very strong and so all objects containing quarks will be strongly interacting with other objects.

All *left* rotating fermions are sensitive to the forces from the $SU(2)$ gauge bosons. Finally, all left rotating, and all electrically charged right rotation fermions feel the $U(1)$ force fields.

Now the standard model also contains a spin 0 particle, the "Higgs particle". It also transmits a force, but an important difference with the gauge boson forces is that if a fermion exchanges a Higgs boson it has to flip from left rotating to right rotating or *vice versa.* Another difference is that the spin 0 particle can disappear straight into the vacuum. This implies that some fermions can make spontaneous transitions form left to right and *vice versa.* Technically, this is the way one may introduce *mass* for these fermions. A particle with mass moves slower than the speed of light and it turns out that for such particles the rotation axis cannot be kept parallel to the velocity vector. So this mass can only be there if the particle has the ability to change its spin direction relative to its velocity vector. One finds that mass of a fermion is proportional to the coupling strength of that fermion to the field of the Higgs particle.

2 SCALING TO HIGHER ENERGIES

The Standard Model is a result of a splendid joint effort of theoreticians and experimentalists alike. But it is clearly not finished. I already mentioned the fact that calculations cannot be done with infinite accuracy, and that there must be energy regions where the model will be hopelessly inadequate. There are other deficiencies. A very fundamental difficulty is that our model is just one choice out of an infinite class of logical possibilites. We do not know of any compelling theoretical argument as to why Nature chose precisely the scheme we derived from observations, and why the various interaction strengths are what they have been found to be. Our present theories gave us only the general framework, but within this framework there still is a huge number of options. We would like to know what forceful principle caused Nature to select just one out of these.

If we consider energy values beyond 1 TeV (= 10^{12} electron Volts) the choice Nature made, according to our model, seems to be very *unnatural,* as if we had an electronic device with very accurately tuned dials. Most physicists believe for this reason that the standard model as presently known will require amendments already at that energy. This is important because this energy range can in principle be reached in the near future using more powerful machines, such as the SSC. It is in such machines that single constituent particles can be made to collide against each other with relative energies more than 1 TeV each. This is why most of us sincerely hope that our communities will continue to support these efforts with adequate budgets. We know how Nature managed to avoid fine-tuning at energies below 1 TeV; we don't know how Nature manages to resolve this problem above that energy.

Clearly Nature outsmarted the theoreticians here, but let us see how far we can get just using pen and paper. A notion that is crucial for explaining the situation is the so-called renormalization group. It tells us that if we understand the basic dynamics of some model at one particular energy scale (which also corresponds to one particular length scale and time scale), in terms of particular values of masses and interaction parameters of our fundamental particles, then we can *calculate* how the system behaves at some other scale. But the masses and interaction strengths will seem to be somewhat different at the new scale, in a way we can calculate. They are *renormalized* (by a finite correction).

The outcome of this calculation depends very much on the number of dimensions of space-time. It so happens that because our space-time is four dimensional, the dependence on scale transformations is *logarithmic.* This means that our theories hardly change at all when you go to a different scale. The present standard model can be extended to tremendously high energies before it explodes. It is the fine-tuning problem that comes much earlier. The standard model works fine mathematically until many billions of TeV are reached, but because of fine- tuning we don't believe it beyond 1 TeV.

The naturalness problem is most severe for scalar particles. They are the ones that undergo the interactions that give rise to mass, and mass needs fine tuning. Scalar particles give also, by the way, the largest number of freely adjustable interaction parameters. And our standard model contains only one scalar particle, the Higgs, which has not yet been seen.

Most probably our standard model in the TeV range will have to be amended in such a way that either the presently needed scalar particle turns out to be *composite*, or it is directly related to a fermionic particle via some symmetry constraint.

The first possibility would be quite natural to think about. It has at least two precedents in Nature. In a superconductor a scalar field called *order parameter* was needed to explain the physical peculiarities of super conductivity. This would have to be a scalar field, but it turned out that it actually is a bound state of two electrons, each with opposite spin, called "Cooper pair". The second precedent is the Yukawa force that keeps the nuclei together in a nucleus. It is caused by the exchange of a light scalar particle called the pion, and in the first more detailed model (the Gell-Mann Lvy sigma model) the pion field also acted as an order parameter. Now we know that the pion also isn't elementary; it is the bound state of two quarks.

So it is tempting to suspect that, in turn, the object presently called Higgs particle will also turn out to be composed of more fundamental fermionic objects. It is after all very much related to Cooper pairs and pions. We actually know that the Higgs will mix fairly strongly with a top anti-top bound state, but to identify Higgs with top anti-top entirely (as is sometimes suggested) does not solve our problem, because this would neither explain the large vacuum expectation value of the Higgs field nor its very different couplings with the other fermions. A new force is needed to keep the mysterious new building blocks of the Higgs together. An attempt was made to construct a new theory using the moulds of quantum chromodynamics, the force that keeps quarks together in hadrons. It was called "technicolor". But then new forces are again needed to explain the couplings of the Higgs with the other fermions, and these were called "extended technicolor".

Extended technicolor turned out not to be a very successful theory. It may well be possible that our world works this way, but we were unable to guess a really credible scenario. Scalar fields will still be needed at still higher energies, so our problem will be back once again. One firm prediction follows from this idea: there should be a new strong interaction region at the TeV scale, where the series of presently known fermions and bosons will converge into infinite lists of resonances like we saw in hadronic physics.

The alternative possibility is a new symmetry between fermions and bosons, called "sypersymmetry". This would be a radical departure from the path Nature has followed thus far, but it is mathematically more elegant and interesting. Sypersymmetry requires that all presently known particles have "superpartners": the superpartners of the presently known bosons (gauge bosons and Higgs) would be fermions (gaugino's and Higgsino) and those of the fermions (quarks and leptons) would have spin zero or one (squarks and sleptons). None of these super partners has ever been identified. This may seem to be a very worrisome aspect of this theory but we have to realize that whatever causes the apparent breakdown of supersymmetry at low energies must be a universal phenomenon, so that it is reasonable to expect that the mass differences for the various super multiplets are *all* of the same magnitude, one or several TeV.

In a supersymmetric theory the interactions at and beyond the TeV scale do not need to become strong; indeed the theory works best if the interactions stay weak enough to allow the renormalization group to extrapolate to very large scales. Therefore I expect that new accelerators will be able to differentiate between the two scenarios I sketched, technicolor *vs* sypersymmetry. It will be extremely helpful if theorist could get a further hint from the big machines.

3 THE SCENIC ROUTE TO THE PLANCK LENGTH

Both options I gave have one feature in common: the ground rules are those of quantum field theory. Causality and unitarity are built in because at every scale we have locally interacting field variables. The logic is straightforward. It does imply that at every scale we will have to accept a bunch of fundamental interaction parameters, and the constraints of logic at any scale will not tell us how large these freely adjustable parameters should be. But the values of these parameters will follow unambiguously from the parameters at higher energy —smaller distance— scales. If all goes well there should be no unnatural fine tuning at any scale.

This way we could proceed all the way to 10^{19} GeV. Already now, from our present primitive understanding of the basic interactions, we seem to be able to deduce that the various different interaction strengths, corresponding to the mathematical groups $U(1)$, $SU(2)$ and $SU(3)$, converge to one common value at very high energies. It is natural to suspect that somewhere at those energies further unification will take place, for instance involving the group $SU(5)$. But this is by no means the only possibility. An alternative is that the route towards 10^{19} GeV is one with near infinite complexity. If that is so the interesting question may be asked whether we humble human beings will ever be able to disentangle this complexity. I find it very unlikely that experiments can be carried out that will provide us with the required data at energies of, say, beyond 1000 TeV. This will probably not happen in our lifetime.

More likely is that more experiments of an indirect nature can be carried out (proton decay, dark matter astronomy, cosmological data) that may put us on the right track. And then it should be possible to improve our techniques of theoretical analysis. If indeed the ground rules will remain the same until the Planck length we may be able to gain so much experience with field theories that extrapolations over large scale differences may perhaps become possible. Advanced computation techniques may help us to attack the strong coupling problem; I do not believe namely that the end point of sophistication in either hardware or software is anywhere close. In short, I have confidence in human ingenuity.

4 WHERE GRAVITY BEGINS

The series of fields theories with their neat rules for spin 0, $\frac{1}{2}$ and 1 particles will come to an abrupt end. This is when the gravitational force lets itself be felt. The gravitational force probes mass, not charge, and the difference is that as we go to smaller distances the associated energies grow rapidly, and hence the effective masses to which the gravitational fields couple also increase rapidly. There will be an energy range at which the gravitational force becomes strong. This is the so-called Planck scale, about 10^{-33} cm, 10^{-44} seconds and an energy of 10^{19} GeV, corresponding to 21 μg, per particle. If we wish to reproduce the beautiful features of Einstein's theory of general relativity describing gravity at large scales, we must accept that the carrier of this force, the graviton, has spin 2 so that the corresponding theory is not renormalizable.

There is no easy cure to the non-renormalizability of a quantized theory of gravity. The point is that *no* model exists that can describe features of space-time at distance scales shorter than the Planck length. *Only* Einstein gravity could do that, but at that scale Newton's constant is simply too small to do the job. An indication of what one might have to think about was given more than a decade ago. The one possible extension of perturbative quantum field theory that does incorporate gravity is the so-called "string theory". If we replace particles by little stretches of closed strings one can study the perturbative interactions among those. It was shown that for these objects no infinite renormalizations are ever necessary. A further perfection of this elaborate theory was achieved in 1984 when it was found that super symmetry on the string's world sheet could prevent the occurrence of tachyonic solutions, and a large set of internal degrees of freedom could cure a problem of unphysical ghost solutions.

The problem with string theory is that this attack is too indirect; the ground rules that should enable us to link them to the real world are not sufficiently well understood. Also the scheme itself is still shaky, to my mind. String theory is fundamentally a perturbative theory and just as in ordinary field theory this perturbation expansion diverges. This is fine if one plans to view string theory as some effective, approximate model, like we presently view quantized gauge theories. But if one wishes to claim that string theory is a basic theory for everything than the perturbative definitions are unacceptable.

I am unable to infer from string theory what the physical degrees of freedom are at length scales shorter that the Planck lenght. The *suggestion* is that at distance scales shorter than the Planck length the interactions should be described by the analog of a *topological* field theory. There are examples of topological field theories in less than four space-time dimensions, but in four dimensions the construction of such theories has been inhibitingly difficult.

To illustrate the difficulty I can show that no measurement can be made of distances smaller than the Planck length. The basic argument goes as follows. Suppose we wish to locate some structure with an accuracy tinier than the Planck distance. This would require the use of probes with a wave length smaller than the Planck length, which would imply that the energy of the particle beam used should be bigger than the Planck energy. These particles would have to collide with the structure we wish to study with an impact parameter also smaller than the Planck length. This would result in a total energy exceeding the Planck energy, inside a volume smaller than the Planck volume. Thus all conditions for the occurrence of gravitational collapse are fulfilled. A black hole will form. This black hole will be bigger than the Planck length, so it will not differentiate among structures smaller than that. End of argument.

Using similar lines of reasoning one can also deduce that no more than roughly one *bit of information* may be stored inside a Planck volume. More precisely, the total number of bits of information one may store within some closed surface cannot exceed a number that is directly proportional to that surface, in Planck units. This is a very severe constraint which seems to be extremely difficult to realize in any conceivable theoretical scheme.

Clearly then, black holes must be very important entities in any attempt at constructing a viable theory for Planck length physics. We can look upon this this way:

> *if the gravitational force is such a problematic aspect in our theories of matter at the Planck length, then what would be more appropriate than study the strongest possible gravitational fields?*

The strongest possible gravitational fields occur near black holes; the tiniest black holes carry the strongest gravitational fields. It is for these reasons that I advocate the theoretical study of black holes in a quantum mechanical environment. We must do *Gedanken* experiments and impose consistency requirements when we list all possible outcomes. Indeed there are a lot of experiments for which we can guess the result, with more or less accuracy. The only unknowns are the experiments which in no frame of reference fall within the regime of known theories.

Somewhat surprisingly, my investigation of these Gedanken experiments lead me to a realm of mathematics only too familiar. Even though no string theory was assumed at the outset, I ended up with amplitudes that had to be very similar to string theoretical ones. There are important differences, such as an imaginary string constant with consequently resonant states that are extremely unstable as compared to the ones in string theory. But some details such as the occurrence of Koba-Nielsen variables were very similar. Indeed, it is the black hole horizon, a basically two dimensional surface, that takes the place here of a string world sheet. This time the connection to physical reality is evident, but now the mathematical rules are much more obscure. I am inviting anyone with a vivid imagination to put his or her teeth in this problem.

The challenge we are facing here is a magnificent one. It is far too early to speculate upon the outcome of this research. There seem to be tremendous obstacles. Perhaps only one model, perhaps very many models can form consistent theories of Planck length physics. What may conceivable happen is that one model will stand out as the by far most obvious possible consistent construction. That then would be the "Theory of Everything" many of us have speculated about before. I wish to stress here that we must be careful with our public announcements concerning the possibility of "theories of everything". Particle physics among themselves know perfectly well what may be meant by such phrases. But I have experienced that many outsiders take these all too literally. "Theory of everything" surely does not imply that we intend to supplant all sciences by theoretical particle physics, but some people suspect us of cherishing secret desires of such a nature. 'The ultimate master equation of the universe' may be a more appropriate terminology, if it comes with the addition that no one ever will be able to solve it.

In my opinion an ultimate equation will have to be accompanied with a precise formulation of the boundary conditions of the universe, including the initial state. I am convinced that such an ultimate equation will be basically simple and absolute, without any explicit reference to the need of perturbative expansions —they may be introduced at later phases as certain aspects of mathematical approximations to solving the equations. Also I am convinced that the ultimate equation will resolve the present paradoxes concerning the quantum mechanical nature of our world —it may well be that quantum mechanics as we know it is nothing more than some convenient statistical approach to obtain a large scale solution of a basically microscopic set of equations, perhaps indeed the *only* way to obtain macroscopic solutions.

Although an "ultimate master equation" would have little significance for the world outside the physics of fundamental particles, it would certainly be a tremendous achievement if particle physicists discovered such a formulation.

Summary

The "Standard Model" is our present description of all known particles and the forces between them. It turned out to be a reliable and accurate picture, but it cannot be complete. Experiments in the near future should enable us to differentiate among various possible theories that may improve our present picture, and may perhaps also tells us how to extrapolate theoretically towards regimes that will be inaccessible to experimentation for quite some time to come.

The Planck scale is the scale of distances, time intervals and collision energies for individual particles at which the gravitational force among individual particle begins to dominate over all other forces. This scale is as yet way beyond what can be reached in any experiment: 10^{-33} cm, 10^{-44} seconds and the equivalent of 21 micrograms of energy for each particle. The importance of these numbers is that here all presently known laws of physics must break down completely; we have no consistent scheme to describe precisely what happens there.

It is nearly impossible to imagine experiments that could give us direct information about the behaviour of particles in the Planck regime. We will probably have to devise indirect experiments such as the search for proton decay, exotic particles and astronomical and cosmological observations. The most important problem is to describe what happens if particles enter a black hole whose mass is nearly as tiny as the Planck mass. It is here that we can do "thought experiments". By simply requiring the outcomes of such experiments to be in accordance with all presently known theories whenever these apply, we might be able to pin down which of the large number of possible scenarios is the most likely.

Several theorist have speculated that there may exist only one theoretical scheme that can match with all known symmetries and logical requirements one would like to impose. Although such ideas are still highly premature I do agree that the idea of one single "theory of everything" in physics is not entirely crazy. In any case it should be stressed that such a construction would have little or no effect on any science other than the physics of fundamental particles or philosophical theories behind the interpretation of the quantum mechanical nature of our world. A "theory of everything" would actually be something like an "equation for everything", with fairly little significance because no one will ever be able to solve such an equation.

References

[1] I.J.R. Aitchison and A.J.G. Hey, "Gauge Theories in Particle Physics", Adam Hilger, 1989.

More pedestrian:

[2] N. Calder, "The Key to the Universe", BBC, London 1977.

[3] J.D. Barrow, "Theories of Everything: the quest for ultimate explanation", Oxford: Clarendon Press 1991.

More technical:

[4] A.M. Polyakov, "Gauge Fields and Strings", Harwood Acad. Publ. GmbH, Chur, Switzerland, 1987.

[5] "Beyond the Standard Model II", Proceedings of the International Conference on High Energy Physics, Norman, Oklahoma, ed. K.A. Milton *et al.*