SCATTERING MATRIX FOR A QUANTIZED BLACK HOLE

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ABSTRACT. We use as a starting point the requirement that in a fully quantized theory of gravity, also black holes should completely obey the conventional rules set by quantum mechanics. In particular, their formation and evolution should be described in terms of a scattering matrix. This way black holes at the Planck scale become indistinguishable from other particles. It is shown how to apply the presently known laws of physics to derive the main features of this *S* matrix.

1. INTRODUCTION

Elementary Particle Theory has reached an unexpected state of perfection in the second half of the seventies. The discovery of the J/ψ particle allowed for detailed checks and subsequent confirmations of both the electro-weak theory and QCD, which both turned out to be non-Abelian gauge theories, proposed by Yang and Mills and Shaw in 1954. Together they form the back-bone of the Standard Model, and the latter continued to be highly successful in the nineties. The basic structure of this Standard Model is displayed in Table 1.

The boxes here denote the various particle types, arranged according to their multiplet structure. The subscripts L and R indicate the left and right rotating components of these fields. Not indicated in the table are the antiparticles of the leptons and quarks, which have the quantum numbers, as well as the subscripts L and R, reversed. The right handed neutrino components are not necessary for most of the phenomenological aspects of the theory, but nowadays there are several experimental and theoretical indications that they may indeed be there.

Because of the limited number of fields and allowed interactions the model is characterised by a limited number of free parameters. These are:

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Table 1

THE STANDARD MODEL

based on $SU(2)_{weak} \times U(1)_{em} \times SU(3)_{strong}$

	generation I	generation II	generation III
LEPTONS (*spin* ½)	$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L$ $[\nu_e]_R$ e^-_R	$\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L$ $[\nu_\mu]_R$ μ^-_R	$\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$ $[\nu_\tau]_R$ τ^-_R
QUARKS (*spin* ½)	$\begin{pmatrix}u_r & u_g & u_b\\ d_r & d_g & d_b\end{pmatrix}_L$ $(u_r\ u_g\ u_b)_R$ $(d_r\ d_g\ d_b)_R$	$\begin{pmatrix}c_r & c_g & c_b\\ s_r & s_g & s_b\end{pmatrix}_L$ $(c_r\ c_g\ c_b)_R$ $(s_r\ s_g\ s_b)_R$	$\begin{pmatrix}t_r & t_g & t_b\\ b_r & b_g & b_b\end{pmatrix}_L$ $(t_r\ t_g\ t_b)_R$ $(b_r\ b_g\ b_b)_R$
GAUGE BOSONS (*spin* 1)	W^+ Z^0 γ W^- $SU(2) \times U(1)$	$\begin{matrix}g & g & g\\ g & g & g\\ g & g & g\end{matrix}$ $SU(3)$	[GRAVITON (*spin* 2)]
HIGGS SCALAR (*spin* 0)	H^0		

- 3 gauge coupling constants, usually indicated as: α (the fine-structure costant), θ_W (the weak mixing angle), and Λ_s (the strong scale parameter);
- 1 topological angle θ_s, describing instanton effects, relevant only for the strong interactions; as far as is known it is very close to zero.
- 2 self-interaction parameters for the Higgs field. One of these determines the Higgs mass M_H, the other is the Higgs field vacuum expectation value F_H. In combinations with θ_W and α this parameter determines the gauge boson masses.

Then there are several interaction terms between the Higgs field and the fermionic fields (Yukawa terms). Many of them correspond to the masses of the various fermions:

- 3 lepton masses: m_e, m_μ and m_τ.
- 6 quark masses: m_u, m_d, m_c, m_s, m_t and m_b.
- 4 quark mixing angles, one of which determines the Cabibbo angle θ_C, and the others describe charm and bottom decay, as well as *CP* violation.

This adds up to 19 "constants of Nature", which are uncalculable; they have to be determined by experiment. We could then add Newton's constant G_N, but this could be used to fix the as yet arbitrary scale for mass, length and time. Strictly speaking there is also the cosmological coupling constant which is also uncalculable, but it may perhaps be set to be identically zero; it would be a 20th parameter.

If indeed the neutrinos are massive then the right handed neutrino fields come into play, adding at least 7 more unpredictable parameters.

Although it is clear that the Standard Model will need revisions to describe the TeV region, it is widely expected that its general theme, gauge fields and a few scalars, both coupled to fermions, will hold up all the way to the Planck energy, corresponding to a mass of 22 micrograms per particle.

What the revisions of the Standard Model will look like (within the framework set by Gauge Theory) is anybody's guess. At present we have no theoretical principle to go by.

The smallest distance scale one can presently imagine is the Planck length. If there would be a way to figure out what the symmetries and conservation laws are there, we would be able to construct a complete model starting from the Planck length.

This brings us to the problem that has been challenging theoreticians now for many decades: there exists *no model at all* describing interactions at the Planck distance scale such that it is consistent with all laws of Physics that we believe in, in particular Quantum Mechanics and General Relativity.

A promising attempt at constructing such a model is Superstring Theory [1]. Unfortunately there are formidable conceptual difficulties in interpreting the logic of this scheme and applying this to the real world. One reason for these difficulties is the highly intuitive nature of the various arguments that formed the prime motivation for this approach.

It would be a lot safer to *derive* the only possible correct setting of variables and forces, directly from the presently established laws of physics. In these lectures we will argue that it is possible to do this, or at least to make a good start, by doing *Gedanken* experiments with black holes.

Black holes are defined as solutions of the classical, i.e. unquantized, Einstein equations of General Relativity. This implies that we only know how to describe them reliably when they are considerably bigger than the Planck length and heavier than the Planck mass. What was discovered by Hawking [2] in 1975 is that these objects radiate and therefore must decrease in size. It is obvious that they will sooner or later enter the domain that we do not understand.

All we require now is that the (as yet unknown) laws obeyed by these tiny descendants of the black holes should be as strict and unambiguous as all other laws of Nature that we do know. It is not inconceivable that mini-black holes are densely populating the vacuum at Planckian distance scales by way of quantum fluctuations. If *deterministic* behavior were to be ruled out because we want Bell's inequalities to be violated, the least thing to require is the existence of a state vector propagating according to a Schrödinger equation with a given Hamiltonian.

Curiously, large black holes do not even seem to obey a single Schrödinger equation. The thermal nature of Hawking radiation seems to indicate that quantum mechanically pure states transform spontaneously into *mixed* states. It seems that the quantum evolution of a large black hole as a single, pure, quantum state is *incomputable* [3].

Various approaches to deal with this strange situation have been considered. They fall in three categories [4]:

1) A black hole Hawking-radiates until it has Planckian dimensions. A "dead" black hole remnant is left behind, and it has taken away forever all "quantum information" of all objects that fell into the hole during its entire past.

2) A black hole evaporates completely, but due to some "strange" *acausal* propagation the information put in comes back out in the radiated matter.

3) A black hole evaporates completely, but the information disappeared. For this to happen a quantummechanically pure state must have evolved into a mixed state (density matrix).

Option 1 has been considered for some time but is now discarded by a majority.

Option 3 has been adopted by various authors [3,4]. It does imply a fundamental departure from the standard formulation of the rules of quantum mechanics. Of course pure states could evolve into mixed states if we have a system with an *uncertain Hamiltonian*: there is a probabilistic distribution for one or more of the fundamental parameters in the Hamiltonian (for instance a distribution of values for the 20 constants of nature). But in this case one would ultimately be able to specify more precisely what the Hamiltonian and its parameters are, if not by theoretical considerations then at least by doing accurate experiments. Sooner or later one would end up with option 2. Option three is only a realistic alternative if somehow the existence of *any* Hamiltonian is being denied.

Our present proposal is to be conservative: it is unlikely that a black hole as a whole can escape from being described as a more or less ordinary quantummechanical system, forming states in a Hilbert space and evolving according to a Schrödinger equation. So that leaves option 2.

We argue that the derivation of the thermodynamic nature of Hawking radiation *only* holds for large black holes. Because the number of quantum states these can be in is tremendous it may well be fundamentally impossible to tell the difference between a pure, single quantum state from a probabilistic mixture of many quantum states. The impossibility to distinguish pure states from mixed states for large black holes may well be a (*new*) fundamental principle of General Relativity.

More precisely, one may argue that it will not be possible for an observer to fall into the hole and observe its structure within the horizon while in the same time an external observer tries to pin down exactly which quantum state one is dealing with.

If we interpret the Hawking effect this way it turns out that it is not in conflict with our earlier requirement that a unique law exisits. What is even more important is the fact that this law, to some extent, can be derived. More precisely: *the exact quantum behavior at large distance scales* (the distance scales reached in present particle experiments) *can be derived uniquely* [5].

The problem with Option 2 is the apparent acausality. If we apply linearized quantum field theory in the black hole background it is ununderstandable how information put in can reemerge as information in the outgoing states. This is because the outgoing radiation originates at $t = -\infty$ and the ingoing matter proceeds until $t = +\infty$, so the information had to go backwards in time. We simply claim that precisely for this reason linearized quantum field theory is inappropriate here. One *must* take gravitational (if not other) interactions between in- and ouitgoing matter into account.

As will be explained, our starting point is a fundamental principle: it is *assumed* that a scattering matrix exists, and then we derive its properties by demanding that in- and outgoing matter interact just according to the rules set by the Standard Model including gravity. We will find that to a large extent the S matrix is unambiguously fixed by this requirement.

It is of crucial importance to note that what we are deriving is not only the (quantum) behavior of the black hole itself. It is the entire system, black hole *plus* all surrounding particles, that we are talking about. Using our (assumed) knowledge of physics at large distance scales we derive the properties of the black hole *and all other forms of matter* at energies larger than the Planck energy.

In ordinary quantum field systems behavior at small distance, or equivalently, at high energies, determines the behavior at large distances and low energies. In the present case the interdependence goes both ways, or, in other words, the whole construction will be over-determined. We expect stringent constraints of consistency, which, as one might hope, may lead to a single unique theory.

This is the motivation of this work. It may lead to "the unique theory". Even though our work is far from finished, we will be able to show that there will be a remarkable role for the old string theory. The mathematical expressions we derive are so similar to those of string theory that perhaps some of its results will apply without any change. But both the physical interpretation and the derivations will be very different. As a consequence, the mathematics is not identical. One important difference is the string constant (determining the masses of the excitations), which in our case turns out to be imaginary.

In the usual string theory one uses the obvious requirements of unitarity and causality to derive that the string is governed by a local Lagrangean on the string world sheet. To derive similar requirements for the strings born from black holes is far from easy. This is presently holding us back from considerations such as tachyon elimination and anomaly cancellation that so successfully seem to have given us the superstring scenario. What we advertise is a careful though slow process establishing the correct demands for a full black hole/string theory. If successful, one will know exactly the rules of the game and the ways how to select good from false scenarios and models.

2. QUANTUM HAIR

Classical black holes are characterised by exactly three parameters: the *mass* M, the *angular momentum* L, and the *electric charge* Q. If magnetic monopoles exist in nature then there will be a fourth parameter, namely magnetic charge B , and if besides electromagnetism there are other long range $U(1)$ gauge fields then also their charges correspond to parameters for the black hole.

However, the existence of long range $U(1)$ gauge fields other than electromagnetism seems to be rather unlikely. Then, since L , Q (and B) are all quantized, the number of different values they can take is limited, and indeed one can argue convincingly (more about this later) that the black hole can be in much more different quantum states than the ones labled by L and Q (and B), or in other words, M must be a function of much more variables than these quantum numbers alone.

An interesting attempt to formulate new quantum numbers for black holes was initiated by Preskill, Krauss, Wilczek and others [6]. They took as a model field theory a $U(1)$ gauge theory in which the local symmetry undergoes a Higgs mechanism *via* a Higgs field with charge Ne . In addition one postulates the presence of particles with charge e . In such a theory there exist vortices, much like the Abrikosov vortex in a super conductor. These vortices can be constructed as classical solutions with cylindrical symmetry, at which the Higgs field makes one full rotation if one follows it around the vortex.

The behavior of the charge e particles around the vortex is more complicated. One finds that because of the magnetic flux in the Abrikosov vortex the fields of these particles undergo a phase rotation when they flow around the vortex, in such a way that an Aharonov-Bohm effect is seen. The Aharonov-Bohm phase is $2\pi/N$, or, if we take a particle with charge ne , this phase will be $2\pi n/N$.

The importance of this Aharonov-Bohm phase is that it will be detectable for any charged particle, in such a way that we will detect its charge *modulo* N . This is surprising because *there is no long range gauge field present*!

An observer who can only detect large scale phenomena may not be able to uncover the chemical composition of the particle, but he can determine its charge modulo N . All he needs is a vortex, which to him will look just like a Nambu-Goto string.

Even if the particle were absorbed by a black hole, its electric charge would still reveil itself. Thus, charge modulo N is a quantum number that will survive even for black holes. It must be a strictly conserved charge.

One can then formalise the argument using only strings and charges modulo N , without ever referring to the original gauge field. Then there may exist many kinds of strings/vortices, so that the black hole may have a rich spectrum of these pseudo-invisible but absolutely conserved charges.

Will this argument allow us to specify all quantum numbers for a black hole? There are several reasons to doubt this. One is that an extremely large number of different kinds of strings must be postulated, which seems to be a substantial departure from the Standard Model at large distance scales.

Secondly, it is not at all obvious that it will be possible to do Aharonov-Bohm experiments with black holes. One then has to assume *first* that black holes indeed occur in well-defined quantum states, just like atoms and molecules. So this argument that black holes have quantum hair is rather circular. In my other lectures I introduce just this assumption and nothing alse. No large-scale strings are needed.

3. DECAY INTO SMALL BLACK HOLES

Due to Hawking radiation the black hole looses energy, hence also mass. The intensity af the radiation will be proportional to T^4, where T is the temperature, and the total area of the horizon, which for the Schwartzschild black hole is $4\pi R^2$; $R = 2M$. Since one expects[#]

$$T = 1/8\pi M \ , \tag{3.1}$$

the mass loss should obey [7]

$$\frac{dM}{dt} = -C\,T^4\,R^2 = -C'/M^2 \ . \tag{3.2}$$

The constants C , C' depend on the number of independent particle types at the corresponding mass scale, and this will vary slightly with temperature; the coefficients will however stay of order one (as long as M stays considerably larger than the Planck mass).

Ignoring this slight mass dependence of C' , one finds

$$M(t) = C''\,(t_o - t)^{\frac{1}{3}} \ , \tag{3.3}$$

where t_o is a moment where the thing explodes violently. Conversely, the lifetime of any given Schwarzschild black hole with mass M can be estimated to be

$$t_1 = M^3/3C' \ . \tag{3.4}$$

Now this is the time needed for the complete disappearance of the black hole. One may also ask for the average lifetime of a black hole in a given quantum mechanical state, i.e. the agerage time between two Hawking emissions.

A rough estimate reveils that the wavelength of the average Hawking particle is of the order of the black hole radius R ,and that this is also the expected spatial distance between two Hawking particles. Therefore the lifetime of a given quantum state is of order R , i.e. of order $1/M$ in Planck units.

In the language of particle physics this implies that the radiating black hole is a resonance state that in an S matrix would produce a pole at the complex energy value

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[#] As was pointed out by this author, the derivation of this formula requires an assumption concerning the interpretation of quantum wave functions for particles disappearing into the black hole. Though plausible, one can imagine this assumption to be wrong, in which case the black hole temperature will be different from (3.1).

$$E = M - C''' \, i/M \,, \tag{3.5}$$

where C''' is again a constant of order one. This corresponds to the value

$$E^2 = M^2 - 2C''' \, iM_{Pl}{}^2 \tag{3.6}$$

for the Mandelstam variable s . We see that all black hole poles are expected to be below the real axis of s at a universal distance of about one in units of the Planck mass squared.

We conclude from this that it is not altogether unreasonable to assume that a black hole is just a pole in the S matrix like any other tiny physical object. To appreciate this point further, imagine the following over-simplified model.

A toy balloon has the property that the pressure needed to blow air into it is large when it is tiny, and then decreases with size. The reason for this is that although the surface tension of the plastic material it is made of may increase with size, the surface curvature decreases, so that the pressure needed to keep it inflated drops.

In our model the toy balloon has a surface tension independent of size and the thing can be inflated to arbitrary sizes. Gas molecules may enter and leave through the small opening. Just like a black hole, the large balloon will emit particles at a slow rate, and the emission increases with decreasing size.

The point we wish to make is that there are two ways to describe the system. One is by using thermodynamics. We use concepts such as temperature, pressure, entropy, etc. If any attempt is made to compute quantum mechanical corrections to its behavior it will be found that the appropriate description of our balloon is by using a density matrix. This density matrix is directly related to the entropy.

But if the balloon is tiny it may contain only one or two molecules. At that point thermodynamics is no longer applicable. We still have a quantum mechanical description, but now it is advisable to use pure states. The Hamiltonian will have complex eigenvalues because at the boundary there will be particles moving outward only. In principle all eigenvalues can be computed and if we consider our object inside a large but finite box we can give a precise formulation of the Hilbert space.

As a matter of principle this same Hilbert space should also describe the large balloons. In any case, an experimenter cannot distinguish pure states from mixed states, and certainly there are no *transitions* from pure states to mixed states. As long as the parameters of our model are precisely formulated we have a system that in principle does not violate any of the postulates of quantum mechanics.

Our postulate for black holes is that there is absolutely no fundamental difference between black holes and our toy balloon. Curiously, such an apparently trivial starting point has consequences, as we will see.

4. THE DYNAMICAL REGION NEAR THE HORIZON

Black hole investigations often concentrate on the singular region near $r=0$. The horizon, $r = 2M$ is considered to be essentially trivial because in local coordinates space-time is perfectly regular there, so apart from the elementary transformations that give us Hawking radiation it is believed that events at the horizon are completely understood and simple.

This however is far from the truth. Hawking radiation is only easy to understand if we apply linearized field theory near the horizon. If interactions are considered between particles that came in long ago and particles that will come out in the distant future then there are tremendous complications.

The point is that although the region near the horizon can be seen to be essentially in only one particular state of Hilbert space (namely the vacuum state), when viewed by a observer in a local inertial frame, it is the observer in the distant future (at $\mathscr{I}^+$) who wants to compare this with a large sample of basis elements. His counters and photographic plates distinguish between many possibilities of particles absent or present. If we extrapolate these states back to the past we get trouble due to the various interactions with incoming matter. One is then forced to consider not only the Hartle-Hawking and Unruh states near the horizon, but also others which are much less regular.

To appreciate the difficulties one encounters the reader is invited to do with me the following exercise [8].

Consider a black hole in equilibrium with radiating matter outside. Because there are particles present the surrounding medium carries energy and entropy. Consider this in ordinary Schwarzschild coordinates and compute the energy- and entropy density at each point outside the horizon. The result will be that one would be tempted to write total entropy S and energy E as integrals over densities.

The volume element is

$$dV = 4\pi r^2 \mathrm{d}r \,/\, \sqrt{1-2M/r} \ , \qquad (4.1)$$

and the local temperature

$$T = 1 \,/\, 8\pi M\sqrt{1-2M/r} \ . \qquad (4.2)$$

The energy density is proportional to T^4 and the entropy density to T^3 . So the energy will be given by an integral

$$E = \int 4\pi r^2 \left(1-2M/r\right)^{-5/2} \mathrm{d}r \ , \qquad (4.3)$$

and the entropy by

$$S = \int 4\pi r^2 \left(1-2M/r\right)^{-2} \mathrm{d}r \ . \qquad (4.4)$$

Both diverge at the horizon! Does this mean that the vacuum surrounding a black hole has infinite energy and entropy? Of course not.

The energy should not be more than the energy M of the hole itself, and the entropy of the region near the horizon should not exceed the total entropy of the black hole, $4\pi M^2$. The point is that all thermodynamical arguments that gave us the black hole entropy would apply to the *total* entropy, of the black hole with its immediate surroundings included.

Thus we see two things: firstly, both energy and entropy of the black hole can be attributed entirely to matter *outside* the horizon, and secondly, a *cut-off* will be needed very near the horizon [8]. Estimates of this cut-off yield that the cut-off distance from the horizon itself is a dimensionless multiple of the Planck length, as measured by an observer in a local inertial frame [8].

In particular, the black hole "dynamics" may well take place entirely at our side of the horizon, with strange deviations (leading effectively to this cut-off) at a Planck distance from the horizon.

In spite of the misleading simplicity of Hawking radiation we do not understand how to formulate the equations for this black hole dynamics. Just how strange this dynamics may be can be seen from the following observation [9].

Consider an arbitrary volume $V \cong R^3$ in three-space, surrounded by a surface $\Sigma \cong R^2$. We allow V, Σ and R to vary from Planckian to cosmological sizes and then ask: What is the dimensionality of the Hilbert space of all possible configurations inside V, as a function of R?

First consider ordinary matter, an ideal gas for simplicity. We have to ask that the total *energy* of this matter is such that collapse into a black hole is avoided. Otherwise we would have to consider this black hole by itself (we'll do that in just a moment). So we have the constraint

$$E \lesssim R \ . \tag{4.5}$$

Elementary arguments of statistics tell us that the most generic configuration is a gas with temperature T such that

$$E = C_1 \, V \, T^4 \quad ; \quad S = C_2 \, V \, T^3 \ , \tag{4.6}$$

which together with (4.5) gives

$$T \lesssim C_3 \, R^{-\frac{1}{2}} \quad ; \quad S \lesssim C_4 \, R^{3/2} \cong C_4 \sqrt{V} \ . \tag{4.7}$$

The number of states in Hilbert space is now directly related to this entropy. This follows from elementary arguments of thermodynamics:

$$\# \cong \exp C_4 \sqrt{V} \ . \tag{4.8}$$

What if we do allow matter to collapse into one or several black holes? Again, the total energy E should be bounded by R. Imagine that there are several black holes with energies E_i. Then

$$E = \sum E_i \quad ; \quad S = \sum \pi R_i{}^2 = 4\pi \sum E_i{}^2 \ . \tag{4.9}$$

It follows that

$$S \lesssim 4\pi\, E^2 \cong \pi\, R^2 . \qquad (4.10)$$

The entropy is bounded by $\frac{1}{4}\Sigma$. Therefore, the *total* number of possible basis elements in Hilbert space is bounded by the exponent of $\frac{1}{4}\Sigma$. The bound is saturated if there is exactly one black hole, touching Σ.

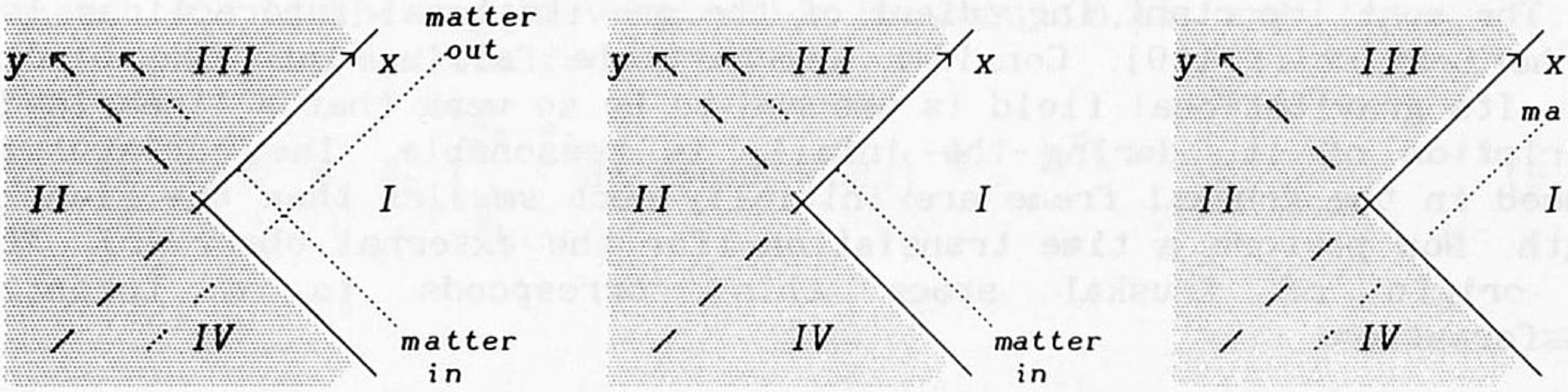
This situation would be mimicked in a model where we have precisely one Boolean degree of freedom σ at every surface element of the horizon with area $\delta\Sigma = 4\,\ell n\, 2$ in Planck units.

It is tempting to think of a black hole covered by spins taking the values ±1 at every surface element $\delta\Sigma$ of the horizon. But what makes this very hard to understand is that these degrees of freedom should *include* the degrees of freedom of all particles and fields in the neighborhood of the black hole.

5. THE *S*-MATRIX ANSATZ AND THE SHIFTING HORIZON

The problem with linearised quantum field theory in the black hole background is that the ingoing particles are independent of the outgoing ones. Hilbert space is a *product* space, $|\psi\rangle = |\psi\rangle_{in} \times |\psi\rangle_{out}$. If we were to describe a black hole that obeys an overall Schrödinger equation then these in- and out-spaces cannot be allowed to be independent of each other. In contrast, one would expect the existence of an S-matrix:

$$|\psi\rangle_{out} \;=\; S\; |\psi\rangle_{in} \quad , \qquad (5.1)$$



a) *b*) *c*)

Fig. 1

a) Linearised quantum field theory produces a Hilbert space that is the product of two factors: $|\psi\rangle = |\psi\rangle_{in} \times |\psi\rangle_{out}$. Ingoing particles form the factor space $\{|\psi\rangle_{in}\}$ (*b*), and outgoing ones form $\{|\psi\rangle_{out}\}$ (*c*). x and y are the Kruskal coordinates for the Schwarzschild metric.

and with this mapping of in- to out-states the degrees of freedom pictured in Fig. 1*a* are replaced by the ones of Fig. 1*b or* Fig. 1*c*.

As stated earlier, the reason why superimposing in- and out-particles as in Fig. 1*a* is incorrect is the breakdown of linearised quantum field theory at distances closer than a Planck length from the horizon. Gravitational interactions there become super strong. We can obtain the black hole representations of Fig. 1*b* and Fig. 1*c* by adopting the following elementary procedure:

i) *Postulate* the existence of an *S* matrix, and

ii) take interactions between in- and out-states into account, in particular the gravitational ones.

We can look upon this procedure as a new and more precise formulation of the general coordinate transformation from Kruskal coordinates to Schwarzschild coordinates, or from flat space-time to Rindler space-time. There is no direct contradiction with anything we know about general relativity or quantum mechanics, but because of the crucial role attributed to the interactions the picture is only somewhat more complicated.

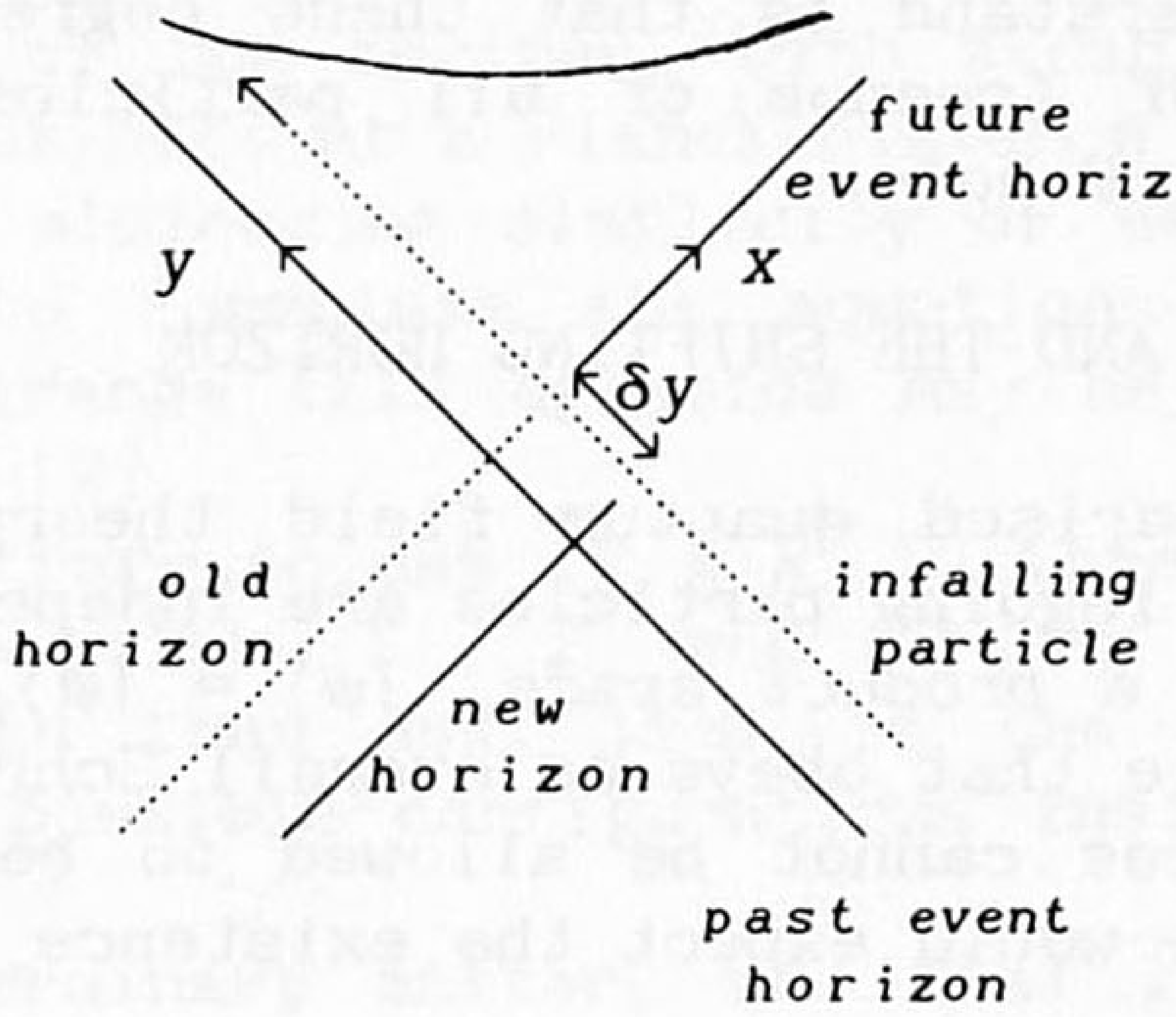


Fig. 2. The horizon displacement.

The most important ingredient of the gravitational interactions is the *horizon shift* [10]. Consider any particle falling into the black hole. Its gravitational field is assumed to be so weak that a linearised description of it, during the infall, is reasonable. The curvatures induced in the Kruskal frame are initially much *smaller* than the Planck length. Now perform a time translation (for the external observer). At the origin of Kruskal space this corresponds to a Lorentz transformation:

$$x \to \gamma^{-1} x \; ; \quad y \to \gamma\, y \quad . \qquad (5.2)$$

The γ factors here can grow very quickly, exponentially with external time. The very tiny initial curvatures soon become substantial, but are only seen as shifts in the y coordinate, because y has expanded such a lot.

The result is a representation of the space-time metric where the incoming particle enters along the y axis, with a velocity that has been boosted to become very close to the speed of light. Its energy in terms of the boosted coordinates has become so huge that the curvature became sizable. It is described completely by saying that two halfs of the conventional Kruskal space are glued together along the y axis with a *shift* δy, depending explicitly on the angular coordinates ϑ and φ.

The calculation of the function $\delta y(\vartheta,\varphi)$ is elementary. In Rindler space, $\delta y(\tilde{x})$ is simply [11]

$$\delta y(\tilde{x}) \;=\; -4G_N p \log(\tilde{x}^2) + C \;, \tag{5.3}$$

where G_N is Newton's constant, p is the ingoing particle momentum, and C is an arbitrary constant.

In Kruskal space the angular dependence of δy is a bit more complicated than the $\tilde{x}$ dependence in Rindler space. It is found by inserting Einstein's equation, which is $R_{\mu\nu} = 0$, everywhere except where the particle comes in. Starting with an arbitrary δy as an Ansatz, one finds Einstein's equation to correspond to

$$(1 - \Delta_{\vartheta,\varphi})\; \delta y(\vartheta,\varphi) \;=\; 0 \;\;, \tag{5.4}$$

where $\Delta_{\vartheta,\varphi}$ is the angular Laplacian. At the angles ϑ_o, φ_o where the particle enters we simply compare with the Rindler result (5.3) to obtain

$$(1 - \Delta_{\vartheta,\varphi})\; \delta y(\vartheta,\varphi) \;=\; \kappa\; p_{in}\; \delta^2(\vartheta,\varphi;\vartheta_o,\varphi_o) \;\;, \tag{5.5}$$

where κ is a numerical constant related to Newton's constant.

This equation can be solved:

$$\delta y(\vartheta,\varphi) \;=\; f(\vartheta,\varphi;\vartheta_o,\varphi_o)\; p_{in}(\vartheta_o,\varphi_o) \;\;;$$

$$f \;=\; \kappa_1 \int_0^\infty \frac{\cos\!\left(\frac{\sqrt{3}}{2}\, s\right) ds}{\left(\cosh s \,-\, \cos\vartheta_1\right)^{\frac{1}{2}}} \;\;, \tag{5.6}$$

where ϑ_1 is the angular separation between (ϑ,φ) and (ϑ_0,φ_o) .

Other expressions for f are

$$f \;=\; \kappa_2 \int_{\vartheta_1}^{2\pi-\vartheta_1} \mathrm{d}z\, \left(\cos\vartheta_1 - \cos z\right)^{-\frac{1}{2}}\, \mathrm{e}^{-\frac{1}{2}\sqrt{3}\, z} \;\;; \tag{5.7}$$

and [12]

$$f \;=\; \frac{\pi\kappa_1}{\sqrt{2}}\; \frac{P_{-\frac{1}{2}\,+\,\frac{1}{2}i\sqrt{3}}\left(-\cos\vartheta_1\right)}{\cosh\left(\frac{1}{2}\pi\sqrt{3}\right)} \;\;, \tag{5.8}$$

where P is a Legendre function with complex index (conical function). From (5.7) one sees directly that for all angles ϑ_1 f is positive.

6. CONSTRUCTION OF THE *S*-MATRIX

We now come to the most important point of these lectures, namely the explicit construction of the *S*-matrix. The horizon shift discussed in the previous section is an essential ingredient in this construction. Without it we would not be able to perform this task. Now we are. The argument goes as follows [5].

1. Consider one particular in-state and one particular out-state. Assume that someone gave us the number obtained by sandwiching the S-matrix between these two states:

$$\langle in | out \rangle \ . \tag{6.1}$$

Both the in- and the out-state are described by giving all particles in some conveniently chosen wave packets. The ingoing wave packets look like

$$e^{-ip^i_{in}x} \, f^{\,i}_{in}(x, \vartheta, \varphi) \quad , \tag{6.2}$$

where i runs over all particles involved, and $f^{\,i}_{in}$ are smooth functions. We assume them to be sharply peaked in the angular coordinates so that we know exactly where the particles enter into the horizon (so the *angular coordinates* and the *radial momenta* of all particles are sharply defined). Similarly the outgoing wave packets are

$$e^{-ip^i_{out}y} \, f^{\,i}_{out}(y, \vartheta, \varphi) \quad . \tag{6.3}$$

Now let us consider a small change in the ingoing state: $|in\rangle \ \rightarrow \ |in'\rangle$. This brings about a sharply defined small change in the distribution of the radial momenta $p_{in}(\vartheta, \varphi)$ on the horizon:

$$p_{in} \ \rightarrow \ p_{in} + \delta p_{in}(\vartheta, \varphi) \quad . \tag{6.4}$$

This δp_{in} now produces an (extra) horizon shift,

$$\delta y(\Omega) \ = \ \int f(\Omega\text{-}\Omega') \, \delta p_{in}(\Omega') \quad , \tag{6.5}$$

where f is the Green function computed in the previous section and Ω stands short for (ϑ, φ) ; $\Omega\text{-}\Omega'$ stands for the angle ϑ_1 between Ω and Ω' .

The horizon shift (6.5) does *not* affect the thermal nature of the Hawking radiation, but it *does* change the quantum states. All out-wave functions are shifted. (6.3) is replaced by

$$e^{-ip^i_{out}(y \, + \, \delta y(\Omega))} \ f^{\,i}_{out}(y, \Omega) \tag{6.6}$$

(the effect of the shift on f is of lesser importance).

We observe that the S-matrix element (6.1) is replaced:

$$\begin{aligned} \langle in' | out \rangle \ &= \ e^{-i\int p_{out}(\Omega) \ \delta y(\Omega) \mathrm{d}^2\Omega} \ \langle in | out \rangle \\ &= \ e^{-i\int\int p_{out}(\Omega) \ f(\Omega\text{-}\Omega') \ \delta p_{in}(\Omega') \ \mathrm{d}^2\Omega \mathrm{d}^2\Omega'} \ \langle in | out \rangle \quad . \end{aligned} \tag{6.7}$$

Here, $p_{out}(\Omega)$ is the total outgoing momentum at the angular coordinates Ω . What we have achieved is that we have been able to compute *another* matrix element of S . Now simply repeat this many times. We then find *all* matrix elements of S to be equal to

$$\langle in|out\rangle \quad = \quad \mathcal{N}\, e^{-i\iint p_{out}(\Omega)\ f(\Omega-\Omega')\ p_{in}(\Omega')\mathrm{d}^2\Omega\mathrm{d}^2\Omega'} \quad , \qquad (6.8)$$

where $\mathcal{N}$ is one common unknown factor. Apart from an overall phase, $\mathcal{N}$ should follow from unitarity.

The derivation of (6.8) ignores all interactions other than the gravitational ones. We will be able to do better than that, but let us first analyse this expression.

What is unconventional in the S-matrix (6.8) is the fact that the in- and out-states must have been characterised *exclusively* by specifying the *total* radial momentum distribution over the angular coordinates on the horizon. If there are more parameters necessary to characterise these states, these extra parameters will not figure in the S-matrix. But this would mean that two different states $|A\rangle$ and $|B\rangle$ would evolve into the same state $|\psi_{out}\rangle$, so these extra parameters will not be consistent with unitarity. *We cannot allow for other parameters than the total momentum distributions* (unless more kinds of interactions are taken into account).

Thus, if for the time being we only consider gravitational interactions, the in-states can be given as $|p_{in}(\Omega)\rangle$ and the out-states as $|p_{out}(\Omega)\rangle$. The operators $p_{in}(\vartheta,\varphi)$ commute for different values of ϑ and φ , and their representations span the entire Hilbert space; the same for $p_{out}(\vartheta,\varphi)$.

The canonically conjugated operators $u_{in}(\Omega)$, $u_{out}(\Omega)$ are defined by the commutation rules

$$[p_{in}(\Omega),u_{in}(\Omega')] \quad = \quad -i\delta^2(\Omega-\Omega') \qquad (6.9)$$

(and similarly for the *out* operators), or,

$$\langle u_{in}(\Omega)|p_{in}(\Omega)\rangle \quad = \quad C\ \exp\ i\!\int\!\mathrm{d}^2\Omega\ p_{in}(\Omega)u_{in}(\Omega) \quad , \qquad (6.10)$$

where C is a normalisation constant.

In terms of the u operators the S-matrix is

$$\langle u_{out}(\Omega)|u_{in}(\Omega)\rangle \quad = \quad \int\!\mathcal{D}p_{out}\mathcal{D}p_{in}\ \exp\bigl(-ip_{in}u_{in}+ip_{out}u_{out}-ip_{out}fp_{in}\bigr) \quad , \qquad (6.11)$$

which is a Gaussian functional integral over the functions p_{out} and p_{in} . Since the inverse f^{-1} of f is $\kappa^{-1}(1-\Delta_\Omega)$, the outcome is

$$\begin{aligned}\langle u_{out}(\Omega)|u_{in}(\Omega)\rangle \quad &= \quad C\ \exp\bigl(-i\kappa^{-1}\!\int\!\mathrm{d}^2\Omega\ u_{in}(\Omega)\ (1-\Delta_\Omega)\ u_{out}(\Omega)\bigr) \quad = \\ &C\ \exp\bigl(-i\kappa^{-1}\!\int\!\mathrm{d}^2\Omega\ (\partial_\Omega u_{in}(\Omega)\ \partial_\Omega u_{out}(\Omega)\ +\ u_{in}(\Omega)\ u_{out}(\Omega))\bigr) \quad .\end{aligned} \qquad (6.12)$$

The last term in the brackets is something like a mass term and becomes subdominant if we concentrate on small subsections of the horizon. Therefore it will often be ignored. Eq. (6.12) seems to be more fundamental than (6.8) because it is local in Ω.

Fourier transforming back we get

$$\langle p_{out}(\Omega)|p_{in}(\Omega)\rangle = \tag{6.13}$$

$$\mathcal{N}\int \mathcal{D}u_{in}\mathcal{D}u_{out}\ \exp\int d^2\Omega\Big(ip_{in}u_{in} - ip_{out}u_{out} - i\kappa^{-1}\partial_\Omega u_{in}(\Omega)\ \partial_\Omega u_{out}(\Omega)\Big)\ ,$$

where the mass term was ignored. We obtain (6.8) written as a functional integral.

It is illuminating to redefine

$$p_{out} = p^+\ \ ;\ \ p_{in} = p^-\ \ ;\ \ u_{out} = x_-\ \ ;\ \ u_{in} = -x^+\ \ , \tag{6.14}$$

so that if we define the transverse components $\tilde{p} \cong 0$ one can write

$$\langle p_{out}(\Omega)|p_{in}(\Omega)\rangle\ = \tag{6.15}$$

$$\mathcal{N}\int \mathcal{D}x^\mu(\Omega)\ \exp\int d^2\Omega\Big(ip^\mu(\Omega)x^\mu(\Omega) - i\kappa^{-1}(\partial_\Omega x^\mu(\Omega))^2 - i\kappa^{-1}(x^\mu(\Omega))^2\Big)\ .$$

This functional integral is very similar to the functional integral for a string amplitude, except for the unusual imaginary value for the string constant:

$$T\ =\ 8\pi G_N i\ \ . \tag{6.16}$$

The similarity is more than superficial. Suppose that the ingoing and outgoing wave packets were not peaked at fixed values for the solid angles Ω but spread with functions $f^i{}_{in,out}$ going like

$$\exp\ i\ \tilde{p}^i\tilde{x}^i\ , \tag{6.17}$$

where $\tilde{x}$ is a transverse coordinate on the horizon. This means that we have all external particles entirely in the momentum representation. Then the functional integral (6.15) has to be convoluted with these extra factors and integrated over $\tilde{x}^i$. This corresponds precisely to the integrations over Koba-Nielsen variables needed to obtain an N particle amplitude.

7. ELECTROMAGNETISM

What happens if more interactions are included? The simplest to handle turn out to be the electromagnetic forces. Suppose that the particles that collapsed to form the black hole carried electric charges. The angular charge distribution was

$$\rho_{in}(\Omega)\ \ . \tag{7.1}$$

As in the previous section, we consider a small change in this setting, so

$$\rho_{in}(\Omega) \rightarrow \rho_{in}(\Omega) + \delta\rho_{in}(\Omega) . \tag{7.2}$$

The $\delta\rho_{in}(\Omega)$ produces an extra contribution to the vector potential at the horizon which is not difficult to compute[1].

$$\delta A_\mu = \frac{1}{r_o^2}\, \delta_{\mu x}\, \delta(x)\, A(\Omega) \quad , \tag{7.3}$$

where r_o is the radius of the horizon, and $A(\Omega)$ must satisfy

$$\Delta_\Omega A(\Omega) = \delta\rho_{in}(\Omega) \quad . \tag{7.4}$$

The field (7.3) is only non-vanishing on the y-$\tilde{x}$ plane, where it causes a sudden phase rotation for all wave packets that go through. An outgoing wave undergoes a phase rotation

$$e^{iQ\Lambda(\Omega)} \quad ; \quad \Lambda(\Omega) = r_o^{-1}\, A(\Omega) \quad . \tag{7.5}$$

This rotation must be performed for all outgoing particles with charge Q. All together the outgoing wave is rotated as follows:

$$|p_{out}(\Omega), \rho_{out}(\Omega)\rangle \;\rightarrow \tag{7.6}$$

$$\exp -i \int d^2\Omega \int d^2\Omega'\, f_1(\Omega-\Omega')\, \rho_{out}(\Omega)\delta\rho_{in}(\Omega') \times |p_{out}(\Omega), \rho_{out}(\Omega)\rangle \quad ,$$

where $f_1(\Omega-\Omega')$ is a Green function that satisfies

$$\Delta_\Omega\, f_1(\Omega-\Omega') = -\kappa_e \delta^2(\Omega-\Omega') \quad . \tag{7.7}$$

κ_e is a numerical constant.

And using arguments identical to the ones of the previous section we repeat the infinitesimal changes to obtain the S-matrix dependence on $\rho_{in}(\Omega)$ and $\rho_{out}(\Omega)$:

$$\langle p_{out}(\Omega), \rho_{out}(\Omega) | p_{in}(\Omega), \rho_{in}(\Omega)\rangle = \tag{7.8}$$

$$N\, e^{-i\iint p_{out}(\Omega)\, f(\Omega-\Omega')\, p_{in}(\Omega')\, d^2\Omega d^2\Omega'} \;\times$$

$$e^{-i\iint \rho_{out}(\Omega)\, f_1(\Omega-\Omega')\, \rho_{in}(\Omega')\, d^2\Omega d^2\Omega'} \quad .$$

Now let us replace $\rho_{out}(\Omega)\rho_{in}(\Omega')$ by

$$-\tfrac{1}{2}\left[\rho_{out}(\Omega)-\rho_{in}(\Omega)\right]\left[\rho_{out}(\Omega')-\rho_{in}(\Omega')\right] \quad . \tag{7.9}$$

---

[1] The unit e of electric charge of the ingoing particle is here included in ρ_{in} .

This differs from the previous expression by two extra terms in the exponent, one depending on $\rho_{out}(\Omega)$ only and the other depending on $\rho_{in}(\Omega)$ only. These would correspond to external "wave function renormalization factors" that do not describe interaction between the in- and the out-state. So we ignore them.

The electromagnetic contribution in (7.8) can then be written as a functional integral of the form

$$\int \mathcal{D}\Phi(\Omega)\ \exp \int d^2\Omega \Big(\frac{-i}{2\kappa_e}(\partial_\Omega \Phi)^2 + i\Phi(\rho_{out}-\rho_{in}) \Big) \quad . \qquad (7.10)$$

Now it may also be observed that the charge distribution ρ is actually a combination of Dirac delta distributions,

$$\rho(\Omega) \ = \ \sum Q_i \delta^2(\Omega-\Omega_i) \quad ; \quad Q_i = n_i e \quad . \qquad (7.11)$$

Therefore, if we add an integer multiple of $2\pi/e$ to the field Φ the integrand does not change. In other words: Φ is a periodic variable.

Adding (7.10) to (6.15) we notice that the field Φ acts exactly as a fifth, periodic dimension. Hence, electromagnetism emerges naturally as a Kaluza-Klein theory.

8. HILBERT SPACE

In this section we briefly recapitulate the nature of the Hilbert space in which these S matrix elements are defined. As stated in Section 4, the states whose momentum and charge distribution over the horizon were given by $p(\Omega)$ and $\rho(\Omega)$ *include* all particles in the black hole's vicinity. But (if we temporarily ignore electromagnetism) we can also form a complete basis in terms of states for which the canonical operators $u(\Omega)$ are given. These $u(\Omega)$ (eq. 6.12) may be interpreted as the coordinates of the horizon. Apparently, *the precise shape of the horizon determines the state of the surrounding particles!*

Furthermore, the *in*-horizon and the *out*-horizon do not commute. Therefore, the positions of the future event horizon and the past event horizon do not commute with each other. If we define a "black hole" as an object for which the location in space-time of the future event horizon is precisely determined, we can define a "white hole" as a state for which the past event horizon is precisely determined. *The white hole is a linear superposition of black holes* (and vice versa); operators for white holes do not commute with the ones for black holes. In our opinion this resolves the issue of white holes in general relativity.

Obviously, it is important that the horizon of the quantized black hole is not taken to be simply spherically symmetric. In a black hole with a history that is not spherically symmetric, the onset of the horizon, i.e. the point(s) in space-time where for the first time a region of spcae-time emerges from which no timelike geodesic can escape to $\mathcal{J}^+$, have a complicated geometrical structure. Their mathematical construction has the characteristics of a caustic. One might conjecture that the topological details of this caustic specify the quantum state a black hole may be in.

The fact that the geometry of the (future or past) horizon should determine the quantum state of the surrounding particles gives rise to interesting problems. In ordinary quantum field theory the Hilbert space describing particles in a region of space-time is Fock space; an arbitrary, finite, number of particles with specified positions or momenta together define a state. But now, close to the horizon, a state must be defined by specifying the *total* momentum entering (or leaving) the horizon at a given solid angle Ω . Apparently we are not allowed to specify further how many particles this were, and what their other quantum numbers were. Together all these possibilities form just one state. So, our Hilbert space is set up differently from Fock space. The difference comes about of course because we have strong gravitational interactions that we are not allowed to ignore.

The best way to formulate the specifications of our basis elements here is to assume a spacial cut-off in the space of solid angles (one "lattice point" for each unit of horizon surface area somewhat bigger than $\delta\Sigma$, the Planck distance squared), and then to specify that there should be *exactly one ingoing and one outgoing particle* at each $\delta\Sigma$. The momenta are given by the operators $p_{in}(\Omega)$ and $p_{out}(\Omega)$ (and the charges by $\rho_{in}(\Omega)$ and $\rho_{out}(\Omega)$). The in- and out-operators of course do not commute.

One may speculate that since $\delta\Sigma$ is extremely small, the totality of all these particles may be indistinguishable from an ordinary Dirac sea for the large-scale observers.

Also one may notice that the way conventional string theory deals with in- and outgoing particles is remarkably similar. Before integrating over the Koba-Nielsen variables the string amplitudes also depend exclusively on the distribution of total in- and outgoing momenta (see concluding remarks in Sect. 6).

9. RELATION BETWEEN TERMS IN THE HORIZON FUNCTIONAL INTEGRAL AND BASIC INTERACTIONS IN 4 DIMENSIONS

In principle one can pursue our doctrine to obtain more precise expressions for our black hole *S* matrix by including more and more interactions that we actually know to exist from ordinary particle theory. We should be certain to obtain a result that is accurate apart from a limitation in the angular resolution, because particle interactions are known only up to a certain energy. In this section we indicate some qualitative results.

The details of the "presently favored Standard Model" (Sect. 1) may well change in due time. We will denote anything used as an imput regarding the fundamental interactions among in- and outgoing particles near the horizon, at whatever scale, by the words "standard model".

Suppose the standard model contains a scalar field. The effects of this field will be felt by slowly moving particles at some distance from the horizon. But at the horizon itself these effects are negligible. Consider namely a particle such as a nucleon, surrounded by a scalar field such as a pion field. Close to the horizon this particle will be Lorentz boosted to tremendous energies. The scalar field configuration will become more and more flattened. But unlike vector or tensor fields, its intensity will not be enhanced (it is Lorentz invariant). So the cumulated effect on particles traversing it will go to zero.

However, one effect due to the scalar field will not go away. Suppose our standard model contains a Higgs field, rendering a $U(1)$ gauge boson massive. This means that the electromagnetic field surrounding a fast electrically charged particle will be of short range only. One can derive that the field equation (7.4) will change into

$$(\Delta_\Omega - M_A{}^2)A(\Omega) \quad = \quad \delta\rho_{in}(\Omega) \quad . \tag{9.1}$$

One may say that the incoming charge density $\rho_{in}(\Omega)$ is screened by charges coming from the Higgs particles.

This implies that the equations for the Φ field in Sect. 7 will obtain a mass term:

$$\int \mathcal{D}\Phi(\Omega) \ \exp \int d^2\Omega \Big(\frac{-i}{2\kappa_e}[(\partial_\Omega\Phi)^2 + M_A{}^2\Phi^2] + i\Phi\ \rho \Big) \quad . \tag{9.2}$$

Note that this mass term breaks explicitly the symmetry $\Phi \rightarrow \Phi + \Lambda$. This explicit symmetry breaking may be seen as a result of the finite and constant value of the Higgs field at the origin of Kruskal space-time.

Next, we may ask what happens if our standard model exhibits confinement. This means that at long distance scales no effect of the gauge field is seen and all allowed particles are neutral.

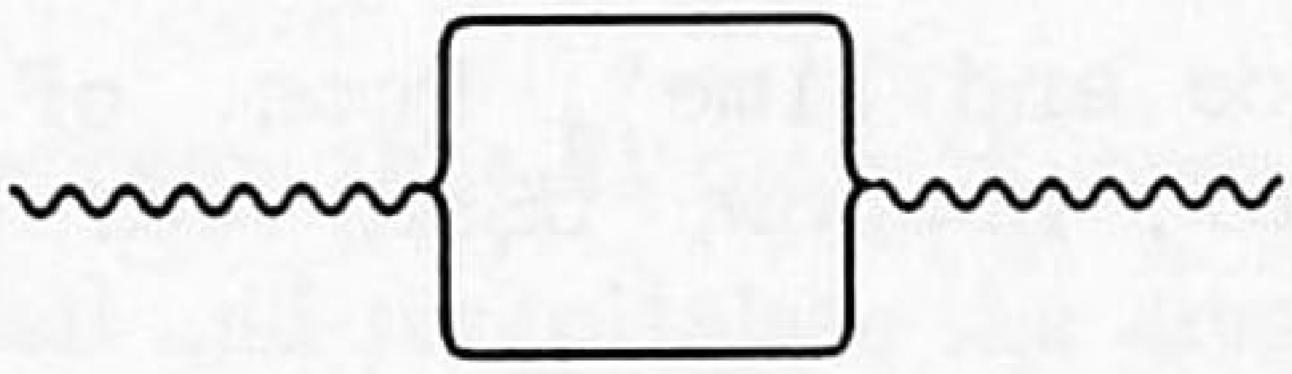
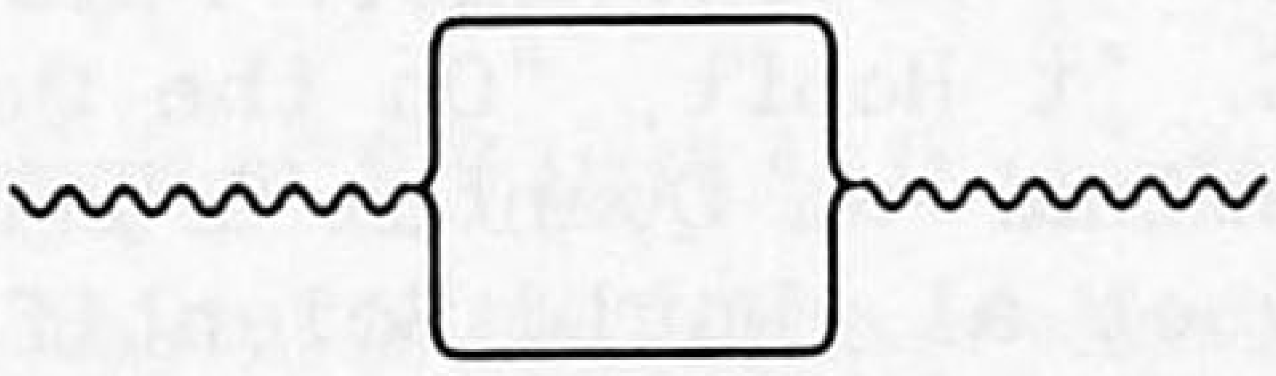
Confinement is usually considered to be the *dually opposite* of the Higgs mechanism: Bose condensation of magnetic monopoles. A magnetic monopole is an object to which the end point of a Dirac string is attatched. A Dirac string is a singularity in a gauge transformation such that the gauge transformation makes one full rotation if we follow a loop around the string.

We must know how to describe the operator field of a monopole at the horizon. Suppose a monopole entered at the solid angle Ω_1. This means that a Dirac string connects to the black hole at that point. The outgoing charged particles undergo a gauge rotation that rotates a full cycle if we follow a closed curve around Ω_1 (an anti-monopole may neutralise this elsewhere on the horizon).

The gauge jump for the vector potential field A can be identified with the periodic field Φ of Sect 7. So adding an entering monopole to the in-state implies that this field Φ is shifted by an amount $\Lambda(\Omega)$ where Λ makes a full cycle when followed over a loop around Ω_1. This is an operation that is called *disorder operator* in statistical physics and field theory. This operator, Φ_D, is dual to the original field Φ. We find that the dual transformation electricity ↔ magnetism corresponds to the duality between Φ and Φ_D.

Thus, if we have confinement, a mass term will result in the equations for Φ_D. It explicitly breaks the symmetry $\Phi_D \rightarrow \Phi_D + C$. But it bars the transformation back to Φ. Therefore, *if confinement occurs, the field* Φ *is no longer well-defined, we have only* Φ_D. Its mass will be the glueball mass.

Table 2.

STANDARD MODEL IN 3+1 DIMENSIONS	INDUCED 2 DIMENSIONAL FIELD THEORY ON BLACK HOLE HORIZON
• *Spin 2:* $g_{\mu\nu}(\mathbf{x}, t)$ local gauge generator: $u^{\mu}(\mathbf{x}, t)$	String variables (*spin 1*): $x^{\mu}(\Omega)$
• *Spin 1:* $A_{\mu}(\mathbf{x}, t)$ local gauge generator: $\Lambda(\mathbf{x}, t)$ *mod* $2\pi/e$	Scalar variable (*spin 0*): $\Phi(\Omega)$ *mod* $2\pi/e$
• *Spin 0:* $\phi(\mathbf{x}, t)$	No field at all
• Higgs mechanism: "spontaneous" mass M_A for vector field	explicit symmetry breaking; $\Phi(\Omega)$ gets same mass M_A .
• Confinement in vector field A_{μ}	Φ must be replaced by disorder op. Φ_D ; its symmetry broken.
• Non-Abelian gauge theory	only scalars Φ_i corresponding to Cartan subalgebra
• *Spin ½:* fermions	no field at all
• *Spin 3/2:* gravitino local gauge generator spin ½	Spin ½ fermion (?)
Vacuum bubble correction to vector propagator due to scalar field: 	vacuum correction due to scalar ghost with derivative coupling, 

In Table 2 we list all peculiarities of the mapping from 4 to 2 dimensions that we found. The *generators* of local symmetry transformations in 4 dimensions correspond to the dynamical variables in 2 dimensions. Thus one expects that if the standard model includes a gravitino (requiring a supersymmetry generator of spin ½) then a fermionic field variable will emerge in 2 dimensions. We even found that certain one-loop corrections to the vector propagator can be reproduced in the 2 dimensional theory by similar loop corrections.

But the above are merely qualitative features. They should be turned into precise quantitative rules and principles, for which further work is needed.

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