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Regime-switching models to study psychological processes

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Abstract

Many psychological processes are characterized by recurrent shifts between different states. To model these processes at the level of the individual, regime-switching models may prove useful. In this chapter we discuss two of these models: the threshold autoregressive model and the Markov switching autoregressive model. We discuss their main features, and propose estimation methods that can handle missing data. We apply these models to daily affect measurements of an individual diagnosed with rapid cycling bipolar disorder.

Regime-switching models to study psychological processes

In the 1980s, two extensions of the linear autogressive (AR) model were introduced: the threshold autoregressive (TAR) model (Tong & Lim, 1980), and the Markov-switching autoregressive (MSAR) model (Hamilton, 1989). Both models allow for recurrent switches between two or more distinct AR processes, each of which is characterized by its own parameters. These models have proved useful in economic research, for instance to model certain features of the business cycle, such as the asymmetry between phases of expansion and decline (Clements & Krolzig, 1998).

Despite their popularity in economics, the TAR model has only seen a few applications in psychological research (Hamaker, Zhang, & Van der Maas, conditionally accepted; Hamaker, 2008; Warren, 2002; Warren, Hawkins, & Sprott, 2003), while the MSAR model has not been applied in psychological research to our knowledge. This is remarkable, because there are many psychological phenomena which are characterized by recurrent shifts between different psychological states, for instance: addiction can involve periods of recovery and relapses; post-traumatic stress disorder is characterized by episodes of intrusion and avoidance; premenstrual syndrome involves physiological and emotional symptoms in the luteal phase, which elevate after the onset of menses; and bipolar disorder is characterized by switches between episodes of mania and depression. Moreover, in cognitive tasks people may switch between different strategies (i.e., psychological states), such as being fast at the cost of making more mistakes, versus being more accurate at the cost of being slower.

In this chapter we refer to such psychological states as regimes. Note that by *regime* we mean “the characteristic behavior or orderly procedure of a natural phenomenon or process” (from the Merriam-Webster dictionary). Hence, in contrast to the more popular use of the term, where regime refers to a form of government that is long-lasting and typically not re-occurring, we use it to refer to a psychological state, which may alternate repeatedly with other psychological states. In this chapter we propose to describe such a psychological state with a particular AR model, and we investigate the possibility that psychological processes that are characterized by recurrent shifts between distinct psychological states can be modeled using regime-switching models such as the TAR and MSAR model.

The purpose of this chapter is to introduce the reader to the TAR model and the MSAR model, and to illustrate the usefulness of these models for gaining more insight in the underlying dynamics of regime-switching processes such as described above. In the first section we present these two models, and discuss their main features and some important extensions. In the second section we present estimation procedures that can be used when there is missing data. In addition we discuss the issue of model selection based on information criteria. The third section is an empirical application in which we compare the TAR and MSAR model to an ordinary AR model (i.e., a model without regime-switching). To this end we make use of daily mood scores obtained with the Positive Affect and Negative Affect Schedule (PANAS; Watson & Clark, 1994) in an individual diagnosed with rapid cycling bipolar disorder. Our aim is to determine whether there are two distinct regimes that represent a manic and a depressed state in this person’s daily affect scores. In addition, we are interested in the actual regime-switching mechanism, that is, whether shifts from depression to mania and vice versa are triggered by affect levels themselves (as in a kind of feedback loop), or whether this is an autonomous mechanism which operates independently of the actual affect levels. We end this chapter with a brief discussion of the potential of these models for psychological research.

Regime-switching AR models

The TAR model and the MSAR model both consist of two or more distinct AR processes, which are referred to as regimes. At each occasion the system is in one of these regimes, meaning that one of the AR processes gave rise to the data. The difference between the TAR model and the MSAR model concerns the process that triggers the regime-switching. In the TAR model, switching is modeled explicitly as a function of some manifest threshold variable. In contrast, in the MSAR model switching is regulated by a hidden Markov process, and has to be inferred from the observed data. This has important implications for the substantive interpretation of these models, as we will see below.

Because we use vector extensions of the TAR and MSAR models in our illustration, we will focus on the multivariate presentation. Clearly, these multivariate models include the univariate model as a specific case. We begin with the vector AR (VAR) model, as it forms the basis of both the vector TAR model and the vector MSAR.

VAR model

If $\mathbf{y}_t = [y_{t,1} \ \dots y_{t,r}]'$ denotes an r-variate observation at occasion t, the VAR(p) process is defined as

$$\mathbf{y}_t = \boldsymbol{\phi}_0^{(j)} + \mathbf{\Phi}_1^{(j)}\mathbf{y}_{t-1} + \cdots + \mathbf{\Phi}_{p^{(j)}}^{(j)}\mathbf{y}_{t-p^{(j)}} + \mathbf{\Omega}\boldsymbol{e}_t, \tag{1}$$

where $\boldsymbol{\phi}_0^{(j)}$ is an r-variate vector with constants, and $\mathbf{\Phi}_1^{(j)}$ to $\mathbf{\Phi}_{p^{(j)}}^{(j)}$ are $r \times r$ matrices with autoregressive parameters on the diagonal and lagged cross-regressive parameters as off-diagonal elements (e.g., Hamilton, 1994, p.257). An autoregressive parameter relates a variable to itself at a preceding occasion, while a lagged cross-regressive parameter relates a variable to another variable at a previous occasion. The last term on the right-hand side of Equation 1 is an r-variate vector with residuals, i.e., the parts of $\mathbf{y}_t$ that could not be predicted from previous realizations of the process. We use the (unconventional) notation $\mathbf{\Omega}\boldsymbol{e}_t$, as this will prove useful when generalizing to the regime-switching models below. It is assumed that the elements of $\boldsymbol{e}_t$ are standard normally distributed, such that the covariance matrix $\boldsymbol{e}_t$ is an identity matrix. As a result, the covariance matrix of the residuals can be denoted as $\mathbf{\Omega}\boldsymbol{e}_t \sim N(\mathbf{0}, \mathbf{\Sigma})$, where $\mathbf{\Sigma} = \mathbf{\Omega}\mathbf{\Omega}'$, such that the residuals may be correlated within an occasion, but are uncorrelated over time. For identification purposes, $\mathbf{\Omega}$ is constrained to be a lower triangular matrix (meaning that all elements above the diagonal are zero).

Vector TAR model

The univariate TAR model was introduced by Tong and Lim (1980), and was extended by Koop, Pesaran, and Potter (1996) and Tsay (1998) to the multivariate case. All TAR models can be described as piece-wise linear models, in which the regime-switching is regulated by a manifest threshold variable z_t. Let $\{A_j\}$ be a nonoverlapping partitioning of the real line such that $A_j = (\tau_{j-1}, \tau_j]$, with $-\infty = \tau_0 < \tau_1 < \cdots < \tau_k = \infty$. Then τ_1 to τ_{k-1} are the thresholds of interest, which are used to distinguish between k distinct regimes. Let $I(z_{t-d} \in A_j)$ denote the indicator function, which takes on value one if z_{t-d} falls in A_j, and is zero otherwise. Note that for a specific occasion t, z_{t-d} will fall in one of the A_j's, meaning that $I(z_{t-d} \in A_j)$ equals one for just one j. We refer to this j as the regime the system is in at occasion t.

The r-variate observation $\mathbf{y}_t$ is generated by a vector TAR$(k, p^{(1)}, \ldots, p^{(k)})$ process if it can be written as

$$\mathbf{y}_t = \sum_{i=1}^{k} \left\{ \boldsymbol{\phi}_0^{(j)} + \boldsymbol{\Phi}_1^{(j)} \mathbf{y}_{t-1} + \cdots + \boldsymbol{\Phi}_{p^{(j)}}^{(j)} \mathbf{y}_{t-p^{(j)}} + \boldsymbol{\Omega}^{(j)} \mathbf{e}_t \right\} I(z_{t-d} \in A_j) \;, \qquad (2)$$

where the vector $\boldsymbol{\phi}_0^{(j)}$ and the matrices $\boldsymbol{\Phi}_1^{(j)}$ to $\boldsymbol{\Phi}_{p^{(j)}}^{(j)}$ play the same role as in Equation 1, except that they now depend on the regime the model is in, as indicated by the superscript (j). Furthermore, $\mathbf{e}_t \sim N(\mathbf{0}, \mathbf{I}_r)$, such that the residuals for regime j are multivariate normally distributed, that is, $\boldsymbol{\Omega}^{(j)} \mathbf{e}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}^{(j)})$, where $\boldsymbol{\Sigma}^{(j)} = \boldsymbol{\Omega}^{(j)} \boldsymbol{\Omega}^{(j)\prime}$.

Most publications on TAR modeling focus on the univariate case, which is obtained from Equation 2 by setting $r = 1$. An important aspect of the TAR model is the threshold variable, for which Tong and Lim (1980) suggested several options. First, one can use the same variable, i.e., y_t. Such a TAR model is referred to as a self exciting TAR (SETAR) model (Tong & Lim, 1980). Characteristic of this model is that there is a feedback loop, which regulates the process when its output y_{t-d} becomes too extreme. In psychological research, Warren and his colleagues used the SETAR model to study the self-regulation in sex offenders (Warren, 2002), and in alcohol addicts (Warren et al., 2003).

Second, one can use an exogenous variable as the threshold variable. Tong and Lim (1980) refer to this as the open loop TAR system (TARSO). This model could prove useful in an experimental setting in which the experimenter continually varies some independent variable to study its effect on the dependent variable of interest within a single subject.

A third version of the TAR model consists of using two variables as each others threshold variable. Such bivariate models have been referred to as a closed loop TAR system (TARSC; Tong & Lim, 1980), and have been used in modeling predator-prey cycles, where the size of the predator population serves as the threshold variable for the prey population and vice versa (Stenseth et al., 1998; Tong & Lim, 1980). In psychological research, TARSCs have been used for modeling dyadic interactions, where each person serves as the other persons' threshold variable (Hamaker et al., conditionally accepted).

Other extensions of the basic (univariate) TAR model consist of: including moving average (MA) relationships (see De Gooijer, 1998 for the TMA model; see Tong, 2003 for the TARMA model; Amendola, Niglio, & Vitale, 2006); adding (lagged) predictors (Tong & Lim, 1980); considering unit roots (Caner & Hansen, 2001); using the change (rather than the variable itself) as the threshold variable (momentum TAR model; Narayan, 2007); and having less sudden changes from one regime to another (smooth TAR model; Chan & Tong, 1986). Many of these extensions could be readily applied to the vector TAR model as well.

MSAR model

Just as the TAR model discussed above, the MSAR model is characterized by recurrent switches between two or more distinct AR processes. But while the regime-switching in the TAR model is governed by an observed variable, the regime-switching process in the MSAR model is governed by a a finite state hidden Markov chain. The sequence of states of this hidden Markov chain is denoted as $\{s_t\}$, where s_t may take on any integer between 1 and k, with k being the total number of regimes. Let $I(s_t = j)$ denote the indicator function, which equals one if s_t equals j, and is zero otherwise (with $j = 1, \ldots, k$).

The multivariate MSAR $(k, p^{(1)}, \ldots, p^{(k)})$ model can be expressed as[1]

$$\mathbf{y}_t = \sum_{j=1}^{k} \left\{ \boldsymbol{\phi}_0^{(j)} + \boldsymbol{\Phi}_1^{(j)} \mathbf{y}_{t-1} + \cdots + \phi_{p^{(j)}}^{(j)} \mathbf{y}_{t-p^{(j)}} + \boldsymbol{\Omega}^{(j)} \mathbf{e}_t \right\} I(s_t = j), \quad (3)$$

where the vectors and matrices have the same dimensions as the ones defined in multivariate TAR model in Equation 2. The only fundamental difference between this MSAR model and the TAR model is that in the latter the indicator function is based on an observed variable, while here is is based on the latent variable s_t.

The hidden Markov chain that governs the switching in the MSAR model is characterized by a matrix with transition probabilities, that is, the probabilities of switching from one regime to another. This matrix can be denoted as[2]

$$\mathbf{p} = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{k1} \\ p_{12} & p_{22} & \cdots & p_{k2} \\ \cdots & & & \\ p_{1k} & p_{2k} & \cdots & p_{kk} \end{bmatrix}. \quad (4)$$

where $p_{ij} = P[s_t = j | s_{t-1} = i]$ (with $i = 1, \ldots, k$ and $j = 1, \ldots, k$), that is, it is the chance of switching to regime j if the system was in regime i at the previous occasion. Here $\sum_{j=1}^{k} p_{ij} = 1$, that is, the conditional probabilities per column have to add to one. Because of this constraint, there are only $k \times (k-1)$ free parameters in the matrix $\mathbf{p}$.

A rather general extension of the MSAR model consists of the Markov-switching state-space (MSSS) model proposed by Kim (1994; Kim & Nelson, 1999). Because many time series models can be formulated in state-space format (e.g., Durbin & Koopman, 2001), this extension allows the researcher to consider a vast variety of models, including models with MA relationships. Another extension of the MSAR model was proposed by Durland and McCurdy (1994). In their duration dependent MSAR model the switching probabilities (i.e., Equation 4) depend on the time spent in a regime.

Model estimation and model selection

Typically, researchers are interested in comparing a number of models that represent rivaling theories. From the previous section it has become clear that the SETAR model and the MSAR model are characterized by different switching mechanisms. To determine which of these switching mechanisms is most likely to have generated the data, or that there actually is no switching mechanism operating in the data, we need to fit a number of different models need (e.g., a linear AR model which represent no switching, an MSAR model with two regimes, etcetera). Subsequently, we have to decide which of these models gives the best description of the data. This procedure is referred to as model selection, and is discussed at the end of this section.

Before we can focus on model selection, we first need to estimate the model.

Hence, we begin this section with discussing parameter estimation. For ease of presentation, we discuss parameter estimation of the multivariate regime-switching models defined in Equations 2 and 3 for the specific case where the order of the VAR processes in each regimes is 1. However, we point out that the methods described here can be readily extended to models with higher order VAR processes.[3]

Estimation of (multivariate) SETAR model

The standard procedure for estimating the parameters of a TAR model is through linear regression (e.g., Tong & Lim, 1980). While this method is simple and fast, a drawback is that it doesn't handle missing data very efficiently: for every occasion that is missing, there are two occasions that cannot be included in the least squares estimation: the occasion that is missing, and the next occasion when the missing occasion serves as a predictor. To overcome this problem, we propose to use an alternative estimation procedure based on the Kalman filter algorithm.

Let the q-th element of $\mathbf{y}_{t-d}$ be the threshold variable, which we denote as $y_{t-d,q}$. For simplicity we assume $d = 1$. Given a particular set of threshold values, this Kalman filter based algorithm consists of the following steps. Set $\mathbf{y}_{1|1} = \mathbf{y}_1$, start at $t = 2$, and

1. Predict $\mathbf{y}_{t|t-1}$ from $\mathbf{y}_{t-1|t-1}$, depending on whether $y_{t-1|t-1,q} \in A_j$

2. Determine the discrepancy $\boldsymbol{\nu}_t = \mathbf{y}_t - \mathbf{y}_{t|t-1}$

3. Update the estimate $\mathbf{y}_{t|t-1}$ by $\mathbf{y}_{t|t} = \mathbf{y}_t$, and set $t = t + 1$

These steps are repeated until the end of the series. Note that the update $\mathbf{y}_{t|t}$ forms the $\mathbf{y}_{t-1|t-1}$ in step 1 of the next occasion. If the observation at t is missing, we cannot determine the discrepancy in step 2, and we cannot update our estimate in step 3. Therefore, we will use $\mathbf{y}_{t|t} = \mathbf{y}_{t|t-1}$, that is the update is the same as the prediction. This update is then used both as the value for the threshold variable, and as the predictor at the next occasion (i.e., $\mathbf{y}_{t-1|t-1}$).[4]

The log likelihood function for this model can be denoted as

$$\log L = \sum_{t=1}^{T} \log f(\mathbf{y}_t|\mathbf{Z}_{t-1}), \tag{5}$$

where $f(\mathbf{y_t}|\mathbf{Z}_{t-1})$ is the likelihood of $\mathbf{y}_t$ given all observed data up to the previous occasion, i.e., $\mathbf{Z}_{t-1} = [\mathbf{y}_1' \; \mathbf{y}_2' \; , \ldots \; \mathbf{y}_{t-1}']'$. In terms of the discrepancies $\boldsymbol{\nu}_{t|t-1}$, the log likelihood can be written

$$\log f(\mathbf{y}_t|\mathbf{Z}_{t-1}) = \sum_{j=1}^{k} \left\{ -\frac{r}{2}\log(2\pi) - \frac{1}{2}\log|\boldsymbol{\Sigma}^{(j)}| - \frac{1}{2}\boldsymbol{\nu}_{t|t-1}'\boldsymbol{\Sigma}^{(j)-1}\boldsymbol{\nu}_{t|t-1} \right\} I(y_{t-1,q} \in A_j). \tag{6}$$

Note that when $\mathbf{y}_t$ is missing, the discrepancy can not be determined, such that this occasion will not contribute to the log likelihood function in Equation 5. However, when $\mathbf{y}_t$ is missing, we can still make a prediction for the next occasion, as we have the update $\mathbf{y}_{t|t} = \mathbf{y}_{t|t-1}$, which we can also use for the threshold value. As a result, a missing observation leads to only one missing occasion in the likelihood function using this procedure, in contrast to estimation based on linear regression.

To obtain estimates of the thresholds τ_1 to τ_{k-1}, we propose an exhaustive search considering all potential threshold values. Note that when we use $y_{t,q}$ as the threshold variable, the thresholds can only be estimated to equal actual observed values of the threshold variable $y_{t,q}$; that is, we can not differentiate between possible threshold values which have not been observed for the threshold variable.

Estimation of MSAR model

A convenient way to estimate the parameters of an MSAR model is through the algorithm proposed by Kim (1994; Kim & Nelson, 1999). This is based on the MSSS model, and consists of a combination of the well-known Kalman filter and the Hamilton filter. Here we present these filters in simplified form, due to the fact that our states and our observations coincide (for a more detailed and general description of this procedure we refer the reader to Kim & Nelson, 1999).

To allow for meaningful comparison to the multivariate SETAR model discussed above, we use the first observation only to regress on, that is, $\mathbf{y}_1^{(j)} = \mathbf{y}_1$ for all j. Start at $t = 2$, and

1. Predict $\mathbf{y}_{t|t-1}^{(i,j)}$ from $\mathbf{y}_{t-1|t-1}^{(i)}$

2. Determine the discrepancy $\boldsymbol{\nu}_t^{(i,j)} = \mathbf{y}_t - \mathbf{y}_{t|t-1}^{(i,j)}$

3. Update the estimate $\mathbf{y}_{t|t}^{(i,j)}$ using the prediction $\mathbf{y}_{t|t-1}^{(i,j)}$and the discrepancy $\boldsymbol{\nu}_t^{(i,j)}$

4. Determine $P[s_{t-1} = i|s_t = j, \mathbf{Z}_t]$

5. Collapse $\mathbf{y}_{t|t}^{(i,j)}$ into $\mathbf{y}_{t|t}^{(j)}$ using $\mathbf{y}_{t|t}^{(j)} = \sum_{i=1}^{k} \mathbf{y}_{t|t}^{(i,j)} P[s_{t-1} = i|s_t = j, \mathbf{Z}_t]$

Note that the collapsed $\mathbf{y}_{t|t}^{(j)}$ forms the $\mathbf{y}_{t-1|t-1}^{(i)}$ at the next occasion. In case of missing data, we cannot determine the discrepancy in step 2. Instead, our update will be equal to our prediction, i.e., $\mathbf{y}_{t|t}^{(i,j)} = \mathbf{y}_{t|t-1}^{(i,j)}$. Moreover, since $\mathbf{Z}_t = \mathbf{Z}_{t-1}$, we propose to set $P[s_{t-1} = i|s_t = j, \mathbf{Z}_t] = P[s_{t-1} = i|s_t = j, \mathbf{Z}_{t-1}]$

The log likelihood function for this model is as defined in Equation 5, but now with

$$
\begin{aligned}
f(\mathbf{y}_t|\mathbf{Z}_{t-1}) &= \sum_{i=1}^{k}\sum_{j=1}^{k} f(\mathbf{y}_t, s_{t-1}=i, s_t=j|\mathbf{Z}_{t-1}) \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} f(\mathbf{y}_t|s_{t-1}=i, s_t=j, \mathbf{Z}_{t-1})P(s_{t-1}=i, s_t=j|\mathbf{Z}_{t-1}), \qquad (7)
\end{aligned}
$$

where

$$
f(\mathbf{y}_t|s_{t-1}=i, s_t=j, \mathbf{Z}_{t-1}) = (2\pi)^{-r/2}|\mathbf{\Sigma}^{(j)}|^{-1/2}\exp\left\{-\frac{1}{2}\boldsymbol{\nu}_t^{(i,j)\prime}\mathbf{\Sigma}^{(j)-1}\boldsymbol{\nu}_t^{(i,j)}\right\}. \qquad (8)
$$

Note that the discrepancy $\boldsymbol{\nu}_t^{(i,j)}$ cannot be computed when $\mathbf{y}_t$ is missing. As a result, this occasion will not contribute to the likelihood function.

Model selection

As indicated at the beginning of this section, researchers may be interested in which model best describes the data at hand. To this end, the researcher has to fit several models to the data, and determine which model fits the best. Typically, model comparison and model selection is done using some sort of test statistic (e.g., the F-test, or a log likelihood difference test). However, the TAR model and the MSAR are not nested, and hence there is no statistical test that can be used to compare these models as rivaling hypotheses. Moreover, although the linear AR model is nested under both the TAR model and the MSAR model (i.e., it can be obtained by setting $k = 1$), comparing these cannot be done using standard tests because of nuisance parameters that are absent (or unidentified) under the null model. For instance, if one wishes to compare a one-regime model with a two-regime model, one can simply constrain all parameters across the two regimes such that there are no differences between the regimes and hence it becomes one regime. However, in that case the threshold parameter (in case of a TAR model), or the switching probabilities (in case of an MSAR model) can take on any value. Hence, these parameters are unidentified under the one-regime model (which is the same as saying they are absent). A similar problem occurs in mixture modeling, if one is interested in comparing models with different numbers of components (see also Frühwirth-Schnatter, 2006, p. 114-115; Strikholm & Teräsvirta, 2006). As a result, the log likelihood difference statistic has an unknown distribution.

Instead, one can make use of information criteria such as the Akaike Information Criterion (AIC; Akaike, 1973) and the Bayesian Information Criterion (BIC; Schwarz, 1978). Both criteria consist of -2 times the log likelihood, plus a penalty for the number of parameters that are estimated in the model. Smaller values point to better models. To facilitate the interpretation of whether a difference between two models should be considered large or small, one can transform the information criteria into model weights. Model weights that are based on the BIC are also referred to as posterior model probabilities (see Burnham & Anderson, 2002, p. 290), and can be interpreted as the probability that this model generated the data, given the set of models that is considered.

When information criteria are used in the TAR literature, typically the threshold parameters are not penalized (e.g., Peña & Rodriguez, 2005; Strikholm & Teräsvirta, 2006), although no rationale for this decision is given. In contrast, in the MSAR literature, the estimated transition probabilities are penalized in the same way as all other model parameters. Hence, in comparing these two models below, we follow Clements and Krolzig (1998), who penalized all model parameters (including the threshold parameters in the TAR model), but the reader is warned that there is no consensus on this issue.

Empirical application

The data that is used in this illustration come from a diary study based on the Positive Affect and Negative Affect Schedule (PANAS; Watson & Clark, 1994). The subject analyzed here was diagnosed with rapid cycling bipolar disorder. The data, consisting of daily self-reported positive affect (PA) and negative affect (NA) are presented in Figure 1. Of the total of 91 days, 17 had missing scores.

Insert Figure 1 about here

We expect this person to switch repeatedly between a manic regime and a depressed regime, and we want to investigate whether this switching is triggered by one of the variables in the system, (i.e., either by PA or NA), or whether it is governed by an autonomous latent process. To this end we consider four models. Model 1 is a linear VAR(1) model, in which there is no regime-switching, which can be denoted as

$$\begin{bmatrix} PA_t \\ NA_t \end{bmatrix} = \begin{bmatrix} \phi_{0,PA} \\ \phi_{0,NA} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} PA_{t-1} \\ NA_{t-1} \end{bmatrix} + \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} e_{t,PA} \\ e_{t,NA} \end{bmatrix}, \qquad (9)$$

where the covariance matrix of the residuals of PA and NA is $\mathbf{\Sigma} = \mathbf{\Omega}\mathbf{\Omega}'$, allowing for a correlation between the residuals at the same occasion. This model implies there are is only one regime, meaning that there are no distinct states that can be recognized as mania and depression in the self-reported affect of this person.

Models 2 and 3 are multivariate SETAR(1) models: in Model 2 PA serves as the threshold variable, while in Model 3 NA is the threshold variable. In these models it is assumed that regime-switching is triggered by affect itself. For instance, if PA is the threshold variable, this model implies that as PA increases and reaches some critical value — the threshold — the system "surrenders" so to say to a different state, which is characterized by a high level of PA. Then, as PA decreases and drops below the threshold value, the system "surrenders" again to the previous state, which is characterized by a low level of PA. Such a feedback mechanism in affect regulation can be understood as maladaptive: Rather then restoring some (non-extreme) equilibrium after the system wanders off too far from equilibrium (as one may expect in a healthy system), the switching mechanism described above implies a system that changes from one extreme to another, without the possibility to settle around some non-extreme equilibrium.

Finally, Model 4 is a multivariate MSAR(1) model. This model consists of two regimes, as is the case in Models 2 and 3, but the switching between the regimes is an exogenous mechanism, which is not influenced by affect itself.

To be able to make use of the AIC and BIC, we have to determine the number of parameters in each of the models. Model 1 contains 9 parameters (two constants, four auto- and cross-regression parameters, three elements in the covariance matrix of the residuals), Models 2 and 3 contain $(2 \times 9 =)18$ regular parameters and 1 threshold parameter, and Model 4 contains $(2 \times 9 =)18$ regular parameters and 2 transition probabilities. The results for these four models are presented in Table 1. Using the AIC and BIC to compare these models, we can conclude there is overwhelming evidence that Model 4 is the most appropriate model among this set of models (cf. Burnham & Anderson, 2002, p. 78). From the model weights presented in the last column it can be concluded that the SETAR models are extremely unlikely to have generated the data. When comparing the linear model to the MSAR model, it can be stated that the latter is over 13 times more likely to have generated the data than the linear model (.9301/.0699=13.3). From this we conclude that there is considerable evidence that the self-reported affect of this rapid cycling bipolar disorder patient can be understood as coming from two distinct regimes. Moreover, switching between these regimes is regulated by an exogenous mechanism, and is not influences by either PA or NA.

To obtain more insight in the process that is inferred by Model 4, and to determine whether we can actually interpret one of the regimes as the manic state and the other as the depressed state, we simulated a time series of 100 000 data points to determine the means and variances of the two variables in each regime. In regime 1, PA has a mean of 29.98 (SD = 7.27), and NA has a mean of 10.49 (SD = 1.30). In regime 2, PA has a mean of 12.67 (SD = 1.84), and NA has a mean of 13.49 (SD = 4.05). From this we can conclude that regime 1 is a positive regime with high scores on PA, and an absence of negative affect, while regime 2 is a negative regime, with very low PA scores and a slight increase in NA scores. Although the slight difference in NA between these regimes may seem curious, these results are in correspondence with other reports, which indicated that PA is more closely related to depression (i.e., low PA is associated with depression), while NA is more closely related to anxiety (Crawford & Henry, 2004; Jacques & Mash, 2004). Hence, we may interpret regime 1 as the manic state, and regime 2 as the depressed state.

It is also interesting to note that PA scores vary a lot in regime 1 but not in regime 2, while for NA the reverse is true. In addition, the correlation between PA and NA is .14 in regime 1, and -.65 in regime 2. The auto- and lagged cross-regressive coefficients are close to zero in both regimes. Finally, the estimated probabilities to remain in the same regime is higher for the positive regime than for the negative regime: .89 for regime 1, versus .71 for regime 2. From this we conclude that for this person the manic episodes (regime 1) are a bit more persistent than the depressed episodes (regime 2).

Finally, we estimated the probability that the person was in regime 1, and plotted these probabilities at the bottom of Figure 1. This shows that at many occasions the probability of being in regime 1 is either very close to one or very close to zero, meaning we can be fairly sure of the regime this person was in at any particular occasion. Only at the end, when there are a lot of missing observations, it becomes more difficult to determine whether the person was in regime 1 or 2.

Discussion

In this chapter we discussed two regime-switching models that can be used to study psychological processes that are characterized by shifts between different psychological states. We have presented estimation procedures for both models which can be used when there is missing data (a common issue in diary data studies), and discussed how these models can be compared using information criteria. In our empirical application we compared multivariate SETAR models — which can be understood as regime-switching processes characterized by maladaptive regulation mechanisms — to a multivariate MSAR model — which is a regime-switching process that is characterized by a latent, exogenous switching process. We compared these three models to a baseline model without regime-switching. The MSAR model proved the best description of the daily affect measurements of an individual with rapid cycling bipolar disorder.

To our knowledge this is the first empirical study that shows that there are indeed two distinct psychological states in the affect of a patient with bipolar disorder. Moreover, we have shown that the switches from mania to depression and vice versa, do not depend on affect itself, but rather depend on an exogenous process. To obtain more insight in the nature of this process, the MSAR model could be extended such that the transition probabilities can be regressed on variables that are assumed important for the switching process. For instance, Frank et al. (2005) indicate that variables such as sleep quality, interpersonal stress, and disruptions in daily routines are crucial in the onset of manic episodes. The current MSAR model could be extended such that the switching probabilities are regressed on such predictors. This would not only include the option of determining whether switching is triggered by these predictors, but it would also allow us to investigate whether switching from the manic regime to the depressed regime depends on different variables than switching from the depressed regime to the manic regime. As such, these models allow for a detailed look at the underlying mood regulating process, and offer the opportunity to test specific hypotheses about this at the level of the individual. In addition, these models could be used to investigate whether mood regulation in other (related) disorders (e.g., borderline personality disorder), is governed by the same factors, or that this is actually a differentiating feature of disorders. If the latter proves the case, investigating the specific regime-switching process in self-reported affect may become a useful tool in diagnostics in the future.

We believe the models discussed in this chapter have strong potential for elucidating underlying dynamics of psychological processes in general, including psychopathology as illustrated here. While the interpretation of these models may (in part) depend on the actual application, we hope that the current presentation is general and detailed enough that it inspires readers to consider the use of these techniques in their own fields of expertise.

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AUTHOR NOTE

The authors gratefully acknowledge Liv Pijck for gathering the data. This work was supported by the Netherlands Organization for Scientific Research (NWO), VENI grant 451-05-012 awarded to E. L. Hamaker, and VENI grant 451-06-013 awarded to R. P. P. P. Grasman.

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Table 1: Results obtained for the linear model (1), a SETAR model with PA as the threshold variable (2), a SETAR model with NA as the threshold variable (3), and an MSAR model (4)

model	-2logL	k	AIC	BIC	MW
1	443.1	9	461.1	481.9	.0699
2	427.1	19	465.1	509.9	$<.0001$
3	436.9	19	474.9	518.7	$<.0001$
4	390.6	20	430.6	476.7	.9301

NOTE: k is the number of parameters that is penalized; AIC = -2logL + $2k$; BIC = -2logL + $\log(n)k$, where n is the number of observations; MW is the model weight based on the BIC.

Notes

[1]In the original presentation of this model by Hamilton (1989), the switching concerned the mean rather than the intercept. This leads to more abrupt switches than is the case in current presentation (see Frühwirth-Schnatter, 2006, pp. 360–361). Here, for comparability to the TAR model, we use the MSAR model with switching intercept instead, and also allow all other model parameters to be regime dependent.

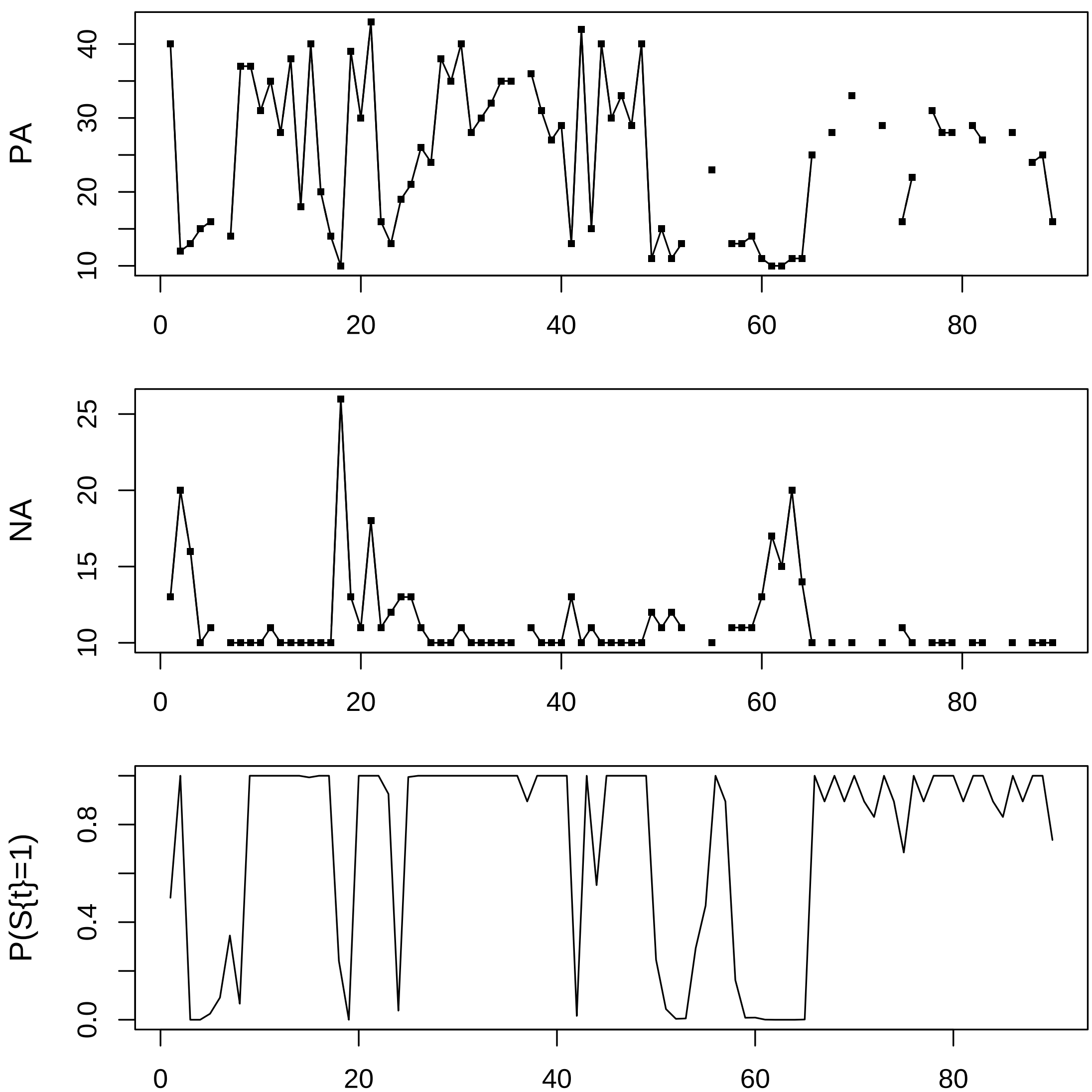
[2]The current presentation is based on Kim and Nelson (1999). In many other presentations of the Markov model, the transpose of this matrix is used, such that the row elements, rather than the column elements sum to 1.

[3]We used our own R code to estimate these models in the subsequent illustration. Currently, we are working on developing an R-package, which is an implementation of the algorithm developed by Kim (1994) for regime-switching state-space models. The latter can be used to estimate MSAR models (see also Kim & Nelson, 1999). For more information contact the first author.

[4]The currently proposed method could be extended by taking the uncertainty of the predicted unobserved values into account. That is, rather then equating $y_{t|t-1,q} = y_{t|t,q}$ we could consider the prediction distribution $N(y_{t|t-1,q}, \sigma_q^{2(j)})$ instead, where j depends on the regime the system is in at occasion t. When a prediction is made using this threshold value, one can make a prediction per regime which is then weighted by the proportion of the prediction distribution of the threshold value that falls in each of the regime areas. Although this would be a more sophisticated approach to the problem of missing data in SETAR models, it is beyond the scope of the current chapter.

Figure Captions

Figure 1. PA and NA scores of a person diagnosed with rapidly cycling bipolar disorder. At the bottom the probability of being in regime 1 as estimated with a MSAR (1) model is presented.


PA
10
20
30
40
0
20
40
60
80
NA
10
15
20
25
0
20
40
60
80
P(S{t}=1)
0.0
0.4
0.8
0
20
40
60
80