"DIAGRAMMAR" AND DIMENSIONAL RENORMALIZATION

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1. INTRODUCTION

There are several possible approaches to quantum field theory. One may start with a classical system of fields, interacting through non-linear equations of motion which are subsequently "quantized". Alternatively, one could take the physically observed particles as a starting point; then define a Hilbert space, local operator fields, and an interaction Hamiltonian. More ambitious, perhaps, is the functional integral approach, which has the advantage of being obviously Lorentz covariant.

All these approaches have one unpleasant and one pleasant feature in common. The unpleasant one is that in deriving the S-matrix for the theory, one encounters infinities of different types. In order to get rid of these one has to invoke a rather *ad hoc* "renormalization procedure", thus changing and undermining the theory halfway. The pleasant feature, on the other hand, is that one always ends up with a simple calculus for the S-matrix: the Feynman rules. Few physicists object nowadays to the idea that these Feynman diagrams contain more truth than the underlying formalism, and it seems only rational to abandon the aforementioned principles and use the diagrammatic rules as a starting point.

It is this diagrammatic approach to quantum field theory which we wish to advertise. The short-circuiting has several advantages. Besides the fact that it implies a considerable simplification, in particular in the case of gauge theories, one can simply superimpose

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the renormalization prescriptions on the Feynman rules. As for unitarity and causality, the situation has now been reversed: we shall have to investigate under which conditions these Feynman rules describe a unitary and causal theory. Within such a scheme many more or less doubtful or complicated theorems from the other approaches can be proven completely rigorously.

An extensive and pedagogical introduction to the diagrammatic approach is given in a CERN report, called "DIAGRAMMAR", to which we shall refer frequently[1]. These notes may be considered as an introduction to DIAGRAMMAR, but we shall treat the dimensional regularization and renormalization procedure in somewhat more detail. We shall repeat the arguments in DIAGRAMMAR in so far as we need them for our purposes.

The Feynman rules will always be defined by a Lagrangian $\mathcal{L}$,

$$\mathcal{L}(x) = \psi_i^*(x)\, V_{ij}\, \psi_j(x) + \tfrac{1}{2}\varphi_i(x)\, W_{ij}\, \varphi_j(x) + \mathcal{L}_I(\psi^*, \psi, \varphi) + J_\psi^*\psi + \psi^* J_\psi + \varphi J_\varphi \,, \tag{1.1}$$

where ψ_i and ϕ_i are complex and real fields respectively, V_{ij} and W_{ij} are space-time operators that must have an inverse. $\mathcal{L}_I$ is a possibly non-local, polynomial function of the fields ψ^*, ψ, and ϕ. The last three terms are source terms, which may be of convenience in defining off-mass-shell Green's functions.

The propagators will then always be defined to be $(-V - i\varepsilon)^{-1}_{ij}$ and $(-W - i\varepsilon)^{-1}_{ij}$, and the vertices will be generated by the interaction terms in $\mathcal{L}_I$. We assume that the reader is more or less familiar with the use of Feynman rules so that we do not need to repeat here the prescriptions for obtaining an S-matrix, starting with the rules for vertices and propagators.

The infinities of quantum field theory can now be removed in different ways. In simple cases one may replace the propagators V^{-1} and W^{-1} by others that converge more rapidly at $k \to \infty$, for instance,

$$-V^{-1} \;\to\; -V^{-1} - (-V + M)^{-1} \,, \tag{1.2}$$

where M is a large number. In gauge theories, however, a replacement such as (1.2) would break gauge invariance, and it becomes extremely difficult to see how gauge invariance can be restored in the limit $M \to \infty$.

For gauge theories one can get rid of the infinities in a better way: the dimensional regularization method. In fact, dimensional regularization can only be formulated for diagrams, not for canonical systems or functional integrals. It is here that the diagrammatic approach is particularly useful.

One of the most important items in DIAGRAMMAR is what we call the canonical transformation. This is the most general field transformation in the Lagrangian (1.1), of the form

$$\begin{aligned} \psi(x) &\to \psi(x) + F(\psi, \psi^*, \varphi, x), \\ \psi^*(x) &\to \psi^*(x) + F^*(\psi, \psi^*, \varphi, x), \\ \varphi(x) &\to \varphi(x) + G(\psi, \psi^*, \varphi, x), \end{aligned} \tag{1.3}$$

where F and G are in general non-local functions of the fields ψ and ϕ. If we decompose the fields into real fields ψ_i, then (1.3) can be written as

$$\psi_i(x) \to \psi_i(x) + F_i(\psi, x). \tag{1.4}$$

The new Lagrangian will again have bilinear and interaction parts. The transformation is also performed in the source terms of (1.1).

Now it is not difficult to verify that the tree diagrams (i.e. diagrams with no closed loops) from the old and the new Lagrangian are exactly equivalent. The reason is that the substitutions in the vertex and source parts of (1.1) exactly cancel the substitutions in the propagator (= bilinear) parts. But the same statement for diagrams with loops does not hold without an additional change in the Lagrangian. As is shown in DIAGRAMMAR, the necessary addition can conveniently be described in terms of a new ghost particle having a complex field $\alpha_i(x)$. Its interaction is defined by the Lagrangian

$$\mathcal{L}^{\text{ghost}} = \alpha_i^*(x)\alpha_i(x) + \int d_4x' \, \alpha_i^*(x) \frac{\delta F_i(\psi, x)}{\delta \psi_j(x')} \alpha_j(x'), \tag{1.5}$$

and by the additional prescription that there should be one more minus sign for each loop of α's (there are only closed loops of α's). So the substitution (1.4) together with the addition of (1.5) leads to a set of rules which is equivalent to the old one, in the sense that for instance vacuum-vacuum transition amplitudes in the presence of sources $\mathcal{J}$ remain the same.

This result justifies the functional integral method, in which field transformations are always accompanied by "Jacobian factors". The addition (1.5) corresponds exactly to the Jacobian for the most general type of transformation. Thus we close the gap between the diagrammatic approach and the functional integral approach for finite theories: a functional integral "proof" of Ward-Takahashi identities[2] can always be substituted by a diagrammatic one or vice versa. But again, if infinities are to be studied, we think the direct diagrammatic description is more accurate.

2. THE CUTTING RULES

The dimensional renormalization procedure is, like most of the others, formulated via an induction process. Counter terms are added order by order in the perturbation expansion, increasing the possible number of closed loops by one at each step*). It is essential to have a profound insight into the nature of the possible infinities arising at each order. To this end we shall derive a relation satisfied by any set of Feynman diagrams. It is called causality relation in DIAGRAMMAR.

The starting point is the decomposition of the propagator into positive and negative energy parts:

$$\Delta_{ij}(x) = \vartheta(x_0)\,\Delta^{+}_{ij}(x) + \vartheta(-x_0)\,\Delta^{-}_{ij}(x), \tag{2.1}$$

$$\Delta^{\pm}_{ij}(x) = \frac{1}{(2\pi)^3}\int d_4k\; e^{ikx}\,\vartheta(\pm k_0)\,\rho(k^2), \tag{2.2}$$

with $x = x_i - x_j$. Here we used the notation

$$\Delta_{ij}(x) = \frac{1}{(2\pi)^4 i}\,(-\nabla - i\varepsilon)^{-1}_{ij}\,(x_i, x_j).$$

Although we shall not really use it here, we take the spectral functions ρ to be real, hence

$$\left.\begin{aligned} &\Delta^{\pm}_{ij} = (\Delta^{\mp}_{ij})^{*}. \\ &\text{Also} \\ &\Delta^{\pm}_{ij} = \Delta^{\mp}_{ji}\ . \end{aligned}\right\} \tag{2.3}$$

Consequently

$$\Delta^{*}_{ij}(x) = \vartheta(x_0)\,\Delta^{-}_{ij}(x) + \vartheta(-x_0)\,\Delta^{+}_{ij}(x)\ . \tag{2.4}$$

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*) Although perfectly correct, this loop expansion method is often redundant, in the sense that more counter terms are added than are strictly necessary. More economic is, for instance, the BPH procedure[3], but we prefer the loop expansion because in gauge theories diagrams are often arranged according to their number of loops.

As usual,

$$\vartheta(x_0) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\tau\, \frac{e^{i\tau x_0}}{\tau - i\varepsilon} = \begin{cases} 1 & \text{if } x_0 > 0, \\ 0 & \text{if } x_0 < 0 \end{cases}$$

$$\vartheta(x_0) + \vartheta(-x_0) = 1 . \tag{2.5}$$

Now consider a diagram with n vertices. Such a diagram represents in coordinate space an expression containing many propagators depending on arguments $x_1, \ldots, x_n$. In DIAGRAMMAR, such an expression is denoted by

$$F(x_1, x_2, \ldots, x_n) .$$

The diagram has a particular topology. For instance we can take the diagram of Fig. 1

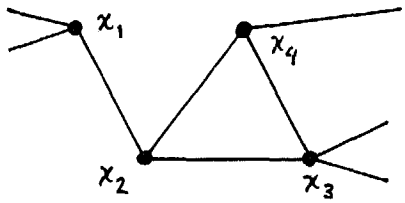


Fig. 1 Diagram for the function $F(x_1, x_2, x_3, x_4)$.

to which may correspond the function

$$F(x_1, x_2, x_3, x_4) = (ig)^4 \Delta_F(x_1 - x_2)\, \Delta_F(x_2 - x_3)\, \Delta_F(x_3 - x_4)\, \Delta_F(x_4 - x_1). \tag{2.6}$$

Belonging to the same topological system, different functions F are now defined, with some of the arguments x_i underlined. They are obtained from the original function F the following way:

i) A propagator Δ_{ki} is unchanged if neither x_k nor x_i are underlined.

ii) A propagator Δ_{ki} is replaced by Δ^{+}_{ki} if x_k but not x_i is underlined.

iii) A propagator Δ_{ki} is replaced by Δ^{-}_{ki} if x_i but not x_k is underlined.

iv) A propagator Δ_{ki} is replaced by Δ^{*}_{ki} if x_k and x_i are underlined.

v) For any underlined x replace one factor i by -i.

Apart from that, the rules for the vertices are unchanged.

Consider F in configuration space. One of the points x in Fig. 1, say x_i, must have the largest time component, $x_{i0} > x_{j0}$ for all j. From (2.1) and (2.4) we then derive

$$F(\ldots, x_i, \ldots) + F(\ldots, \underline{x}_i, \ldots) = 0. \tag{2.7}$$

where the underlining of the other arguments is the same in both terms. The sign is a consequence of def. (v). From Eq. (2.7) another equation follows which is <u>independent</u> of the order of the time components of the points x:

$$\sum_{\text{underlinings}} F(x_1, \underline{x}_2, \ldots, x_i, \ldots, \underline{x}_n) = 0, \tag{2.8}$$

where we sum over all possible ways in which the arguments can be underlined (including no underlinings and all x underlined). To prove (2.8) from (2.7) one merely selects the x with the largest time and arranges the terms in (2.8) in pairs, which are then all zero according to (2.7). Equation (2.8) is the crucial equation to determine whether a certain set of Feynman rules leads to a unitary S-matrix (see DIAGRAMMAR).

Now again consider $F(x_1, \ldots, x_n)$ and take any two variables, say x_i and x_j. Let us suppose that the time component of x_j is larger than that of x_i. Then the following equation holds independently of the order of the other time components:

$$\sum_{\substack{\text{underlinings} \\ \text{except } x_i}} F(x_1, \ldots, x_i, \ldots x_n) = 0 \qquad \text{if } x_{i0} < x_{j0}. \tag{2.9}$$

A similar equation holds if $x_{j0} < x_{i0}$, and so we have

$$F(x_1, x_2, \ldots, x_n) + \vartheta(x_{j0} - x_{i0}) \sum_i{}' F(x_1, \ldots x_i, \ldots, \underline{x}_n) + \vartheta(x_{i0} - x_{j0}) \sum_j{}' F(x_1, \ldots, x_j, \ldots, \underline{x}_n) = 0. \tag{2.10}$$

The prime indicates absence of the term without underlined variables. The index i implies absence of terms with x_i underlined.

Now we can also take together those terms with neither x_i nor x_j underlined:

$$F(x_1, x_2, \dots, x_n) + \sum_{ij}{}' F(x_1, \dots, x_n)$$
$$+ \vartheta(x_{j_0} - x_{i_0}) \sum_{\substack{j \text{ underlined,} \\ i \text{ not}}} F(x_1, \dots, x_i, \underline{x}_j, \dots, x_n)$$
$$+ \vartheta(x_{i_0} - x_{j_0}) \sum_{\substack{i \text{ underlined,} \\ j \text{ not}}} F(x_1, \dots, \underline{x}_i, x_j, \dots, x_n) = 0. \tag{2.11}$$

This equation can be visualized in a figure if we use a shaded line to divide the diagrams into two regions: the underlined vertices are in the shaded region at the right, the ones not underlined are at the left (Fig. 2).

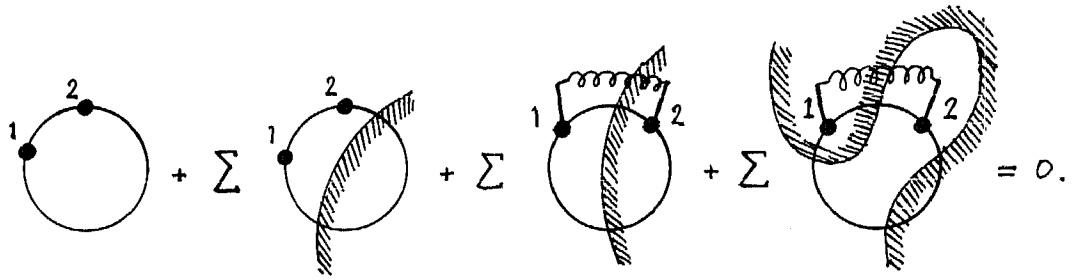


Fig. 2 Equation (2.11) in diagrams.

The blob stands for any diagram or collection of diagrams. The points 1 and 2 indicate two arbitrarily selected vertices. The "self-inductance" denotes the θ function (Fig. 3).

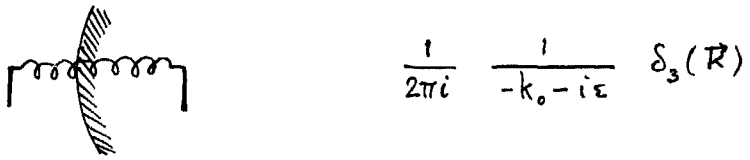


Fig. 3 The θ function in momentum configuration.

The summation is over all possible ways in which the diagrams can be cut with the points 1 and 2 in the given position. Note that lines going through the cutting line are on-mass-shell, and integrations through the cutting line are phase-space integrations.

The reason why Eq. (2.11) is called causality relation is that the same relation can again be applied to that part of the second

diagram in Fig. 2 which is in the unshaded region. Repeating the procedure a number of times we find that the original diagram can be expressed completely as a sum of two terms, one proportional to $\theta(x_{20} - x_{10})$ and having only positive energy flow from 1 to 2, and one vice versa.

The causality equation is particularly useful for renormalization if the regularization procedure does not violate the decomposition properties (2.1)-(2.4) of the propagator.

Suppose we have an irreducible diagram containing ℓ loops. Suppose further that all infinities of subgraphs with $\ell - 1$ or less loops have been removed by means of local counter terms (there is no subgraph with ℓ loops). Then we shall prove that the infinity of our diagram can be removed by a local counter term, whose dimension can be obtained by power counting. A local counter term is a vertex that is a pure polynomial in terms of the momenta.

Let $x_1, \ldots, x_r$ be the external vertices, that are the vertices to which external lines of the graph are attached. The external momenta are $p_1, \ldots, p_{r-1}, -(p_1 + \ldots + p_{r-1})$. Let us apply Fig. 2, with 1 and 2 replaced by 1 and r. The first term is the diagram of interest. The second term has at least one loop cut through because the original diagram was irreducible. The second term therefore consists of a phase-space integral of diagrams with less loops and is hence finite because of the induction assumption. The third and similarly the fourth term in Fig. 2 contain one additional integration, which we denote explicitly:

$$\int dk_0 \frac{1}{2\pi i} \frac{1}{k_0 - i\varepsilon} G(\vec{p}_1, p_{10} + k_0, p_2, \cdots, p_{r-1}), \qquad (2.12)$$

where G stands for the third blob in Fig. 2 but without the θ function self-inductance. For the momentum p_1 we denoted space and time components explicitly. Note that G is again a finite function.

This equation is an ordinary dispersion relation that might diverge in the limit $M \to \infty$, or $n \to 4$, or whatever our regularization procedure was. But of course, before taking such a limit, one can make subtractions such as:

$$\begin{aligned}\int dk_0 \frac{1}{2\pi i} \frac{1}{k_0 - i\varepsilon} \Big\{ & G(\vec{p}_1, p_{10} + k_0, p_2, \ldots) \\ & - G(\vec{p}_1, k_0, p_2, \ldots) - p_{10} G'(\vec{p}_1, k_0, p_2, \ldots) - \ldots \Big\} \\ & + C_0(\vec{p}_1, p_2, \ldots p_{r-1}) + p_{10} C_1(\vec{p}_1, p_2, \ldots) + \ldots, \end{aligned} \qquad (2.13)$$

until the integral converges. The number of necessary subtractions can be deduced from power counting. Then after the limit $M \to \infty$ or

$n \to 4$ the only place where an infinity can develop is in the unknown terms C_0, $p_{10}C_1$, ..., which are polynomials in p_{10}. By extending the argument to the other external momenta, and by Lorentz invariance, we see that the infinity can only be a polynomial in all external momenta, which was the thing to prove.

From this theorem we may conclude the following. If the regularized theory also respects Eqs. (2.1)-(2.4) for finite values of the regularization parameter, then the perturbation series for the S-matrix can be made finite by adding, order by order in the perturbation series, local counter terms.

3. DIMENSIONAL REGULARIZATION, INTUITIVELY

Consider the following integral,

$$I = \int d_n p \, \frac{1}{(p^2 + 2kp + m^2 - i\varepsilon)^\alpha} \,. \tag{3.1}$$

If n is a positive integer and $2\alpha > n$ it is well defined, and not difficult to calculate:

$$I = \frac{i\pi^{n/2}}{(m^2 - k^2)^{\alpha - n/2}} \, \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)} \,. \tag{3.2}$$

The dimensional regularization procedure consists of attributing to the integral (3.1) a formal meaning also if

i) n is not a positive integer

ii) $2\alpha < \mathrm{Re}\, n$.

Formula (3.2) will then hold for general n and α, as we shall see more explicitly in the next section. At first sight this may not seem to be much of an improvement, because if we let $\alpha < n/2$ and $n \to$ integer, then $\Gamma(\alpha - n/2)$ in Eq. (3.2) will exhibit a pole and also the "formal" integral will be infinite. But now we have a convenient regularization parameter n.

Take as an example $\alpha = 1$. The infinite part as $n \to 4$ is

$$I \to i\pi^2(m^2 - k^2) \, \frac{-1}{2 - n/2} + \text{finite as } n \to 4. \tag{3.3}$$

In connection with the theorem in the previous section it is not very astonishing that this is a polynomial in terms of k and m^2. We shall add to the amplitude the effect of a local counter term,

$$i\pi^2 (m^2 - k^2) \frac{2}{4-n} . \tag{3.4}$$

We then get

$$I^{\text{renormalized}} = \lim_{n\to 4} \left(\frac{i\pi^{n/2}}{(m^2-k^2)^{1-n/2}} \frac{\Gamma(1-n/2)}{\Gamma(1)} + \frac{2i\pi^2(m^2-k^2)}{4-n} \right)$$

$$= i\pi^2 (m^2-k^2)\left(\log(m^2-k^2) + C \right) , \tag{3.5}$$

with

$$C = \lim_{n\to 4} \left(\pi^{n/2-2} \Gamma(1-n/2) + \frac{2}{4-n} \right) .$$

The constant C can be removed by finite renormalizations. In fact, C depends on our initial choice of the unit of mass, simply because the dimension of the counter term (3.4) differs from that of the amplitude (3.2) at $n \neq 4$, and therefore should also contain a factor

$$\mu^{n-4} ,$$

with μ some arbitrary unit of mass.

The result (3.5) is exactly the same as what one would get from dispersion relations or more conventional regularization techniques. The dimensional renormalization procedure now consists of taking the "bare" masses and coupling constants to be dependent on n (in the perturbation expansion they will have poles at $n = 4$), in such a way that the limit $n \to 4$ is finite and makes sense. The program, and some of its features, is pictured schematically in Table 1.

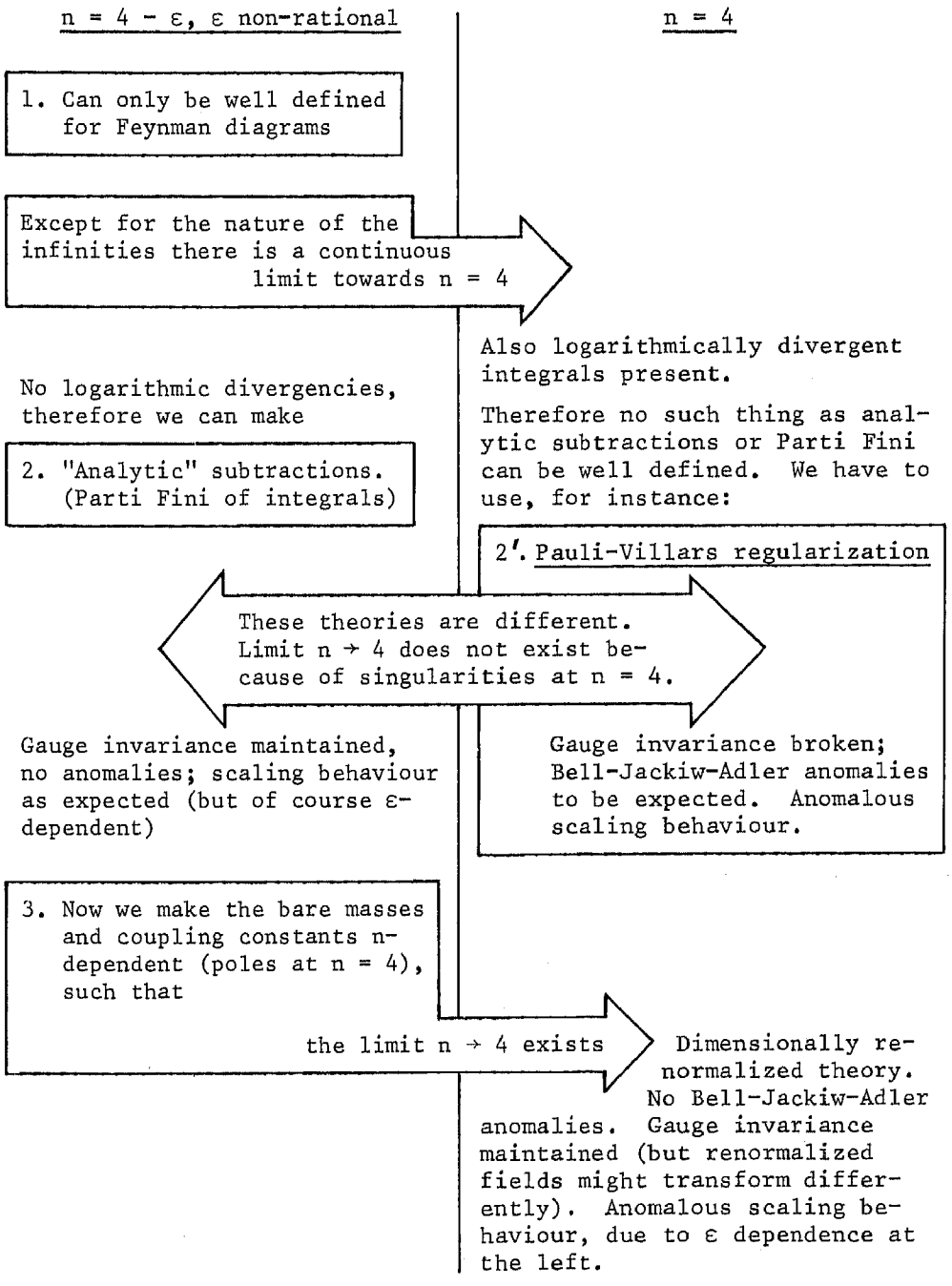
4. RIGOROUS DEFINITIONS FOR THE DIMENSIONAL REGULARIZATION PROCEDURE

In this section we first consider regularization with which we denote the procedure to make the theory finite in terms of some cut-off parameter, here $n - 4$. The renormalization program concerns the limit towards the physical situation, here $n \to 4$ and shall be considered later (step 3 in Table 1).

First of all, according to Table 1, we must define what we mean by taking a non-integer number of dimensions. This is simply done by requiring the formula

$$\int d^n k \, f(k^2) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2})} \int_0^\infty f(r^2) r^{n-2} \, dr^2 , \tag{4.1}$$

Table 1 The dimensional renormalization procedure



to be valid also for non-integer (even negative or complex) n. This formula can be applied if we have a finite number of vector integration variables. First we perform all integrations over the angles, and then the remaining ones using the formula (4.1). So our definition is good for Feynman diagrams with a finite number of loops, but it seems to be impossible to extend it to Hilbert spaces or path integrals. Note that there is not yet any discontinuity in formula (4.1) at positive integer n.

Next the ultraviolet divergent integrals are considered. We propose a subtraction method which does not work for logarithmically divergent integrals, so we are forced to go first to non-integer, or even non-rational, n. Our subtraction method which we call "analytic" subtraction, is probably equivalent to taking the "Parti Fini" of an integral[4] and can be formulated in various ways. One way is to decompose the integral into parts which are all convergent in a certain region of n, and then continuate analytically; or one may redefine the integral by making partial integrations and subsequently neglecting the (infinite or zero) boundary terms. This has been defined very precisely in DIAGRAMMAR and elsewhere[5]. Perhaps the most convincing and unambiguous way is to decompose integrands in k-space into finite terms (when integrated), and terms of the form

$$f(k_1 - a_1, k_2 - a_2, \dots), \tag{4.2}$$

where f has the property

$$f(\lambda p_1, \lambda p_2, \dots) = \lambda^\alpha f(p_1, p_2, \dots), \tag{4.3}$$

and a_1, a_2, ... are arbitrary. The integrals over the functions f are then defined to be zero, so that we are left with the finite integrations ("Parti Fini"). This definition is unambiguous because the above decomposition is unambiguous. It only does not work for logarithmically divergent integrals because a logarithmically divergent function f satisfying (4.3) is divergent both at the origin and at infinity, so we cannot compensate logarithmic infinities at infinity only, without creating new ones at some point a.

The formula in the previous section can be deduced from these definitions, and indeed we notice the singularity at $n \to 4$ develop, where we would have to deal with a logarithmically divergent part of the integral.

The importance of the analytic subtraction procedure is that summations, differentiations, and integrations commute, and that the result is independent of the choice of the integration variables. [Remember that Bell-Jackiw-Adler anomalies[6] are a consequence of the ambiguity of the integration variable. They evidently cannot

occur in 4-ε dimensions!] This implies that all formal manipulations with Feynman diagrams in Sections 1 and 2 are also strictly legal for analytically subtracted diagrams.

As for the cutting rules (Section 2), it is only with respect to the time direction that Fourier integration and decomposition of the propagator are necessary. Dimensional regularization is therefore applied with respect to space-dimension. The cutting rules can exhibit infra-red divergent threshold singularities if n is taken to be too small (see DIAGRAMMAR) but this is of no influence to our argument here because we only need to consider the vicinity of n = 4.

From the theorem of Section 2 we therefore conclude that order-by-order renormalization by means of local counter terms (polynomials in k, but singular in n = 4) leads to a completely finite perturbation theory.

5. SCALE TRANSFORMATIONS

Quantum field theories behave quite remarkably under scale transformations. The clue needed to calculate this behaviour is the Callan-Symanzik equation[7]. In first approximation the scaling behaviour is governed by the one-loop corrections to the amplitudes. The leading terms of these integrals are rather easy to calculate, but the next to leading parts seem to be rather ugly.

An alternative way to calculate the scaling behaviour is to use the dimensional renormalization procedure. The advantage of that method, which has been published elsewhere[8], is perhaps its elegance, and moreover the fact that for the leading terms one never needs to calculate real integrals, but only their "infinite" part [compare Eq. (3.3)]. The central idea is that in 4-ε dimensions the analytically subtracted amplitudes obey the "naïve" scaling rules, which can easily be obtained by dimension-counting. So we merely need to examine closely the $\varepsilon \to 0$ limit, keeping in mind that the original Lagrangian has been altered by counter terms such as (3.4) in order to obtain a physically interesting limit, and the observation in Section 3 that these counter terms must be given the correct dimension by means of some unit of mass, μ.

In perturbation theory one can choose freely in which of the perturbation terms to put the finite part of such counter terms. Exploiting this freedom we derive equations for the scaling behaviour on the one hand, and for the higher-order singularities at n = 4 on the other. In fact, the coefficients in front of the k^{th} order poles can be expressed in terms of the coefficients in front of the single poles. In other words, the parameters in the analytically subtracted theory, λ_B^i, must have an n-dependence of the form

$$\lambda_B^i = \mu^{D^i(n)} \left[\lambda_R^i + \frac{a_1^i(\lambda)}{n-4} + \frac{a_2^i(\lambda)}{(n-4)^2} + \cdots\right], \qquad (5.1)$$

where μ is a unit of mass and $D^i(n)$ is the dimension of λ^i as a function of n; we find how to express the Callan-Symanzik coefficient β in terms of $a_1^i(\lambda)$, and we find relations that express the coefficients a_2^i, a_3^i, ... in terms of a_1^i. These relations are sufficiently simple to enable us to sum the series (5.1). A more thorough discussion of the resulting expressions for coupling constants is given by Symanzik[9].

One can take full advantage of the fact that only the infinities (i.e. poles at $n = 4$) of the integrals enter in our expressions for β. These infinities can be obtained by means of a simple algebra, deirectly from the original Lagrangian of the theory[10].

A surprise emerges if one applies the above-sketched techniques to field theories of different kinds. A renormalizable field theory always turns into itself by a scale transformation, but with different coefficients. All conventional theories, like ϕ^4 or QED, have the property that the coupling constants thus become large if we scale towards the far ultraviolet region, which implies that the short-distance behaviour is described by a badly divergent perturbation expansion and is therefore uncomputable; on the other hand, the infra-red divergencies can often be summed to give something finite. The opposite, however, is the case in pure non-Abelian gauge theories, or non-Abelian gauge theories with a limited number of fermions. These theories have a well-computable short-distance behaviour but have an unpredictable infra-red behaviour.

The scaling behaviour of the theory of quantum electrodynamics can be visualized the following way: if we consider an electric charge and its electric field as a function of the distance, then we observe that the charge is "screened" by the virtual pairs created by the electric field. So at a large distance the effective charge is smaller than at a small distance: the effective coupling constant e increases if we decrease the length scale.

The question now rises why the above heuristic argument does not apply to the non-Abelian gauge bosons themselves*). An argument -- but a not very convincing one -- might be the following. From Ref. 10 it can be deduced that the opposite effect stems mainly from the same terms that determine the (quite large) magnetic moment of the gauge bosons. Indeed, if we take a magnetic dipole and consider its field at a certain distance, then it is conceivable that virtual boson pairs tend to align their spins (anti-) parallel to the magnetic field (ferromagnetism) thus amplifying instead of screening

*) We thank L. Susskind for discussions on this point.

the field. By Lorentz invariance, moving virtual pairs should also amplify electric fields, thus giving rise to a negative screening effect. Because the magnetic moment is relatively large, this effect is much stronger than the positive screening due to their electric charge. We can assign to each particle a number denoting their contribution to the screening. For SU(2), the relative numbers are:

gauge bosons (including possible ghosts)		$-1\frac{1}{6}$
spin $\frac{1}{2}$ fermions,	isospin $\frac{1}{2}$	$+\frac{1}{6}$
	" 1	$\frac{2}{3}$
	" $\frac{3}{2}$	$\frac{5}{3}$
	" 2	$\frac{10}{3}$
spin 0 bosons,	isospin $\frac{1}{2}$	$\frac{1}{24}$
	" 1 (real fields)	$\frac{1}{12}$
	" $\frac{3}{2}$	$\frac{5}{12}$
	" 2 (real fields)	$\frac{5}{12}$

To find the total screening, one simply adds these numbers for all occurring particles. If the number accidently adds up to zero then the sign of the screening will be determined by higher-order effects that have not been calculated.

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