BLACK HOLES AS CLUES TO THE PROBLEM OF QUANTIZING GRAVITY

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Abstract Black holes are considered as extreme forms of matter that probably play a key role in the interplay of elementary particles at extreme energies. Our present understanding of their quantum properties is reviewed, and it is explained in what way the inevitable formation of black holes at high energy densities poses restrictions on any theory of quantum gravity. It appears that even space and time themselves have to be quantized.

1. INTRODUCTION

Gravitation is the weakest and the strongest force known in physics. When considered as a force between two single electrons, it is nearly 43 orders of magnitude weaker than the electro-magnetic force. But gravity works collectively: when an amount of matter, somewhat more than the mass of our Sun, is allowed to cool and compress under its own weight, then sooner or later a complete collapse has to take place. No other force can then overcome the gravitational one. The process of collapse can be computed using well-established laws of physics, and few physicists doubt on the final outcome: a black hole[1].

Black holes form an extremely interesting theoretical laboratory. A quite surprising result was found by S. Hawking[2] in 1975. Applying quantum field theory to the region surrounding a black hole he discovered that a black hole must radiate matter of all sorts, behaving like an ideal radiating black body with a temperature T given by*

$$k_B T = 1/8\pi M \ , \tag{1.1}$$

where k_B is Boltzmann's constant and M the black hole mass, in units where $\hbar=c=\kappa=1$ (κ is Newton's gravitational constant.)

This result implies that black holes must loose energy, becoming ever lighter. Is there a limiting value for their mass? If so this can only be when either quantum field theory or general relativity, or both, cease to be valid. The fundamental principles of these theories are unobjectionable as long as the gravitational force can be quantized perturbatively, and this is the case when all distance scales used are considerably larger than

$$\sqrt{\frac{\kappa\hbar}{c^3}} = 1.6 \cdot 10^{-33} \text{ cm} \ , \tag{1.2}$$

called the *Planck length*. At this length scale we have energies of the order

$$\sqrt{\frac{\hbar c^5}{\kappa}} = 1.22 \cdot 10^{22} \text{ MeV} \ , \tag{1.3}$$

the Planck energy. One could expect that the lightest black hole has a mass of this order, but at this energy black holes may well become indistinguishable from elementary particles. After all, elementary particles with such a mass would be surrounded by the same gravitational field, so they also can be seen as "collapsed objects", and it may well be that all particles at such energies are unstable, emitting radiation much like black holes do while they decay.

Plausible as this picture may seem (and indeed we will adopt it), there are problems with it. It suggests namely that black holes should form a discrete spectrum just like

*Although this is the most widely accepted value, its derivation is not free from assumptions. An alternative theory yielding a different value can be formulated[3].

elementary particles, to be characterized by quantum numbers. Also larger black holes could then, at least in priciple, be characterized by discrete quantum numbers, and this is not what one gets applying Hawking's techniques.

Either something is wrong with our conventional picture of what matter is like at the Planck scale, or something is missing in the theory leading to Hawking radiation, or both.

To find out what the possible resolution of this dilemma is we will study Hawking radiation. Assuming that quantum mechanics in some sense will continue to be exactly valid at the Planck scale, we will concentrate on the assumptions that went into the derivation of the Hawking radiation, and ask ourselves how one can improve the procedure so that no contradiction arises. As we will see some very fundamental aspects of the quantum theory are at stake; we claim that a "deterministic" theory for the evolution of variables at the Planck length scale, i.e. a theory without the interference effects typical for ordinary quantum mechanics, is among the possibilities. What we call quantum mechanics today is then nothing but a statistical treatment of the highly chaotic solutions of these microscopic equations.

2. THE BLACK HOLE

A black hole is a solution of the classical Einstein equations for the space-time metric, and the equation of state for matter, such that space-time can be seen to be divided in two regions, "region *I*" and "region *III*" (called that way for no particularly good reason), which are defined as follows:

i) All points in *I* can be connected to the outside world by a timelike geodesic directed into the future, and

ii) None of the points in *III* can be connected that way to the outside world.

We have to explain what is meant by "outside world". This concept only means something if we have a non-compact asymptotically flat region surrounding the "black hole" solution at $t \to \infty$. In contrast, region *III* can at all times be enclosed inside a closed surface with an area never exceeding a certain number Σ . For a black hole with an infinite lifetime the question whether or not a signal will be able to escape from region *III* will be unambiguous.

The (lightlike) surface dividing the two regions is called the future event horizon, or horizon for short[†]. Within the horizon, in region *III*, all light cones are directed inwards, so that no information can escape. In the center we usually have some sort of *singularity*. The singularity is essentially a divergence, where presumably our known laws of physics break down. Remarkably however, the singularity will not play a significant role in our discussion. This is because it is well shielded from the observable world; we don't see it, so its precise nature is irrelevant. In these lectures we'll mainly focus on the horizon.

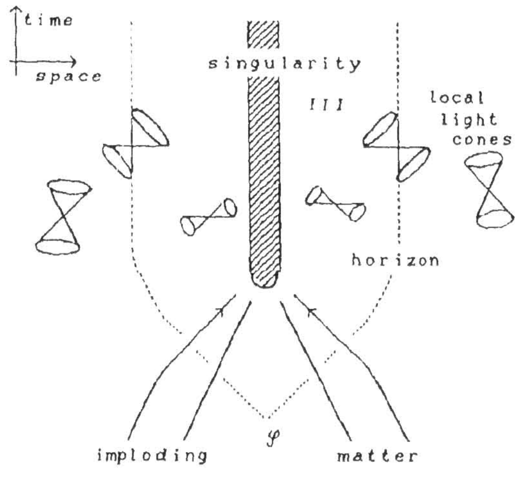

Figure 1

The black hole

As can be seen from Figure 1, a black hole is expected to rapidly settle for a stable, stationary configuration. In a particular coordinate frame the space-time metric can then be written as

[†]The well-informed reader, knowing that we expect black holes to have only a finite lifetime, might wonder how this affects our definition of the horizon. Indeed, it makes the horizon somewhat "fuzzy". However, the uncertainty this induces to the location of the horizon will be far less than other fluctuations that we will discuss.

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2 \quad ;$$
$$d\Omega^2 = d\vartheta^2 + \sin^2\vartheta \, d\varphi^2 \quad . \tag{2.1}$$

where M is the mass of the black hole in natural units. This we refer to as the Schwarzschild solution.♪

In this representation there may seem to be a singularity at $r = 2M$, but this is a coordinate artefact. There is a better set of coordinates, called "Kruskal coordinates", in which we see that (2.1) in fact describes part of a larger space-time[4]. Replace r and t by x and y , defined by

$$xy = (1 - r/2M)\, e^{r/2M} \quad ;$$
$$x/y = -\, e^{t/2M} \quad . \tag{2.2}$$

Then in terms of x, y, ϑ and φ we have

$$ds^2 = -2A(x,y)\,dx\,dy + r^2 d\Omega^2 \quad , \tag{2.3}$$

with $d\Omega^2$ as in eq. (2.1) and

$$A(x,y) = \frac{16M^3}{r}\, e^{-r/2M} \quad , \tag{2.4}$$

which is not singular at $r = 2M$. Notice now that for every r and t we have in general *two* solutions for x and y , differing from each other by a sign. This inplies that our universe is smoothly connected to another, equal, universe, and we obtain regions *II* and *IV* , see Figure 2.

♪For simplicity we limit ourselves to the non-rotating chargeless Schwarzschild solution. The more general Kerr-Newman solution (having charge and angular momentum) has a different structure at $r=0$ and a somewhat different horizon structure[4]. Since we are mainly concerned about the horizon our arguments can be extended to this more general case.

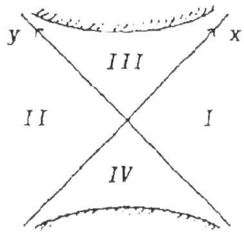

Figure 2

The Kruskal coordinates

If the Kruskal coordinates are used to describe the black hole of Fig. 1 then the regions *II* and *IV* in there are unphysical. This is because the infalling matter that created the black hole was ignored. Region *IV* would be at the infinite past only if the black hole is "eternal": it existed already before the universe was there[ℱ]. Region *II* can only be reached by travelling faster than the speed of light. In realistic black holes (the ones formed by a collapsing object) regions *II* and *IV* are replaced by flat space. On the *y*-axis ($x=0$) one then has the imploding material which gives a right-hand side to Einstein's equations such that this hybrid configuration (regions *I* and *III* glued onto flat space) is a solution.

3. FIELD THEORY IN RINDLER SPACE

Let us now concentrate on the region $r \cong 2M$. Write

$$x = t_1+z_1 \ ; \quad y = t_1-z_1 \ ; \quad r/2M - 1 = \zeta^2 \ ; \quad t/4M = \tau \quad . \tag{3.1}$$

We look at a small angular region so that ϑ and φ can be replaced by two transverse coordinates $\tilde{x}_1 = (x_1 \ , y_1)$. Then close to the horizon ($r=2M$) we have the coordinate mapping

$$z_1 = \zeta \cosh \tau \quad ; \quad t_1 = \zeta \sinh \tau \quad . \tag{3.2}$$

We call $\tilde{x}_1$, z_1 and t_1 the locally regular coordinates. ζ and τ are the Rindler coordinates[5]. We see that they are directly related to the Schwarzschild coordinates r and t. In terms of the regular coordinates space-time is not at all

[ℱ] More about "eternal" black holes in sect. 7.

singular near the horizon.

Note now that a shift in the τ parameter (or equivalently in the Schwarzschild time t) corresponds to a Lorentz boost in x_1 -space. All equations of physics are invariant under such boosts and therefore invariant under shifts in Rindler (or Schwarzschild) time.

A field theory - any local field theory - can be defined by giving the Hamiltonian as the integral of a Hamilton density over 3-space. In ordinary flat coordinates this is

$$H_{reg} = \int \mathcal{H}(\mathbf{x})\ d^3\mathbf{x} \quad . \tag{3.3}$$

The Hamiltonian is the operator that generates shifts in time t_1 . Now at $\tau=0$ an infinitesimal shift $\delta\tau$ in τ corresponds to a shift

$$\delta t_1 = \zeta\delta\tau \quad , \tag{3.4}$$

in t_1 , as one can easily see from (3.2). Thus, the generator for an infinitesimal translation in τ is

$$H_{Rin} = \int \mathcal{H}(\mathbf{x}) z d^3\mathbf{x} = H_I - H_{II} \quad , \tag{3.5}$$

with

$$H_I = \int_{z>0} H(\mathbf{x}) z d^3\mathbf{x} \quad ; \quad H_{II} = \int_{z<0} H(\mathbf{x}) |z| d^3\mathbf{x} \quad . \tag{3.6}$$

We recognise (3.5) as the generator of a Lorentz transformation.

In most field theories it is not difficult to check that

$$[H_I, H_{II}] = 0 \quad , \tag{3.7}$$

because the integrands in (3.6) vanish at $z=0$.♫ Indeed, one expects (3.7) because no signal can be transmitted between regions I and II .

To describe physical processes at the visible part of a black hole one needs only H_I . Would particles described by H_I alone "bounce" against the horizon? To see what happens it is instructive to consider a field theory first in its Lagrange form. The simplest Lagrangian is

$$\mathcal{L} = -\tfrac{1}{2}(\partial_z\varphi)^2 + \tfrac{1}{2}(\partial_t\varphi)^2 - \tfrac{1}{2}(\tilde{\partial}_x\varphi)^2 - \tfrac{1}{2}m^2\varphi^2 \quad , \qquad (3.8)$$

and if we write $\zeta = e^{\sigma}$ then this becomes in the Rindler coordinates

$$\mathcal{L}_{kin} = \mathcal{L}\,\frac{dz\,dt}{d\sigma\,d\tau} = -\tfrac{1}{2}(\partial_\sigma\varphi)^2 + \tfrac{1}{2}(\partial_\tau\varphi)^2 + e^{2\sigma}\left(-\tfrac{1}{2}(\tilde{\partial}_x\varphi)^2 - \tfrac{1}{2}m^2\varphi^2\right). \qquad (3.9)$$

We see that at a given transverse momentum $\tilde{k}$ all wavelike solutions will satisfy

$$\partial_\sigma^2\varphi = \partial_\tau^2\varphi \quad , \quad \text{at} \quad \sigma \to -\infty \ , \qquad (3.10)$$

$$\varphi = \varphi_{out}(\tau-\sigma) + \varphi_{in}(\tau+\sigma) \quad . \qquad (3.11)$$

This means that the boundary $\sigma = -\infty$ is open! An infinite world of particles, on their way in or out, exists in the region $\zeta \cong 0$.

One would conclude from this that black holes are in a fundamental way different from other soliton like configurations in gauge theories such as magnetic monopoles: even if we would enclose them in a finite space surrounding the black hole, particles near the black hole will form a continuous spectrum because they occupy a non-compact space.

It should be clear however that particles near $\sigma \to -\infty$ probe the infinitely small distance regime. Therefore, since

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♫The only contribution to (3.7) could come from the origin. Now the commutator $[\mathcal{H}(x),\mathcal{H}(y)]$ contains at most one derivative of a Dirac deltafunction, whereas there are two factors z in eq. (3.7). This is why, after partial integration, one gets zero.

quantum gravity is non-renormalizable, there will be values for σ beyond which the above analysis fails. Can gravitational forces turn the spectrum into a discrete one? The author believes this to be the case. But first we will explain what Hawking's result implies for this boundary.

4. THE HAWKING EFFECT

In this chapter we briefly review the derivation of the Hawking effect[2]. Everything can be understood as a feature of the central region in Kruskal space, and indeed all we need is the Rindler coordinate transformation (3.2). Only non-rotating, non-interacting particles are considered; other cases are more complicated but yield the same results.

A scalar field in the regular coordinates r_1 can be written as

$$\Phi(\mathbf{r}_1,t_1) = \int\frac{d^3\mathbf{k}}{\sqrt{2k_0(\mathbf{k})V}}\left(a(\mathbf{k})e^{i\mathbf{k}\mathbf{r}_1-ik_0t_1} + a^\dagger(\mathbf{k})e^{-i\mathbf{k}\mathbf{r}_1+ik_0t_1}\right),$$

$$\dot\Phi(\mathbf{r}_1,t_1) = \int\frac{d^3\mathbf{k}\ (ik_0)}{\sqrt{2k_0(\mathbf{k})V}}\left(-a(\mathbf{k})e^{i\mathbf{k}\mathbf{r}_1-ik_0t_1} + a^\dagger(\mathbf{k})e^{-i\mathbf{k}\mathbf{r}_1+ik_0t_1}\right). \qquad (4.1)$$

Here, $V = (2\pi)^3$, and we have

$$[a(\mathbf{k}),a^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k}-\mathbf{k}')\ , \qquad (4.2)$$

and

$$[\dot\Phi(\mathbf{r}),\Phi(\mathbf{r}')] = -i\delta^3(\mathbf{r}-\mathbf{r}')\ , \qquad (4.3)$$

etc. First we make the transition to lightcone coordinates,

$$u = \tfrac{1}{2}(t_1-z_1)\ ,\quad v = \tfrac{1}{2}(t_1+z_1)\ ,$$

$$k_+ = k_0+k_3\ ,\quad k_- = k_0-k_3\ . \qquad (4.4)$$

In Rindler time these evolve as

$$v \to ve^{\tau} \ , \quad u \to ue^{-\tau} \ . \tag{4.5}$$

And we define new annihilation operators a_ℓ as

$$a(\mathbf{k})\sqrt{k_0} = a_\ell(\tilde{k}, k_+)\sqrt{k_+} \ , \tag{4.6}$$

which, because

$$\partial k_+/\partial k_3 \ \Big|_{\tilde{k}} = k_+/k_0 \ , \tag{4.7}$$

are now normalized by

$$[a_\ell(\tilde{k}, k_+), a_\ell^\dagger(\tilde{k}', k_+'))] \quad = \quad \delta^2(\tilde{k}-\tilde{k}') \ \delta(k_+-k_+') \ . \tag{4.8}$$

So we can write

$$\Phi(r_1, t_1) = A(\tilde{r}, u, v) + A^\dagger(\tilde{r}, u, v) \quad ; \tag{4.9}$$

$$A(\tilde{r}, u, v) = \int_{k_+>0} \frac{d\tilde{k}\ dk_+}{\sqrt{2Vk_+}} \ a_\ell(\tilde{k}, k_+) \ e^{i\tilde{k}\tilde{r} \ -ik_+u \ -ik_-v} \ . \tag{4.10}$$

Now, in Rindler time, $\tilde{k}$ is constant and

$$k_+ \to k_+e^{\tau} \quad , \quad k_- \to k_-e^{-\tau} \ . \tag{4.11}$$

If we Fourier transform the field Φ with respect to τ then we may expect to get annihilation and creation operators corresponding to definite amounts of energy for the Rindler observer. Therefore we now choose to Fourier transform a_ℓ with respect to $\log k_+$:

$$a_\ell(\tilde{k},k_+)\sqrt{k_+} = (2\pi)^{-1/2}\int_{-\infty}^{\infty} d\omega a_R(\tilde{k},\omega)\ e^{-i\omega\ \ln\ (k_+/\mu)}\ , \tag{4.12}$$

where

$$\mu^2 = \tilde{k}^2+m^2 = k_+k_-\ , \tag{4.13}$$

and the new annihilation operators a_R are normalized as

$$[a_R(\tilde{k},\omega), a_R^\dagger(\tilde{k}',\omega')] = \delta^2(\tilde{k}-\tilde{k}')\delta(\omega-\omega')\ . \tag{4.14}$$

The inverse of eq. (4.12) is

$$a_R(\tilde{k},\omega) = \int_0^\infty dk_+(2\pi k_+)^{-1/2}a_\ell(\tilde{k},k_+)\ e^{i\omega\ \ln\ (k_+/\mu)}\ . \tag{4.15}$$

The Hamiltonian density for this theory in the regular coordinates was

$$\mathcal{H}_{reg}(\mathbf{r}_1,t_1) = \tfrac{1}{2}\dot{\Phi}^2 + \tfrac{1}{2}(\partial\Phi)^2 + \tfrac{1}{2}m^2\Phi^2\ , \tag{4.16}$$

A straightforward calculation now yields

$$H_{Rin} = \int z\mathcal{H}_{reg}(\mathbf{r}_1,0)d^3\mathbf{r}_1 = \int d^2\tilde{k}\int_{-\infty}^{\infty} d\omega\ \omega\ a_R^\dagger(\tilde{k},\omega)a_R(\tilde{k},\omega)\ , \tag{4.17}$$

so indeed in Rindler space a_R behaves as an annihilation operator corresponding to a Rindler energy ω .

Nevertheless, a_R is *not* the annihilation operator we want to work with, because we want to split H into H_I and H_{II} . How do the fields Φ in regions I and II depend on a_R ?

Let us write the field A in eq. (4.9) in terms of a_R :

$$A(\mathbf{r}_1,t_1) = \int_{-\infty}^{\infty} d\omega \int \frac{d^2\tilde{k}}{\sqrt{4\pi V}}\ K(-\omega,\mu u,\mu v)\ e^{i\tilde{k}\tilde{r}}\ a_R(\tilde{k},\omega)\ . \tag{4.18}$$

Here, u and v are the coordinates (4.4) and K is an integration kernel, which turns out to be

$$K(\omega,\alpha,\beta) = \int_0^\infty \frac{dx}{x}\, x^{i\omega} e^{-ix\alpha - i\beta/x} . \tag{4.19}$$

It is useful to know some properties of K. When $\alpha < 0$ and $\beta > 0$ then the integrand in (4.19) converges rapidly if $\mathrm{Im}(x) \geq 0$. Therefore we may rotate the integration contour by

$$x \to x\, e^{i\varphi}, \quad 0 \leq \varphi \leq \pi . \tag{4.20}$$

Taking $\varphi=\pi$ gives us the identity

$$K(\omega,\alpha,\beta) = \int_0^\infty \frac{dx}{x}\, x^{i\omega}\, e^{-\pi\omega}\, e^{ix\alpha + i\beta/x} = e^{-\pi\omega}\, K^*(-\omega,\alpha,\beta) ,$$

$$\text{when} \quad \alpha < 0 \quad \text{and} \quad \beta > 0 . \tag{4.21}$$

When $\alpha > 0$ and $\beta < 0$ we have, using a similar shift,

$$K(\omega,\alpha,\beta) = e^{\pi\omega}\, K^*(-\omega,\alpha,\beta) , \quad \text{if} \quad \alpha > 0 \quad \text{and} \quad \beta < 0 . \tag{4.22}$$

We now split the integral (4.18) into two integrals for positive ω. If $(\mathbf{r}_1, t_1)$ is in the region I we have $u < 0$ and $v > 0$. Therefore,

$$A(\mathbf{r}_1, t_1) = \int_0^\infty d\omega \int \frac{d^2\tilde{k}}{\sqrt{4\pi V}}\, e^{i\tilde{k}\tilde{r}} \Big(K(-\omega,\mu u,\mu v)\; a_R(\tilde{k},\omega) \; +$$

$$+ \; e^{-\pi\omega}\, K^*(-\omega,\mu u,\mu v)\; a_R(\tilde{k},-\omega) \Big) , \tag{4.23}$$

and

$$A^{\dagger}(\mathbf{r}_1, t_1) = \int_0^{\infty} d\omega \int \frac{d^2\tilde{k}}{\sqrt{4\pi V}} e^{i\tilde{k}\tilde{r}} \Big(K^*(-\omega, \mu u, \mu v)\; a_R^{\dagger}(-\tilde{k}, \omega) +$$

$$+ e^{-\pi\omega} K(-\omega, \mu u, \mu v)\; a_R^{\dagger}(-\tilde{k}, -\omega)\Big) . \tag{4.24}$$

Substituting these expressions now in eq. (4.9) for Φ , we get, if $(\mathbf{r}_1, t_1) \in I$,

$$\Phi(\mathbf{r}_1, t_1) = \int_0^{\infty} d\omega \int \frac{d^2\tilde{k}}{\sqrt{4\pi V}} e^{i\tilde{k}\tilde{r}} \Big[K(-\omega, \mu u, \mu v) \big(a_R(\tilde{k}, \omega) + e^{-\pi\omega} a_R^{\dagger}(-\tilde{k}, -\omega)\big) +$$

$$K^*(-\omega, \mu u, \mu v) \big(a_R^{\dagger}(-\tilde{k}, \omega) + e^{-\pi\omega} a_R(\tilde{k}, -\omega)\big)\Big] . \tag{4.25}$$

This suggests that we should consider the operator

$$a_I(\tilde{k}, \omega)\sqrt{1 - e^{-2\pi\omega}} = a_R(\tilde{k}, \omega) + e^{-\pi\omega} a_R^{\dagger}(-\tilde{k}, -\omega) . \tag{4.26}$$

Clearly, if $(\mathbf{r}_1, t_1) \in I$ then $\Phi(\mathbf{r}_1, t_1)$ only depends on a_I and its Hermitean conjungate. Similarly, in region II we only need a_{II} defined by

$$a_{II}(\tilde{k}, \omega)\sqrt{1 - e^{-2\pi\omega}} = a_R(\tilde{k}, -\omega) + e^{-\pi\omega} a_R^{\dagger}(-\tilde{k}, \omega) . \tag{4.27}$$

The normalization factors are needed to get the commutation rules

$$[a_I(\tilde{k}, \omega), a_I^{\dagger}(\tilde{k}', \omega')] = \delta^2(\tilde{k} - \tilde{k}')\delta(\omega - \omega') ; \tag{4.28}$$

similarly for $[a_{II}, a_{II}^{\dagger}]$, and furthermore we have:

$$[a_I, a_I] = [a_{II}, a_{II}] = [a_I, a_{II}] = [a_I, a_{II}^{\dagger}] = 0 . \tag{4.29}$$

Transformations that mix creation and annihilation operators such as (4.26) and (4.27) are called Bogolyubov transformations.

Since $\mathcal{H}$ in region I depends only on Φ in region I, which only depends on a_I, it is not surprising to see that H_I only depends on a_I and H_{II} on a_{II}:

$$H_I = \int_0^\infty \mathrm{d}\omega \int \mathrm{d}^2\tilde{k}\ \omega\ a_I{}^\dagger a_I + C\ ;$$
$$H_{II} = \int_0^\infty \mathrm{d}\omega \int \mathrm{d}^2\tilde{k}\ \omega\ a_{II}{}^\dagger a_{II} + C\ , \qquad (4.30)$$

where C is a common, irrelevant constant coming from the ordering process. It cancels in H_R, eq. (3.5).

In the previous chapter we stated that an observer in region I only works with H_I. Therefore he has only a_I to his disposal, not a_{II}. Now suppose that in terms of the *regular coordinates* r_1, t_1 an observer would see a vacuum in the region around the origin. This would be a state $|\Omega\rangle$ in Hilbert space defined by

$$a|\Omega\rangle = a_\ell|\Omega\rangle = a_R|\Omega\rangle = 0\ , \quad \text{for all } \tilde{k},\ \omega\ . \qquad (4.31)$$

Now let us choose a new basis for the Hilbert space which for each $\tilde{k}$, ω have given values for the quanta

$$n_I = a_I{}^\dagger a_I\ , \quad n_{II} = a_{II}{}^\dagger a_{II}\ . \qquad (4.32)$$

Let us call these basis elements $|n_I, n_{II}\rangle$. Clearly,

$$\prod_{\tilde{k},\omega} |0,0\rangle \neq |\Omega\rangle\ . \qquad (4.33)$$

To express $|\Omega\rangle$ in our Rindler basis we use, from eqs (4.26) and (4.27),

$$a_I(\tilde{k},\omega)|\Omega\rangle - e^{-\pi\omega}a_{II}^{\dagger}(-\tilde{k},\omega)|\Omega\rangle = 0 \; ;$$
$$a_{II}(\tilde{k},\omega)|\Omega\rangle - e^{-\pi\omega}a_{I}^{\dagger}(-\tilde{k},\omega)|\Omega\rangle = 0 \; , \tag{4.34}$$

so that, when acting on $|\Omega\rangle$, we have

$$a_I^{\dagger}a_I = e^{-\pi\omega}a_I^{\dagger}a_{II}^{\dagger} = e^{-\pi\omega}a_{II}^{\dagger}a_I^{\dagger} = a_{II}^{\dagger}a_{II} \; . \tag{4.35}$$

Consequently, $|\Omega\rangle$ only consists of states with $n_I = n_{II}$:

$$|\Omega\rangle = \sum_n f_n|n,n\rangle \; . \tag{4.36}$$

We find f_n from eq. (4.34):

$$\sum_n f_n\sqrt{n}|n-1,n\rangle = e^{-\pi\omega}\sum_n f_n\sqrt{n+1}|n,n+1\rangle \; ; \tag{4.37}$$

$$f_{n+1} = e^{-\pi\omega}f_n \quad , \tag{4.38}$$

Conclusion:

$$|\Omega\rangle = \prod_{\tilde{k},\omega}\sqrt{1-e^{-2\pi\omega}}\sum_{n=0}^{\infty}e^{-n\pi\omega}|n,n\rangle_{\pm\tilde{k},\omega} \quad . \tag{4.39}$$

where the square root is a normalization factor. Notice that eq. (4.36) implies

$$H_R|\Omega\rangle = 0 \; , \tag{4.40}$$

which just means that $|\Omega\rangle$ is Lorentz-invariant.

If $\mathbb{H}_I$ is the Hilbert space in region I and $\mathbb{H}_{II}$ in region II then we see that $|\Omega\rangle$ is a superposition of states in $\mathbb{H}_I \otimes \mathbb{H}_{II}$. The expectation value of any operator

$\mathcal{O}$ in $\mathbb{H}_I$ is

$$\langle\mathcal{O}\rangle = \sum_{\{n_I\},\{n_{II}\},\{n_I'\}} \langle\Omega|\{n_I,n_{II}\}\rangle\, \mathcal{O}_{\{n_I\},\{n_I'\}}\, \langle\{n_I',n_{II}\}|\Omega\rangle =$$

$$= \Big[\prod_{\tilde{k},\omega}\sum_{n_I(\tilde{k},\omega)}\Big]\mathcal{O}_{\{n_I(\tilde{k},\omega)\},\{n_I(\tilde{k},\omega)\}}\prod_{\tilde{k},\omega}(1-e^{-2\pi\omega})e^{-2\pi\omega n_I(\tilde{k},\omega)} . \tag{4.41}$$

This one could write as

$$\langle\mathcal{O}\rangle = \mathrm{Tr}\,\rho\,\mathcal{O} \quad , \tag{4.42}$$

where ρ is a density matrix:

$$\rho = \prod_{\tilde{k},\omega}\rho_{n,n'} \quad ; \qquad \rho_{n,n'} = (1-e^{-2\pi\omega})e^{-2\pi\omega n}\,\delta_{n,n'} \quad . \tag{4.43}$$

This is the density matrix for a thermal system at temperature given by

$$k_B T = 1/2\pi \; . \tag{4.44}$$

Thus, assuming that the most energetic particles near the point $\zeta = 0$ are absent when measured by a freely falling observer, the Rindler observer who uses τ as his time parameter sees particles emerging from his horizon corresponding to radiation with this temperature. Noticing the relation (3.1) between τ and Schwarzschild time t, one derives that in proper units the temperature with which the black hole radiates is given by

$$k_B T = 1/8\pi M \; , \tag{4.45}$$

the Hawking temperature. In our way of dealing with Rindler space one sees that this result corresponds to information concerning the boundary condition at $\sigma=-\infty$, or $r=2M$.

It is important to try to understand this boundary condition. The physics generated by the Lagrangian (3.9) is not altered. So one still seems to have a continuous spectrum. All we get is the statement that particles come back from the horizon at random, with weight factors equal to the Boltzmann factor corresponding to the Hawking temperature. This random behavior is different from what one would get if there were some given scattering matrix reflecting particles against the horizon. How different?

5. THE BLACK-HOLE SPECTRUM AND THERMODYNAMICS

The existence of an infinite spectrum of plane waves to and from the horizon, described by eq. (3.10), suggests that the spectrum of black holes is continuous. Yet, a more compelling reason exists to suggest that the spectrum should be discrete. A continuous spectrum would imply that an infinite number of mutually orthogonal states exists that can be stored within the volume defined by the horizon, with energies E between M and $M+dM$, where M is the black hole mass. A small region of space would allow an infinite amount of information to be stored in there, at the cost of virtually no energy. This, we think, sounds unlikely.

Suppose for a moment that the density of black holes at mass M (and charge Q and angular momentum L), is given by some finite number $\rho(M)$ (or $\rho(M,Q,L)$). Let us now compare the absorption process of an object with energy ΔE by a black hole with mass M , with the Hawking emission process for the same object by a hole with mass $M+\Delta E$:

$$(M) + (\Delta E) \Leftrightarrow (M+\Delta E) . \qquad (5.1)$$

The absorption process has a cross section σ given approximately by

$$\sigma \simeq \pi R^2 , \qquad (5.2)$$

where R is the black hole radius, $R=2M$. The emission probability W is approximately

$$W \propto \pi R^2\ \rho_1(\Delta E)\ e^{-\beta_H \Delta E}\ , \tag{5.3}$$

where β_H is the inverse Hawking temperature, $\beta_H = 8\pi M$ (putting the gravitational constant κ and Boltzmann's constant k_B equal to one). $\rho_1(\Delta E)$ is the density of states for the objects radiated out with energy ΔE .

If there were a quantum mechanical theory for the black hole, the same quantities could be expressed in terms of transition amplitudes, using the "golden rule":

$$\sigma = \left|\langle M{+}\Delta E|\mathcal{T}|M,\Delta E\rangle\right|^2 \rho(M{+}\Delta E)\ ; \tag{5.4}$$

where $\mathcal{T}$ is the transition matrix, and

$$W = \left|\langle M,\Delta E|\mathcal{T}|M{+}\Delta E\rangle\right|^2 \rho(M)\rho_1(\Delta E)\ . \tag{5.5}$$

By virtue of PCT invariance, the matrix elements in (5.4) and (5.5) would be each other's conjungates, and therefore we find

$$\rho(M{+}\Delta E)/\rho(M) \ = \ \rho_1(\Delta E)\ \sigma/W \ = \ e^{\beta_H \Delta E}\ ; \tag{5.6}$$

this should hold for a range of values for ΔE as long as $\Delta E \ll M$, and with $\beta_H = 8\pi M$ we find

$$\rho(M) = C\ e^{4\pi M^2}\ , \tag{5.7}$$

where the universal constant C is the only unknown. Note that the exponent is one quarter of the area of the horizon, this is what one also finds in more general cases.

C could be finite, in which case we indeed have a finite spectrum density, or C is infinite, but in this case equations (5.4) and (5.5) could be considered for the lightest state in the continuum. Either this would be an absolutely stable thing ($W{=}0$), in which case *virtual pair creation* of these things would give infinite contributions to graviton self-energy diagrams, or the cross section σ would tend to infinity. Neither of these latter options sound physically very attractive, which is why we suspect C to be finite. We must realize however that very large numbers are indigenous in quantum gravity. It could be that C is of the order of 10^{40} or 10^{-40}.

It is unlikely that C is exactly constant. There may well be subdominant corrections either in the exponent, or in the form of powers in front of the exponent.

There is a different way to derive the same expression (5.7) for $\rho(M)$. This is by applying *thermodynamics* to the black hole[6]. The free energy F is defined by

$$e^{-\beta F} = \int_M^{M+dM} \rho(M)dM \; e^{-\beta_H M} \;\simeq\; \rho(M)\; e^{-8\pi M^2} , \qquad (5.8)$$

where the integral could be suppressed because the leading term is given by the exponent anyhow. We inserted Hawking's value for the temperature and used natural units for the mass (the unit is the Planck mass, 21.7 μg).

The expectation value for the energy M is

$$\langle M \rangle = \frac{\partial}{\partial\beta}(\beta F) = \beta/8\pi \;; \qquad (5.9)$$

$$\beta F = \beta^2/16\pi = 4\pi M^2 , \qquad (5.10)$$

and we conclude that

$$\rho = 4\pi M^2 . \qquad (5.11)$$

The philosophy used is that black holes behave just like little containers with some gas or liquid inside. If these communicate thermally with the outside world, one can deduce information concerning the total number of states from these reactions. The only thing slightly unusual about the thermal black holes is their negative specific heat:

$$c = \frac{\partial}{\partial T}\langle M \rangle = -\beta^2 \frac{\partial}{\partial\beta}(\beta/8\pi) = -\beta^2/8\pi = -8\pi M^2 . \qquad (5.12)$$

This implies that black holes cannot be completely in equilibrium with their environment. Martinez and York[7] speculate that this may have deep implications for the probabilistic interpretation of quantum mechanical expressions concerning black holes.

If indeed $\rho(M)$ is finite then this has also consequences for the symmetry aspects of a quantum gravity theory: we cannot have any absolute global conservation laws such as baryon number conservation![8] The reason for this is that one could imagine dropping an unlimited amount of baryons into the hole, waiting each time for an equal amount of energy to reemerge in the form of Hawking radiation. Our derivation implies that equal numbers of baryons and anti-baryons should come out, on the average. So one could increase the hole's baryon number indefinitely. If ρ is finite there is however only a finite number of black hole states so that baryon number sooner or later becomes an ill-specified quantum number. Any preference of the hole to emit baryons rather than antibaryons would contradict our derivations of sect. 4.

Now this remark implies that applying quantum field theory alone to the black hole horizon will not yield us further details concerning these spectral states. We could have global symmetries in this quantum field theory, and they will be standing in the way! something fundamental is missing.

Our preliminary standpoint will be the following: our present understanding of the laws of physics is imprecise[9]: applying them gives us only statistical information about the black hole states. Since this statistical answer was a consequence of our applying quantum mechanics this may well mean that the usual statistical interpretation of quantum mechanics may be a consequence of an incomplete description. Imagine just a slight "uncertainty" in the Hamiltonian,

$$H \rightarrow H + \delta H \ , \tag{5.13}$$

where δH has some *probabilistic* distribution. In that case the solutions to the Schrödinger equation,

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = -iH\psi \ , \tag{5.14}$$

will have a "thermal" contribution, to be described by a density matrix, just as what one gets as Hawking radiation, eq. (4.43).

6. THE HORIZON'S DISPLACEMENTS

Our problem can be traced back to the plane wave solutions (3.11) of eqs. (3.10): there will be mutually independent states of ingoing and outgoing particles. In reality we expect that the outgoing radiation will be *determined* by what comes in *via* an S-matrix. The first thing to suspect is that it was wrong to neglect gravitational interactions between ingoing and outgoing radiation. What will these interactions be like? Fortunately, they can be computed rather precisely.

Consider again Rindler space, described by the coordinates $\tilde{x}$, ζ and τ. We ignore matter that fell in long ago. At time $\tau=\tau_1$ a light particle is dropped in. See Figure 3.

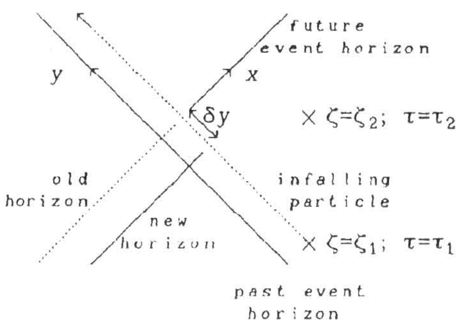

Figure 3

The horizon displacement

Now a translation in τ corresponds to a Lorentz boost in the regular coordinates. Therefore, at Rindler time $\tau=\tau_2$, $\tau_2 \gg \tau_1$, the infallen particle appears to be boosted to an enormous energy. Sooner or later the gravitational fields due to this energy become important.

To describe its gravitational field the particle may be considered massless, moving with light velocity. The field is easily found[10] by first taking a light particle at rest and then boosting it together with its Schwarzschild field. The result is a field that takes the form of a "shock wave" (called "inpulsive wave" in the literature,) not unlike the sonic boom of an airplane moving with the speed of sound. Both in front of the particle and behind it we have flat space-time, and at the space-time points x with

$$p_1 \cdot (x - x_1(t)) = 0 \quad , \tag{6.1}$$

where p_1 is the particle's 4-momentum and $x_1(t)$ its trajectory, the two flat spaces are seamed together, shifted by an amount δx that depends on the transverse distance $\tilde{x}$:

$$\delta x = 2\kappa p_1 \log(1/\tilde{x}^2) \quad , \tag{6.2}$$

which, because of its nonlinear $\tilde{x}$ dependence produces a delta-distributed curvature.

Now (6.1) and (6.2) hold for a particle in a flat space-time background. How do they generalise when the particle moves in(to) a black hole? This turns out to be straightforward[11], if at finite τ_1 the particle had little energy. The situation at late τ_2 is sketched in Figure 3, which must now be seen as representing a black hole in its Kruskal coordinates. Two regular Schwarzschild solutions, described in their Kruskal coordinates, are glued together at the line $x=0$, shifted by an amount δy in the y direction, where δy now depends on the angles $\Omega = (\vartheta, \varphi)$. The functional dependence of δy on ϑ and φ is determined by the equation

$$(1-\Delta_\Omega)\ \delta y(\Omega) = 4\pi\kappa p_{in}\ \delta^2(\Omega, \Omega_1) \quad , \tag{6.3}$$

where Δ_Ω is the angular Laplacian and Ω_1 is the set of angles at which the particle dropped in. The solution of eq. (6.3) can be written as

$$\delta y(\Omega) = f(\Omega, \Omega_1)\ p_{in} \quad , \tag{6.4}$$

where $f(\Omega, \Omega_1)$ is a Green function approaching the logarithm of eq. (6.2) for large black holes. It is uniquely determined by (6.3) because $1-\Delta_\Omega$ has a unique inverse.

Eq. (6.3) is found by writing the space-time metric obtained from gluing the two Schwarzschild solutions together as an *Ansatz* and then imposing Einstein's equations. The Green function f can be given in an integral form but this is not very illuminating. One finds that $f(\Omega, \Omega_1) > 0$ for all Ω, Ω_1 , and f diverges logarithmically when $\Omega \to \Omega'$.

A consequence of these observations is that if we drop a particle into the black hole, the position of the horizon at times τ *before* the particle fell in, changes, as drawn in Fig. 3. This change is barely perceptible at times $\tau \lesssim \tau_1$, but at times $\tau_2 \gg \tau_1$ the change is large. An observer there sees Hawking radiation that now originated in a *different* region of space-time than it would have if the particle had not been thrown in.

Is this consequence of any importance? What does it matter if the Hawking radiation originated somewhere else? It will certainly look the same as before.

We will argue in the next section that only in a quantum theory that is detailed enough to give us a scattering matrix instead of a density matrix, shifting horizons will be relevant. Indeed, important constraints on the scattering matrix will be found.

7. THE SCATTERING MATRIX. BLACK HOLE - WHITE HOLE DUALITY

Suppose a scattering matrix exists. This means that if we have completely specified the state $\{p_1, p_2, \dots\}$ of all particles that ever went into the black hole, the outgoing matter should be in one well-specified state $|\psi\rangle_{out}$. A basis for $|\psi\rangle_{out}$ is the set of states where all outgoing particles have well-specified momenta at a certain time $\tau=\tau_0$. Now at $\tau=\tau_1>\tau_0$ we drop a light particle into the hole, with momentum p_{in} (in regular coordinates) at solid angle Ω_1. The change this induces for the outgoing wave is now determined primarily by the horizon shift (if other, non-gravitational interactions may be ignored). Thus, the new state will now be

$$|\psi\rangle_{out} \to e^{-i\int P_{out}(\Omega)\delta y(\Omega)d^3\Omega}\,|\psi\rangle_{out}\,, \qquad (7.1)$$

where $P_{out}(\Omega)$ is the operator that generates a shift in the configurations at the solid angle Ω. It is, of course, also the total momentum emerging at solid angle Ω. In here, we can now substitute eq. (6.4) for δy.

Now this means that if we know $|\psi\rangle_{out}$ at one stage, then $|\psi\rangle_{out}$ can, in principle, be determined after allowing any number of particles to fall in. If we may ignore non-gravitational interactions, we see that all states $|\psi\rangle_{out}$ ever to be produced by the black hole are generated by the operator $P_{out}(\Omega)$ from one single state. Therefore, $|\psi\rangle_{out}$ must be generated by the *algebra* of these operators. Similarly, the *ingoing* particles are only distinguished by the *total* momentum $p_1(\Omega_1)$ at each solid angle Ω_1 .

We find the following important result[12]. For the incoming wave functions one may diagonalize the operators $P_{in}(\Omega_2) = p_{in}(\Omega_1)$, and for the outgoing states we diagonalize $P_{out}(\Omega)$. Eq. (7.1) then tells us how a change in p_{in} affects the outgoing state. Up to a proportionality factor, the complete transformation rule for ingoing states into outgoing states should be generated by this equation. This rule is not difficult to find:

$$\langle\{p_{out}(\Omega)\}|\{p_{in}(\Omega')\}\rangle = N\ e^{-i\int d^2\Omega\ d^2\Omega'\ p_{out}(\Omega) f(\Omega,\Omega') p_{in}(\Omega')} , \tag{7.2}$$

where N is a normalization factor.

Eq. (7.2) is the S-matrix we wanted. *If* an S-matrix exists, and if we may ignore other than the longitudinal gravitational forces, it must be this one.

The problem with it is the algebra that generated the basis in which it is defined. We have the following commutation rules

$$[p_{in}(\Omega), p_{in}(\Omega')] = 0 \quad ; \quad [p_{in}(\Omega),\ x_{in}(\Omega')] = -i\delta^2(\Omega,\Omega') \quad ; \tag{7.3}$$

$$[p_{out}(\Omega), p_{out}(\Omega')] = 0\ ;\ [p_{out}(\Omega),\ y_{out}(\Omega')] = -i\delta^2(\Omega,\Omega') \quad , \tag{7.4}$$

and we have the relation

$$y_{out}(\Omega) = \int d^2\Omega_1\ f(\Omega,\Omega_1)\ p_{in}(\Omega_1) \quad . \tag{7.5}$$

This implies

$$[x_{in}(\Omega), y_{out}(\Omega')] = if(\Omega,\Omega') \quad , \tag{7.6}$$

so that we have also

$$x_{in}(\Omega) = -\int d^2\Omega' \; f(\Omega,\Omega') \; p_{out}(\Omega') \quad . \qquad (7.7)$$

The operators x_{in} and y_{out} could be interpreted as coordinates of "particles", but then there should be *exactly one particle at every value of* Ω. This is where this Hilbert space differs from ordinary Fock space, where we may have any number of particles (mostly this will be zero) at every mode. This is also why it will be difficult to interpret our *S*-matrix directly as a matrix describing scattering of familiar particles.

More appropriately, x_{in} and y_{out} are to be seen as the position operators for the *past* and the *future* horizon. We then recognise an important consequence of our description of Hilbert space: past and future horizons cannot both be localized accurately; these obey an uncertainty relation. Indeed, they are each other's dual conjungates, much in the same way as coordinates are dual to momenta.

We claim that this also does away with a question considered often in the literature: does the time-reversed black hole (called "white hole") exist? Does the "eternal black hole" (one with a past white hole that was already there before the universe began) exist? The questions are not appropriate if our *S*-matrix exists: white hole coordinates are ill-specified once a black hole was localised.

The difficult but perhaps exciting picture that emerges is that the exact shape of either the past or the future horizon may completely determine the particle content of the black hole's vicinity. It should be possible to refine this picture by incorporating gravitational forces in the transverse direction, and non-gravitational forces. It is not hard to take the electromagnetic force into account. Here, the electric charge density operator $\rho(\Omega)$ and the gauge phase operator $\phi(\Omega)$ are each other's duals. Electromagnetic shock waves (Čerenkov radiation) surrounding charged massless particles are very similar to gravitational shock waves[13].

As will be explained in the next section, we expect a cut-off in Ω space. Probably the transverse forces are responsible for that. Surely, if coordinates in the transverse direction would be specified with accuracies better than the Planck length (implying $\delta\Omega \lesssim M_{Pl}/M$, where M is the black hole mass), then momenta in the transverse direction exceed the Planck energy so that also shifts in the transverse direction will arise that are bigger than $\delta\Omega$. The simplest cut-off would be a lattice in Ω space, but reality will probably be more complicated.

8. DISCRETE PHYSICS

Imagine a volume $V = L^3$ in three-dimensional space. Suppose that one could define a quantum (field) theory of all phenomena (particles, black holes) inside this volume, in terms of a Hilbert space. We ask: how many dimensions (basis elements) $\mathcal{D}$ does this Hilbert space have as V grows? In ordinary field theories the answer is strictly infinite. Even a single particle can occupy infinitely many states, so surely a second-quantized theory will have an infinite-dimensional Hilbert space. Only if we would introduce a rigid lattice cut-off, and accept only fundamental fermions in our theory, the total dimensionality would be finite. It would grow like

$$\mathcal{D} \cong O(e^{\Lambda^3 L^3}) \quad , \tag{8.1}$$

where Λ is the inverse lattice size.

But this changes if the gravitational force is taken into account. In that case we cannot allow the total energy to exceed a certain value depending on V :

$$E_{tot} \leq L/2\kappa \quad , \tag{8.2}$$

because otherwise a black hole would form with a horizon that stretches beyond the edges of V .

Most quantum field theories have only

$$\mathcal{D} \cong O(e^{C\kappa^{-3/4} L^{3/2}}) \tag{8.3}$$

basis elements with energy less than that (C is some constant). If we allow black holes we expect more states. Suppose there are N black holes with labels $i=1, \dots ,N$. Each black hole has a density of states of the order of

$$\mathcal{D}_i \cong e^{4\pi M_i^2} \quad , \tag{8.4}$$

(leaving again $M_{Pl}^2 = \kappa = 1$), and the total system has

$$\mathcal{D} = \prod_i \mathcal{D}_i = e^{4\pi\sum_i M_i^2} \leq e^{4\pi(\sum_i M_i)^2} \quad ; \qquad (8.5)$$

whereas the total energy is

$$E_{\rm tot} = \sum_i M_i \leq \tfrac{1}{2}L \quad . \qquad (8.6)$$

Therefore, allowing black holes we find

$$\mathcal{D} \lesssim e^{L^2\pi} \quad , \qquad (8.7)$$

which, surprisingly, grows exponentially with the surface area L^2 rather than the volume L^3 .

We claim that in any "complete" quantum theory of the world the total dimensionality of Hilbert space inside a volume L^3 must be finite and approximately given by eq. (8.7). Theories where this dimensionality does not grow exponentially with the volume but with the surface area are not easy to construct. It could mean that there is some sort of additional constraint to be imposed on all states in the "physical part" of Hilbert space, whose solutions would have the dimensionality given by (8.7).

There is an interesting class of finite-dimensional theories. These are the so-called cellular automata. A cellular automaton is a system containing a definite number of completely discrete and limited variables located at "cells". The contents of each cell are continuously updated according to a given arithmetic rule depending on the previous values and the values inside neighboring cells. They are ideal for computer simulations.

A cellular automaton may either be completely deterministic or quantum mechanical. In a quantum mechanical cellular automaton the updating is prescribed by some unitary operator U defined in Hilbert space. At each tick of the clock we have

$$|\psi(t+1)\rangle = U\,|\psi(t)\rangle \quad . \qquad (8.8)$$

This equation is the direct discrete analogon of the Schrödinger equation. The model is *deterministic* if a basis exists in terms of which the operator U happens to be also a permutation operator:

$$U\,|n_1,\ n_2,\ \dots\rangle \ = \ |\mathbf{P}(n_1,\ n_2,\ \dots)\rangle \ . \tag{8.9}$$

If ever we find the operator U describing the real world it might not be easy to establish whether or not it can be seen to be a deterministic one, because the search for the corresponding basis may be difficult. It is very tempting to *suspect* that U should be deterministic[14]. It then remains to be seen how we can understand that nevertheless features typical for quantum mechanics govern the macroscopic world. Experiences with computer simulations with cellular automata show that they often behave *chaotically*. This means that there is no simple way to describe the macroscopic world in terms of deterministic laws in spite of local determinism. Perhaps the *only* way to describe the macroscopic world is *via* the Schrödinger equation (8.8) or in other words: perhaps our world is a cellular automaton but it allows only a statistical description. The mathematics of this statistics may happen to be that of conventional quantum mechanics[15].

There are numerous difficulties with such a picture but we think it cannot be ruled out. It would be a neat way to resolve the philosphical difficulties usually caused by quantum mechanics. In particular we will have to face the Einstein Podolski Rosen paradox[16] and the question how one can understand the violation of Bell's inequalities[17] by quantum mechanical interference effects. We stress however that strictly speaking there is no logical contradiction here at all because the *vacuum state* $|\varnothing\rangle$ will have to be a superposition of all basis elements, in particular if we use the deterministic evolution operator P of eq. (8.9). Since all our experimental set-ups are always surrounded by a vacuum we should not be surprised to find "interference effects".

In attempting to improve our calculation of the S matrix elements (7.2) we found that the horizon can probably not be kept topologically as simple as an S_2 sphere. Indeed at the point $\mathscr{S}$ in Fig. 1 the topology must be very involved. One might speculate that the topological structures at $\mathscr{S}$ are denumerable and this could be a starting point for a completely discrete theory for the black hole horizon, and, with that, for space-time itself.

9. CONFORMAL FIELD THEORY FOR THE BLACK HOLE HORIZON

The commutation relations among the operators that are defined on the two-dimensional surface spanned by the *intersection* of past and future horizon, as in Section 7, can be seen to generate a veritable two-dimensional conformal quantum field theory very closely related[18] to string theory. While these notes were written much of this theory was still under construction. We here give a very brief preliminary account.

The theory describes states in Hilbert space by giving the coordinates in space *and* time for the horizon intersection surface $x^{\mu}(\sigma_1,\sigma_2)$. We can identify, following the equations of Section 7,

$$\tilde{x} = (x_1, x_2) \propto (\vartheta, \varphi) \quad ; \qquad x^+ = x_{in} \quad ; \qquad x^- = x_{out} \quad . \tag{9.1}$$

For simplicity we limit ourselves to large black holes so that the horizon is approximately flat and $\tilde{x}$ can be treated as two flat coordinates. However, let us rewrite the equations of Section 7 in such a way that they become covariant under general two dimensional coordinate transformations. Equation (7.6) can be seen to be in the special gauge

$$\sigma_1 = x_1 \quad , \quad \sigma_2 = x_2 \quad . \tag{9.2}$$

We have

$$[x^+(\tilde{\sigma}),\ x^-(\tilde{\sigma}')] = if(\tilde{\sigma},\tilde{\sigma}') \ , \tag{9.3}$$

with

$$\partial^2_{\sigma}\ f(\tilde{\sigma},\tilde{\sigma}') = -4\pi\kappa\delta^2(\tilde{\sigma}-\tilde{\sigma}') \quad . \tag{9.4}$$

Now this is probably only an approximation that holds when

$$x^{\mu}(\sigma_1,\sigma_2) \simeq (\tilde{x}_1,\tilde{x}_2,0,0) \quad , \tag{9.5}$$

so (9.3) can be rewritten as

$$\begin{aligned} [x^{\mu}(\tilde{\sigma}),\ x^{\nu}(\tilde{\sigma}')] &= i f^{\mu\nu}(\tilde{\sigma},\tilde{\sigma}') \ ; \\ f^{\mu\nu}(\tilde{\sigma},\tilde{\sigma}') &\cong \frac{2\kappa}{\sqrt{g}}\, \varepsilon^{\mu\nu\rho\lambda}\varepsilon^{ab}\partial_a x^{\rho}\partial_b x^{\lambda}\ \log|\tilde{\sigma}-\tilde{\sigma}'| \quad , \end{aligned} \tag{9.6}$$

where g is the determinant of the induced metric in σ space:

$$g_{ab} = \partial_a x^{\mu}\partial_b x^{\mu} \quad . \tag{9.7}$$

The function $f^{\mu\nu}$ can be seen to be defined more precisely by:

$$\sqrt{g}\ g^{ab}D_a D_b f^{\mu\nu}(\tilde{\sigma},\tilde{\sigma}') = -8\pi\kappa\delta^2(\tilde{\sigma}-\tilde{\sigma}')\varepsilon^{\mu\nu\rho\lambda}\left(\frac{\partial x^{\rho}}{\partial\sigma_1}\right)\left(\frac{\partial x^{\lambda}}{\partial\sigma_2}\right) \ , \tag{9.8}$$

where D_a is the covariant derivative. The first part of eq. (9.6) and eqs. (9.7), (9.8) can be seen to be entirely covariant under general coordinate transformations in σ space and *special* coordinate transformations in x space[♯].

When we make the transition to a conformal gauge,

$$g_{ab} = \lambda\delta_{ab} \quad , \tag{9.9}$$

[♯] Indeed we described effects close to the horizon in a space-time metric $g_{\mu\nu}$ that is locally normalized to stay close to $\delta_{\mu\nu}$. The transition to general $g_{\mu\nu}(x)$ may well be as in string theories.

we find an algebra that should form the basis of a fundamental conformal theory for black holes. The resulting theory resembles very much the string theories, but seems to be much more complicated because the x^{μ} do not commute (when x is large the commutators become negligible, so we are dealing with a small-distance effect here. A further difference with string theories is that the string constant comes out to be imaginary[12,19], but note that this makes little difference when the zero-slope limit is considered. Thus, the apparent success of string theories in explaining some qualitative phenomenological aspects of the standard model will perhaps also be shared by this theory.

The author thanks Duke University, Durham NC, for its hospitality.

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