On the Quantization of Space and Time

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ABSTRACT

None of the regularization schemes of quantum gravity that have been tried sofar (such as lattice regularization or the addition of R^2 terms, etc.) are sufficient to remove the large redundancy of quantum states in a black hole. If we impose apparently physically reasonable conditions on the quantization of black holes then a more drastic two-dimensional quantization of space-time itself seems to be necessary.

We suggest that the dynamics of a black hole is in fact determined by the geometry of an orbifold at the onset of the event horizon. A simple model exposing some of the features of the kind of Hilbert space we expect near a black hole is presented. A quantum-deterministic spectrum of states near the horizon is obtained.

1. INTRODUCTION

That quantum gravity should somehow eradicate the infinities of quantum field theory is being suspected for a long time. An early attempt to understand how this might happen was made by Salam, Lehmann, and others[1)], who suggested to reorder perturbation expansion obtaining exponentiated propagators. It was hoped that these "super propagators" would make Feynman diagram integrations finite. Later various versions of space-time lattices were proposed[2)]. Since we realized that black hole formation would make it impossible to separate two points less than one Planck length unit apart, it seemed natural to suggest that perhaps there is no more than one space-time point in every four-dimensional unit of volume with Planckian dimensions.

Nowadays it is claimed that the best candidate theory that may produce only finite, well-behaved amplitudes including gravitational interactions is the superstring theory. Unfortunately, the number of states a superstring can be in is horrendously infinite. One could hope that this infinity is a long-distance ("infrared") infinity only, and that the number of quantum states per Planck 3-volume may still be limited. However, if we look at the standard construction of the theory we still notice that every string configuration in x-space corresponds to a different quantum mode, or basis vector in Hilbert space, so that it will seem impossible to avoid an infinity of states within a given Planck volume; a situation that may only become worse if a Kaluza-Klein picture is adopted.

Now surely the above arguments counting the number of ways a string can curl up inside a given volume might be called naive. But a more sophisticated formulation of the theory telling us that the actual number is indeed finite does not yet exist and would seem to be a rather fundamental departure from the established picture.

In this paper we will not only argue that the number of independent basis vectors for the quantum mechanical Hilbert space coming from a given space-time volume should be finite; it should be much smaller even than one could expect from any of the conventional regularization schemes, much less than one per Planckian volume. The argument goes as follows.

Consider some compact, simply connected, three-dimensional subspace V of $\mathbb{R}^3$. We suppose its volume to be L^3, where the length L may be anything varying from the Planck length to cosmic distance scales. The boundary of V is taken to be some surface σ with total area L^2. Suppose that we take this surface fixed and simply ask how many mutually orthogonal basis elements Hilbert space may have, considering all possible states of matter (and indeed space-time itself) inside our volume V.

In a quantum field theory excluding gravity the answer is of course infinite: every single particle has an infinity of momentum eigenstates to choose from. A regularized quantum field theory might give a finite answer, for instance if we have only fermions regularized by a lattice: each lattice point could correspond to $2^4 = 16$ basis elements. If the total volume is $V = L^3$ and the lattice distance is a, then the number of points N is $(L/a)^3$ and the dimensionality D of Hilbert space

$$D = (16)^{(L/a)^3} = \exp(\Lambda^3 L^3) , \qquad (1.1)$$

where $\Lambda = \mathcal{O}(1/a)$ is the cut-off parameter. It will be clear that any conventionally regularized theory may at best produce an upper bound on D that increases exponentially with the volume V.

Now enter the gravitational interactions. It is then important to watch the total energy E inside our system. We should make sure that the corresponding Schwarzschild radius does not exceed $\frac{1}{2}L$, so

$$2\ GE < \tfrac{1}{2}L , \qquad (1.2)$$

where G is Newton's constant. The total number of orthogonal states in Hilbert space with total energy bounded by (1.2) is easy to estimate in any quantum field theory. It is approximated by

$$D = \exp S , \qquad (1.3)$$

where S is the entropy of the system at thermal equilibrium (where it is maximal).

Let us for a moment ignore the possibility of gravitational collapse, and assume a temperature T. For a more or less conventional particle system that contains (a number of) massless particles, we find at thermal equilibrium

$$E \approx C_1\ V\ T^4 , \qquad (1.4)$$

$$S \approx C_2\ V\ T^3 , \qquad (1.5)$$

(where $C_{1,2}$ are constants of order one). Together with (1.2) this gives $T \propto L^{-\frac{1}{2}}$ and

$$S < C_3\ L^{3/2}\ G^{-3/4}\ . \tag{1.6}$$

This however cannot be a completely stable configuration: if not directly then at least by tunneling the system could collapse into one or several black holes. Let each of these have energies E_i with $\Sigma\ E_i = E$. Then we assume that Hawking's treatment of quantum field theory close to a horizon is correct, if not exactly then at least by approximation, so that each black hole carries an entropy

$$S_i = C_4\ G\ E_i^2\ , \tag{1.7}$$

$$S = C_4\ G \sum E_i^2 < C_4\ G\ E^2 \tag{1.8}$$

(the highest entropy is obtained by a state with a single black hole.) Again we assume (1.2), so now (all C_i are constants of order one)

$$S < C_5\ L^2/G\ , \tag{1.9}$$

to be compared with (1.6). As soon as $L^2 >> G$ the Planck length squared) the single black hole is the state with largest entropy. We must assume that the surface σ stays outside the Schwarzschild horizon otherwise σ would not be observable!

In most quantummechanical systems the entropy is directly linked to the total number D of mutually orthogonal states in the given energy domain, via the third law of thermodynamics, so one is tempted to write

$$D \approx \exp(\Lambda^2 L^2)\ , \tag{1.10}$$

where Λ is roughly the inverse Planck length. Now we are well aware of the possibility that the third law does not hold for black holes; in eq. (1.10) one is free to insert a further arbitrary constant, which in principle could be infinite. Indeed this is what many researchers in this field suspect[3], namely that the number of modes of a black hole is unbounded. Now this would imply an infinite degeneracy of Hilbert space at already quite low energies E. Although certainly we are unable to disprove such a possibility, in our view it would be a rather ugly situation, not at all supported by any evidence from Elementary Particle Theory. In short, we will henceforth assume that the constant of proportionality in eq. (1.10) is finite, of order one, and therefore negligible as soon as L in eq. (1.10) becomes large. Only then we may hope to obtain a rather appealing picture of this world, where black holes are the tiniest construction elements of matter, in no way different from other elementary particles. After all, any other heavy elementary particles such as the heavier excited modes of some (super-)string, should have finite Schwarzschild radii just as well, and therefore any distinction between these and "real" black holes would be highly artificial.

Comparing (1.10) with (1.1) we see clearly our problem: lattice - or indeed any of the conventional - regularization schemes give us far more states than we can really accept, in particular when the volume V tends to infinity.

It may be amusing to observe that the "most probable", or rather "generic", state of matter inside our volume is the one containing a black hole whose horizon coincides with the surface σ, and that according to (1.10) the total number of states is that of some "regularized" quantum field theory on σ. We are led to suspect that whenever someone tries to formulate the complete set of all excited states inside a volume V then he obtaines this 2+1 dimensional "field theory" on σ instead. States which do not have a black hole inside V only form a very tiny subspace of this Hilbert space, in spite of the fact that when gravity can be ignored the number of states, as given by eq. (1.1), may seem to be much larger. In reality we know that the exponent in (1.1) is smaller than in (1.6), since gravity can only be ignored if the total energy is kept sufficiently small, and this is only possible if the lattice cut-off Λ is taken to be many orders of magnitude smaller than the Planck mass.

However, things are perhaps not quite this simple. If we have a black hole whose horizon just touches σ, we must also expect a heavy traffic of particles from the black hole into its environment and vice versa. An outside observer who studies these particles must be assumed not to sit in a generic state but in a special state, extremely close to the vacuum. This is a problem. We wanted to study "all possible modes" of á given system. We are now forced to limit ourselves to all possible modes within some volume V, allowing one special states, "close to the vacuum", for the world outside V, in which the observer may take place.

We notice in passing that apparently the true vacuum plays a crucial role in our description of all other states. In ordinary quantum field theories the true vacuum becomes irrelevant when it comes to describing excited modes (such as field theory at some very high temperature). For these highly excited states it makes no difference whether or not the true vacuum respects a symmetry or not. In the case of quantum gravity the situation may be very different. Although it is too early to draw any useful conclusions, we think that indeed the vacuum must be a very special state in any theory of quantum gravity, because of the vanishing cosmological constant.

2. BLACK HOLES AND WHITE HOLES

The most central assumption in our work is that a black hole is simply the most compact, and in some sense the most general, object with a given total energy. In all other respects black holes are assumed to obey the ordinary rules of quantum mechanics. This then implies that when the state of all particles that made up the black hole in some implosion process is completely specified, then also the state of all outgoing particles should be well determined, probably as a complicated linear superposition of many different "decay modes". In particular, it should not be a mixture of different states in a density matrix ρ unless

$$\mathrm{Tr}\ \rho^2 = \mathrm{Tr}\ \rho = 1 \ , \qquad (2.1)$$

which means that it can be seen as a single pure state, if the initial state was pure as well.

Now this is not what one finds when applying standard quantum field theory in the vicinity of the black hole horizon. In fact, one finds a thermal spectrum[4]:

$$\rho = C\, e^{-\beta H} \quad , \quad \beta = 8\pi M \ , \tag{2.2}$$

to be normalized by

$$\mathrm{Tr}\ \rho = 1 \ , \tag{2.3}$$

so that eq. (2.1) cannot be obeyed. What is wrong in standard quantum field theory near a horizon?

First of all we note that the difference between pure states and mixed states will be more and more difficult to detect as the black hole becomes larger. The final state could be pure but so complicated that for all practical purposes it can be handled as a thermodynamical mixture. By the time a black hole is so large that observers can be sent in nobody will ever detect the difference.

For small black holes however the question of quantum mechanical purity is extremely important. It seems then that (2.2) can only be an "approximation". Is there a way to replace it by a single realistic wave function?

What was ignored in the standard derivation was the gravitational interaction between ingoing and outgoing matter. But this interaction is crucial. Suppose a particle goes in with momentum p_1 at time $t = t_1$. After a long time, $t = t_2 \gg t_1$, we look again at the black hole and observe a Hawking particle with momentum p_2 coming out. At some time t_o, roughly halfway between t_1 and t_2 the two particles must have met, that is, they were at the same distance from the horizon, one entering, the other leaving. Both were accompanied by a gravitational shock wave[5]. Particles and shock waves collide at $t = t_o$. The center-of-mass enegery with which this collision takes place can be easily estimated:

$$E_{c.m.}^2 = -s = \mathcal{O}[p_1 p_2\, e^{(t_2 - t_1)/4M}] , \tag{2.4}$$

increasing far beyond control when

$$t_2 - t_1 \gg 4M . \tag{2.5}$$

In reality collisions with these energies do not take place, or rather, they are not seen. The ingoing particle does not see the outgoing particle but experiences a surrounding vacuum. However, as soon as we introduce a detector at $t = t_2$ that can distinguish different modes of outgoing particles, we have trouble at $t = t_0$, because the detector has split up the wave function in pieces that may contain these objects, hitting the ingoing particles with tremendous center-of-mass energies.

A black hole can be in a pure state if all particles that produce the hole in some distant past were in a pure state. We now notice that when observations are made at much larger times, we are tempted to use elements of Hilbert space that are extremely singular in the past, and to expand the ingoing states as quantum superpositions of outgoing modes will be made extremely difficult because of this. Since the standard derivation of the Hawking effect ignores these gravitational self-interactions, we believe that the resulting density matrix ρ of eq. (2.2) cannot be trusted completely.

Consider a black hole that was formed by a collapse at $t = t_1$, and an observer at $t = t_2$ has decomposed the wave function as just described, by looking at particles emerging from the hole at various angles. If we follow these particles back in the past, we see that for a while they stick to the "past horizon", but then, at $t \approx t_1$, they are released. By that time they have energies roughly described by eq. (2.4), and, coming from different directions they collide. The Schwarzschild radius corresponding to (2.4) is large, and hence a space-time singularity at $t = t_1$ is now unavoidable. Thus there will be a time-reflected black hole in the past. This is sometimes called a "white hole".

Considering some Cauchy surface at $t = t_0$, we see that quantum mechanical superpositions must be allowed between states that contain different kinds of black hole singularities both in the future and in the past. We also see that the distinction between "primordial" black holes and black holes that have been formed a relatively short time ago disappears, and our view upon black holes is entirely symmetric under time-reversal.

Since the arguments presented in this section are essentially independent of the assumption mentioned in the Introduction we believe that they provide further support to the idea that a black hole can decay entirely into "ordinary" particles. The concept of a naked remnant singularity[6)] does not fit very well in this picture.

3. BLACK HOLES AND ORBIFOLDS

With the considerations of the previous section it has not become easier to set up some Hilbert space for black holes. In sects. 4 and 5 we will attempt to construct a preliminary model. First we must ask in what way such a model will have to deviate from standard dogma. Since standard background field theories do not provide us with any clue as to what to do with the space-time singularities mentioned in the previous section (in particular how to superimpose them), we will have to look at the information stored away in the black hole differently.

Suppose a black hole, formed at $t = t_1$, is observed at t_2, where now t_2 is allowed to tend to infinity. The wave function at $t = t_1$ will then be completely distorted, and a huge singularity will completely screen all of space-time at earlier times. What is obtained is a space-time with a natural boundary. The particles moving along this boundary are only seen at $t = t_2$, but the shape of our space-time boundary will be completely determined by the way infalling particles have affected the space-time metric. Of course, our boundary coincides with the future event horizon, which is now well defined just because we allowed t_2 to go to infinity.

Here we think we may have a clue on how to recover information that disappeared into the black hole: perhaps the "shape" of the horizon determines the wave functions of the outgoing particles. How do we detect this shape?

Consider the locus $\mathfrak{R}$ of all points in space-time from which information can escape to infinity. The boundary of $\mathfrak{R}$ is the future event horizon $\mathfrak{S}$. On generic points of $\mathfrak{S}$ we have the situation that information (light) can escape in only one direction. This is the zero eigenvector of the induced metric on $\mathfrak{S}$. On these points $\mathfrak{S}$ is a light-like surface. However, there are singular regions on $\mathfrak{S}$. These are the points from which light can escape in two directions (a two-dimensional subset of $\mathfrak{S}$ to be called $\mathfrak{T}$), and there will be a one-dimensional set $\mathfrak{U}$ of singular points (for instance boundary points) in $\mathfrak{T}$, where the situation may be even more complicated.

When we look at a black hole at a late time t_2, we are actually selecting one of these light rays on $\mathfrak{S}$, which form a two-dimensional space $\mathbb{Q} = \mathfrak{S} | \mathbb{R}^+$. Now our subset $\mathfrak{T}$ of $\mathfrak{S}$ connects in a unique way two light rays in $\mathfrak{S}$. This means that $\mathfrak{T}$ induces a mapping of $\mathbb{Q}$ into itself in the sense that pairs of points of $\mathbb{Q}$ are being identified. The subset $\mathfrak{U}$ of $\mathfrak{T}$ may give some triple identifications. Thus, with exceptions at the singular points , we have a mapping of the form Z_2. If, from a certain moment t on, nothing is thrown into the black hole, this mapping will remain unaltered, and we imagine that this Z_2 may specify the particular state our hole is in. Notice that

$$\mathfrak{T} = \mathbb{Q} | Z_2 , \tag{3.1}$$

so that $\mathfrak{T}$ is an orbifold. It is this $\mathfrak{T}$ that depends uniquely on all information that went into the black hole. One might conjecture that the dynamics of this orbifold determines the black hole's fate.

In an earlier paper[7)] this author speculated that the dynamics on $\mathfrak{S}$ could be related to string theories. Indeed we have a two-dimensional world here, and the equations for the fluctuations on $\mathfrak{T}$ resemble the string equations very much. Unfortunately, $\mathfrak{T}$ is Euclidean, whereas the string's world sheet has the Minkowski signature. Originally we thought that this was a harmless distinction, to be made undone by some Wick rotation, but if a correct formalism exists that connects our $\mathfrak{T}$ space with some (super-)string world sheet it has not yet been found.

4. A NEW HILBERT SPACE

In what way will the shape of the orbifold $\mathfrak{T}$ be represented in the wave functions of outgoing particles? The details of this will be difficult to guess, but a few observations may help.

Let us compare two states in Hilbert space. The second is the same as the first, except for one extra particle going in at $t = t_1$. That portion of the horizon $\mathfrak{H}$ that corresponds to times $t < t_1$ in these two states is not in exactly the same position. The displacement can be accurately calculated[5]:

$$\delta u(\Omega) = f(\Omega,\Omega')\, p_{in}(\Omega') \,, \tag{4.1}$$

$$(2M)^{-2}\,(1-\Delta_\Omega)\, f(\Omega,\Omega') = 8\pi G\, \delta(\Omega.\Omega') \,, \tag{4.2}$$

$$\Omega = (\theta,\varphi) \quad ; \quad \Delta_\Omega = \partial_\theta^2 + \cot\theta\, \partial_\theta + \frac{1}{\sin^2\theta}\, \partial_\varphi^2 \,, \tag{4.3}$$

and p_{in} is the momentum of a particle coming in at solid angles Ω', in some suitable units; u is a Kruskal coordinate (the other Kruskal coordinate will be called v). $\delta(\Omega,\Omega')$ is a two-dimensional Dirac delta.

Let now p_{out} be the momentum of an abserved outgoing particle. Then naturally the displacement (4.1) will give its wave function an extra factor

$$e^{-ip_{out}(\Omega)\,\delta u(\Omega)} = e^{-ip_{out}(\Omega)\, f(\Omega,\Omega')\, p_{in}(\Omega')} \,. \tag{4.4}$$

If we suppose that the shift δu is all information we'll ever get back from the ingoing particle, then (4.4) must give the amplitude for the process. What have we found out?

The fact that $\mathfrak{T}$ is an orbifold rather than a manifold has not been used here, but let us first describe the Hilbert space in which (4.4) is an acceptable amplitude. We have functions $p_{in}(\theta,\varphi)$ describing the ingoing momenta, and $p_{out}(\theta,\varphi)$ for the outgoing momenta. These are operators depending on θ and φ. The angles θ and φ are continuous, but we'll quickly replace them by a dense but discrete lattice, much in the spirit of our findings in sect. 1, but it will also be necessary to make our expressions well-defined. Thus, the Dirac delta will have to be thought of as a Kronecker delta on some dense lattice.

In the usual Hilbert space of particles we expect

$$[p_{in}(\Omega), p_{in}(\Omega')] = 0 \ , \ . \tag{4.5}$$

and the same for the outgoing particles. The conjugated operators are $v_{in}(\Omega)$, $u_{out}(\Omega)$, with

$$[p_{in}(\Omega), v_{in}(\Omega')] = -i\delta(\Omega,\Omega') \ , \tag{4.6}$$

Now we see that eq. (4.4) suggests

$$u_{out}(\Omega) = -\int f(\Omega,\Omega')\, p_{in}(\Omega')\, d^2\Omega' \ , \tag{4.7}$$

$$v_{in}(\Omega) = \int f(\Omega,\Omega')\, p_{out}(\Omega')\, d^2\Omega' \ , \tag{4.8}$$

obtaining

$$\langle p_{out}(\Omega')|p_{in}(\Omega)\rangle = N\, e^{-i\int p_{out}(\Omega)\, f(\Omega,\Omega')\, p_{in}(\Omega')\, d\Omega\, d\Omega'} \tag{4.9}$$

where N is a normalization factor.

Notice that (4.9) is an entirely acceptable unitary "scattering matrix". Unfortunately, the Hilbert space generated by (4.5) and (4.6), in which this matrix acts, is quite unnatural. It resembles a bit the Fock space of in- and out-particles in a mixed coordinate-momentum representation where the transverse coordinates Ω and the longitudinal momenta p_r are specified for each particle. What is unusual about it is that at <u>every</u> pair of values for the angles θ and φ we must have exactly <u>one</u> particle!

As physicists we might not be too much worried about this situation. After all, we already suspected that the black hole will be surrounded by particles, possibly swimming in a Dirac sea. Suppose we had a dense lattice in Ω-space. Why not reshuffle those particles a little bit so that there is exactly one for each point in this Ω lattice? The answer is that it is not so easy to link this special Hilbert space to the space of real particles in the real world. In particular the situation far away from the black hole will be difficult to handle. Also, any such procedure will depend delicately upon the Ω cut-off procedure used.

There is another reason why the need for an Ω cut-off should not surprise us. Tiny values $\Delta\Omega$ can only be detected by particles with large transverse momentum. A difficulty here is that these shifts will produce more complicated forms of curvature in space-time. As long as we cannot handle this situation precisely we will stick to more crude Ansätze for a lattice cut-off.

5. A SIMPLE MODEL

In order to get some idea about the consequences of a reaction matrix such as (4.9) for a black hole, we present here a very simple model. We'll worry about more realistic ones later.

Let us replace the Ω by just one point. In this "approximation" in- and outgoing particles behave as spherically symmetric shells of matter. We just have one set of momentum variables p_{in} and p_{out}, and one set of coordinates v_{in} and u_{out}. We have an ingoing wave function $\psi_{in}(v_{in})$ and an outgoing wave function $\psi_{out}(u_{out})$.

We now take (omitting the redundant subscripts for u and v)

$$\psi_1(v) = \psi_{in}(v) = [\tfrac{g}{2\pi}]^{\frac{1}{2}} \int du\, e^{-iguv}\, \psi_2(u) \;, \tag{5.1}$$

and

$$\psi_2(u) = \psi_{out}(u) = [\tfrac{g}{2\pi}]^{\frac{1}{2}} \int dv\, e^{iguv}\, \psi_1(v) \;, \tag{5.2}$$

where g is "Newton's constant". u and v were Kruskal coordinates but in our model we treat them as Rindler coordinates.

Note that

$$\frac{\partial}{\partial u}\psi_1 = \frac{\partial}{\partial v}\psi_2 = 0 \;. \tag{5.3}$$

It is tempting to write this as a Dirac equation:

$$\{(1+\sigma_3)\partial_u + (1-\sigma_3)\partial_v\}\ \psi = 0\ . \tag{5.4}$$

In our model we now introduce a non-trivial behaviour far from the black hole by adding a mass term to (5.4):

$$\{(1+\sigma_3)\partial_u + (1-\sigma_3)\partial_v + i\sigma_1 m\}\ \psi = 0\ , \tag{5.5}$$

which is just the massive Dirac equation in Rindler space.

Now this is incompatible with (5.1) and (5.2) unless we take these to be the <u>boundary condition</u> at u = 0 and v = 0:

$$\psi(0,v) = \begin{pmatrix} \psi_1(v) \\ 0 \end{pmatrix}\ , \quad \psi(u,0) = \begin{pmatrix} 0 \\ \psi_2(u) \end{pmatrix}\ . \tag{5.6}$$

We now have a complete set of equations which we will now solve approximately.

If we write

$$\psi \underset{\mathrm{def}}{=} e^{\frac{1}{2}i\omega\, \ell n|v/u|} \begin{pmatrix} \varphi_1/\sqrt{|v|} \\ \varphi_2/\sqrt{|u|} \end{pmatrix}\ , \tag{5.7}$$

$$\varphi = \varphi(r)\quad , \ r^2 = |uv|\ , \tag{5.8}$$

then ω can be interpreted as the energy of the particle as seen by the outside observer.

For φ we get the equations

$$\partial_r\varphi - \frac{i\omega}{r}\sigma_3\ \varphi + m\sigma_2\varphi = 0\ , \tag{5.9}$$

$$[(r\partial_r)^2 + rm\sigma_2 + (\omega^2 - r^2m^2)]\ \varphi = 0\ . \tag{5.10}$$

Taking $k(r) = \sqrt{\omega^2 - r^2m^2 + rm\sigma_2}$, we find by W.K.B. approximation,

$$\varphi(r) \approx [k(r)]^{-\frac{1}{2}}\ \{\sin[\int_r^{r_0} k(r')\frac{dr'}{r'} + \pi/4]\}\ \varphi_0\ , \tag{5.11}$$

where r_o is the positive solution of $k(r) = 0$.

Let

$$\int_r^{r_o} k(r',\sigma_2)\, dr'/r' \underset{\text{def}}{=} I(r,\sigma_2) \quad , \quad \sigma_2 = \pm \, , \tag{5.12}$$

Then by contour integration we find

$$\lim_{r \to 0} \; (I(r,+) - I(r,-)) = \pi/2 \, , \tag{5.13}$$

$$I(r,-) \approx \omega\, (\ell n(\omega/rm) + \mathcal{F}) \quad , \quad \mathcal{F} = \mathcal{O}(1) \, . \tag{5.14}$$

At the boundaries we assume

$$\psi_1(0,v) = |v|^{-\frac{1}{2}}\, e^{i\omega\, \ell n\, |v|}\, (A\theta(v) + B\theta(-v)) \, , \tag{5.15}$$

$$\psi_2(u,0) = |u|^{-\frac{1}{2}}\, e^{-i\omega\, \ell n\, |u|}\, (C\theta(u) + D\theta(-u)) \, , \tag{5.16}$$

and the W.K.B. approximation yields

$$\begin{pmatrix} A \\ B \end{pmatrix} = -ie^{-2i\omega\, (\ell n(\omega/m + \mathcal{F})} \begin{pmatrix} D \\ C \end{pmatrix} , \tag{5.17}$$

whereas the boundary condition (5.6) gives

$$\begin{pmatrix} A \\ B \end{pmatrix} = e^{-i\pi/4 + i\omega\, \ell n\, g}\, \frac{\Gamma(-i\omega+\frac{1}{2})}{\sqrt{2\pi}} \begin{pmatrix} e^{-\pi\omega/2} & ie^{\pi\omega/2} \\ ie^{\pi\omega/2} & e^{-\pi\omega/2} \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} . \tag{5.18}$$

These equations can be solved together to give constraints on the possible values of ω. We find the spectrum

$$\omega_N \approx 2\pi N/\ell n(Ng/m^2) \, , \tag{5.19}$$

for large N.

6. DISCUSSION

The model of the previous section describes a single particle bouncing back and forth against the black hole, occupying energy levels as given by eq. (5.19). For various reasons the model is not yet very realistic. First of all it is essential that there is one and only one particle present. Second quantization is not allowed here because then we would have various different states with all the same set of values for the momentum $p(\Omega)$. To describe more particles we must introduce Ω dependence as explained in sect. 4. Secondly we notice that all four regions of Rindler space play a role in the model of section 5, whereas what we really want is to describe what an observer sees who can only reach the region $u > 0$, $v < 0$.

But we do see that a quantum deterministic black hole might be feasible, because what our model is describing here, is not really a particle but the oscillating horizon itself.

REFERENCES

1) R. Delbourgo, A. Salam and J. Strathdee, Trieste Preprints 1970, 1971; H. Lehmann and K. Pohlmeyer, Comm. Math. Phys. 20 (1971), 101.
A. Salam, Proceedings of the Amsterdam Conf. on Elementary Particles, 1971, A. G. Tenner, M.J.G. Veltman, eds., North Holland Pub. Comp.
2) G. 't Hooft, in "Recent Developments in Gravitation", Cargese 1978, M. Levy and S. Deser, eds., Plenum, New York and London;
T.D. Lee, "Physics in Terms of Difference Equations, Talk given at the Niels Biohr Centenerary Symp., Copenhagen, October 1985.
L. Bombelli et al., "Spacetime as a causal set", Syracuse, Princeton, prepr., March 1987.
P. Menotti and A. Pelissetto, Pisa prepr., Oct. 1986.
3) R.M. Wald, Phys. Rev. D20 (1979) 1271; G. Gibbons, private communication.
4) S.W. Hawking, Comm. Math. Phys. 43 (1975) 199.
J.B. Hartle, S.W. Hawking, Phys. Rev. D13 (1976) 2188.
W.G. Unruh, Phys. Rev.D14 (1976) 870.
S.W. Hawking and G. Gibbons, Phys. Rev. D15 (1977) 2738.
S.W. Hawking, Comm. Math. Phys. 87 (1982) 395.
5) T. Dray and G. 't Hooft, Nucl. Phys. B253 (1985) 173; Comm. Math.

Phys. 99 (1985) 613; Class. Quantum Grav. 3 (1986) 825.

6) See for instance: M.A. Markov, "On the Stability of Elementary Black Holes", Trieste Preprint IC/78/41, 1978: Progr. Theor. Phys., Suppl. Extra Number, 85 (1965); ZHETF, 51, 878 (1966); id., "Micro-Macro Symmetric Universe", Acad. of Sciences of the USSR, Inst. of Nucl. Research, Moscow, Prepr. 1985.

7) G. 't Hooft, Physica Scripta T15 (1987) 143.