# BREAKDOWN OF UNITARITY IN THE DIMENSIONAL REDUCTION SCHEME

Ruud van DAMME and Gerard 't HOOFT
*Institute for Theoretical Physics, Princetonplein 5, P.O. Box 80006, 3508 TA Utrecht, The Netherlands*

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$\beta$-functions of any field theory using different regularization schemes should obey the physical rule that they can be transformed into each other by a finite transformation of the renormalized coupling constants in the theory. The dimensional reduction scheme does not obey this rule. The cause is that unacceptable counterterms had to be used where overlapping divergencies occur, so that unitarity is violated. Supersymmetry (or at least the $N = 2$ and $N = 4$ supersymmetric gauge theories and all supersymmetric theories not containing a vector field) turns out to be insensitive to this discrepancy, because the so-called "$\epsilon$-scalar" renormalizes the same way as the scalar, fermion and vector fields.

*1. Introduction.* Let us consider a field theory with coupling constants $\lambda_i$. We take $\lambda_i$ such that they have the dimensions of either a scalar field or a gauge coupling constant squared, so that the conventional perturbation expansion is in single powers of $\lambda_i$.

The conventional dimensional regularization scheme [1] respects unitarity and local gauge invariance but not supersymmetry. We denote this scheme by "minimal subtraction". [Strictly speaking a distinction should be made on whether one takes a factor $(2\pi)^{-4}$ or $(2\pi)^{-n}$ in $n$ dimensions, and whether or not Euler's constant is added in the one-loop expressions. The decision taken here is not of relevance to our further discussion.] The number of dimensions is $n = 4 - \epsilon$ and the bare parameters of the theory are

$$\lambda_i^B = \lambda_i^R + \epsilon^{-1}\Delta^{(1)}\lambda_i + \epsilon^{-2}\Delta^{(2)}\lambda_i + \ldots, \qquad (1.1)$$

where $\lambda_i^R$ and $\Delta^{(j)}\lambda_i$ are taken to be finite for all $\epsilon$. At increasing $j$ the $\Delta^{(j)}\lambda_i$ are of increasing order in $\lambda_k$. Since $\Delta^{(i)}\lambda$ are again functions of $\lambda^R$ we prefer to write this as

$$\lambda_i^B = \lambda_i^R + \epsilon^{-1}A_i(\lambda^R) + \epsilon^{-2}B_i(\lambda^R) + \ldots, \qquad (1.2)$$

with

$$A_i = A_i^{jk}\lambda_j^R\lambda_k^R + A_i^{jkl}\lambda_j^R\lambda_k^R\lambda_l^R + \ldots,$$

$$B_i = B_i^{jkl}\lambda_j^R\lambda_k^R\lambda_l^R + \ldots. \qquad (1.3)$$

Often we will write

$$A_{ij} = \partial A_i/\partial\lambda_j^R, \quad \text{etc.} \qquad (1.4)$$

By considering the behaviour of the system under space–time scaling two important theorems were derived in ref. [2]. Let $\mu$ be the mass scale of external fields and/or momenta. The $\beta$ functions are defined by

$$\frac{\mu \mathrm{d}}{\mathrm{d}\mu}\lambda_i^R = \beta_i(\lambda). \qquad (1.5)$$

*Theorem 1.* The $\beta$ functions can be expressed in terms of $\Delta^{(1)}\lambda_i$, as follows:

$$\beta_i(\lambda) = A_{i(1)} + 2A_{i(2)} + \ldots, \qquad (1.6)$$

where $A_{i(k)}$ are the $k$-loop contributions to $\Delta^{(j)}\lambda_i$ (the $k$th term in (1.3)).

*Theorem 2.* The higher order poles in $\epsilon$ can be expressed in the first order poles, e.g.

$$2B_{i(2)} = A_{k(1)}A_{ik(1)}. \qquad (1.7)$$

These theorems are derived for the case that $\lambda_i^R$ and $A_{i(k)}$ are essentially independent of $\epsilon$. If $\lambda_i^R$ is chosen to vary slightly with $\epsilon$, then $A_{i(k)}$ must vary accordingly since they are functions of $\lambda^R$.

Clearly, minimal subtraction gives just one choice

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for the finite parts of the infinite counter terms. Other subtraction schemes may correspond to redefining the renormalized coupling constants. Their $\beta$ functions in general differ from the previous ones starting at the two-loop level. Let us define

$$\bar{\lambda}_i^{\rm R} = \lambda_i^{\rm R} + \delta\lambda_i^{\rm R} , \qquad (1.8)$$

where $\delta\lambda_i^{\rm R}$ are of order $\boldsymbol{\lambda}^2$:

$$\delta\lambda_i^{\rm R} = \alpha_{ijk}\lambda_j\lambda_k + \dots . \qquad (1.9)$$

The coefficients $\alpha$ depend on the scheme used. If we define

$$\frac{\mu \mathrm{d}}{\mathrm{d}\mu}\bar{\lambda}_i^{\rm R} = \bar{\beta}_i(\bar{\lambda}^{\rm R}) , \qquad (1.10)$$

then we find, substituting (1.5),

*Theorem 3.*

$$\bar{\beta}_i(\lambda) - \beta_i(\lambda) = \left(\frac{\partial}{\partial\lambda_k}\,\delta\lambda_i^{\rm R}\right)\cdot\beta_k - \delta\lambda_k^{\rm R}\,\frac{\partial}{\partial\lambda_k}\,\beta_i$$
$$+ \text{higher orders} . \qquad (1.11)$$

In this paper we will be mainly concerned with the lowest non-trivial order, so, with theorem 1, we have:

$$\bar{\beta}_i(\lambda) - \beta_i(\lambda) = \frac{\partial\delta\lambda_i^{\rm R}}{\partial\lambda_k}\cdot A_k - \frac{\partial A_i}{\partial\lambda_k}\,\delta\lambda_k^{\rm R} . \qquad (1.12)$$

*2. Dimensional reduction.* Just like the minimal subtraction scheme, the dimensional reduction scheme prescribes a computation of amplitudes at $n = 4 - \epsilon$ dimensions. However the various polarizations of the particles must be as in 4 dimensions in order to preserve supersymmetry. The following prescription [3] now appears to preserve supersymmetric relations, as was verified by explicit calculations [4].

(1) Consider the spinor and Lorentz indices of an amplitude (with loops) to be computed. Take those to be *4-dimensional* and do all the algebra, until only external indices occur and inner products of momenta to be integrated over.

(2) Express these inner products in a Lorentz-invariant way, such that now extension towards $n = 4 - \epsilon$ dimensions is possible.

(3) Do the momentum integration(s) in $n$ dimensions. In general one then encounters poles in $\epsilon$.

(4) Subtract these poles minimally, as in the minimal subtraction scheme.

(5) Subtract the poles for the divergent subgraphs.

Careful consideration of what this means in practice reveals that, except for step (5), the difference with minimal subtraction can be expressed in terms of contributions due to the so-called $\epsilon$-scalars. The vector field index $\mu$ is allowed to point into the $\epsilon$-dimensional space orthogonal to the $n$-dimensional space of the integration parameters $p_\nu$. These couple via corresponding $\gamma$ matrices with the fermions.

Because of these properties the field components $A_\mu$ with $\mu$ in the $\epsilon$ direction are called "$\epsilon$-scalars". The Feynman rules for the $\epsilon$-scalars are just like those for ordinary scalars except that the multiplicity $E$ of these scalars is put equal to $\epsilon$.

The $\epsilon$-scalars only give some contribution to the amplitudes where the factor $\epsilon$ is balanced by a pole term of the form $1/\epsilon$. A finite piece is expected that depends polynomially on the external momenta and therefore can be seen as a redefinition of the form (1.8). In the unitarity relation $SS^\dagger = \mathrm{I}$ the effect of the $\epsilon$-scalars is expected to vanish proportionally with $\epsilon$ and therefore one expects unitarity to be respected.

Clearly, this argument is valid only if the interactions of the $\epsilon$-scalars with the other fields and each other are kept finite even at higher orders. This implies that one must carefully choose also those counterterms that balance the infinities of the interactions of these unphysical particles. We now claim that that criterion is not met in the conventional procedure, where the $\epsilon$-scalar interactions are given counterterms as if they were not scalars but components of vector particles $A_\mu$.

Such an "error" is not easy to cure by simply replacing the wrong counterterms by the correct ones, as we will argue further in section 5.

*3. Comparing the different schemes.* For the sake of clarity we now consider four "different" systems, all starting with the same gauge model lagrangian $\mathcal{L}(\boldsymbol{\lambda})$. The finite, renormalized coupling constants are all defined differently, so we call them $\boldsymbol{\lambda}^{\rm R}(k)$, $k = 1, \dots, 4$.

*System 1.* In this system, all $\lambda_i^{\rm R}(1)$ are defined using conventional minimal subtraction. The lagrangian is written as $\mathcal{L}(\boldsymbol{\lambda}(1))$.

*System 2*. In this system, the lagrangian is

$$\mathcal{L} = \mathcal{L}(\boldsymbol{\lambda}(2)) + \mathcal{L}_\phi(\boldsymbol{\lambda}(2), \boldsymbol{\lambda}_E) , \tag{3.1}$$

where $\mathcal{L}_\phi$ is the lagrangian of the $\epsilon$-scalars. $\phi$ is the $\epsilon$-scalar. However, its multiplicity is defined to be some number $E$, being unrelated to $\epsilon$. We then define $\boldsymbol{\lambda}(2)$ and $\boldsymbol{\lambda}_E$ by ordinary minimal subtraction. Since the $E$-scalars always have finite couplings, given by $\boldsymbol{\lambda}_E$, system 2 is equal to system 1 in the limit $E \to 0$. We choose $\boldsymbol{\lambda}_E = \boldsymbol{\lambda}(2) + O(\lambda^2)$.

*System 3*. The lagrangian in this system is just as (3.1). However now we choose

$$E = \epsilon , \tag{3.2}$$

and subsequently choose the renormalized couplings $\boldsymbol{\lambda}(3)$ to be $\epsilon$-independent. Subtraction of the poles in $\epsilon$ are done as in eq. (1.1). The result is

$$\boldsymbol{\lambda}(3) \neq \boldsymbol{\lambda}(2) , \tag{3.3}$$

because of the extra $\epsilon$ dependence from (3.2).

*System 4*. This is the "theory" obtained by the dimensional reduction prescription. It differs from system 3, as we will show, for the $\epsilon$-scalars are treated as if they were vectors. The difference will further be discussed in section 5.

The bare coupling constant in system 1 is

$$\lambda_i^{\mathrm{B}} = \lambda_i^{\mathrm{R}}(1) + \epsilon^{-1} A_i^{(1)}(\lambda^{\mathrm{R}}(1)) + \epsilon^{-2} B_i^{(1)}(\lambda^{\mathrm{R}}(1)) \,... , \tag{3.4}$$

and in system 2 up to the relevant orders

$$\lambda_i^{\mathrm{B}} = \lambda_i^{\mathrm{R}}(2) + \epsilon^{-1} [A_i^{(2)}(\lambda^{\mathrm{R}}(2)) + E A_i^{(2E)}(\lambda^{\mathrm{R}}(2))] + \epsilon^{-2} [B_i^{(2)}(\lambda^{\mathrm{R}}) + E B_i^{(2E)}(\lambda^{\mathrm{R}})] + ... . \tag{3.5}$$

We have $A_i^{(1)}(\lambda) = A_i^{(2)}(\lambda)$ and $B_i^{(1)}(\lambda) = B_i^{(2)}(\lambda)$ since system 2 approaches system 1 in the limit $E \to 0$.

In system 3 we rewrite this as

$$\lambda_i^{\mathrm{B}} = \lambda_i^{\mathrm{R}}(3) + \epsilon^{-1} A_i^{(3)}(\boldsymbol{\lambda}^{\mathrm{R}}(3)) + \epsilon^{-2} B_i^{(3)}(\boldsymbol{\lambda}^{\mathrm{R}}) + ... , \tag{3.6}$$

with

$$E = \epsilon . \tag{3.7}$$

Up to the relevant orders (one-loop correction terms to $\lambda^{\mathrm{B}}$ of order $\epsilon^{+1}$ or $E^{+1}$ are not relevant for our problem but in the general case one should be careful whether or not such terms may be ignored) this means

$$\lambda_i^{\mathrm{R}}(3) = \lambda_i^{\mathrm{R}}(2) + A_i^{(2E)} , \tag{3.8}$$

$$A_i^{(3)}(\lambda^{\mathrm{R}}(3)) = A_i^{(1)}(\lambda^{\mathrm{R}}(2)) + B_i^{(2E)}(\lambda^{\mathrm{R}}) , \tag{3.9}$$

$$B_i^{(3)} = B_i^{(1)} . \tag{3.10}$$

In the limit $E \to 0$ we have the relation

$$\lambda_i^{\mathrm{R}}(3) = \lambda_i^{\mathrm{R}}(1) + A_i^{(2E)}(\lambda) , \tag{3.11}$$

which is a relation of the form (1.8) considered in section 1. Eq. (3.11) should be the relation between the normalized coupling constants of the minimal subtraction and the dimensional reduction scheme.

Let us check whether the other relations obey the theorems of section 1. Substituting (3.8) into (3.9) gives

$$A_i^{(3)}(\lambda^{\mathrm{R}}) + A_{ik}^{(1)} A_k^{(2E)} = A_i^{(1)}(\lambda^{\mathrm{R}}) + B_i^{(2E)} . \tag{3.12}$$

Now $B_i^{(2E)}$ follows from our theorem 2 of section 1:

$$B_i^{(2E)} = \tfrac{1}{2}(A_k^{(2E)} A_{ik}^{(2)} + A_k^{(2)} A_{ik}^{(2E)}) . \tag{3.13}$$

Therefore

$$A_i^{(3)}(\lambda^{\mathrm{R}}) = A_i^{(1)}(\lambda^{\mathrm{R}}) + \tfrac{1}{2}(-A_k^{(2E)} A_{ik}^{(2)} + A_k^{(2)} A_{ik}^{(2E)}) . \tag{3.14}$$

The difference is a two-loop term. Therefore, with theorem 1,

$$\beta_i^{(3)}(\lambda^{\mathrm{R}}) = \beta_i^{(1)}(\lambda^{\mathrm{R}}) - A_k^{(2E)} A_{ik}^{(2)} + A_k^{(2)} A_{ik}^{(2E)} . \tag{3.15}$$

This indeed agrees with eq. (1.11) and (3.8). Theorem 3 applies with

$$\delta\lambda_i^{\mathrm{R}} = A_i^{2E} . \tag{3.16}$$

*4. The parameter shift*. Our problem now is the last term in eqs. (3.14) and (3.15). The two-loop $\beta$ functions were computed directly using dimensional reduction in a toy model (see section 6), but the result was not in accordance with these equations.

The index $i$ in (3.14), (3.15) refers to a physical coupling constant. The coefficients $A_{ik}^{(2)}$ vanish unless $k$ is also a physical coupling constant (a physical coupling constant refers to a coupling among physical particles, not $\epsilon$-scalars). This is because $A_{ik}^{(2)}$ is the $E$-independent part of the coefficients. But the index $k$ in the last term also refers to the new coupling constants

$\lambda_E$ in eq. (3.1). Indeed this is the dominant contribution in $A_{ik}^{(2E)}$. So we need the $A_k^{(2)}$ for unphysical $k$ values. The shift (3.16) is formally correct, but the term $A_i^{(2E)}$ depends critically on the unphysical coupling constants $\lambda_E$. This would have been relatively harmless if $\lambda_E$ had the same one-loop $\beta$ functions as $\lambda$, but this is not the case: $A_k^{(2)}$ changes when $k$ is replaced by this unphysical value.

$A_k^{(2)}$ refers to one-loop counterterms. The fact that the one-loop counterterms for the $\epsilon$-scalars and those for the vector fields $A_\mu$ do not coincide may not be obvious. After all, the $\epsilon$-scalars are just those components of $A_\mu$ which point in the $\epsilon$-direction. Are they not related by Lorentz-invariance?

In fig. 1 two diagrams are compared. One might suspect that they are equal by Lorentz-invariance but that is not the case. Consider the diagrams where the internal lines are vector or scalar lines. Then $A_\mu$ may couple to an internal momentum $p_\mu$ that is integrated over. But the $\epsilon$-scalar can only couple to $p_\epsilon$ which is put equal to zero. This is most easily seen when one realizes that steps (1)–(3) of the dimensional reduction scheme displayed in section 2 correspond to the replacement

$$\int d^4p \to \int d^4p\, \delta(p_\epsilon)\,, \tag{4.1}$$

rather than

$$\int d^4p \to \int d^{4-\epsilon}p\,, \tag{4.2}$$

as in the minimal subtraction scheme. In (4.2) the loop momenta are truly $4-\epsilon$-dimensional; in (4.1) they are four-dimensional, implying a four-dimensional index algebra, except that only $4-\epsilon$ components are integrated over, the $\epsilon$ remaining components are put equal to zero.

Indeed, for scalar and vector particles inside the

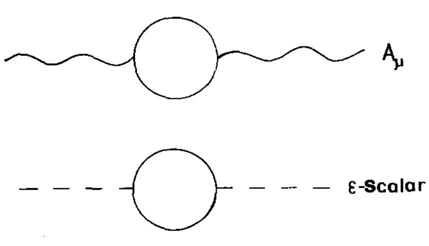


Fig. 1.

loop, the second diagram of fig. 1 vanishes, whereas the first does not.

The conclusion of this section is that if we wish to compare system 3 with system 1 supplemented by a shift of its parameters $\lambda$, described by eq. (3.16) where the index $i$ only refers to physical coupling constants, then eq. (3.14) does not imply (3.16). Instead, one would get (3.16) only if (3.14) were replaced by

$$A_i^{(3)}(\lambda^R) = A_i^{(1)}(\lambda^R) + \tfrac{1}{2}(-A_k^{(2E)}A_{ik}^{(2)} + A_k^{(2)}A_{ik}^{(2E)}) + \tfrac{1}{2}(A_{\underline{k}}^{(2)} - A_k^{(2)})A_{ik}^{(2E)}\,, \tag{4.3}$$

where $A_{\underline{k}}^{(2)}$ is the set of counterterms for the physical coupling constants only: the replacement of an index $k$ by $\underline{k}$ is defined by substituting all unphysical values (those corresponding to the coupling constants $\lambda_k^E$) by the associated physical values (corresponding to the coupling constants $\lambda_k$).

*5. System 4, a discrepancy*. We now ask which of the eqs. (3.14) and (4.3) is reproduced by the dimensional reduction recipe. The answer is neither, which we now explain.

Clearly the correction terms in (3.14) or (4.3) are due to the presence of a loop of $\epsilon$-scalars in a two-loop diagram. So the whole diagram has a factor $E$ in the numerator. Since we are looking at the $1/\epsilon$ pole for the case $E = \epsilon$, we really are only interested in the double pole contribution. In a Feynman diagram double poles come about as follows.

Consider a diagram with an overlapping divergent subgraph. The subgraph typically behaves as

$$k^{-\epsilon}/\epsilon - 1/\epsilon \tag{5.1}$$

times a canonical factor, where $k$ is a typical external momentum. Inserted in a two-loop integral this becomes one of the type

$$\int d^{4-\epsilon}k/(k^2+\mu^2)^2\,(k^{-\epsilon}/\epsilon - 1/\epsilon)\,, \tag{5.2}$$

which gives a double pole singularity

$$1/2\epsilon^2 - 1/\epsilon^2\,. \tag{5.3}$$

The first term of these stems from the two-loop integral, the second from the counterterm of the one-loop subgraph. The latter is twice as large (and of opposite sign) as the former. So the cancellation goes exactly half-way. This is the same of all two-loop double pole expressions.

We now observe that in the dimensional reduction scheme the choice of the one-loop counterterm is dictated by Lorentz-invariance: it has the form of a diagram insertion, as in the unperturbed classical theory. Now it has a factor two compared with (4.3). The reduction scheme therefore produces

$$A_i^{(4)}(\lambda^{\mathrm{R}}) = A_i^{(1)}(\lambda^{\mathrm{R}}) + \tfrac{1}{2}(-A_k^{(2E)}A_{ik}^{(2)} + A_k^{(2)}A_{ik}^{(2E)}) + (A_{\underline{k}}^{(2)} - A_k^{(2)})A_{ik}^{(2E)} \,. \tag{5.4}$$

The factor 2 compared to (4.3) follows from (5.3): the complete integral yields $-\frac{1}{2}A_{\underline{k}}^{(2)}$, the counterterm $+A_{\underline{k}}^{(2)}$, with a relative factor $-2$ as in (5.3).

The deviation from (4.3) is obvious. Eq. (3.6) apparently will not produce the correct shift.

This rather odd-looking factor 2 discrepancy when overlapping divergencies occur was indeed found to be the source of the mismatch in the numerical calculations of ref. [4]. The term $A_{\underline{k}}^{(2)} - A_k^{(2)}$ is the coefficient of the surviving infinity in the couplings of the $\epsilon$-scalar, causing unitarity to be violated (see section 2).

*6. A toy model.* To give an example to show that the factor of 2 is genuine we use the toy model which was introduced in ref. [4]. (Here the $\beta$-functions could not be transformed into each other.)

$$\begin{aligned}\mathcal{L} = &-(1/4g)(G_{\mu\nu}^a)^2 - \tfrac{1}{2}(\mathrm{D}_\mu\phi^a)^2 - \bar\psi^a \not{D}\psi^a \\ &- \mathrm{i}\bar{y}\epsilon^{abc}\bar\psi^a\phi^b\psi^c - \tfrac{1}{8}\lambda[\phi^2]^2 - \tfrac{1}{2}(\mathrm{D}_\mu\chi_\epsilon^a)^2 \\ &- \tfrac{1}{2}\rho_1(\chi_\epsilon^2)(\phi^2) + \tfrac{1}{2}\rho_2(\chi_\epsilon^a\phi^a)^2 - \bar\rho_3\epsilon^{abc}\bar\psi^a\chi_\epsilon^b\gamma_\epsilon\psi^c \\ &- \tfrac{1}{4}\rho_4[\chi_\epsilon^2]^2 + \tfrac{1}{4}\rho_5(\chi_\epsilon^a\chi_\tau^a)^2 \,.\end{aligned} \tag{6.1}$$

Here $\chi_\epsilon$ stands for the $\epsilon$-scalar, $\rho_i$ are the unphysical parameters (which become equal to $g$, the gauge coupling, at the end). Furthermore barred coupling constants have dimension $\hbar$, unbarred coupling constants have dimension $\hbar^2$, and $g = \bar{g}^2$; $y = \bar{y}^2$, etc. The $\beta$-functions are not hard to compute at the one-loop level:

$$\begin{aligned}&\beta(\lambda) = (4\pi)^{-2}(11\lambda^2 - 24g\lambda + 24g^2 + 16y\lambda - 32y^2)\,,\\ &\beta(y) = (4\pi)^{-2}(16y^2 - 24yg)\,,\\ &\beta(g) = (4\pi)^{-2}(-\tfrac{26}{3}g^2)\,.\end{aligned} \tag{6.2}$$

The two-loop differences for the $\beta$-functions for the two different schemes give [4]

$$\begin{aligned}&\beta_{\mathrm{red}}(\lambda) - \beta_{\mathrm{min}}(\lambda) = (4\pi)^{-4}(-224\lambda g^2 - \tfrac{256}{3}g^3)\,,\\ &\beta_{\mathrm{red}}(y) - \beta_{\mathrm{min}}(y) = (4\pi)^{-4}(16yg^2)\,,\\ &\beta_{\mathrm{red}}(g) - \beta_{\mathrm{min}}(g) = 0\,.\end{aligned} \tag{6.3}$$

The last one is a general property of all models. Using the notation of the preceding chapters we only need the following ingredients:

$$\begin{aligned}&A^{(2E)}(\lambda) = (4\pi)^{-2}(12\rho_1^2 + 4\rho_2^2 - 8\rho_1\rho_2)\,,\\ &A^{(2E)}(y) = 0\,,\\ &A^{(2E)}(g) = (4\pi)^{-2}(\tfrac{2}{3}g^2)\,,\\ &A^{(2)}(\rho_1) = (4\pi)^{-2}(4g\lambda - 8gy - 2g^2)\,.\end{aligned} \tag{6.4}$$

It turns out that the only $A^{(2)}$ coefficient needed is $A^{(2)}(\rho_1)$, in the limit that all unphysical parameters equal $g$. Using (5.4) this leads to

$$\begin{aligned}\beta_{\mathrm{red}}(y) - \beta_{\mathrm{min}}(y) &= (4\pi)^{-4}[0 - (-24\cdot\tfrac{2}{3}g^2y) + 2\cdot 0]\\ &= (4\pi)^{-4}(16yg^2)\,,\end{aligned}$$

$$\begin{aligned}\beta_{\mathrm{red}}(\lambda) - \beta_{\mathrm{min}}(\lambda) &= (4\pi)^{-4}\{[16g(4g\lambda - 8gy - 2g^2)\\ &\quad - 8g^2(22\lambda - 24g + 16y) - (-24\lambda + 48g)(\tfrac{2}{3}g^2)]\\ &\quad + 2[16g(-\tfrac{26}{3}g^2) - 16g(4g\lambda - 8gy - 2g^2)]\}\\ &= (4\pi)^{-4}[g^3(-\tfrac{256}{3}) + g^2\lambda(-224)]\,.\end{aligned} \tag{6.5}$$

Of course the gauge coupling renormalization is unaffected.

Note the necessity of the unitarity breaking factor of 2 in (5.4) to get eq. (6.5) in accordance with the numerical results in eq. (6.3).

Another model we checked was scalar QED. The results fitted. But as this theory has only one coupling beside the gauge coupling (the self interaction of the scalar field), it is possible to see that up to two loops, there exists a transformation among the coupling constants that would reproduce the $\beta$-function computed by dimensional reduction from the correctly computed $\beta$-function. Therefore, this model would be less suitable to demonstrate that dimensional reduction gives wrong results in non-supersymmetric theories.

*7. Discussions.* It has been shown that for ordinary gauge theories the regularization scheme by dimensional reduction, proposed by Siegel [3] breaks unitarity already at the two-loop level. The only way out is that the $\epsilon$-scalar renormalizes exactly the same way as the vector field in the theory.

In a previous version of this paper we claimed that also in supersymmetric models unitarity is broken by the same mechanism discussed here. We now understand that a calculational error gave $A_{\bar{k}}^{(2)} \neq A_k^{(2)}$ without violating the other equations in this chapter, so that an apparently consistent "anomaly" appeared. In agreement with findings of other authors [5] we now find $A_{\bar{k}}^{(2)} = A_k^{(2)}$ for all supersymmetric models that we checked, such as $N = 2$ and $N = 4$ super Yang–Mills theories.

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