Symmetry Breaking through Bell-Jackiw Anomalies*

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In models of fermions coupled to gauge fields certain current-conservation laws are violated by Bell-Jackiw anomalies. In perturbation theory the total charge corresponding to such currents seems to be still conserved, but here it is shown that nonperturbative effects can give rise to interactions that violate the charge conservation. One consequence is baryon and lepton number nonconservation in $V-A$ gauge theories with charm. Another is the nonvanishing mass squared of the η.

When one attempts to construct a realistic model of nature one is often confronted with the difficulty that most simple models have too much symmetry. Many symmetries in nature are slightly broken, which leads to, for instance, the lepton and quark masses, and CP violation. Here I propose to consider a new source of symmetry breaking: the Bell-Jackiw anomaly.

My starting point is the solution of classical field equations given by Belavin *et al.*[1] in four-dimensional (4D) Euclidean gauge-field theories. The solution is obtained from the vacuum by mapping SU(2) gauge transformations onto a large sphere in Euclidean space. Taking the new, gauge-rotated, vacuum as a boundary condition, they obtain a nontrivial solution inside the sphere, characterized by a topological quantum number. If the Lagrangian is

$$\begin{aligned} \mathcal{L}^{\mathrm{YM}} &= -\tfrac{1}{4} G_{\mu\nu}{}^{a} G_{\mu\nu}{}^{a}, \quad a = 1,2,3, \\ G_{\mu\nu}{}^{a} &= \partial_\mu A_\nu{}^{a} - \partial_\nu A_\mu{}^{a} + g\epsilon_{abc} A_\mu{}^{b} A_\nu{}^{c}, \end{aligned} \tag{1}$$

then the topological quantum number is

$$n = (g^2/32\pi^2) \int G_{\mu\nu}{}^{a} \tilde{G}_{\mu\nu}{}^{a}\, d^4x, \tag{2}$$

with

$$\tilde{G}_{\mu\nu}{}^{a} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta} G_{\alpha\beta}{}^{a}. \tag{3}$$

n is an integer for all field configurations in Euclidean space that have the vacuum (or a gauge transformation thereof) at the boundary. In Minkowsky space n would be i times an integer (if we take d^4x and ϵ_{1234} to be real and A_4, $\partial/\partial x_4$ imaginary).

The solution with $n=1$ in Euclidean space is

$$A_\mu{}^{a}(x)^{\mathrm{cl}} = \frac{2}{g} \frac{\eta_{a\mu\nu}(x-x_0)^\nu}{(x-x_0)^2 + \lambda^2}. \tag{4}$$

Here, x_0 is free because of translation invariance and λ is a free scale parameter; η is a tensor that maps antisymmetric representations of SO(4) onto vectors of one of its two invariant subgroups SO(3):

$$\begin{aligned} &\eta_{a\mu\nu} = \epsilon_{a\mu\nu} \quad (a,\mu,\nu = 1,2,3), \\ &\eta_{a4\nu} = -\delta_{a\nu} \quad (a,\nu = 1,2,3), \\ &\eta_{a\mu4} = \delta_{a\mu} \quad (a,\mu = 1,2,3), \\ &\eta_{a44} = 0. \end{aligned} \tag{5}$$

Thus isospin is linked to one of the SO(3) subgroups of SO(4). The solution has

$$S = \int \mathcal{L}\{A^{\mathrm{cl}}\}\, d^4x = -8\pi^2/g^2. \tag{6}$$

Since we have a 4D rotational symmetry, the solution is not only localized in three-space, but also instantaneous in time. I shall refer to such objects as "Euclidean-gauge solitons,"[2] EGS for short.

There is a simple heuristic argument that explains why these solutions of the Euclidean field equations are relevant for describing a tunneling mechanism in real (Minkowsky) space-time, from one vacuum state to a gauge-rotated vacuum (a gauge rotation that cannot be obtained via a series of infinitesimal gauge rotations). Consider an ordinary quantum mechanical system with a potential barrier V larger than the available energy E, which I put equal to zero. Then the leading exponential of the tunneling amplitude is $\exp(-\int p\, dx)$, with

$$p^2/2m = V - E.$$

This corresponds to the classical equations of motion, except for a sign difference. Thus the leading exponential is obtained by replacing in the equations of motion t by it and computing the action S for a path from one to the other vacuum. [Note that both in Euclidean and in Minkowsky space the gauge group is the compact group SU(2).]

Suppose now that we have in addition N massless fermion doublets coupled to the gauge field:

$$\mathcal{L}^{\mathrm{fermion}} = -\sum_{t=1}^{N} \bar{\psi}^t \gamma_\mu D_\mu \psi^t, \tag{7}$$

where

$$D_\mu \psi_i{}^t = \partial_\mu \psi_i{}^t - \tfrac{1}{2} i \tau_{ij}{}^a A_\mu{}^a \psi^t. \tag{8}$$

I will call the SU(2) index i "color" and the index $t = 1, \ldots, N$ "flavor." The vector currents

$$J_\mu{}^{st} = i\bar{\psi}^s \gamma_\mu \psi^t, \tag{9}$$

and the traceless part of the axial vector current

$$J_\mu{}^{5st} = i\bar{\psi}^s \gamma_\mu \gamma_5 \psi^t, \tag{10}$$

are all conserved without anomalies. Thus we have the exact chiral flavor symmetry $SU(N)_L \otimes SU(N)_R \otimes U(1)$. But the current

$$J_\mu{}^5 = \textstyle\sum_t J_\mu{}^{5tt}$$

has an anomaly[3]

$$\partial_\mu J_\mu{}^5 = -i(Ng^2/16\pi^2) G_{\mu\nu}{}^a \tilde{G}_{\mu\nu}{}^a. \tag{11}$$

Let us now compare this with Eq. (2).

A configuration in Minkowsky space with $n = 1$ would be associated with a violation of axial charge conservation:

$$\Delta Q^5 = 2N. \tag{12}$$

To calculate the amplitude for such an event directly in Minkowsky space one needs more understanding of the quantum mechanical tunneling from one vacuum to the gauge-rotated vacuum. In practice it is much easier to make use of the explicit solution in Euclidean space. Let us assume then that all Green's functions in Minkowsky space can simply be obtained from the Euclidean ones by analytic continuation.

Let us consider the vacuum-to-vacuum amplitude in Euclidean space, first without, and then with source insertions in the Lagrangian[4]:

$$\langle 0|0\rangle = \int \mathfrak{D}A \mathfrak{D}\psi \mathfrak{D}\varphi \exp\int \{\mathcal{L}^{\text{gauge}}(A) + \mathcal{L}^{\text{fermion}}(A,\psi) + \mathcal{L}^{\text{fix}}(A) + \mathcal{L}^{\text{ghost}}(A,\varphi)\} d^4x, \tag{13}$$

where $\mathcal{L}^{\text{fix}}$ fixes the gauge and $\mathcal{L}^{\text{ghost}}$ is the corresponding Faddeev-Popov ghost term; φ is the ghost field. We perform the perturbation expansion around those values of the fields where the exponent is stationary. The solution of Eq. (3) is such a stationary point. Collective coordinates[5] must be introduced for x_0 and λ. The first will lead to energy-momentum conservation in an obvious way; the second might at first sight lead to infinities at both ends of the scale, but there are natural cutoffs, as we will see later.

The arguments that follow now must be considered as a summary of a series of mathematical manipulations needed to compute the wanted amplitudes. Let us expand

$$A_\mu{}^a = A_\mu{}^{a\,\text{cl}} + A_\mu{}^{a\,\text{qu}}, \quad \mathcal{L}^{\text{gauge+fermion+ghost}} = \mathcal{L}\{A^{\text{cl}}\} - A^{\text{qu}} M_1 A^{\text{qu}} - \bar{\psi} M_2 \psi - \varphi^* M_3 \varphi$$
$$+ \text{ higher orders in the quantum fields.} \tag{14}$$

It will be very convenient[6] to use the so-called "background gauge":

$$\mathcal{L}^{\text{fix}} = -\tfrac{1}{2}(D_\mu{}^{\text{cl}} A_\mu{}^{\text{qa}})^2, \quad D_\mu{}^{ac,\text{cl}} = \partial_\mu \delta^{ac} + g\epsilon_{abc} A_\mu{}^{b,\text{cl}}. \tag{15}$$

Because we introduced collective coordinates, we may restrict the quantum fields to be orthogonal to those values that generate pure translations or dilatations of the classical solution. The amplitude (in the one-EGS sector) is now formally

$$\langle 0|0\rangle = \int d^4x_0 \int d\lambda\, (\det J)(\det M_1)^{-1/2} (\det M_2)(\det M_3) \exp\int \mathcal{L}(A^{\text{cl}}) d^4x. \tag{16}$$

Here $(\det J)$ is the Jacobian following from our transition to collective coordinates. If the background gauge is used it turns out to be finite and proportional to λ^{-1}. From Eq. (6) it follows that the exponent equals

$$\exp(-8\pi^2/g^2), \tag{17}$$

which explains why we get results that are unobtainable through ordinary perturbation expansions with respect to g^2.

The other determinants are in principle obtained by solving the equations

$$M_1 A^{\text{qu}} = E_1 A^{\text{qu}}, \quad M_2 \psi = E_2 \psi, \quad M_3 \varphi = E_3 \varphi. \tag{18}$$

Now M_1 and M_3 have some zero eigenvalues that neatly cancel. But there are also solutions to

$$M_2 \psi = 0, \quad \psi = (1 + r^2)^{-3/2} u, \tag{19}$$

where u is a fixed tensor with Dirac and isospin indices. There is one such solution for each of the N flavors. They are chiral solutions, very much like the fermion bound states described by Jackiw and Rebbi[7] in one and three spacelike dimensions (but stationary in time). These zero eigenvalues are not canceled by anything, so

$\det M_2 = 0$, and the amplitude (13) vanishes. I interpret this result as being a consequence of Eq. (12): We must not sandwich the functional integral expression between two vacuum states, because the initial and final chiral charges Q^5 should be different.

Let us now insert a source term $\psi_i^{-s} J_{(x)}{}^{st} \psi_i^t$ into the Lagrangian, where $J(x)$ may contain flavor indices and γ matrices, but must be gauge invariant. Now the lowest eigenvalues will become different from zero:

$$M_2 \psi_{(i)}{}^s + J^{st}(x) \psi_{(i)}{}^t = E_{2(i)} \psi_{(i)}{}^s, \quad i = 1, \ldots, N. \tag{20}$$

Using lowest-order perturbation expansion we find that

$$\int d^4x\, \psi_{(i)}{}^{*s} J^{st}(x) \psi_{(j)}{}^t = E_{(j)} \int \psi_{(i)}{}^{*s} \psi_{(j)}{}^s d^4x = E_{(i)} \delta_{ij}, \quad i, j = 1, \ldots, N,$$

where $\psi_{(i)}{}^s = \psi_0 a_{(i)}{}^s$, and ψ_0 is the zero-"energy" solution for any flavor, and $a_{(i)}{}^s$ are coefficients. Thus

$$\det(M_2 + J) = \prod_{(i)} E_{(i)} \simeq \det_{st} \int \psi_0{}^* J^{st}(x) \psi_0 d^4x \times \{\int \psi_0{}^* \psi_0 d^4x\}^{-1}. \tag{21}$$

Substituting the known form of ψ_0, Eq. (19), we find that Eq. (21) goes like

$$\prod_{i=1}^{N} (x_i - x_0)^{-6} J(x_i)$$

for large distances, and only the $1 - \gamma_5$ part of J^{st} is selected out. We ask now for an effective vertex that could mimic the same amplitude (neglecting the finite size of the EGS) and we find

$$\mathcal{L}^{\text{eff}} = C g^{-8} \exp(-8\pi^2/g^2) \mathcal{L}^f + \text{H.c.}, \tag{22}$$

where $\mathcal{L}^f$ is a $2N$ fermion interaction that has the chiral transformation properties of

$$\det_{st} \bar{\psi}^s (1 + \gamma_5) \psi^t$$

but the isospin indices are arranged in a more general way. The factor

$$\prod_{i=1}^{N} (x_i - x_0)^{-6},$$

is exactly reproduced by the $2N$ propagators that connect the sources with the EGS (Fig. 1).

Note that the sources have to switch chirality. This explains why the instanton gives $\Delta Q^5 = 2N$. I have found that the constant in Eq. (22) can be computed analytically to zeroth order in g. The calculation is lengthy and will be discussed in a separate publication.

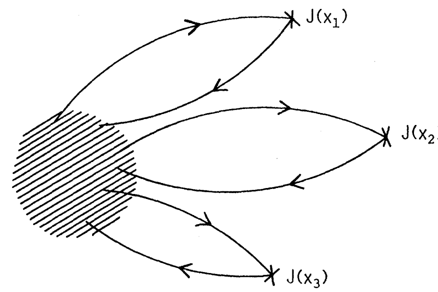

FIG. 1. The case of three flavors. The sources J turn the axial charge Q^5 into $-Q^5$ for each flavor. The amplitude goes like J^3.

The integration over the collective coordinate $x_{0\mu}$ in Eq. (16) simply implies that the effective action is Eq. (22) integrated over space and time, so that we get energy-momentum conservation. The integration over λ is finite if the Callan-Symanzik β function for g is negative and if also the ir divergence is cut off by the Higgs mechanism.

A notable application is the case of the Weinberg-Salam SU(2)⊗ U(1) model in an often cited form. The leptons are e_R, $(\nu_L{}^e, e_L)$, μ_R, $(\nu_L{}^\mu, \mu_L)$; and the quarks are

$$\begin{matrix} u_R{}^r, & u_R{}'^r, & (u_L, d_L{}^C)^r \\ u_R{}^r, & s_R{}^r, & (u_L{}', s_L{}^C)^r \end{matrix} \tag{23}$$

where r denotes red, yellow, or blue, and C denotes Cabibbo rotation. All currents that are coupled to gauge fields are anomaly free, hence the model is renormalizable. But the baryon and lepton currents have anomalies. We find that one Euclidean-gauge soliton gives

$$\begin{aligned} &\Delta E = \Delta M = 1, \\ &\Delta u + \Delta d^C = 3, \\ &\Delta u' + \Delta s^C = 3, \end{aligned} \tag{24}$$

where E is the electron number, M is the muon number, and u, u', d^C, and s^C are the numbers of the corresponding (Cabibbo rotated) quarks. Thus, because of the Cabibbo rotation, a proton and a neutron (two baryons equal six quarks) may annihilate to form two antileptons, one of electron and one of muon type.

The factors $\exp(-16\pi^2/g^2) = \exp(-4\pi \times 137 \times \sin^2\theta_W)$ in the cross sections and lifetimes will make none of the predicted effects observable if the Weinberg angle is not very small.

In a color gauge theory for strong interactions with two massless quark triplets, Eq. (22) is an effective four-fermion interaction with exactly the chiral quantum numbers of a mass term for the η particle. Comparison with the actual value of the η mass is not yet possible because of the infrared divergences in that theory.

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Note added.—In Eq. (22) the isospin indices have been suppressed. The isospin structure of this expression, however, is more complicated than the compact notation suggests. There is also a power of the coupling constant g in front of the exponent. The full details will be published soon.

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Evidence for Primordial Superheavy Elements*

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Microscopic crystalline monazite inclusions showing giant halo formation in biotite mica have been analyzed by the method of proton-induced x-ray emission. The observed x-ray energy spectra are best explained by the presence of a number of superheavy elements.

Radiation damage induced by alpha-particle decay of uranium and thorium isotopes and daughter products can generate spherical halos that image the known energies of the alpha groups when such materials are contained in microscopic inclusions in transparent materials such as mica.[1] While intensive studies of giant and other halos in certain micas[1] suggest a chemical origin for some, giant halos (GH) have been found which exhibit three-dimensional structure. This implies a radioactive origin, in which case the halo radii would require α energies up to about 14 MeV.

This work reports recent investigations of normal uranium/thorium halos and giant halos through the use of ion-induced x-ray analysis with low-energy proton beams.[2-4] The experiment was de-