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Questioning the answers or Stumbling upon good and bad Theories of Everything

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Summary

The last two decades witnessed a new breakthrough of fundamental physics in its struggle towards a better understanding of the World of the Small. This gave rise to new speculations concerning the idea that there may exist a single ultimate Law of Physics underlying all particles and forces, and therefore also everything made out of these particles held together by these forces, which could include us and the universe. Discovering this Law would be a tremendous achievement, but would not at all imply our understanding of most of its consequences. In this chapter I focus on two themes:

(1) Speculations that such a fundamental Law exists and that there is a possibility that we humble human beings may in due time be able to find this Law and give a detailed and precise formulation of it – including boundary conditions and initial conditions if necessary – are far from ridiculous (Section two), in spite of lessons from the past. Our present knowledge indicates that the possibility is real (Section three).

(2) Claims made by several theoretical physicists that they are coming very close towards actually realizing this hope are highly premature and naive. 'Superstring theory' is almost certainly not the answer. The dispute concerning the physical interpretation to be given to the formalism called 'quantum mechanics' will have to be continued, and it is as yet impossible to predict the vastness of the undoubtedly formidable obstacles that are still in our way (Sections four and five).

Practical applications of our allegedly expected understanding of a Universal Law will be very limited and will certainly have no effect on our daily life.

But philosophical implications will be far-reaching. I will try to explain what I mean by the complete merging of theoretical physics with mathematics.

1 What could possibly be the meaning of the words 'Theory of Everything'?

Science, as we know it at present, is based on empirical knowledge. In our perpetual quest for a better understanding of the patterns and forces that govern our world, we have learned how to devise all sorts of machinery for making subtle observations, and to perform experiments by which theories concerning these patterns and forces can be checked. Experimental observations are never perfect, and theories can never be absolutely confirmed. In the best of circumstances a theory can either be falsified or given further support. In view of this it would seem to be silly to ever expect that we can brew a *perfect* theory. There is an infinity of phenomena still waiting to be studied by us, and so one should expect also an infinity of theories, some better than others, but none perfect, let alone *one single* theory that could accommodate everything at once. If the words 'Theory of Everything' were to be interpreted literally it would be quite obvious that such a concept will never be fabricated by mortal human beings.

What one can speculate about in a more fruitful manner is something that could be called an 'Ultimate Law for Basic Dynamics'. What I mean by this is best illustrated by formulating an example for a basic Law of Dynamics that could be used to generate an entire universe. The example I have in mind is an idea that originated in the early 1970s when physicists and mathematicians began to play various apparently nonsensical games with computers. When the first personal computers arrived somewhat later, it became very popular. The idea was called 'Conway's Game of Life', and it went as follows (see Fig. 1).

On a rectangular infinite lattice we have 'cells', each of which carries one 'bit' of information: the cell is said to be 'alive' if this bit of information is a one; it is 'dead' if it is a zero. Then there is a clock. At every tick of the clock the contents of each cell is being updated. For each cell the new status, at time $t + 1$, depends on the contents of itself and its nearest eight neighbors at time t (Fig. 2).

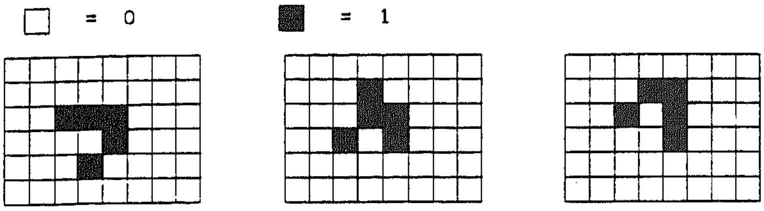

Fig. 1. Evolution of a particular pattern in Conway's Game of Life, at three consecutive times. It will propagate diagonally over the lattice.

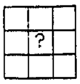

Fig. 2. The cell in the center is updated depending on what was there before, and on what was in the eight surrounding cells drawn here.

The rule is as follows:

- *If exactly two neighbors are alive, the cell in the center will stay as it was.*
- *If exactly three neighbors are alive, the cell in the center will live.*
- *In all other cases the cell in the center will die.*

In Fig. 1, I show a pattern that, after a while, returns into itself, but at a different place: it moves! One can start with an initial configuration where several of these moving patterns are sent towards each other so that they will collide, and then study what happens. The results may become rather complex, and without a computer hopeless to calculate.

The point I want to make is that this system is a 'model universe'. Imagine that, given a large enough lattice and a patient enough computer, an 'experiment' is run for a sufficiently long time. Regular, recognizable structures may appear again and again, satisfying their own 'laws of physics'. In principle these laws of physics should be derivable from the original rules mentioned above. We could call these structures 'atoms'. Atoms themselves will form 'molecules', and so on. Eventually, one might find 'intelligent' creatures built out of these building blocks. We might call them 'humans'. They will investigate the world they are in, and perhaps ultimately discover the three fundamental 'Laws of physics' on which their universe is based.

It is unlikely that the rules given above will do all that. But the question one may nevertheless ask now is: could it possibly be that the universe we are in ourselves has a similar simple structure, based on such a simple universal Law, a Law that is invariable and absolute? Is it conceivable that we will ever be able to discover that Law if it exists? The first thing one notices is that, although Conway's world is at best only a poor caricature of our real world, there are striking similarities with the real world. Conway's world does contain different kinds of 'atoms', of which the moving pattern shown above is only one example. There are many more. These particles can collide against each other and interact. Furthermore, something else is needed before a complete description of the Universe is obtained: one should also formulate the *boundary conditions*. Not only should we state how large the lattice is, and whether it closes into itself like a sphere or a torus (a doughnut shape); one must also specify what the *starting configuration* was: the boundary condition at time $t = 0$. There are even versions of models of this sort that can describe an 'expanding universe', much like our own. The boundary conditions may then be extremely simple and yet the details of the structures obtained at later times may be extremely complicated.

It is interesting to speculate that in a model of Conway's type various particles might form super structures such as 'atoms' and 'molecules', and all large scale structures one thus obtains, and all their properties, will be sole consequences of the single three Laws that I formulated in the beginning. Just imagine that a community of 'intelligent' creatures could emerge in a Conway-like system. By patiently performing numerous series of experiments, constructing and rejecting one theory after the other, these creatures might eventually stumble upon the conjecture that their universe is generated by just three basic Laws. All of their experiments will be in agreement with these Laws, although these creatures can never be sure that the formulae they found are the ultimate ones. It could always be conjectured that exactly once every 1 048 576 ticks of the clock the rule is different: if the closest 624 neighbors are all in one particular given pattern you get a one instead of a zero. But after elaborate tests one could rule out such proposals one by one, and a moment will arrive when most of the inhabitants will be convinced that the Ultimate Law has been found.

Of course, by guessing the Ultimate Law not all phenomena in Conway's world would be immediately explained. In fact it is only then that at last physicists could begin explaining the phenomena they see one by one, and the ultimate 'explanation' of phenomena as complex as life, let alone intelligent life, will almost certainly never come. It is an Ultimate Law that physicists are seriously thinking about, not a theory of everything in the literal sense. The name 'Theory of Everything' instead of 'Ultimate Law' or 'Basic Law' was a bad commercial.

In summary, what I would like to discuss here, and what I will indicate admittedly inaccurately by the misnomer 'Theory of Everything', is a set of laws of physical dynamics with the following properties:

(1) The laws will determine with infinite accuracy the evolution of all physical dynamical variables at a local level, and should also include a description of the 'boundary' of the universe, as well as its initial state.
(2) There exists no closely resembling alternative theory. This means that any slight change brought about in the rules would make the theory unlikely or inelegant. The theory will be a 'package deal': take it, or leave it. This should hold both for the local laws and for the boundary conditions.
(3) Evolution according to these laws will give rise to a nearly infinite complexity, a complexity sufficiently extensive to include the marvelously perplexing wonders abounding in our universe – the emergence of life and intelligence being only a few of these.

The phenomenon of Complexity is familiar among physicists. A well-known example is the weather system in our atmosphere. In combination with another complex phenomenon, namely the formation of continents on Earth, it gave our planet the enormously rich structure it has. But for non-physicists it is often difficult to imagine how even much simpler basic laws can sometimes produce equally or even more complex behavior. I believe that the reason why many of its opponents intuitively reject the notion of a theory of everything is that they underestimate the effects of complex behavior. More about this in Section five.

2 The present theory of elementary particles and forces

How does our present understanding of elementary particles compare with a 'Theory of Everything'? In certain respects we have made great progress in precisely that direction, but on the other hand it is quite clear that the gaps separating us from any 'ideal' theory are still tremendous. Let me discuss these two observations in turn.

In the 1970s a development took place in quantum field theory that allowed us to formulate a model for all known particles and forces between them. This model became known as 'The Standard Model'. It is a set of equations which were found to give a remarkably accurate description of precisely all the particles and forces ever detected in any laboratory experiment. The model is fairly complicated, and it cannot predict the reaction of particles and fields under all conceivable circumstances – after all, in our fantasy we could easily imagine experimental setups that are completely impossible to realize in practice. But it seems to be a very good description of what is happening nearly everywhere in our universe. I will now give a rough description of this model and its laws.

The basic entities are *Dirac fermions*, elementary particles that show a certain amount of spinning motion. This amount of spinning motion is measured in multiples of a fundamental constant of Nature, Planck's constant divided by 2π: these particles are said to have 'spin $\frac{1}{2}$'. We start with particles that can only move with the speed of light. For these particles the *axis of rotation* can be seen to be always parallel to their velocity vector, and one can derive that this statement is unique and independent of the velocity of the observer. This is why one can distinguish unambiguously particles that spin 'to the left' from particles that spin 'to the right', with respect to this axis. From a mathematical point of view there is some resemblance between particles of this type and the zeros and ones in Conway's model.

The forces these fermions exert on each other are now described by introducing another set of particles, the *gauge bosons*. These are particles with spin 1. They can either be considered as the 'energy quanta' of various kinds of electric and magnetic fields, or one can view these particles themselves as the transmitters of these forces. When that picture is used one describes the force as being the consequence of an *exchange* of a gauge boson between two fermions: one fermion emits a boson and the other fermion absorbs it. If the mass of the boson is negligible then the efficiency of this exchange process is inversely proportional to the square of the distance: the Coulomb force law. If the boson has a certain amount of mass when at rest, then the force it transmits will range only up to a distance inversely proportional to that mass, and decrease very rapidly beyond that distance.

An important feature of the gauge boson force between two fermions is that before and after the emission of a gauge boson the fermion keeps the *same helicity*. This means that a left rotating particle remains left rotating, and a right rotating particle remains right rotating. This is of special importance for the neutrinos: neutrinos *only* exist as left rotating objects. Right handed neutrinos have never been observed (at least not for sure). In contrast, *anti*-neutrinos only come rotating towards the right, not left. By emitting a gauge boson a neutrino may turn into an electron, but then the electron must rotate towards the left.

We can now make a listing of all fermions and gauge particles (see Fig. 3). Here we see that the fermions are divided into *leptons* and *quarks*. The left rotating objects are indicated by an '*L*' and the right rotating ones by an '*R*'. For the anti-leptons and anti-quarks,

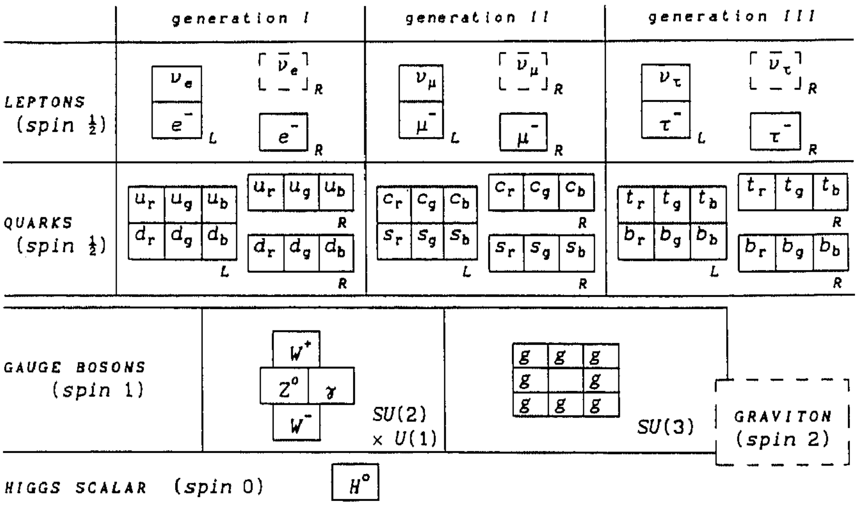

Fig. 3. The Standard Model based on $SU(2)_{\text{weak}} \times U(1)_{\text{em}} \times SU(3)_{\text{strong}}$. Right handed neutrinos, which may exist, are indicated in dotted boxes.

which are not shown, the L and R are interchanged. Of all fermions only the quarks are sensitive to the forces of the eight gauge bosons in the box called '$SU(3)$'. This force is very strong and so all objects containing quarks will be strongly interacting with other objects.

All *left* rotating fermions are sensitive to the forces from the $SU(2)$ gauge bosons. Finally, all left rotating, and all electrically charged right rotating fermions feel the $U(1)$ force fields.

Now the Standard Model also contains a spin 0 particle, the *Higgs particle*. It also transmits a force, but an important difference between the latter and the gauge boson forces is that if a fermion exchanges a Higgs boson it has to flip from left rotating to right rotating or vice versa. Another difference is that the spin 0 particle can disappear straight into the vacuum. This implies that some fermions can make spontaneous transitions from left to right and vice versa. Technically, this is the way one may introduce *mass* for these fermions. A particle with mass moves slower than the speed of light and it turns out that for such a particle the rotation axis cannot be kept parallel to the velocity vector. So this mass can only be there if the particle has the ability to change its spin direction relative to its velocity vector. One finds that the mass of a fermion is proportional to the coupling strength of that fermion to the field of the Higgs particle.

I should emphasize that this theoretical understanding of the behavior of fundamental particles in terms of relatively simple equations is due to a huge combination of efforts by theoretical and experimental physicists alike. Without numerous ingenious experiments, sometimes carried out in very sober conditions but often in enormous multinational collaborations in gigantic laboratories, the present insights would perhaps never have been obtained.

The *gravitational force* could be added to our description of the Standard Model by postulating a spin 2 particle, the *graviton*. The rules for this force are very strictly prescribed by Einstein's General Theory of Relativity, but only in as far as it acts collectively on many particles. The details of multiple graviton exchange between individual particles are not understood, mainly because any effects due to such exchanges would be so tremendously weak that no experimental verification will be possible within any foreseeable future.

The above is a qualitative description of the rules according to which the particles in the Standard Model move. To formulate these rules in more precise mathematical terms, as we were able to do for Conway's Game of Life, would lead us way beyond the scope of this chapter. In some sense the rules are nearly as precise.

Allow me to point out the similarities between this Standard Model and an ideal Theory of Everything: all material objects in the Universe have to move according to its rules. In some sense the standard model is nearly as esthetic and elegant as Conway's model. But of course the standard model is far from perfect. One reason is that we expect the model not to provide the equations of motion for matter under all conceivable circumstances. One can imagine energies, temperatures and matter densities that are so large that the situation cannot be mimicked in any accelerator. Although our model could in principle be used to calculate what will happen, there are several indications that one should not take such predictions seriously. A second reason is that the *mathematics* of the model is less than ideal. Even if we had infinitely powerful computers at our disposal, some phenomena cannot be computed with any reasonable precision. One has to rely on certain perturbative approximation schemes that sometimes fail catastrophically.

One of the most significant distinctions between the present Standard Model and a Theory of Everything as defined in the previous section is that our interactions depend on a number of 'constants of Nature', which are to be given as real numbers. The precision with which these real numbers are known will always be limited. There are essentially 20 independent numbers:

- 3 gauge coupling constants, corresponding to the strength with which the various kinds of gauge bosons couple to the fermions. The $SU(3)$ coupling constant is much larger than the one for $SU(2)$ and $U(1)$. Then there are several interaction terms between the Higgs field and the fermionic fields (Yukawa terms). Many of them correspond to the masses of the various fermions:
- 3 lepton masses: m_e, m_μ and m_τ.
- 6 quark masses: m_u, m_d, m_c, m_s, m_t and m_b.
- 4 quark mixing angles, determining further details of the decay of exotic particles.

- 1 topological angle θ_s, a peculiarity relevant only for the strong interactions; as far as is known it is very close to zero.
- 2 self-interaction parameters for the Higgs field. One of these determines the Higgs mass M_H, the other determines the Higgs-to-vacuum transitions. In combinations with the other constants it produces the gauge boson masses.

This adds up to 19 'constants of Nature', which are incalculable; they have to be determined by experiment. We could then add Newton's gravitational constant G, but this could be used to fix the as yet arbitrary scale for mass, length and time. Strictly speaking there is also the so-called *cosmological coupling constant* which is also incalculable, but it may perhaps be set to be identically zero; it would be the 20th parameter.

The presence of these constants of Nature violates condition (2) for a Theory of Everything to be acceptable. This is because with this model we will always have the option to consider an arbitrarily tiny change in one or more of these numbers, and thus come forward with an 'alternative theory' that is equally as probable as the original one and cannot be distinguished from it by any experimental test. Experimental tests always have limited precision. A good Theory of Everything should not contain any freely adjustable constants of Nature that take the form of real numbers.

There must be three constants of Nature that are not adjustable and therefore acceptable in any theory. These are the *scale determining constants*, for which we usually take:

- the speed of light, $c = 2.997\ 924 \times 10^{8}$ m s^{-1}
- Planck's constant, $\hbar = h/2\pi = 1.054\ 588 \times 10^{-34}$ J s
- Newton's gravitational constant, $G = 6.672 \times 10^{-11}$ m^{3} kg^{-1} s^{-2}

Together, these determine a fundamental unit of length,

$$\sqrt{\frac{G\hbar}{c^3}} = 1.6 \times 10^{-35}\ \mathrm{m}$$

one unit for time,

$$\sqrt{\frac{G\hbar}{c^5}} = 5.4 \times 10^{-44}\ \mathrm{s}$$

and a unit for mass:

$$\sqrt{\frac{\hbar c}{G}} = 22\ \mu\text{g}$$

with a corresponding energy of 1.2×10^{22} MeV (or 2.0×10^{9} J).

These units are called the Planck length, Planck time and Planck mass or energy. One could use these to dispose of the arbitrariness of the 'meter', the 'second' and the 'kilogram' as units for length, time and mass. In terms of the Planck units the three corresponding constants of Nature are just one, and therefore not adjustable.

Just because the gravitational constant is extremely tiny at the level of elementary particles, these resulting 'natural' time and length scales are tremendously small, and, compared to particles, the mass is extremely large.

It is for structures at these distance, time and mass scales that our Standard Model is known to be desperately inadequate. We know that our present formalism of quantum field theory is not suitable to describe the gravitational force among individual particles. What would be needed is a theory that combines our present knowledge of both quantum mechanics and Einstein's general relativity into one scheme, and in spite of vigorous attempts from theorists such a scheme has not been found. It is generally believed that the present formalisms all have to be completely revised before they can be combined. We do not know how this can be done, and this is why there will be room for speculations.

The Standard Model is elegant and esthetic, but not elegant and esthetic enough; it is universally valid, but not universally enough, and so on. And then, still very little is known about possible boundary conditions of our universe. The science of cosmology, that should tell us the answers to these questions is making progress, but should still be regarded as being in its infancy.

3 Is there any evidence for a universal law?

The evidence in favor of the existence of a universal law was clearly visible to the physicists of the previous century. If Conway's model and personal computers had existed then, they would immediately have pointed to the striking similarities between that model and the real world. Only religious arguments or mystical reflections could be raised against the idea, and it is to be suspected that most scientists would reject such counter arguments. Anyone with some experience in the universal nature and mathematical strictness of the laws of physics would be very much tempted to believe that there exists a completely general scheme according to which everything moves.

And there are now also elements in the most recent theories that lend further support to this view. The existence of the Planck length and time scale suggests that at that scale the laws are fundamental and absolute. The requirement that these laws must agree with the very restrictive postulates of both quantum mechanics and general relativity has up to now proved to be so difficult to realize in any physical model that one is tempted to suspect that not more than one model will exist at all which agrees with all this. More significantly, studies of the quantum mechanical laws in the vicinity of black holes indicate that the total amount of *information* one can store in any given volume is limited. This is precisely what we also have in Conway's model!

The idea of a Theory of Everything was given further new impetus in 1984 when a new theory of particles was formulated that includes the gravitational force. It became known as 'superstring theory'. This idea became very popular because it seemed to obey requirement (2): it contained no freely adjustable constant of Nature. Unfortunately, the mathematics could never be given such a concise form as in Conway's model, and at present a majority of particle physicists have lost most of their high expectations of this scheme. String theory is a mathematical device, not a complete theory of particles and forces.

We can play with toy models of an ultimately mathematical universe, but we cannot ignore an overwhelmingly important piece of evidence *against* the existence of any local law of dynamics. It is an aspect of the theory of quantum mechanics itself. When the fundamental laws of quantum mechanics became known, in the 1920s, it was also realized that there is an enormous disparity between this theory and any system based upon local and deterministic dynamical laws. By 'local' we mean to say that the evolution of some physical quantity should only depend on physical quantities in its immediate neighborhood, so that there is no action at a distance. Action at a distance was already excluded by Einstein's theory of relativity: interactions over some distance can only occur if some sort of 'field' propagates through the region in between. 'Deterministic' means that one single initial configuration can lead to one and only one possible configuration later. If locality and determinism were fundamental requirements of a Theory of Everything then quantum mechanics would be at odds with such a theory. And the evidence that quantum mechanics correctly describes all tiny objects such as atoms, molecules and all other small particles, is overwhelming.

In contrast, according to the laws of quantum mechanics any initial configuration may lead to many different possible outcomes, each with a precisely calculable probability. An atomic physicist equipped with the theory of quantum mechanics can never predict all details of the outcome of experiments, but only probabilities for many different outcomes, as if throwing dice.

Attempts to remedy this situation, replacing quantum mechanics by specially constructed deterministic models containing some statistical elements, were unsuccessful. In fact, it could be *proven* that many such models fail to reproduce the statistical predictions from a quantum mechanical calculation. Einstein, Rosen and Podolsky constructed a 'Gedanken' experiment of which quantum theory gives a precise prediction (after an easy calculation) that cannot be reproduced unless one has action at a distance. Bell later formulated rigorous mathematical theorems with the same outcome.

In my opinion (but I stress that this is a minority view), there may nevertheless be a compromise. This is that there is no direct action at a distance, but there is some sort of 'conspiracy'. With this I mean that the 'state' of Nature that we now call 'vacuum' is actually a very complicated dynamical solution of the equations of motion, showing correlations over space-like distances. Einstein, Rosen, Podolsky and Bell never took such correlations completely into account. With correlations one can have apparently impossible 'coincidences' spreading faster than the speed of light, but which are not in conflict with the requirement of special relativity that *information* cannot spread faster than the speed of light. You may ignore this deviating view because it is not essential for the main theme of this chapter. I will come back to it, briefly, later.

More in line with standard interpretations of quantum mechanics would be the idea that we might still have something resembling Conway's Game of Life, at the Planck scale of time and distance, but that it is a 'quantum mechanical' version of the model. A slight complication might be that the number of 'quantum-Conway' models seems to be much larger than that of classical, deterministic ones. On the other hand, requiring such a model to include the postulates of general relativity may, as stated earlier, reduce the number of possible schemes to perhaps only one. Today, no such model is known at all.

4 What are the prospects of discovering the basic laws?

No working model that combines all we know at present concerning the basic laws of physics is known at all. Yet there is ample evidence that Nature itself did produce such a model, namely our present universe. So we should not despair: a mathematical scheme exists; it just remains to be uncovered by us. Will we ever succeed in doing this?

The lessons from recent and more distant history are instructive. A number of times in the past physicists have expressed as their opinion that the ultimate laws of the world had 'nearly' been found. They were always proven wrong. Is it not evidently more reasonable to expect only piecewise improvements in our understanding of Nature's secrets, and that the series of false alleys and surprises awaiting us will be strictly endless? Or that progress will eventually slow down and come to an end just because we will never be able to afford the formidable expenses needed to build the next generation machines, before enough information were obtained from which we could deduce the basic laws?

Yes, it may be feared that history will repeat itself, but history never repeats itself in a predictable manner. So the fact that claims in the past turned out to be premature does not imply that future claims of this sort will be premature also. The recent claim by superstring theorists that the Theory of Everything was close at hand enjoyed the echo of a considerable number of cheer-leaders, but I did not belong to them. This theory did not have the characteristics of any basic theory that would be acceptable to me. The mathematical formalism was far too complex and indirect to serve as a universal law. It was and is not understood how to work with this theory under all circumstances, and such is an absolutely obligatory requirement for any theory that boasts to be an ultimate one.

So, no, I must emphasize, we are not even close to the ultimate theory at present. We should keep in mind that the obstacles waiting for us are indeed formidable. The Planck scale corresponds to an environment where particles feel each other's gravitational forces. These forces under normal conditions are extremely feeble. Experiments in which these forces dominate the outcome of the measurements will probably never be possible. It is not unlikely that improvement over the present concepts can gain us a few orders of magnitude, but to raise the energies and/or accuracies of the measurements by 16 orders of magnitude is, it seems, a hopeless task. Information concerning this area of physics will have to be obtained by extrapolation, making optimal use of our theoretical and calculational ingenuities.

It is our human ingenuity on which I am counting. Ultimately I believe that clever ideas will be found enabling us to deduce how Nature works. If a simple fundamental law exists, it will be found, sooner or later. If the law is found, we will all recognize it as being very likely the truth. I have no doubt there. I also have no doubt that regardless how convincing the evidence for a new super theory will be, people will continue to raise questions and objections. There never will be an ultimate proof of the ultimate theory.

5 Aspects of complexity

It is very clear that objections can be raised against the very notion of a single all-embracing theory for the tiniest dynamical units in this world. The idea of one single and simple mechanism that is universally valid might be a complete illusion never to be applicable to the real world. I am not blind to the valid objections. Many people however may raise intuitive objections which one can put aside. One objection that I can imagine would be that a dynamical law as simple as that of

Conway's model could never give rise to all the riches of the real Universe, its immense beauty, diversity and magnitude, in particular the emergence of life, let alone intelligent life.

This is clearly an objection against the idea that simple initial laws could give rise to infinite complexity. In the lecture I paid little attention to the notion of complexity, for lack of time (I refer to John Barrow's chapter), but let me dwell on it, a little, in this written version.

Complexity is a quite common feature both in physical and in mathematical systems. A simple physical example is 'chaotic behavior', displayed in many systems that themselves are described by simple and straightforward laws. Consider a gas such as air. The laws for the motion of tiny volumes of air, the so-called Navier–Stokes equations, are fairly simple. To understand the behavior of air one may often ignore even the fact that it consists of molecules. We may treat it as an 'ideal' gas. Yet air may give rise to *turbulence*. Given a simple shape such as a perfect sphere, one can calculate, and also observe directly, how air flows around it. Under certain conditions the solutions of the relatively simple equations become extremely complicated. That is when turbulence shows up. We call this phenomenon 'chaos'. The complexity may very quickly reach the very limits of even the most powerful computers, and so, even if we have the full equations, there will be uncertainties when we try to apply them. The reason is not only that we do not know the equations precisely enough, but also that we can never control the initial state precisely enough, and that our mathematical formulations would never be sufficiently accurate, and finally that there are many more physical degrees of freedom than we can handle, even with the most powerful computers. Nature itself will always act as a computer much more powerful than anything we will be able to construct to mimic her.

This fact will imply that the questions we are addressing here, and whatever answers we come up with, will have little effect on everyday life. A Theory of Everything of the type we are discussing here will never be a theory of everything in the literal sense. Chaotic behavior will prevent us from computing just 'anything we want to know' at a very early stage. The question we are addressing is the question whether or not there exists an ultimate law and whether we will be able to discover this law. The question is not whether we will ever be able to use this law to explain whatever we like to have explained. Almost certainly, our dynamical laws will be useless for that, due to the tremendous amount of complexity, presumably already at scales a little bit larger than the Planck scale.

Conversely, one may easily use these observations to counter the objection that our universe is too complex to be described by one single dynamical law. We know that single dynamical laws may show infinite complexity in their solutions. Anyone who can program a computer can demonstrate this for him- or herself. Take a cellular automaton such as Conway's Game of Life, or some variation on this scheme. Under favorable conditions one can start with an extremely simple law and an extremely simple initial state, and find that its behavior grows infinitely complex.

Not only physical models, but even purely mathematical systems can easily become infinitely complex. One example is the series of prime numbers, which at extremely high orders becomes more and more difficult to predict. Another one is the famous mathematical function called the 'zeta function', of which some mathematical aspects are not completely understood because of its erratic behavior in the complex plane.

The beauty of our real world is that there is some equilibrium in its degrees of order and degrees of complexity. In my personal theory for the interpretation of the laws of quantum mechanics, it is complexity that dominates the structure of the 'vacuum', yet this complexity is far from random. There are correlations, and the only correct way to describe these is by what is now known as quantum mechanics. The laws of quantum mechanics as we know them contain a statistical element, just because the complexity of the vacuum is far too immense to allow it to be described in any other way.

I have not been able to explain why these vacuum correlations are stable and why the vacuum has the symmetry properties that we observe in practice, nor have I been able to reproduce all these properties in mathematical models, but I don't see why the general idea should be wrong. It could be that there exists only one such model, the one that describes the real world . . .

6 Just suppose . . .

Allow me for a short while to speculate that there is indeed a simple dynamical law, waiting to be discovered. As I explained and emphasized earlier, the practical implications would hardly be noticeable. It is almost certain that such a law will rapidly produce chaotic behavior, so that even the most elementary calculations using this law as a starting point will be cumbersome at best, impossible in most cases.

But from a more philosophical viewpoint the implications would be immense. We would have to conclude that what we used to call 'Nature' is actually something like a mathematical processor, much like the processors we have inside our computers, just quite a bit bigger and faster.

It is not obvious that Nature's own mathematical processor works just like ordinary ones. Most physicists would argue that, since we have quantum mechanics, Nature's processor will have to obey 'quantum logic' instead of ordinary logic. This is a widespread belief which, as I stated earlier, I do not share. What physicists at present call 'quantum logic' may well be nothing but the best representation of our present understanding of the laws of physics, but may not necessarily be the ultimate truth. From a philosophical point of view it could not have been otherwise than that our first attempts at formulating the laws of physics should contain statistical elements; all those dynamical degrees of freedom or variables that we do not understand at present just look like random noise to us. Experts will no doubt recognize in these words the 'conjecture of the hidden variables'. Since several simplistic versions of such theories have been ruled out in the past, the notion of hidden variables is no longer very popular in theoretical physics. So I will try not to emphasize my personal belief in some more advanced version of hidden variables, but I won't hide it either.

Whatever the logic, quantum or classical, random or deterministic, let us suppose that there exists a well-formulated physical law – and even that is not an obvious fact at all. Then I claim that there should also exist some prescription concerning the order in which the law –

or laws – should be applied to calculate how the numerous physical degrees of freedom evolve. It is this order that I would identify with 'time'. One must *first* calculate how 'early' degrees of freedom evolve, and only *then* one has the necessary data to calculate how 'later' degrees of freedom will behave. Without a good prescription of the logical order the laws will not be unambiguous. This inevitable fact seems to me to imply the necessary existence of *time* and *time ordering*. Going backwards in time, or following 'closed timelike loops' will be contradictions in terms.

I can even go a step further. If our universal law is sufficiently simple, one could just as well conclude that the entire universe is nothing but an enormous series of mathematical combinatorial 'theorems'. The mathematical theorems are time ordered. Time ordering is indeed also the order of logic: first come the theorems that were relatively easy to prove, then come the theorems that require the previous theorems to be proven first. Now many mathematical theorems are in some sense 'equivalent': they require tremendously complex calculations for their proofs, but one theorem follows directly if we know the other. For the universe this could mean that there may exist many equivalent descriptions of its dynamics.

A simple example of an infinite series of 'theorems' is the list of prime numbers. To prove that a large number is a prime number you first need to know the smaller prime numbers. Thus, the series of prime numbers in some sense form their own universe – indeed, one can represent the prime numbers as a four-dimensional table just like our real universe! If mathematicians could prove that all combinatorial theorems in number theory ultimately are connected to knowing the complete series of prime numbers, then our universe 'is' the series of prime numbers! This is what I referred to in the summary as the complete merging of theoretical physics with pure mathematics.

A very important problem then remains: the problem of identifying our present position in this universe! Present theories of cosmology indicate that galaxies should have formed from initial tiny fluctuations in a very early, very hot universe. These fluctuations will be extremely difficult, if not impossible, to calculate. One can imagine in the distant future a computer program that works out the equations and finds the rough shapes of the galaxies. Maybe we will recognize our own!

I mention this admittedly rather crazy idea as an introduction to something more important: the so-called 'anthropic principle'. According to this principle the Universe 'is as it is' because 'if it weren't, we would not be here to discuss it!' This principle is sometimes used to argue why certain constants of Nature (the ones in the Standard Model for instance) have the values they have. The problem with that is, in my opinion, the following.

The anthropic principle seems to work reasonably well to explain why certain constants are what they are, up to some degree of accuracy. It is probably true that if the fine structure constant α were only slightly different from the known value, 1/137.036 ... nuclear and atomic physics would give some substances quite different chemical properties and consequently also different abundances, such that human life would probably have become impossible. But would this also be true for the 24th decimal place of α? And what about the 10 000th decimal place? Now I admit that physicists would have a hard time even to define this constant to such an accuracy, but it should be true that at least some phenomena in this universe do depend on such small details. But does the existence of humanity depend on all decimal places of α? Probably not. So the anthropic principle only works up to a point.

In contrast, there is a version of the anthropic principle that is obviously correct: we are living on Earth, not Venus, Mars or the Moon. The reason is obvious: those places are (as yet) uninhabitable. Our domicile has a breathable atmosphere. The anthropic principle does explain why our atmosphere is breathable, and I guess we all accept this explanation. The difference between these two anthropic principles can easily be formulated in strictly mathematical terms. Stars and planets can be enumerated – that is, one could make a finite list of all of them in our present universe. The acceptable version of the anthropic principle asks us to choose one item of this denumerable list. The version that is not acceptable to me asks us to choose a physical constant out of a set of real numbers. These numbers are strictly innumerable. So I propose to accept the 'discrete anthropic principle' but to reject the 'continuous anthropic principle'.

The discrete anthropic principle states that there could be many Conway games, they form a discrete – denumerable – list. The principle allows us to pick the model we can live in as being our universe.

7 Perennial doubt

Thank you for allowing me to day-dream. The universal law does not – not yet? – exist. Until that time arrives we can just speculate about it. But, of course, we can also try to find out if there are no further compelling counter arguments.

One thing seems to be clear. Perhaps someone will come along with a proposal for a universal law. Perhaps this proposal will pass a certain number of tests. But all experiments have limited accuracy. How can we ever be sure that the theory is correct? Of course we can't. One could always keep the suspicion that the proposed law is nearly but not quite always and everywhere valid. Every now and then Nature could choose to deviate, either following a different law, or just following no law at all.

I would then argue that this is implausible. This is not Nature as we physicists came to know her. Nature's laws are basically simple, straightforward and absolute. There are no dispensations. You could call this 'professional experience' or 'intuition' or just 'faith', but completely convincing it will never be.

More serious is the fundamental denial of the necessity of any law at all. Many would argue that every now and then there will be 'divine intervention'. My personal belief is that 'God', or whoever it is out there, has ample opportunity to play tricks with the chaotic solutions of the equations so that suspension of the equations is not at all necessary if he (or she) wishes to divinely intervene.

The most difficult objection to counter is the denial that a physical law can be simple or defined at all at a local level and scale. It could be, as is indeed favored by many researchers of quantum theory, that there exists some untraceable action at a distance. There may exist laws that cover large objects or beings separately, even laws that give to people 'minds' and 'souls'. Laws could exist that allow for telepathic communications. And so on. If you believe any of this, you will have to reject our Theories of Everything, which leave no room whatsoever for such frivolities.

Physicists who search for their universal theory are no missionaries. We have not the least desire to convince anyone against his or her will into believing our theory. All we wish to do is marvel at Nature's beauty and simplicity. We have seen and tasted the beauty, simplicity and universality of our latest theories and models of the fundamental particles and the cosmos. We are now trying to uncover more of that. It is our belief that there is more. It is a challenge we cannot ignore. And whatever we find, there will be questions and challenges that will be left unanswered.