Fundamental aspects of Quantum Theory related to the Problem of Quantizing Black holes

Gerard 't Hooft

Institute for Theoretical Physics
Princetonplein 5, P.O. Box 80.006
3508 TA UTRECHT, The Netherlands

Abstract

It is shown that the scattering amplitudes for elementary particles against a black hole can be deduced to some extent from the mere assumption that such amplitudes should exist and be in agreement with low energy field theory and general relativity. It automatically follows that the underlying field theory has a stringlike structure, albeit an unconventional one.

The fact that the black hole's entropy is finite implies that it has a discrete spectrum and this in turn suggests a discrete structure of Hilbert space, not only for the black hole but also for the field theory as a whole. Such a fundamental discreteness of Hilbert space allows a new interpretation of quantum mechanics that we call "evolutionary determinism": the local laws of physics could be deterministic while superposition of states and wave functions is still allowed and indeed necessary for deriving the low energy limit of these laws to obtain something resembling the standard model.

1. The Black Hole Spectrum

The standard Kerr-Newman solution[1] describing a black hole depends on three parameters, the mass M, charge Q and angular momentum L. All three are in principle continuous, but a particle physicist could convincingly argue that Q and L must be quantized. More speculatively one could suspect that also M is not continuous. Now, as is well-known[2], a black hole is expected to emit radiation. Therefore, once it is in a state (M, Q, L), it will not stay there very long. It is not difficult to estimate that because of Hawking radiation the masses M will be complex:

$$M^2 \Rightarrow M^2 - iB \ , \tag{1}$$

where B is of the order of the Planck mass squared for all black holes.

But how dense is the spectrum of values (1)? The level density $\rho(M,Q,L)$ can be estimated the following way[3], where for simplicity we ignore the dependence on Q and L. Consider the reversible process where a black hole of mass M absorbs a particle with energy δM, becoming a black hole with mass $M + \delta M$. The absorption cross section σ is approximately

$$\sigma \ \cong \ \pi R^2 \ , \tag{2}$$

where R is the black hole radius, $R = 2M$. Conversely, a black hole with mass $M + \delta M$ can emit the same particle as Hawking radiation. The emission probability W is approximately

$$W \simeq \pi R^2 \ \rho_1(\delta M) \ e^{-\beta_H \delta M} \ , \tag{3}$$

where β_H is the inverse Hawking temperature, $\beta_H = 8\pi M$ (putting the gravitational constant κ and Boltzmann's constant k_B equal to one). $\rho_1(\delta M)$ is the density of states for the objects radiated out with energy δM.

If there were a quantum mechanical theory for the black hole, the same quantities could be expressed in terms of transition amplitudes, using the "golden rule":

$$\sigma = |\langle M{+}\delta M|\mathcal{T}|M,\delta M\rangle|^2 \rho(M{+}\delta M) \ ; \tag{4}$$

where $\mathcal{T}$ is the transition matrix, and

$$W = |\langle M,\delta M|\mathcal{T}|M{+}\delta M\rangle|^2 \rho(M)\rho_1(\delta M) \ . \tag{5}$$

By virtue of PCT invariance, the matrix elements in (4) and (5) should be each other's conjugates, and therefore we find

$$\rho(M{+}\delta M)/\rho(M) \ = \ \rho_1(\delta M)\ \sigma/W \ = \ e^{\beta_H \delta M} \ ; \tag{6}$$

this should hold for a range of values for δM as long as $\delta M \ll M$, and with $\beta_H = 8\pi M$ we find

$$\rho(M) = C\ e^{4\pi M^2} \ , \tag{7}$$

where the universal constant C is the only unknown. Note that the exponent is one quarter of the area of the horizon; this is what one also finds in more general cases.

C could be finite*, in which case we indeed have a finite spectrum density, or C is infinite, but in this case equations (4) and (5) could be considered at the lower end of the continuum. Let $|M+\delta M\rangle$ be in the continuum but $|M\rangle$ one of the discrete states directly underneath. Either $|M+\delta M\rangle$ would be an absolutely stable thing ($W=0$), in which case *virtual pair creation* of these things would give infinite contributions to graviton self-energy diagrams, or the cross section σ for collisions against $|M\rangle$ would tend to infinity. Neither of these latter options sound physically very attractive, which is why we suspect C to be finite. We must realize however that very large numbers are indigenous in quantum gravity. It could be that C is of the order of 10^{40} or 10^{-40}.

2. The Gravitational Back Reaction

Although Hawking radiation of black holes naturally leads to a picture of a discrete black hole spectrum, one does not find a discrete spectrum when one attempts to derive it more explicitly from Field Theory in the black hole background, in contrast with for instance the spectrum of magnetic monopoles that one could derive this way. The reason why one finds a continuous spectrum for black holes is technically that in any state of Hilbert space there seems to be no relation (in the form of some boundary condition at the horizon) between waves of incoming particles and waves of outgoing particles.

This cannot be right. If black holes can be seen as any form of matter there should be an S-matrix relating out states to in states. Now however we make the fundamental observation that it is possible to *assume* that an S-matrix *does* exist. To treat the black hole as a static background and subsequently rely on background field theory *ignoring gravitational interactions between in- and outgoing particles* may well be fundamentally incorrect, or at least imprecise. For any given in state $|in\rangle$, Hawking's derivation gives a *probabilistic distribution* of out states:

$$|in\rangle \Rightarrow \{|out\rangle,\, W_{out}\}\ , \tag{8}$$

where W_{out} is the probability to find the state $|out\rangle$. Hawking's original interpretation of this result was that pure states may make transitions into mixed states, but it is more likely that the probabilities must be interpreted as "the probability that the calculation was right"; the calculation did not yield one single Hamiltonian for the black hole, but a probabilistic distribution of Hamiltonians. The true Hamiltonian is just one of these.

Can we devise a formalism that gives one single S-matrix, or one single Hamiltonian, in stead of a probabilistic distribution? We claim that it exists[4]. First, one must understand how infalling matter interacts gravitationally with outgoing Hawking radiation.

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*It is unlikely that C is exactly constant. There may well be subdominant corrections either in the exponent, or in the form of powers in front of the exponent.

Consider a representation of the Schwartzschild solution in terms of its Kruskal coordinates[5]:

$$xy = -(r/2M - 1)e^{r/2M} \quad ; \qquad (9)$$

$$x/y = e^{(t-t_0)/2M} , \qquad (10)$$

where t_0 is some arbitrary reference time. In these coordinates space-time is regular at the future and past horizons, which are given by $y = 0$ and $x = 0$, respectively. A boost in time t corresponds to a Lorentz transformation in x and y around the origin $(x=y=0)$. Suppose a particle falls in around the time t_0, and a Hawking particle leaves the hole around time $t \gg t_0$. We see that at the horizon these two particles cross each other with a tremendous center-of-mass energy, given by the Lorentz boost factor $\exp((t-t_0)/M)$. Because of this large relative energy the exchange of gravitational interactions may not be ignored.

This interaction is easy to compute in the limit where we neglect the particles' rest masses. An infalling particle causes a *shift* in the metric: two ordinary Schwartzschild metrics are glued together along the surface $x = 0$ with a relative shift δy in the y coordinate[6], which depends on the angles $\Omega = (\vartheta, \varphi)$. This is not unlike a "sonic boom" or Čerenkov radiation. The functional dependence of δy on Ω is determined by the equation

$$(1-\Delta_\Omega)\, \delta y(\Omega) = 4\pi\kappa p_{in}\, \delta^2(\Omega, \Omega_1) , \qquad (11)$$

where Δ_Ω is the angular Laplacian and Ω_1 is the set of angles at which the particle dropped in, with momentum p_{in} with respect to the Kruskal coordinates.

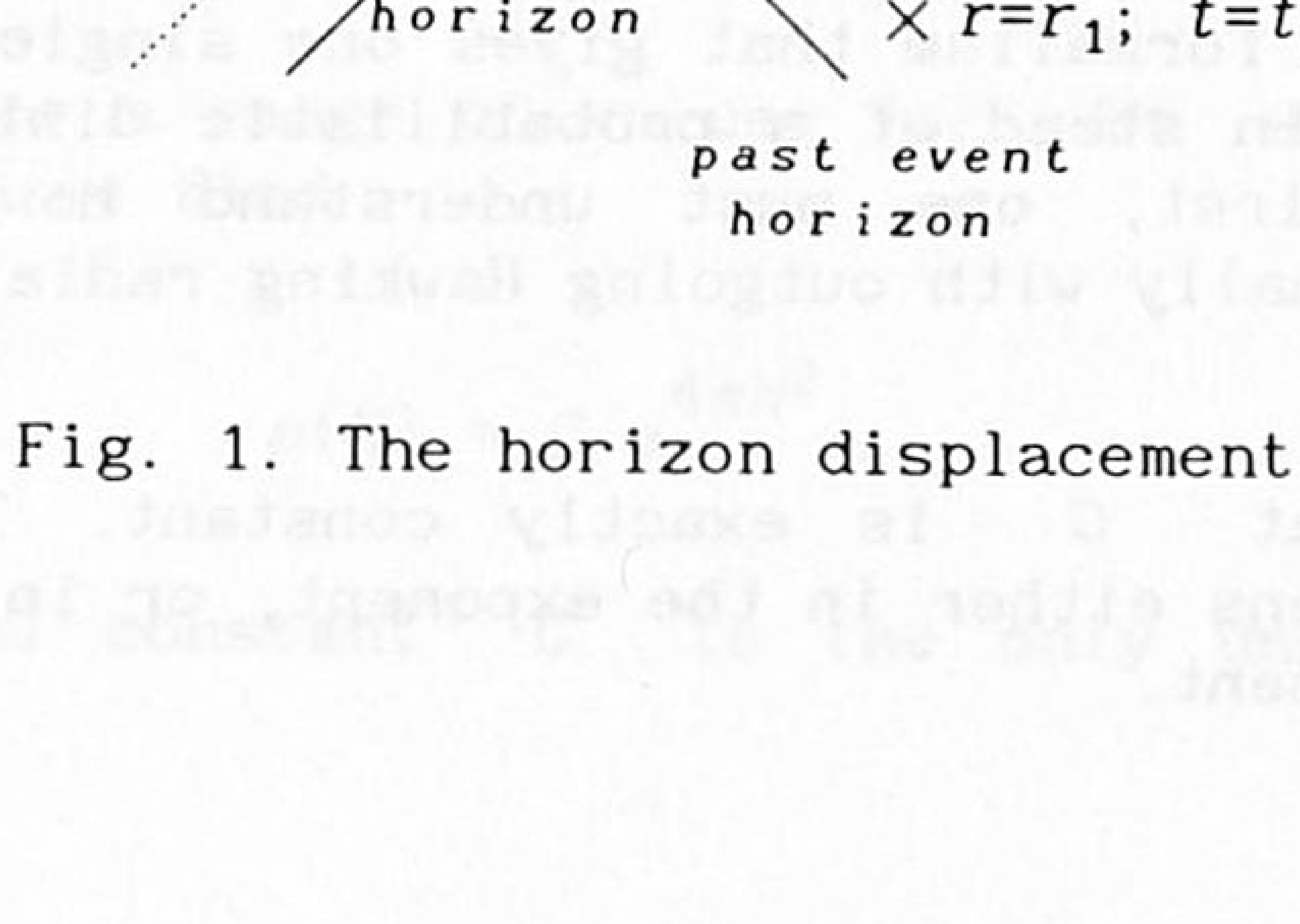

Fig. 1. The horizon displacement.

The solution of Eq. (11) can be written as

$$\delta y(\Omega) = f(\Omega,\Omega_1)\, p_{in} , \qquad (12)$$

where $f(\Omega,\Omega_1)$ is a Green function. It is uniquely determined by Eq. (11) because $1-\Delta_\Omega$ has a unique inverse.

This solution of Einstein's equations is found by writing the space-time metric obtained from gluing the two Schwarzschild solutions together as an *Ansatz* and then imposing Einstein's equations. The Green function f can be given in an integral form but this is not very illuminating. One finds that $f(\Omega,\Omega_1) > 0$ for all Ω, Ω_1 , and f diverges logarithmically when $\Omega \to \Omega_1$.

A consequence of these observations is that if we drop a particle into the black hole, the position of the horizon at times t *before* the particle fell in, changes, as drawn in Fig. 1. This change is barely perceptible at times $t \lesssim t_1$, but at times $t_2 \gg t_1$ the change is large. An observer there sees Hawking radiation that now originated in a *different* region of space-time than it would have if the first particle had not been thrown in.

Is this consequence of any importance? What does it matter if the Hawking radiation originated somewhere else? It will certainly look the same as before.

We will argue in the next section that only in a quantum theory that is detailed enough to give us a scattering matrix instead of a density matrix, shifting horizons will be relevant. Indeed, important constraints on the scattering matrix will be found. As was argued in ref[7] the no-hair theorem is *not* true for the quantum black hole. In ref[7] it was found that finite, discrete quantum numbers may exist for black holes; we will go one step further: we expect one 'quantum hair follicle' (one Boolian degree of freedom) per Planckian unit of surface on the horizon; this would give the right spectral density.

3. Constructing the S-matrix

Suppose a scattering matrix exists. This means that if we have completely specified the state $\{p_1, p_2, \ldots\}$ of all particles that ever went into the black hole, the outgoing matter should be in one well-specified state $|\psi\rangle_{out}$. A basis for $|\psi\rangle_{out}$ is the set of states where all outgoing particles have well-specified momenta at a certain time $t=t_0$. Now at $t=t_1>t_0$ we drop a light particle into the hole, with momentum p_{in} (in Kruskal coordinates) at solid angle Ω_1 . The change this induces for the outgoing waves is now determined primarily by the horizon shift (if other, non-gravitational interactions may be ignored). Thus, the new state will now be

$$|\psi\rangle_{out} \to e^{-i\int P_{out}(\Omega)\delta y(\Omega) d^3\Omega}\, |\psi\rangle_{out} , \qquad (13)$$

where $P_{out}(\Omega)$ is the operator that generates a shift in the configurations at the solid angle Ω . It is, of course, also the total momentum emerging at solid angle Ω. In here, we can now substitute

Eq. (12) for δy.

Now this means that if we know $|\psi\rangle_{out}$ at one stage, then $|\psi\rangle_{out}$ can, in principle, be determined after allowing any number of particles to fall in. If we may ignore non-gravitational interactions, we see that all states $|\psi\rangle_{out}$ ever to be produced by the black hole are generated by the operator $P_{out}(\Omega)$ from one single state. Therefore, $|\psi\rangle_{out}$ must be generated by the *algebra* of these operators. Similarly, the *ingoing* particles are only distinguished by the *total* momentum $p_1(\Omega_1)$ at each solid angle Ω_1.

We find the following important result[4,8]. For the incoming wave functions one may diagonalize the operators $P_{in}(\Omega_1) = p_{in}(\Omega_1)$, and for the outgoing states we diagonalize $P_{out}(\Omega)$. Eq. (13) then tells us how a change in p_{in} affects the outgoing state. Up to a proportionality factor, the complete transformation rule for ingoing states into outgoing states should be generated by this equation. This rule is not difficult to find:

$$\langle\{p_{out}(\Omega)\}|\{p_{in}(\Omega')\}\rangle = N\, e^{-i\int d^2\Omega\, d^2\Omega'\, p_{out}(\Omega) f(\Omega,\Omega') p_{in}(\Omega')} \tag{14}$$

where N is a normalization factor†.

Eq. (14) is the *S*-matrix we wanted. *If* an *S*-matrix exists, and if we may ignore other than the longitudinal gravitational forces, it must be this one. It can be rewritten as (neglecting the 1 and the $4\pi\kappa$ in Eq. (11)),

$$\langle out|in\rangle = \int \mathcal{D}x^+(\Omega)\mathcal{D}x^-(\Omega) e^{i\int d^2\Omega(-\partial_\Omega x^+ \partial_\Omega x^- + p_{out}x^- - p_{in}x^+)} \tag{15}$$

It is here that we see a strikingly close resemblance to string theory[4]. As in string theory, Eq. (15) should be universal. The amplitude is nearly the same as the one used as a starting point in string theory; only the string coupling constant comes out being imaginary[4,8]. If we replace the usual string amplitudes (for which after all no direct physical motivation can be found) by Eq. (15) or possible refinements[8] of Eq. (15), then the *spectrum of massless states at the zero-slope limit* will remain the same. Thus we imagine that the qualitative successes of string phenomenology can also be attributed to this amplitude.

However, our amplitudes were directly motivated by consistency requirements for black holes, and in our implementation of these requirements a number of approximations were made. And it is not hard to argue that Eq. (15) cannot be exactly correct.

4. Horizon operator algebra

The problem with it is the algebra that generated the basis in which it is defined. We have the following commutation rules

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†If other properties of the in- and outgoing particles are taken into account besides their momenta, (e.g. electric charge) then N becomes a unitary matrix. In the case of electric charge this matrix represents the contribution of a fifth, compactified, dimension[8].

$$[p_{in}(\Omega), p_{in}(\Omega')] = 0 \quad ; \quad [p_{in}(\Omega),\ x_{in}(\Omega')] = -i\delta^2(\Omega,\Omega') \quad ;$$
$$[p_{out}(\Omega), p_{out}(\Omega')] = 0 \ ; \ [p_{out}(\Omega),\ y_{out}(\Omega')] = -i\delta^2(\Omega,\Omega') \quad , \tag{16}$$

and we have the relation

$$y_{out}(\Omega) \ = \ \int d^2\Omega_1\ f(\Omega,\Omega_1)\ p_{in}(\Omega_1) \quad . \tag{17}$$

This implies

$$[x_{in}(\Omega), y_{out}(\Omega')] \ = \ if(\Omega,\Omega') \quad , \tag{18}$$

so that we have also

$$x_{in}(\Omega) \ = \ -\int d^2\Omega'\ f(\Omega,\Omega')\ p_{out}(\Omega') \quad . \tag{19}$$

The operators x_{in} and y_{out} could be interpreted as coordinates of "particles", but then there should be *exactly one particle at every value of* Ω . This is where this Hilbert space differs from ordinary Fock space, where we may have any number of particles (mostly this will be zero) at every mode. This is also why it will be difficult to interpret our *S*-matrix directly as a matrix describing scattering of familiar particles.

But in spite of the unusual way in which the dynamical variables are represented we do believe that this description of the Hilbert space surrounding the black hole can be defended. Imagine a lattice-like cut-off on the horizon, where the lattice length is of the order of the Planck length. At every lattice site Ω there is exactly one particle. This leaves us more than enough particles to reconstruct ordinary Hilbert space. Ordinary particle physics is at the low-energy limit, where we never need to know what happens when two or more particles sit at exactly the same site Ω .

Thus, on the one hand we have ordinary particles, but alternatively, x_{in} and y_{out} can be seen as the position operators for the *past* and the *future* horizon. We then recognise an important consequence of our description of Hilbert space: past and future horizons cannot both be localized accurately; these obey an uncertainty relation. Indeed, they are each other's dual conjungates, much in the same way as coordinates are dual to momenta.

We claim that this also does away with a question considered often in the literature: does the time-reversed black hole (called "white hole") exist? Does the "eternal black hole" (one with a past white hole that was already there before the universe began) exist? Our discovery is that these questions are not appropriate if our *S*-matrix exists: white hole coordinates are ill-specified once a black hole was localised.

The difficult but perhaps exciting picture that emerges is that the exact shape of either the past or the future horizon may completely determine the particle content of the black hole's vicinity.

It should be possible to refine this picture by incorporating gravitational forces in the transverse direction, and non-gravitational forces. It is not hard to take the electromagnetic force into account. Here, the electric charge density operator $\rho(\Omega)$ and the gauge phase operator $\phi(\Omega)$ are each other's duals. Electromagnetic shock waves (Čerenkov radiation) surrounding charged massless particles are very similar to gravitational shock waves[9].

As is discussed further in Ref.[8], we expect a cut-off in Ω space. Probably the transverse forces are responsible for that. Surely, if coordinates in the transverse direction would be specified with accuracies better than the Planck length (implying $\delta\Omega \lesssim M_{Pl}/M$, where M is the black hole mass), then momenta in the transverse direction exceed the Planck energy so that also shifts in the transverse direction will arise that are bigger than $\delta\Omega$. The simplest cut-off would be a lattice in Ω space, but reality will be more complicated. What we expect actually to happen is that the Hilbert space algebra itself will produce a cut-off. The operator algebra is worked out further in Ref.[8].

We should not expect to get discrete representations of this algebra directly, because our Hilbert space still includes particles at infinite distances from the black hole, which certainly form a continuum♪. But if we erect a laboratory wall at several black hole radii then the bulk of all entropy resides near the hole, and the total number of states then *is* determined by the hole's surface: one flip-flop degree of freedom par Planckian unit of surface area. The necessity of a laboratory wall presumably implies that we should impose further boundary restrictions at infinity. This may explain why we actually do find a discrete representation, which however is not unitary[8].

Let us summarise what was found in Ref.[8]. The algebra is replaced by a Lorentz covariant algebra, assuming that this way also gravitational shifts in the transverse direction will be incorporated. One then discovers representations of this algebra as follows. On the black hole's horizon surface area we imagine a lattice built from surface elements Σ_i, $i=1, \ldots, N$. Any such lattice corresponds to a representation. At each surface element Σ_i a 3-vector operator $L^a{}_{(i)}$, $a=1,2,3$, is defined, satisfying the commutation rules of angular momenta:

$$[L^a{}_{(i)}, L^b{}_{(j)}] = i\delta_{ij}\varepsilon^{abc}L^c{}_{(i)} . \tag{20}$$

Further,

$$L_{(i)}{}^2 = \tfrac{1}{2} \text{ or } 1 , \tag{21}$$

to guarantee that they represent minimal surface elements. Angular momenta corresponding to larger agglomerations of surface segments are obtained by adding the corresponding L operators in the usual way. A representation would then be given by the set of numbers

$$|\{m_{(i)}\}\rangle , \tag{22}$$

which of course is discrete.

Unfortunately, because the operators $L^a{}_{(i)}$, from their definition, are not Hermitean, the states (22) are not orthonormal.

♪I thank S. Coleman for a discussion on this point.

5. Discrete Quantum Mechanics

It is very interesting to study the quantum mechanics of systems that are entirely discrete such as the sets of numbers (22). Although (22) is a Heisenberg state, not evolving in time, we could imagine a similar configuration obeying a Schrödinger equation,

$$\mathrm{d}|\psi\rangle/\mathrm{d}t \quad = \quad -iH|\psi\rangle \quad . \tag{23}$$

At first sight one might think that this is fundamentally different from a "classical" discrete system, evolving in a jumpy way,

$$\{m_{(i)}\}_{t_1} \;\Rightarrow\; \{m'_{(i)}\}_{t_2} \;\Rightarrow\; \{m''_{(i)}\}_{t_3} \;\Rightarrow\; \dots\,, \tag{24}$$

but the transition "classical" ⇒ "quantum mechanical" for discrete systems is very different from the continuous case.

The evolution (24) is a *special case* of the *more general* quantum mechanical law of evolution (23). There is no need for a limiting procedure $\hbar \to 0$.

Even if a discrete system is classical, one can formally introduce a Hilbert space of states, $|\{m_{(i)}\}\rangle$, defined to be orthonormal. In terms of these states the law of evolution (24) is

$$|\{m_{(i)}\}\rangle_{t_1} \;\Rightarrow\; |\{m'_{(i)}\}\rangle_{t_2} = U(t_2,t_1)\,|\{m_{(i)}\}\rangle \quad , \tag{25}$$

where $U(t_2,t_1)$ is a matrix containing ones and zeros, corresponding to the permutation $m_{(i)} \Rightarrow m'_{(i)}$. If we are dealing with a true permutation U is unitary and an operator H can be found so that

$$U(t_2,t_1) \quad = \quad e^{-iH(t_2-t_1)} \quad . \tag{26}$$

Not all operators H satisfy an equation of the form (26) such that U is a permutation operator. These Hamiltonians form a subclass of all Hamiltonians.

If a basis $\{|n\rangle\}$ *and a discrete set of time variables* t_i *exists such that* $U(t_i-t_j)$ *in this basis are genuine permutation operators we define this system to have "evolutionary determinism".*

In practice we will also require that the definition of this special basis (called "primordial basis" in Ref. [10]) is in some sense local, so that projection operators selecting out certain basis elements are physically observable.

Our evolutionary determinism is to be distinguished from other forms of determinism in the sense that we do not object against transformations to some other basis in Hilbert space in terms of which U can no longer be seen to be an obvious permutator. The initial state may well be chosen to be some linear superposition. Actually we believe that linear superpositions are absolutely necessary if we want to deduce macroscopic, low energy laws of physics from the microscopic ones.

6. Examples

Consider an atom with a large angular momentum ℓ . Its Hilbert space is finite dimensional, and usually we chose as a basis the states

$$|m\rangle \quad , \quad m = -\ell, \ \ldots \ , +\ell \ . \tag{27}$$

Let's have a magnetic field B in the z-direction so that the Hamiltonian is given by

$$H|m\rangle \ = \ \mu B \ m|m\rangle \ ; \tag{28}$$

where for simplicity we will take $\mu B = 1$. Of course the evolution operator U is

$$U(t)|m\rangle \ = \ e^{-imt}|m\rangle \ . \tag{29}$$

Now choose the new basis $\{|g\rangle\}$, with

$$|g\rangle = \frac{1}{\sqrt{2\ell+1}} \sum_m e^{\frac{2\pi i mg}{2\ell+1}} |m\rangle \ , \quad g = 0, \ \ldots, \ 2\ell . \tag{30}$$

Writing $2\ell+1 = N$ we see

$$U(t)|g\rangle \ = \ \frac{1}{\sqrt{N}} \sum_m e^{-2\pi i mg/N \, -imt}|m\rangle \quad , \tag{31}$$

and if we limit ourselves to time intervals $t = 2\pi k/N$, k integer, then we see

$$U(t)|g\rangle \ = \ |g + k \ (\mathrm{mod} \ N)\rangle \ , \tag{32}$$

and this is exactly a (cyclic) permutation. We observe that the spinning atom in a homogeneous magnetic field has evolutionary determinism.

Evolutionary determinism does not forbid us to introduce non commuting operators such as L_x and L_y . Indeed, they may be called "observables" in the usual sense. In fact, the Hamiltonian H itself is non diagonal in $\{|g\rangle\}$, and we accept it as a truly observable variable. Treating the spinning atom as a model of the Universe, our philosophy will be that at the beginning of the Universe a single state $|g\rangle$ was selected, so that the evolution is completely deterministic. But we may decide to select eigenstates of L_x, L_y, or H if that comes out handy to formulate the (apparently quantum mechanical) "macroscopic" laws of physics.

A slightly less trivial example is a one space - one time dimensional model. Imagine an infinite series of cells x , $-\infty < x < \infty$, each of which may or may not contain one particle going to the right and/or one particle going to the left. These states can be characterized as

$$|\psi\rangle \ = \ \prod_{x=-\infty}^{\infty} |r_x, \ \ell_x\rangle \quad ; \qquad r_x = 0 \text{ or } 1 \ ; \ \ell_x = 0 \text{ or } 1 \quad . \tag{33}$$

There is a clock ticking in definite time intervals. The law of evolution is simple: at each tick of the clock the left movers make one step to the left and the right movers make one step to the right.

To write the evolution operator in a quantum mechanical way we introduce annihilation operators:

$$\sigma_\ell^-(x)|r_x,\ 1\rangle = |r_x,\ 0\rangle \ ; \ \sigma_\ell^-(x)|r_x,\ 0\rangle = 0 \ ;$$
$$\sigma_r^-(x)|1,\ \ell_x\rangle = |0,\ \ell_x\rangle \ ; \ \sigma_r^-(x)|0,\ \ell_x\rangle = 0 \ . \tag{34}$$

These operators do not satisfy nice (anti-)commutation rules. Therefore we introduce the Jordan Wigner transformation

$$\psi_\ell(x_1) = (-1)^{\sum_{x<x_1}\ell_x} \cdot \sigma_\ell^-(x_1) \ ;$$
$$\psi_r(x_1) = (-1)^{\sum_x \ell_x + \sum_{x<x_1} r_x} \cdot \sigma_r^-(x_1) \ . \tag{35}$$

These satisfy

$$\{\psi_i(x),\psi_j(x')\} = 0 \quad ; \quad \{\psi_i(x),\psi_j^\dagger(x')\} = \delta_{ij}\delta_{xx'} \ . \tag{36}$$

They have been constructed carefully in order to have

$$\psi_\ell(x,t) = \psi_\ell(x+t,0) \quad ; \quad \psi_r(x,t) = \psi_r(x-t,0) \ . \tag{37}$$

Let us now introduce their Fourier transforms,

$$\psi_i(x) = (2\pi)^{-\frac{1}{2}} \int_{-\pi}^{\pi} dk\ e^{-ikx}\ \hat{\psi}_i(k) \ . \tag{38}$$

One finds

$$\hat{\psi}_i(k,t) = e^{ik\sigma_3 t}\ \hat{\psi}_i(k,0) \quad ; \quad \sigma_3\psi_{\ell \atop r} = \pm\,\psi_{\ell \atop r} \ . \tag{39}$$

Using the commutation rules we find that this evolution is generated by the evolution operator

$$U(t) = e^{-iHt} \quad ; \quad H = -\int_{-\pi}^{\pi} k\ \hat{\psi}^\dagger(-k)\sigma_3\hat{\psi}(k)\ dk \ . \tag{40}$$

Notice that in the continuum limit this Hamiltonian tends to

$$H \Rightarrow \int dx\ \psi^\dagger(x)\ i\sigma_3 \frac{\partial}{\partial x}\ \psi(x) \ , \tag{41}$$

which is Dirac's Hamiltonian for a one-dimensional massless fermion gas.

This gas clearly also has evolutionary determinism. The model is less trivial than the previous example because we can also consider its more usual description. Filling up the Dirac sea we can describe the lowest energy levels as left or right moving "neutrinos". The field $\psi(x)$ however creates a superposition of a neutrino and an antineutrino, so the "macroscopic" law shows quantum mechanical interference effects. On the other hand, there is no "Zitterbewegung" because the neutrinos naturally move with the speed of light; most of the usual quantum paradoxes are still absent in this model.

To construct less trivial models, in particular models that can show how the fundamental paradoxes of quantum mechanics such as the EPR paradox can emerge, seems to be difficult. *Formally*, every deterministic discrete model *corresponds to* a quantum system. We imagine that extremely non-trivial cellular automata[11] are equivalent with equally non-trivial quantum field theories, but we have not been able to show this in a detailed construction. A difficulty is that the quantum mechanical Hamiltonian H does not follow uniquely from the evolution operator U if we only know it at discrete time intervals δt . All Eigenvalues of H are only determined modulo $2\pi/\delta t$. This has two difficult consequences:

i) One discrete classical model may correspond to many apparently different quantum field theories, and

ii) The vacuum state, defined to be the state with lowest energy, depends on how many times we choose to add $2\pi/\delta t$ to the various eigenvalues.

7. Discussion

We believe that black holes shed a new light upon the question what to think about the quantum mechanical nature of our laws of physics. They suggest namely that at the Planck level everything is discrete. The set of discrete quantum mechanical models has a subset featuring evolutionary determinism. This is a form of determinism fundamentally different from the "hidden variables" theory, because none of the variables in our states are hidden from direct observation. There is determinism at the microscopic level which however needs not lead to determinism at a macroscopic level. The reason why one can say this is that the state we call "vacuum state" needs not be one of the "primordial" basis elements but may well be a superposition of these. If this is the case then quantum mechanical superposition will be a natural phenomenon in macroscopic physics, since all experiments we do are surrounded by vacuum.

The way to visualize what is actually happening is that our world could be something like a cellular automaton, such that the state we call "vacuum" is a "chaotic" solution. It should be called a miracle that such a system should show any regular effective laws of physics at all at macroscopic scales. At best one would expect certain forms of statistical regularities. This may be what we call quantum mechanics today.

Physicists have learned to work with these quantum mechanical rules in order to be able to make predictions concerning the future behaviour of a system. The truth is (could be) that these predictions cannot be better than statistical in nature because of the ubiquitous chaos in all but a few of our physical variables.

The macroscopic "effective" laws of quantum mechanics can only be formulated in terms of observables that are non diagonal in the primordial basis. This includes the laws according to which our accelerator devices and measuring apparatus, including our own eyes and our own brains, work. We can only say "there is an electron here, with spin up" when we assume these laws to describe absolute truths. In reality we may be talking about some minute statistical fluctuations around the chaotic oscillations of the vacuum. If we say "it is sent towards the moon", we really mean that we know from experience that the chaotic vacuum oscillations bombarding us from all directions in outer space will propagate this fluctuation towards the moon. The best way for us to describe these oscillations is the remarkable discovery that they are nearly perfectly described by the zero energy eigenstate of the Hamiltonian, which is a superposition of all basis elements.

We do not claim that all or even some of the usual paradoxes and other difficulties with the quantum mechanical nature of our world are resolved this way. One problem, among many others, is the question why our Universe should be so close to the vacuum state. Our main contribution to the discussion is this one proposal, namely that the microscopic laws of physics show some form of evolutionary determinism.

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