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Reswitching and capital models

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Abstract

Reswitching of techniques was shown to be possible in Leontief's input–output model with flow and stock matrices. The finding is unearthed and related to the capital debate based on Sraffian models.

Keywords: Cambridge capital debate, reswitching, dynamic input-output model, fixed capital, Leontief, Sraffa

1 Introduction

A new initiative of the International Input–Output Association is the collection and digitization of materials going back to the "pioneers" and their students. Thus, at the suggestion of Bert Steenge, I checked my files and found a correspondence between Wassily Leontief and Thijs ten Raa on the reswitching of techniques in 1985–1986. Reswitching was the focal point of the capital debate between Cambridge, USA, and Cambridge, UK. The Americans represented the neoclassical school, according to which the factor distribution of income between capitalists and workers is determined by the marginal productivities of capital and labor.

Neoclassical economists base their theory on a macroeconomic production function that maps quantities of the two-factor inputs—capital and labor—into a quantity of output. The production function has the usual property of diminishing marginal products. The greater the labor force, the lesser their productivity, and likewise for capital. Equilibrium with the supply of labor and capital determines the factor prices. A serious foundation of such a macroeconomic production function had been given by Houthakker (1955). He argued that if the microeconomic production units have fixed input coefficients, but the coefficients differ across the production units, then the aggregate production function features substitutability of capital and labor, as an increase in the wage rate will bankrupt labor-intensive production units, and likewise for an increase in the rate of return to capital. In fact, Houthakker demonstrated that if the input coefficients follow a Pareto distribution, then the aggregate production function is Cobb–Douglas. Of course, different input distributions across production units will generate different aggregate production functions; see ten Raa (2019) for a review.

The Cambridge, UK, economists represented the classical school, according to which the factor distribution of income between capitalists and workers is not determined by marginal productivities, but by other mechanism, such as class struggle. They argued

that the neoclassical assumption of a decreasing relationship between factor intensity and factor reward need not hold. The source of the trouble is that capital consists of produced commodities, such as machinery and equipment. There is an aggregation issue, which neoclassical economists assume away. A symptom of the aggregation issue is that as the rate of return on capital increases, a capital-intensive technology may be substituted out—as neoclassical economists would predict since the rate of return is the price of capital for entrepreneurs—but the capital-intensive technology may reemerge at a still greater rate of return. This phenomenon of reswitching of techniques contradicts the construct of a macroeconomic production function and its use in the distribution of income. Loud criticaster and defender of the neoclassical school of thought were Joan Robinson and Bob Solow, respectively; for a review of the capital controversies, see Cohen and Harcourt (2003).

However, the best intellectuals among the neoclassical opponents and their opponents, Paul Samuelson and Luigi Pasinetti, respectively, were basically in agreement. In fact, in an early paper Samuelson himself provided an example of reswitching. Samuelson (1966) did so in the context of an Austrian circulating-capital model, where capital is input that is in the pipeline of production. Labor-intensive and time-intensive techniques coexist, and reswitching may occur.

In the simplest input–output analytic terms the Austrian model of circulating capital can be based on $x(t)=Ax(t+1)+y(t)$, where t is time, x is the gross output vector, y the net output vector, and A the matrix of input–output coefficients. This equation subsumes a single technique for every product, with a unitary lag of production. To produce output, the inputs must be available one unit ahead of time. Distributive lags in production have been analyzed and estimated by ten Raa (1986), who solves for the path of gross outputs that sustains a path of net outputs, and ten Raa et al. (1989), who estimate different production lags. Alternative techniques can be introduced by working directly with the use and make tables (which underlie the A matrix): $Vs=Us+y$, where s is the vector of activity levels, U the use table and V the make table. U and V are rectangular, reflecting that there may be more activities than products, a source of substitution. Production time lags may vary as in ten Raa et al. (1989) and this modification would yield a full-blown input–output model of Austrian circulating capital.

Leontief, however, modeled capital not as circulating but as fixed, using a matrix of capital coefficients B. The row vector of product prices fulfills $p=pA+rpB+l$, normalizing the price of labor at 1, denoting the rate of return on capital by r, and assuming perfect competition (zero profit).

Leontief (1986) considered the possibility of reswitching an empirical issue and investigated if the phenomenon occurs in the U.S. economy. In this paper Leontief mentioned that a numerical test example can easily be constructed of a simple, say, three sector input–output system in which the cooking recipes are such that some recipes would drop out if the rate of return goes up from 5% to say 10%, but reappear again when it rises to 15%. He did not spell out an example. We corresponded on this paper. I wrote that reswitching can be shown in an even simpler input–output system, with only two sectors. He wrote the paper was rejected by "our leading economic journal" (the *American Economic Review*). "It might amuse you to know that the editor rejected it explaining that it would be of no interest to its readers."

In this note I present the example and position Leontief's model in the capital literature.

2 The example

Leontief's input–output system consists of a matrix of input–output coefficients, A, a matrix of capital coefficients, B, and a row vector of labor coefficients, l. Matrices A and B have dimension $n \times n$ and row vector l has dimension $1 \times n$, where n is the number of sectors. The system (A, B, l) constitutes a technology. An alternative technology is indicated by (α, β, λ). My example consists of $A = 0$, $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, $l = (1\ 1)$ and $\alpha = 0$, $\beta = \begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix}$, $\lambda = (1\ 0)$. Only sector 2 features technical change. The first economy is decomposed, with sector 1 using labor and sector 2 using labor and home-made capital. The second economy is integrated, with sector 2 using capital produced by sector 1.

Solving for the product prices, $p = l(I - A - rB)^{-1}$. In the first economy this reduces to $p = (1\ \ 1)\begin{pmatrix} 1 & 0 \\ 0 & 1-r \end{pmatrix}^{-1} = (1\ (1-r)^{-1})$. In the second economy this reduces to $p = (1\ \ 0)\begin{pmatrix} 1 & -5r \\ 0 & 1 \end{pmatrix}^{-1} = (1\ 5r)$. The change of technology (in sector 2) does not impact the price of product 1 (as sector 1 uses no product of sector 2, neither intermediate input nor capital). Therefore, the second technology will be adopted if and only if the second price of product 2 is less than the first price, $5r < (1-r)^{-1}$. This condition for technical change is fulfilled for rates of return on the interval (0 0.28) and the interval (0.72 1) (rounded). At intermediate rates of return the first technology is adopted. The relationship between the rate of return on capital and the choice of technique is not monotonic in the example. Reswitching is possible. An empirically relevant example of reswitching has been found by Han and Schefold (2006).

The choice of technology depends on the product prices. Since the product prices are continuous with respect to the input–output coefficients, the reswitching result holds for positive input–output coefficients. Hence reswitching is not only possible for perhaps rather unusual numerical values.

3 Alternative models and their connection

The Sraffian literature and the recent input–output literature on reswitching, e.g., Zambelli (2018), are based on a different price equation, namely $p = pA + rpA + l$. In words, Leontief assumes capital is fixed, while Sraffa c.s. assume capital is circulating. The two points of view are consistent if the flow coefficients (A) are proportional to the stock coefficients (B). Remarkably, this is precisely Bródy's (1970) capital condition. Moreover, ten Raa (1997) has shown that Bródy's capital condition is required for Leontief's dynamic input–output model to be consistent with a temporal distributed use–make framework.

Anyway, reswitching has been established in Sraffa's model and in Leontief's model. In the simplest case of an economy with two production sectors this has been

accomplished by Samuelson (1966) for Sraffa's model and in this note for Leontief's model.

4 Concluding remark

The Cambridge–Cambridge capital controversy was purely theoretical. Cambridge, UK, won on points. Leontief (1986) admitted that the reswitching of techniques may happen in the dynamic input–output model with fixed capital but found no evidence of the phenomenon. This note presents the simplest example of reswitching and positions Leontief's model in the capital literature. As Pasinetti (2003, p. 228) concluded later:

> *"Samuelson closed the 1966 Symposium with a few wise words (p. 583): "If all this causes headaches for those nostalgic for the old time parables of neoclassical writing, we must remind ourselves that scholars are not born to live an easy existence. We must respect, and appraise, the facts of life." Current neoclassical economists do not seem to like Samuelson's "facts of life.""*

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