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MAGNETIC CHARGE QUANTIZATION AND FRACTIONALLY CHARGED QUARKS

G. 't HOOFT
Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands

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If magnetic monopoles with Schwinger's value of the magnetic charge would exist then that would pose serious restrictions on theories with fractionally charged quarks, even if they are confined. Weak and electromagnetic interactions must be unified with color, leading to a Weinberg angle θ_W close to 80°.

1. Introduction

Recently an experimental group [1] claimed to have detected a magnetic monopole with magnetic charge

$$h = \frac{4\pi}{e} = 137e\,. \tag{1.1}$$

This is the value predicted by Schwinger [2], and twice the minimum value predicted by Dirac [3]. The mass was reported to be beyond 600 times the proton mass.

Certain models of weak and electromagnetic interactions permit soliton solutions with a calculable magnetic charge and mass. One simple model due to Georgi and Glashow [4] gives a Schwinger monopole with mass between 1000 m_p and 9000 m_p [5], but the model is probably excluded because it contains no neutral currents.

In this paper we will disregard the considerable scepticism against the claim that a monopole was seen. We here consider an important question that arises when we try to extend models with monopoles to hadronic interactions. We wish to describe quarks with electric charge $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$ that are confined by some color confinement mechanism. But these third-integer charges apparently cannot coexist with magnetic monopoles, unless

$$h = 2\pi n/\tfrac{1}{3}e = \tfrac{3}{2}n \times 137e\,, \tag{1.2}$$

with n integer, contrary to the findings of ref. [1].

Observation of Schwinger monopoles would exclude free, fractionally charged, quarks. Does it also exclude quarks that are confined? We can only answer that question if we assume that quarks are confined by a color confinement mechanism, and that magnetic monopoles are the soliton solutions of a gauge field theory. The answer will be yes, except when the color group SU(3) and the e.m. group U(1) are both subgroups of one larger group like SU(4).

There is an obvious necessary condition for these soliton solutions to exist: it must be impossible to construct representations of the gauge group that would yield particles with other than (half)-integer electric charge. For instance, in Weinberg's model [6] it is possible to add to the theory a doublet of particles with electric charges xe and $(x-1)e$, where x is arbitrary. Clearly Weinberg's model can have no soliton solutions with magnetic charge as long as this doublet is not forbidden by the structure of the underlying gauge group.

In the colored quark model, quarks are a triplet representation of the color-gauge group SU(3) and they have electric charge $\frac{2}{3}e$ or $-\frac{1}{3}e$. If in addition triplet representations with no electric charge, or singlet representations with charge $\frac{2}{3}e$ or $-\frac{1}{3}e$ would not be forbidden by the group structure of the theory, then solitons with Schwinger's value of the magnetic charge could not exist. Let us call such representations "exotic".

In all the simple Higgs models, as far as we would check, the necessary condition, the impossibility of non-integer charges, turns out to be sufficient also for the existence of magnetically charged solitons whose magnetic charge quantum h is then given by the Dirac quantization rule

$$hq = 2\pi\,, \tag{1.3}$$

if q is the lowest possible electric charge.

We now claim that in a color-confinement theory the necessary and sufficient condition for the existence of a soliton with magnetic charge h is the impossibility of representations that are singlets under color-SU(3) and have an electric charge which is not an integer times $2\pi/h$. In our case then $2\pi/h = \frac{1}{2}e$. Triplet SU(3) representations may have U(1) charges as small as $\frac{1}{6}e$.

We can only imagine one realistic gauge group structure where exactly this situation is realized. That is if

$$SU(3)^{\text{color}} \times U(1)$$

(where U(1) is the only invariant Abelian subgroup of the usual weak and electromagnetic interactions) are in one subgroup SU(4) of the complete gauge group $\mathcal{G}$. So we can have for instance that

$$SU(3)^{\text{color}} \times [U(1) \times SU(2)]^{\text{weak and e.m.}}$$

is a subgroup of

(a) $SU(4) \times SU(2)$ or (b) $SU(4) \times SU(2)^{\text{left}} \times SU(2)^{\text{right}}$.

This means that the quark triplets p_i, n_i, λ_i, ... each must have a fourth component. It is appealing to assume that these fourth components are nothing but the leptons, as has been proposed by Pati and Salam [7]. In their 4 × 4 quark model the gauge group has separate left and right handed gauge groups [case (b)]. The other gauge group (a) is possible in for instance 6 × 4 quark schemes [8].

Another way to implement the gauge group is to extend our SU(4) to SU(5), therefore also the model of ref. [9] satisfies our criterion.

The U(1) charges of the right-handed parts of the p and n quartets are both

$$(\tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, -\tfrac{1}{2}) .$$

Since the real photon is a mixture of the pure U(1) photon and the neutral component of the SU(2) vector bosons, the real electric charges of the two quartets are

$$(\tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3}, 0), \qquad (-\tfrac{1}{3}\ -\tfrac{1}{3}\ -\tfrac{1}{3}, -1) .$$

The fact that the total sum of these charges must always be zero explains why SU(3) triplets must have one third of the charge quantum of the corresponding SU(3) singlets.

We must make sure that also the left-handed part of the gauge group is compact; that is why we are forced to add either heavy fermions that are coupled through V + A currents to the other particles, or add a separate $SU(2)^{\text{left}}$ gauge group.

So, our condition is satisfied if (the isoscalar part of) electromagnetism is unified with strong interactions: (the isoscalar part of) electromagnetism is the fourth color. This implies that at energies of the order of the mass of the monopole, the U(1) coupling constant must be equal to the strong color-coupling constant. Now the strong coupling-constant g_c can be estimated from the asymptotically free color gauge theories:

$$g_c^2/4\pi \simeq \tfrac{1}{4} , \tag{1.4}$$

at energies of several GeV. If the strong interaction is, as expected, asymptotically free, then the strong coupling constant g_c will decrease logarithmically at higher energies. If unification takes place between 100 and 1000 GeV then we expect that at those energies

$$g_c^2/4\pi \to \tfrac{1}{8} \quad \text{to} \quad \tfrac{1}{12} .$$

It will be related to Weinberg's U(1) coupling constant g' as

$$g_c^2 = \tfrac{2}{3} g'^2 . \tag{1.5}$$

Since

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} , \tag{1.6}$$

the SU(2) coupling constant $g \simeq e$. This would mean that the Weinberg angle θ_w would be given by

$$\mathrm{tg}\,\theta_\mathrm{w} = g'/g \simeq \sqrt{\tfrac{3}{2} \times 137\, g_\mathrm{c}^2/4\pi} = 4 \text{ to } 5\,,$$

$$\theta_\mathrm{w} \simeq 77^\circ\,. \tag{1.7}$$

A smaller Weinberg angle would imply much higher unification energies and extremely heavy monopoles. Here we suppose that a relatively light monopole was seen.

Thus the neutral vector boson Z will be much heavier than the charged vector boson. Perhaps a more complex Higgs mechanism [10] can account for an apparent quantitative contradiction with experiments on neutral current pure leptonic interactions, whereas semi leptonic and pure hadronic weak interactions may be more complicated in this model due to the presence of colored vector bosons.

2. A simple model

In order to show that solitons with Schwinger's value for the magnetic charge can really coexist with fractionally charged, but permanently bound quarks we construct a simple model of strong, electromagnetic and weak interactions, based on SU(4) × SU(2). We neglect the parity breaking part of the weak interactions. There are two gauge coupling constants, g_c and g. Here g_c governs the color gauge field interactions and part of the electromagnetic interactions and is close to one. The other, g, is close to e and governs electromagnetic and weak interactions.

The Higgs field Q is a 4 × 2 representation of this group and its vacuum expectation value in a convenient gauge is

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ F & 0 \end{pmatrix}\,. \tag{2.1}$$

This leaves $\mathrm{SU(3)^{color}} \times \mathrm{U(1)^{em}}$ as an exact symmetry: color and electromagnetism. The rest is spontaneously broken. This is the only Higgs field necessary in this model. Among the fifteen SU(4) gauge fields A_μ^a, the components $A_\mu^9, ..., A_\mu^{14}$ obtain a mass

$$M_A^2 = \tfrac{1}{2} g_\mathrm{c}^2 F^2\,, \tag{2.2}$$

and among the three SU(2) gauge fields W_μ^a, the components $W_\mu^{1,2}$ (the intermediate vector bosons) get a mass

$$M_\mathrm{w}^2 = \tfrac{1}{2} g^2 F^2\,. \tag{2.3}$$

Finally, A_μ^{15} and W_μ^3 mix according to

$$Z_\mu = W_\mu^3 \cos\theta_W - A_\mu^{15} \sin\theta_W ,$$
$$A_\mu^{em} = W_\mu^3 \sin\theta_W + A_\mu^{15} \cos\theta_W , \tag{2.4}$$

where

$$\mathrm{tg}\,\theta_W = \sqrt{\tfrac{3}{2}}\, g_c/g . \tag{2.5}$$

Here A_μ^{em} is the physical photon and Z_μ is a neutral vector boson with mass

$$M_Z^2 = (\tfrac{3}{2} g_c^2 + g^2) F^2 = M_W^2 / \cos^2\theta_W . \tag{2.6}$$

The value of the electric charge e of an electron

$$e = \frac{g g_c}{\sqrt{g_c^2 + \frac{2}{3} g^2}} = g \sin\theta_W . \tag{2.7}$$

We now consider the arguments of ref. [5]. If we pull a closed contour C over the centre of a soliton solution, (that is the soliton travels through the contour) then a gauge rotation which started as a complete $U(1)^{em}$ rotation, must go continuously towards a constant rotation. Or, we must find an element $\Omega(\theta, \phi)$ of the gauge group, as a continuous function of the angles θ and ϕ, such that at $\theta \to 0$, we have one complete rotation as ϕ runs from zero to 2π, while at $\theta \to \pi$ we must have that Ω becomes independent of ϕ. If we let this Ω act on a configuration with constant Higgs field, we get a configuration where a magnetic charge is trapped at the centre. The value of the magnetic charge is determined by the number of U(1) rotations at $\theta \to 0$.

Now in an arbitrary quark model it should be possible to let Ω act on a quark triplet with charges $(-\frac{1}{3}e, -\frac{1}{3}e, -\frac{1}{3}e)$, and since Ω must be single-valued, this implies that we must have at least an integer times 6π rotations at $\theta \to 0$, so the magnetic charge quantum would be at least $\frac{3}{2} \times 137e$.

But in our model we may perform color rotations as well. So instead of taking at $\theta \to 0$

$$\Omega(0, \phi) = e^{i\Lambda^{15}\phi} ,$$

with

$$\Lambda^{15} = \sqrt{6}\,\lambda^{15} = \begin{pmatrix} 1 & & & \emptyset \\ & 1 & & \\ & & 1 & \\ \emptyset & & & -3 \end{pmatrix} , \tag{2.8}$$

we may take

$$\Omega(0, \phi) = e^{i\tilde{\Lambda}\phi} ,$$

with

$$\tilde{\Lambda} = \tfrac{1}{3}\sqrt{6}\,\lambda^{15} - \frac{1}{\sqrt{3}}\,\lambda^8 = \begin{bmatrix} 0 & & & \emptyset \\ & 0 & & \\ & & 1 & \\ \emptyset & & & -1 \end{bmatrix} . \tag{2.9}$$

Because $\Omega(\theta, \phi)$ may be any element of SU(4) we can continuously shift this rotation towards a constant as $\theta \to \pi$.

However, the choice (2.9) is not yet a pure $SU(3)^{color} \times U(1)^{em}$ rotation as $\theta \to 0$. This is necessary. So, in order to eliminate the elements of the broken parts of the gauge group we multiply with

$$e^{i\tau^3\phi} , \tag{2.10}$$

where τ^3 is the SU(2) rotation $\left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$. This way we get at $\theta = 0$ a gauge rotation that leaves the Higgs field (2.1) invariant.

It now must be shifted towards a constant as $\theta \to \pi$. Thus, the complete $\Omega(\theta, \phi)$ may be constructed as follows:

$$\Omega(\theta, \phi) = (\cos \tfrac{1}{2}\theta\ e^{i\tilde{\Lambda}\phi} + \lambda^{13} \sin \tfrac{1}{2}\theta)(\cos \tfrac{1}{2}\theta\ e^{i\tau^3\phi} + \tau^1 \sin \tfrac{1}{2}\theta) , \tag{2.11}$$

where

$$\lambda^{13} = \left[\begin{array}{ccc|c} & & & 0 \\ & 0 & & 0 \\ & & & 1 \\ \hline 0 & 0 & 1 & 0 \end{array}\right] , \qquad \tau^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} .$$

In this way we get the product of (2.9) and (2.10) at $\theta \to 0$ and a constant at $\theta \to \pi$, while Ω is continuous and unitary everywhere.

If we let this Ω act on the Higgs field we get

$$Q = F \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ e^{i\phi} \cos \frac{1}{2}\theta \sin \frac{1}{2}\theta & \sin^2 \frac{1}{2}\theta \\ \cos^2 \frac{1}{2}\theta & e^{-i\phi} \cos \frac{1}{2}\theta \sin \frac{1}{2}\theta \end{bmatrix} . \tag{2.12}$$

We claim that if this angle dependence is chosen as a boundary condition for the Higgs field at large distance from the origin, then a magnetic charge sits at the origin. It has the Schwinger value because we were forced to take the combined rotation (2.9) and (2.10) at $\theta \to 0$, which is a double $U(1)^{em}$ rotation. After all, our model does not prohibit half-integer charges, which could occur for instance in a 4×1 or in a 1×2 representation of $SU(4) \times SU(2)$.

3. Calculation of the magnetic charge

We will now calculate explicitly the magnetic charge at the origin, if the Higgs field far away from the origin is given by (2.12). To do that we must first give the correct definition of the Maxwell field $F_{\mu\nu}$: it must coincide with $\partial_\mu A_\nu^{\mathrm{em}} - \partial_\nu A_\mu^{\mathrm{em}}$ as soon as the Higgs field is

$$Q = Q_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ |Q| & 0 \end{bmatrix} . \tag{3.1}$$

We observe that if $Q = Q_0$ then

$$\frac{Q^* D_\mu Q}{Q^* Q} = \tfrac{1}{2} i \left(-\frac{3}{\sqrt{6}} g_{\mathrm{c}} A_\mu^{15} + g W_\mu^3 \right) = \frac{1}{2} i \frac{g}{\cos\theta_{\mathrm{W}}} Z_\mu . \tag{3.2}$$

So we now define

$$Z_\mu = \frac{-2i\cos\theta_{\mathrm{W}}}{g} \frac{Q^* D_\mu Q}{|Q|^2} , \tag{3.3}$$

which is gauge invariant. Z_μ is in fact the field of the heavy neutral intermediate vector boson.

Next, we find a quantity $W_{\mu\nu}$ which corresponds to $\partial_\mu W_\nu^3 - \partial_\nu W_\mu^3$ in the gauge $Q = Q_0$,

$$W_{\mu\nu} = \frac{1}{|R|} R^a G_{\mu\nu}^a - \frac{1}{g|R|^3} \epsilon_{abc} R^a D_\mu R^b D_\nu R^c , \tag{3.4}$$

where

$$G_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\,\epsilon_{abc} W_\mu^b W_\nu^c , \tag{3.5}$$

$$R^a = Q^* \tau^a Q . \tag{3.6}$$

In the gauge $Q = Q_0$ we have

$$R^a = |R|\,(0, 0, 1) . \tag{3.7}$$

As usual D_μ stands for the covariant derivative.

Eliminating A_μ^{15} in eq. (2.4) we find, in the gauge $Q = Q_0$,

$$A_\mu^{\mathrm{em}} = \frac{1}{\sin\theta_{\mathrm{W}}} W_\mu^3 - \cot\theta_{\mathrm{W}} Z_\mu ,$$

$$F_{\mu\nu} = \frac{W_{\mu\nu}}{\sin\theta_{\mathrm{W}}} - \cot\theta_{\mathrm{W}} (\partial_\mu Z_\nu - \partial_\nu Z_\mu) . \tag{3.8}$$

Together with (3.3) and (3.4) this is a gauge-invariant definition of the Maxwell field. We apply this definition now to the case that the Higgs field satisfies (2.12).

A possible source for the magnetic components of $F_{\mu\nu}$ can only come from the first term in (3.8). Indeed, the vector R^a, if the Higgs field satisfies (2.12), is

$$R^a = F^2 (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) = \frac{F^2}{r} (x, y, z) . \tag{3.9}$$

By continuity then R^a must have a single zero at the origin and we know from ref. [5] that then the magnetic part of $W_{\mu\nu}$ (eq. 3.4) has a source with strength $4\pi/g$, at the point where $R = 0$. Therefore $F_{\mu\nu}$ contains a magnetic field with source strength

$$4\pi/g \sin\theta_{\mathrm{W}} = 4\pi/e \qquad (3.10)$$

(compare eq. 2.7).

We herewith verified that in this model any field configuration that satisfies the boundary condition (2.12) carries a magnetic charge with the value $137e$.

4. On the color-magnetic charge

There is a problem connected with our way of constructing a magnetic monopole in the color model. That is that at $\theta = 0$ we not only performed a $U(1)^{em}$ rotation but at the same time an $SU(3)^{color}$ rotation as well. Consequently the object inside the sphere is not only a magnetic monopole in the electromagnetic sense, but also with respect to color: it is also the source of a "color magnetic" field. The question is whether this color magnetic field is or is not observable and whether it is of long range.

It is difficult to answer this question without a detailed theory of color confinement but we think that the color magnetic field will be screened completely by color gluons and therefore will be of short range only.

In particular, we do *not* believe in the second possibility: that this monopole would be confined by the same mechanism that confines quarks. Confinement of either the quarks, or our magnetic monopoles, is sufficient for Dirac's quantization condition to be satisfied, whereas confinement of both would be superfluous. For that we would need that both electric and magnetic quantized flux lines exist. That this is not the case, we derive from our "dual equivalence" theory [11]: the color confinement mechanism works just as the Nielsen-Olesen theory [12] of vortex-like magnetic field configurations in a Higgs model, or a superconductor, but we must interchange the words "electric" and "magnetic". Quarks with a color-electric charge are confined in the same way as magnetic monopoles would be confined if they occur inside a superconductor.

Color-magnetic monopoles are then to be compared with electrically charged objects inside a superconductor: they will be screened completely. There are no electric quantized flux lines in a superconductor.

Another way of looking at the problem is to consider these color magnetic monopoles in Wilson's lattice approximation of the color theory [13]. The monopole would be the end point of a color-Dirac string. A color-Dirac string is a region of space and time that differs from the ordinary vacuum by a gauge rotation that is singular on the string. But in Wilson's lattice theory we must do the (functional) integration over all gauge rotations, including those that would yield Dirac strings. Thus the Dirac string would not be noticed and would not carry any energy. In other words: the only long range interactions in Wilson's theory are the color-electric ones. If we believe that the lattice theory describes the most essential long-range features of the theory then the color part of the magnetic charge will be screened at hadronic distances of the order of one GeV^{-1}. Since the typical parameter that determines the size of those classical solutions that describe monopoles is much less than 1 GeV^{-1}, the calculation of the monopole mass is not appreciably affected by this screening mechanism.

5. Conclusion

Confirmation of the existence of a magnetic monopole with Schwinger's value of the magnetic charge will have vast theoretical implications, also in the domain of conventional high-energy physics. If the mass comes out to be smaller than 1000 proton masses then it will be difficult to find any explanation in terms of soliton solutions of gauge theories. The typical mass parameter for these is

$$4\pi M_{\mathrm{W}}/e^2 = 137\, M_{\mathrm{W}}\ , \tag{5.1}$$

or 5000 GeV.

If the mass is within the range $10^3 - 10^5$ proton masses it can be incorporated in a unified gauge theory. However, if we insist on theories with fractionally charged, but confined quarks, then there is only a very restricted set of possibilities. At energies comparable with the monopole mass, electromagnetism and color must be unified. This most probably corresponds to unification of quarks and leptons [7]. Now the unit of electric charge, e, is small and the color coupling constant g_{c} is large, so we are forced to mix the photon with yet another set of gauge fields, most likely the weak intermediate vector bosons. To get $e \ll g_{\mathrm{c}}$ the mixing angle, θ_{W}, must be close to 90°. This is only compatible with experiments on neutral currents if we assume a spontaneous symmetry breaking mechanism that is more complex than the simple Higgs mechanism [10].

In this scheme one inevitably gets colored intermediate vector bosons with quark and lepton quantum numbers, but baryons need not be unstable.

An unwanted consequence of the new vector bosons is that they give additional contributions to semileptonic weak interactions by their exchange in the crossed channels. This is just one more reason to consider our model as preliminary. Perhaps a more realistic one can be obtained through a more general Higgs mechanism, or a more clever scheme of cancellations.

The gauge group may not contain an invariant U(1) subgroup. All electric charges in one multiplet must add up to zero. That implies that also the left-handed parts of the fermions must be assembled in larger multiplets than the SU(4) quartets alone. The apparent absence of V + A transitions can then be explained either by assuming new heavy fermions [8] or by adding a separate weak gauge group $\mathrm{SU(2)}^{\mathrm{left}}$, more heavily broken than its right-handed partner. The latter solution however would require either a very small coupling constant g^{left} or a very massive vector boson W^{left}, both of which could make the mass of a magnetic monopole much higher than in the other case. A "light" magnetic monopole would therefore favor the scheme proposed by Harari, Fritzsch, Gell-Mann and Minkowsky [8], consisting of six or more quarks (each being a color-triplet) and an equal number of leptons. We can then obtain a left-right symmetry only broken by mass terms.

If the monopole mass is much more than 10^5 GeV then there will be many different models which will be difficult to check.

Our model still admits half-integer electric charges, unless the group SU(4) × SU(2) will be unified to SU(8). In that case the Dirac monopole will also be possible but much heavier than the Schwinger one.

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