Journal of Hydrology 635 (2024) 131145

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

Gisser-Sánchez revisited: A model of optimal groundwater withdrawal under irrigation including surface–groundwater interaction

Marc F.P. Bierkens [a,b,*], L.P.H. Rens van Beek [a], Niko Wanders [a]

[a] Department of Physical Geography, Utrecht University, Utrecht, The Netherlands
[b] Unit Subsurface and Groundwater Systems, Deltares, Utrecht, The Netherlands

ARTICLE INFO

Keywords:
Groundwater
Hydroeconomic
Optimal withdrawal
Depletion
Gisser-Sánchez effect

ABSTRACT

We revisit the classic problem of determining economically optimal groundwater withdrawal rates for irrigation. The novelty compared to previous mathematical analyses is the inclusion of non-linear groundwater-surface water interaction that allows for incorporating the impact of capture, i.e. the fact that all or part of the pumped groundwater comes out of reduced surface water flow or increased recharge. We additionally included the option to internalize environmental externalities (e.g. streamflow depletion) and maximize social welfare rather than farmer's profit. This analysis results in a fixed optimal groundwater withdrawal rate q_{opt} when withdrawal q remains smaller than some critical withdrawal rate (maximum capture) q_{crit} and provides depletion trajectories, either under competition or optimal control, if q is larger than q_{crit}. Based on the relative value of q, q_{crit} and q_{opt} it also yields four quadrants of distinct withdrawal strategies. Using global hydrogeological and hydroeconomic datasets we map the global occurrence of these four quadrants and provide global estimates of optimal groundwater withdrawal rates and depletion trajectories. For the quadrants with groundwater depletion ($q > q_{crit}$) we derive and compare depletion trajectories under competition, optimal control and optimal control including environmental externalities, and assessed globally where the differences between these depletion modes are small, which is known as the Gisser-Sánchez effect. We find that the Gisser-Sánchez effect is globally ubiquitous, but only if environmental externalities are ignored. The inclusion of environmental externalities in optimal control withdrawal result in notably reduced groundwater decline and larger values of social welfare in many of the major depletion areas.

1. Introduction

Over the last few decades, the increase in population numbers, economic development and dietary changes have caused a steep increase in the demand for food and water (Godfray et al., 2010). To meet these increasing demands, irrigated agriculture has expanded into regions with limited precipitation and surface water availability (Siebert et al., 2015), leading to an ever-increasing dependence of crops on groundwater irrigation (Wada et al., 2012). This trend, in turn, has caused the steady increase of non-renewable groundwater use, i.e. when groundwater is taken out of storage that will not be replenished in human time scales, and which is associated with high rates of aquifer depletion around the globe (Konikow and Kendy, 2005; Wada et al., 2010; Gleeson et al., 2012; Döll et al., 2014; Rodell et al., 2018). Estimates of current groundwater withdrawal rates range between 600–1000 km^3/yr, resulting in groundwater depletion rates of 150–400 km^3/yr (Wada, 2016).

In areas subject to groundwater depletion, the withdrawal of groundwater has a finite time horizon since the progressive lowering of groundwater levels or hydraulic heads induces increasing pumping costs that would eventually exceed the revenues from irrigated crop production. Previous economic analyses have focused on the question how withdrawal rates should develop over time until the moment of economic depletion, when the goal is to maximize profit. These analyses provide economically optimal withdrawal trajectories that result in a maximum net present value (NPV) of the profits obtained from groundwater use and the associated groundwater decline (Burt, 1964; Burt, 1967; Domenico et al., 1968; Brown and Deacon, 1972). Such "optimal control" withdrawal trajectories require full cooperation between the farmers that pump groundwater from the same aquifer, or the existence of a water authority that enforces withdrawal rates at all times. If farmers that pump water from the same aquifer have a myopic attitude

* Corresponding author at: Department of Physical Geography, Utrecht University, Utrecht, The Netherlands.
E-mail address: m.f.p.bierkens@uu.nl (M.F.P. Bierkens).

https://doi.org/10.1016/j.jhydrol.2024.131145
Received 16 May 2023; Received in revised form 9 December 2023; Accepted 10 March 2024
Available online 9 April 2024

and are therefore non-cooperative, they are in competition and individually try to maximize profit each year instead of accumulated over time. This occurs because they cannot expect the groundwater that they leave in storage to be available to them next year, since it may have been used by a neighboring farmer. In theory, the case of optimal control withdrawal would result in less groundwater depletion and more accumulated profits (in terms of net present value NPV) from the depletion of groundwater then withdrawal under competition. Gisser and Sánchez (1980a) however showed mathematically that both modes of withdrawal yield similar results in case the aquifer storage capacity is large, confirming a previous empirical result (Gisser and Sánchez,1980b). Koundouri (2004) provided an extensive review of later studies that looked at this so-called Gisser–Sánchez effect and showed that it seems to hold in most cases. Exceptions are cases where pumping costs become very large as the aquifer is close to total depletion (Koundouri, 2000), where the relationship between pumping costs and head is very nonlinear (Worthington et al., 1985; Foster et al., 2015) or in case additional benefits of saving groundwater are included (MacEwan et al., 2017). Examples of the latter include the benefits of using groundwater as a drought-risk reserve (the ability to irrigate high-valued crops under drought) and avoiding capital costs associated with stranded assets, i.e. dry wells (Jasechko and Perrone, 2021). Additionally, Esteban and Albiac (2011) showed that the Gisser–Sánchez effect does not necessarily hold if the optimal control also includes environmental externalities, i.e. the costs associated with the negative impacts on the environment that occur due to groundwater withdrawal.

In most of this previous work, particularly the analytical treatments of the optimal groundwater withdrawal problem, the interaction between groundwater and surface water is not considered. This leads to two related issues. First, by avoiding groundwater-surface water interaction, it is wrongly assumed that all pumped groundwater comes out of storage, while a considerable part may result from a decrease of

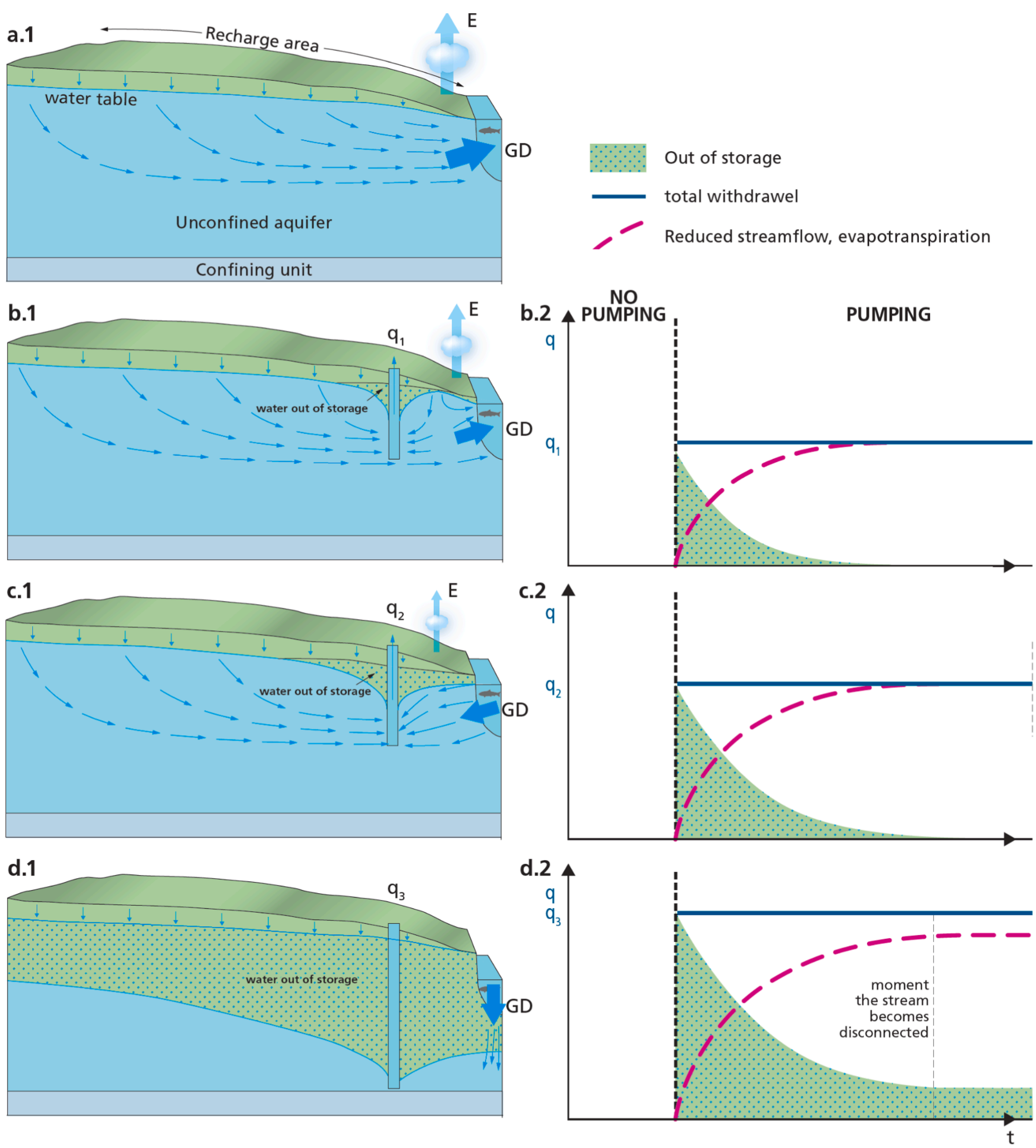

Fig. 1. Schematic overview of groundwater-surface water dynamics affected by groundwater withdrawal. a: gaining stream, natural conditions; b: gaining stream, limited withdrawal q_1. c: connected losing stream, higher withdrawal rate q_2. d: disconnected losing stream, even more intense groundwater withdrawal rate q_3; b and c are regimes with physically stable withdrawal rates leading to some equilibrium groundwater decline, while d is a regime with physically non-stable withdrawal rates if q_3 is larger than recharge over the depression cone and stream infiltration; Figures a.1, b.1, c.1 and d.1 show schematic cross-sections of the groundwater-surface water interaction; Figures b.2, c.2 and d.2 portray the relative contributions of storage change and capture to total withdrawal; modified from Alley et al (1999) and Konikow and Leake (2014); credit to the United States Geological Survey.

groundwater discharge to streams, a decrease in evaporation and infiltration of streams into groundwater aquifers. This is illustrated in Fig. 1 taken from Bierkens and Wada (2018), who in turn have based the figure on the classic paper by Theis (1940) and subsequent explanations by e.g., Alley et al (1999), Bredehoeft (2002) and Konikow and Leake (2014). Fig. 1 shows that for lower withdrawal rates (Fig. 1b, c), initially, right after the commencement of withdrawal, the pumped groundwater comes mostly out of storage (see the contribution of storage change in b.2 and c.2). However, as sustained withdrawal leads to a reduction of the water table, this induces a negative feedback by decreasing groundwater discharge to the stream (or even reversing the flow to streambed infiltration, Fig. 1c.1) and reducing evaporation, together called "capture". This, in turn, limits further water table decline to a point that a new equilibrium is reached where all water comes out ca pture and, aside from seasonal and inter-annual variation, no further decline of the water table occurs. If withdrawal rates are so high that they exceed maximum capture, i.e., the sum of the maximum recharge over the depression cone and the stream infiltration (Fig. 1d), the water table will become disconnected from the stream (Brunner et al., 2009). The negative feedback from increased capture no longer exists, while the withdrawal rate in excess of the maximum capture will come out of storage leading to a persistent decline in groundwater levels (Fig. 1d.2). Thus, the impact of groundwater withdrawal on groundwater level decline is non-linear.

Second, the impact of groundwater-surface water interaction is that there are two distinct regimes of withdrawal: withdrawal rates below some critical threshold q_{crit} (maximum capture) where groundwater decline is limited and groundwater withdrawal is physically stable, i.e. can be maintained over time (Bierkens and Wada, 2019), and withdrawal rates above this limit that lead to persistent aquifer depletion. These regimes are subject to two distinct types of hydroeconomic analysis, in that for the physically stable or equilibrium regime an optimum withdrawal rate may be sought that maximizes yearly profit, akin to renewable resources such as restricted fishing grounds and forestry (Halvorsen and Layton, 2015), while for the physically non-stable or depleting regime, methods from the economics of non-renewable resources (Hotelling, 1931; Halvorsen, 2018) are applicable that aim for intertemporal efficiency by finding depletion trajectories that maximize the net present value of profits over time (Burt, 1964; Burt, 1967; Domenico et al., 1968; Brown and Deacon, 1972).

Note that in reality the change of a groundwater level-dependent to a groundwater-independent groundwater-surface water flux is less abrupt than assumed here. When the water table is just below the river bottom, negative pressure heads occur below the riverbed while the soil is fully or partly saturated (Brunner et al., 2009, 2011). Wang et al. (2016) show experimentally and theoretically that a full disconnection, i.e. the water table has no impact on the infiltration flux, occurs only when the depth of the groundwater table below the stream becomes larger than the stream water depth.

In this paper, we revisit the classic problem of determining economically optimal groundwater withdrawal rates for irrigation. We present a hydrogeological-hydroeconomic model that can be used as a quick analytical tool to determine, at first order, optimal groundwater withdrawal strategies at large scales. The lumped-conceptual nature of the model ensures tractability and results in closed form analytical solutions. The novelty of this model compared to previous mathematical analyses is the inclusion of non-linear groundwater-surface water interaction that allows for incorporating the impact of capture and leads to different forms of hydroeconomic optimality for the physically stable (equilibrium) and physically non-stable (depleting) regimes. The model also allows for the inclusion of environmental externalities. In this case "optimal withdrawal" pertains to a wider definition of optimality, i.e. changing it from maximizing farmers' profits to maximizing social welfare. Another novelty is that we apply the analytical model globally, based on global agroeconomic datasets (e.g. FAO (2021a, 2021b); World Bank (2020)) and parameters and outputs from a global hydrological model (Sutanudjaja et al., 2018), allowing us to: 1) assess for the current groundwater irrigated areas which optimality regime applies; 2) for the areas with equilibrium regimes (currently no groundwater depletion) how current withdrawal rates differ from optimum withdrawal rates and how this relates to reduced profit; 3) and, for the areas where groundwater depletion occurs, the difference of maximum groundwater decline and NPV under competition, optimal control and optimal control including environmental externalities, thereby testing the applicability of the Gisser–Sánchez effect globally.

The remainder of this paper is set up as follows. First, we briefly describe a lumped conceptual model of large-scale groundwater withdrawal that includes groundwater–surface water interaction as previously introduced by Bierkens et al. (2021). Second, we introduce the hydroeconomic model, which is built on the Bierkens et al. (2021) representation and which is used to derive the optimal groundwater withdrawal rate for the physically stable (equilibrium) regime and the full competition and optimal control depletion trajectories for the physically non-stable (depleting) regime. Third, we describe the datasets that are used for a global application of the hydroeconomic model. Next, we show the results of the global application, where we elect to show global results as relative differences rather than absolute values. This is to stress that, due to the many uncertainties in parameters at the global scale, we are confident to use the hydroeconomic model to show relative magnitudes, but not predicting absolute quantities. Additionally, we further explore the robustness of the global results using a sensitivity analysis. Finally, we end with a discussion and the main conclusions.

2. Conceptual model of large-scale groundwater withdrawal with groundwater–surface water interaction

Bierkens et al. (2021) introduced a lumped conceptual model of large-scale groundwater withdrawal including groundwater-surface water interaction (Fig. 2). Here we provide short summary of their model description. The following assumptions underly this lumped conceptual model (also named "hydrogeological model" hereafter): 1) groundwater withdrawal is limited to irrigation only, and we limit our analyses to irrigated agriculture; 2) The aquifer is represented as a single cell ("bathtub") whose size is unknown and whose properties are captured by an average specific yield and a drainage resistance parameter, effectively lumping aquifer properties determining groundwater-surface water interaction. As a consequence, the model neglects

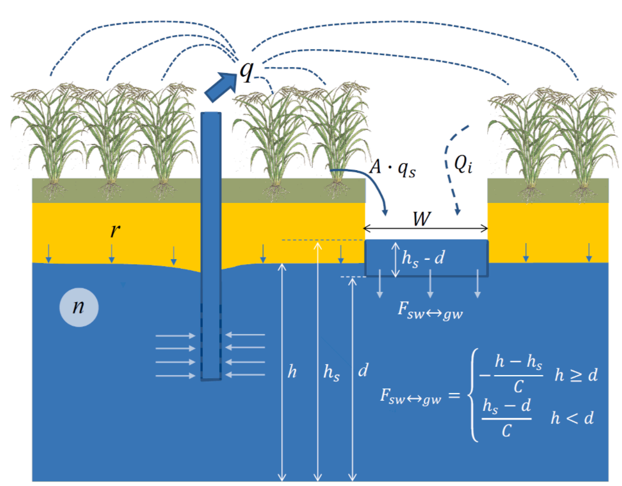

Fig. 2. Lumped conceptual model of groundwater withdrawal for irrigation from an aquifer recharged by diffuse recharge and riverbed infiltration (also referred to as "hydrogeological model" in the text); the symbols are explained in the text; note that the slight depression cone shown close to the well is actually not in the model, since we assume the withdrawal to be a diffuse sink representing the collective withdrawal of many farmers.

(lateral) groundwater flow processes within the aquifer; 3) the total rate of groundwater withdrawn sums up the withdrawal rates of a large number of landowners that all draw water from the same aquifer that can hence be seen as a common pool resource. This entails that withdrawal is treated as a diffuse sink; 4) access to the aquifer is limited to the landowners sitting on top of the aquifer; 5) groundwater recharge is the sum of diffuse recharge from precipitation and concentrated recharge from riverbed infiltration. The river discharge that is the source of concentrated recharge comes from local surface runoff and inflow from upstream areas outside the area of interest; 6) All boundary fluxes, i.e., withdrawal rate, surface runoff and upstream river discharge are constant in time, neglecting seasonal variations that usually occurs due to variation in weather and crop water demand.

We note that assumption 1 limits the model application to aquifers where groundwater is predominantly exploited for irrigated agriculture. Assumptions 2–4 are reasonable in case the analyses are limited to larger (regional or aquifer) scales and in case there are many landowners and thus many wells evenly spread across the aquifer that all withdraw groundwater from this common resource. It is unclear what the errors are that are related to assumption 6, since this depends on how temporal variability of boundary conditions interacts with the non-linear threshold behaviour of the groundwater-surface water interaction. It would lead to biased estimates of the mean groundwater level decline in case groundwater levels switch between the equilibrium and the depleting regimes, either seasonally or inter-annually. If groundwater level variation at seasonal and inter-annual time scales tends to reside in one regime and regime shifts only occur under long term changes in withdrawal rates, the system is quasi-linear and allows for reasonable estimates of its long-term mean behaviour with mean annual boundary conditions. The results of the lumped conceptual hydrogeological model were extensively compared to dynamic global hydrological model results and GRACE gravity anomalies at the global scale and with the results of groundwater flow models at the continental to regional scales with satisfactory results (see the Supplement to Bierkens et al., 2021).

Based on the simplifications described above, the change of groundwater level h (Fig. 2) can be represented by a simple linear differential equation of the total aquifer mass balance:

$$n\frac{dh}{dt} = r + F_{gw\leftrightarrow sw}(h) - q \tag{1}$$

with

h: groundwater head (m) with respect to some reference level (e.g. the bottom of the aquifer);

n: specific yield (−);

q: withdrawal rate per area (m^3 m^{-2} yr^{-1});

$F_{gw\leftrightarrow sw}$: surface water infiltration (or drainage) flux density (m^3 m^{-2} yr^{-1}).

The groundwater – surface water flux (positive towards groundwater) is modelled as follows:

$$F_{gw\leftrightarrow sw}(h) = \begin{cases} -\dfrac{h-h_s}{C} & h \geq d \\ \dfrac{h_s-d}{C} & h < d \end{cases} \tag{2}$$

with

h_s: the surface water level (m);

d: elevation of the bottom of the water course (m); this is used as the absolute reference level of the model;

C: drainage resistance (years) which pools together all the parameters of surface-water groundwater interaction, i.e., the density or area fraction of surface waters, surface water geometry and river/lake-bed conductance and the transmissivity of the aquifer.

The surface water level itself is a variable which is related to the surface water discharge Q (m^3 yr^{-1}) and the groundwater level as follows:

$$Q = Wv(h_s - d) = Q_i + q_sA - F_{gw\leftrightarrow sw}(h)A \tag{3}$$

with

A: The area over the (sub-)aquifer considered (m^2);

q_s: surface runoff (m yr^{-1}); also including shallow subsurface storm runoff;

Q_i: influx of surface water from upstream (m^3 yr^{-1}); accounts for aquifers in dry climates where the surface water system is fed by wetter upstream areas, e.g., rainfall or snow from mountain areas;

W: Stream width (m);

d: Bottom elevation stream (m);

v: Stream flow velocity (m yr^{-1}).

Equations (1)-(3) together describe the coupled surface water-groundwater system where all parameters and boundary fluxes (q, r, Q_i, q_s) are assumed constant over time and groundwater head h and surface water levels h_s change over time as a result of groundwater withdrawal only. In Bierkens et al. (2021) expressions are derived for $h(t)$ and $h_s(t)$ and derived variables as streamflow $Q(t)$ and the fraction of groundwater pumped that comes out of capture and storage. These expressions are different for the equilibrium regime, where groundwater withdrawal rate is smaller than maximum capture ($q < q_{crit}$) and groundwater head decline is limited and the depleting regime ($q > q_{crit}$) where groundwater heads keep falling.

Appendix A provides a table with the derived results and an explanation of the variables involved. For the remaining part of the paper the following quantities are important

Composite variables α (m) and β (−). They have been derived in Bierkens et al. (2021) and link the surface water level to the groundwater level in case of a two-way interaction that occurs when the stream is still connected ($h > d$), i.e. $h_s(t) = \alpha + \beta h(t)$. These composite variables are used for brevity of the expressions and defined as:

$$\alpha = \frac{Q_iC + q_sAC + WvdC}{WvC + A} \tag{4}$$

$$\beta = \frac{A}{WvC + A} \tag{5}$$

The critical withdrawal rate q_{crit}, also called maximum capture, separates withdrawal rates that lead to an equilibrium regime or a depleting regime is given by:

$$q_{\mathrm{crit}} = r + \frac{Q_i + q_sA}{WvC + A} \tag{6}$$

The equilibrium groundwater decline or drawdown s (m) for the equilibrium regime ($q < q_{crit}$) is:

$$s = h(0) - h(\infty) = \frac{Cq}{1-\beta} \tag{7}$$

with $h(0)$ the groundwater head under natural circumstances (without withdrawal) and $h(\infty)$ the final groundwater head after equilibrium decline has been reached.

The groundwater decline $s(t)$ for the groundwater depleting regime, which is valid after the water table is disconnected from the surface water and persistent decline sets in ($q > q_{crit}$ and $h(t) < d$) (see Table A1 and Appendix A):

$$s(t) = \frac{rC + \alpha}{1-\beta} - h(t) \tag{8}$$

with $h(t)$ being obtained from the following differential equation:

$$\frac{dh}{dt} = \left[\frac{Ar + Q_i\beta + Aq_s\beta}{nA}\right] - \frac{q}{n} \quad h(0) = d \tag{9}$$

3. Hydroeconomic model for optimal groundwater withdrawal rates

After listing its assumptions and limitations, we present the final closed form equations of the hydroeconomic model, separately for the equilibrium regime and for depleting regime. The derivations leading to these equations are given in Appendices B, C and D.

3.1. A hydroeconomic model: assumptions and limitations

We present an analytical hydroeconomic model of optimal groundwater withdrawal rates for irrigation including groundwater surface water interaction. It is based on the following assumptions: 1) we consider an aquifer system with groundwater surface water interaction that is modelled with the lumped conceptual model of (Bierkens et al., 2021) (Section 2); 2) the groundwater can only be accessed by the farmers sitting on top of it. So, we assume exclusive access; 3) we assume that a farmer irrigates a single crop; 4) crop prices follow a linear inverse demand function (constant elasticity); 5) crop yield is assumed to be only dependent on irrigation water applied following a linear production function. This implies that all other production factors are assumed optimal; 6) the costs of groundwater withdrawal are proportional to withdrawal rate and the depth of the water table (m^3 m^{-1}); 7) all other production costs are lumped into a single factor that reduces the unit price of the crops; i.e. if p' is the original crop price (USD kg^{-1}), and f is the fraction of non-water related production costs, we use the value $p = (1-f) \bullet p'$ in our calculations; 8) farmers are either in competition (i.e. myopic or fully non-cooperative) trying to maximize their own profit at every time or fully cooperative or optimally controlled, i.e. maximizing profit or social welfare across time; 9) safe for changing the withdrawal rates, farmers are non-adaptive when the costs of groundwater withdrawal increase; 10) when included, environmental externalities are represented as additional unit withdrawal costs (USD m^{-3}), i.e. proportional to withdrawal rate q.

We note that the majority of these assumptions are similar to the hydroeconomic representations of Burt, 1964, 1967; Domenico et al., 1968; Gisser and Sánchez, 1980a, Esteban and Albiac, 2011. They serve the tractability of the model to arrive at closed form solutions to the hydroeconomic optimization problems. We will elaborate on the disparities between these assumptions and reality and possible generalizations of the model in the Discussion section at the end of the paper.

3.2. The equilibrium regime (q and $q_{opt} < q_{crit}$): Competition for a renewable resource

The optimal withdrawal rate q_{opt} that maximizes profit from using water for irrigation is derived assuming that withdrawal rates are smaller than maximum capture ($q < q_{crit}$; equilibrium regime) and additionally that the optimal withdrawal rate is also in the equilibrium regime ($q_{opt} < q_{crit}$).

As stated above, we assume that the aquifer can only be accessed by the farmers sitting on top of it. However, even though there is a limited number of farmers with access to the aquifer's groundwater, they all suffer from pumping externalities: the development of groundwater depth over time and the associated costs of extraction depends on the withdrawal rates of all the farmers. In the absence of any regulation, the total groundwater withdrawal over the aquifer is expected to be the outcome of the myopic decisions of the individual farmers, each one maximizing their own profits. The collective optimal withdrawal rate q_{opt} that maximizes profit can then be obtained by finding the withdrawal rate for which marginal costs equal marginal benefits.

In accordance with a free market economy, we assume the crop prices to follow an inverse demand function (assumed linear here):

$$p = p_0 - \frac{Y}{k} \tag{10}$$

with p (USD kg^{-1}) the net crop price, which is defined as the producer's price (price the farmer gets for produce that leaves his property) minus the sum of all unit production costs except water (e.g., land, labour, fertilizer etc.), p_0(USD kg^{-1}) the intercept of the inverse demand function, Y crop yield (kg m^{-2} yr^{-1}) and k the elasticity of the demand (kg^2 USD^{-1} m^{-2} yr^{-1}).

To relate yield to water use we assume a Cobb-Douglass type production function:

$$Y(q) = aq^b \tag{11}$$

with $0 \le b \le 1$. We further assume that $b = 1$, which entails that the pre-factor a (kg m^{-3}) can be interpreted as the water productivity for the given crop.

The cost of groundwater withdrawal is assumed to be proportional to the groundwater head decline s (m) and the abstraction rate q (m^3 m^{-2} yr^{-1}):

$$C_q(s,q) = p_p s q \tag{12}$$

with p_p (USD m^{-3} m^{-1}) the pumping costs per m^3 water per m groundwater depth. With equation (7) for the final decline that occurs under an equilibrium regime associated with renewable groundwater use we have:

$$C_q(h,q) \equiv C_q(q) = \frac{p_p C q^2}{1-\beta} \tag{13}$$

With equations (10, 11 and 13) the optimum withdrawal rate that maximizes profit can be found by equating marginal costs to marginal benefits (Appendix B) to find:

$$q_{opt} = \frac{k a p_0 (1-\beta)}{a^2(1-\beta) + 2k p_p C} \tag{14}$$

$$\pi_{\max} = \frac{k a^2 p_o^2 (1-\beta)}{2a^2(1-\beta) + 4k p_p C} \tag{15}$$

with q_{opt} the optimal withdrawal rate (m^3 m^{-2} yr^{-1}) and $\pi_{\max}$ (USD m^{-2} yr^{-1}) the maximum attainable profit.

Note that as long as $q_{opt} < q_{crit}$ and no other costs or benefits are included, we have that the optimal withdrawal rate found under competition (Equation 14) would be the one that also maximizes social welfare. It is, however, not guaranteed that the optimal withdrawal rate is smaller than the critical withdrawal rate, i.e., that $q_{opt} < q_{crit}$. In case $q_{opt} > q_{crit}$ economic incentives would drive the aquifer system from an equilibrium regime to a state of persistent aquifer depletion. Thus, for the equilibrium regime to be possible we should in fact require that: $q < q_{crit}$ and $q_{opt} < q_{crit}$. In that case, maximizing profits while maintaining a stable withdrawal regime requires some form of cooperation or control.

Also note that if Q_i and q_s are zero (See Appendix A) and the optimal withdrawal rate is close to the critical withdrawal rate this will lead to a complete capture of all the recharge and a streamflow close to zero. Thus, what may be deemed optimal in terms of profit maximization may not be maximizing social welfare. For this, all costs and benefits, including environmental externalities, i.e. the loss of streamflow by capture, should be included in the optimization. The inclusion of environmental externalities can for instance be done by adding the cost C_{dQ} of streamflow loss $dQ = Q(q) - Q(q=0)$. Since at equilibrium all pumped water comes out of capture, these costs are proportional to the withdrawal rate q: $C_{dQ} = \gamma q$ (with the unit cost γ in USD m^{-3} pumped). If we include these costs in the profit function (see Equation B7) and we assume that farmers now maximize social welfare in the same way, thus profit maximization but with including these extra costs (which may be internalized in the form of cooperation or enforced by e.g. taxes), the optimal pumping rate and maximum profit including the costs of externalities then become:

$$q_{opt,e} = \frac{k(ap_0 - \gamma)(1-\beta)}{a^2(1-\beta) + 2kp_pC} \quad (16)$$

$$\pi_{\max,e} = \frac{k(ap_o - \gamma)^2(1-\beta)}{2a^2(1-\beta) + 4kp_pC} \quad (17)$$

Equation (17) shows that the farmer's profit will decrease. However, if we compare the value of social welfare including environmental externalities $\pi_{max,e}$ with the social welfare of optimal withdrawal without these externalities $\pi_{max} - \gamma q_{opt}$, we find that $\pi_{max,e} \geq \pi_{max} - \gamma q_{opt}$, i.e. social welfare is larger if environmental externalities are included (Appendix B).

Using this formulation, it is also possible to calculate the value of γ that ensures that the streamflow remains above a given environmental flow requirement Q_{env} or the value of γ that prevents the optimal groundwater withdrawal from exceeding the critical withdrawal rate q_{crit} (see Equation B22).

To illustrate the hydroeconomic model we show a case that represents an aquifer with an area of 1000 km^2 in a semi-humid climate, with surface runoff and recharge of approximately 180 mm yr^{-1}, an upstream discharge of the stream of 50 m^3/s, an effective river-bed width of 20 m, a river flow velocity of 1 m s^{-1}, a specific yield of 0.3 and a C parameter of 1000 days (parameters in SI units Table 1). The C value represents a medium density of perennial streams and/or a medium to large transmissivity. Under this hydrogeological and climatological setting, we assume hydroeconomic parameter values that are thought to be typical for wheat production in the U.S. (FAO, 2021a, b). This case results in a critical withdrawal rate q_{crit} of 83 mm yr^{-1}, while the optimal groundwater withdrawal rate q_{opt} is 81 mm yr^{-1}, just small enough to ensure that the equilibrium case state can persist.

In Fig. 3 we show the impact of hydrogeological and climatological setting on q_{crit} and q_{opt}. On the x-axis we show a pre-factor representing climate dryness/wetness as compared the standard case in Table 1. So, the factor 0.2 on the left represents a drier climate where rainfall is thought to be only 20 % of the standard precipitation and the factor 2 on the righthand side a wetter climate with twice as much precipitation. Accordingly, we scale the surface runoff and recharge with the same factor and the C parameter with the inverse of that factor. The latter is based on the notion that density of perennial streams decreases with dryness, and the parameter C, representing groundwater-surface water interaction, increases. One can thus imagine moving from a wetter climate to a drier climate from right to left, for instance, moving from north to south on the High Plains Aquifer.

Fig. 3 shows that q_{crit} (blue line) is much more sensitive to hydrogeological parameters than q_{opt} (orange line). It also shows that moving from a humid to a semi-arid climate results in q_{opt} to become larger than q_{crit}, which means that farmers that try to optimize their profits in such climates end up in a situation with persistent groundwater depletion. Fig. 3 also shows what happens if we take environmental externalities into account by taxing water withdrawal with $\gamma = 0.05$ USD m^{-3} (dashed orange line). The reduced optimal withdrawal rates make that profit maximization and an equilibrium (physically stable) regime remain possible under drier conditions. Of course, the water tax negatively impacts the farmers' profits, reducing these for the entire area of 1000 km^2 from 176 million to 130 million USD yr^{-1}. However, if we calculate total social welfare, (profits of optimal pumping rates minus the environmental costs), not taking externalities into account results in a smaller value of 127 million USD yr^{-1} than including environmental externalities: 130 million USD yr^{-1}.

Table 1
Hydrogeological and hydroeconomic parameters used in the example shown in Fig. 3. See the text for an explanation of the symbols.

Hydrogeological parameters			Hydroeconomic parameters (Wheat)		
A	1.000 10^9	m^2	p_0	0.18	USD kg^{-1}
q_s	0.185	m yr^{-1}	k	11	kg^2 USD^{-1} m^2 yr^{r-1}
Q_i	1.578 10^9	m^3 yr^{-1}	a	2	kg m^{-3}
d	295	m	p_p	0.001	USD m^{-1} m^{-3}
W	20	m			
v	3.078 10^7	m yr^{-1}	γ	0.05	USD m^{-3}
C	2.74	yr			
n	0.3	–			
r	0.187	m yr^{-1}			

3.3. Groundwater withdrawal for a depletion withdrawal regime ($q > q_{crit}$, $h < d$) under competition and optimal control

In case the withdrawal rate exceeds maximum capture ($q > q_{crit}$) groundwater head h will eventually fall below the bottom of the stream network ($h < d$), after which persistent depletion sets in. From the analysis above, it also follows that in case $q < q_{crit}$, but at the same time $q_{opt} > q_{crit}$, it is likely that withdrawal rates when driven by economic optimization increase until they exceed the critical rate, again leading to persistent depletion. As stated before, under a depleting regime there are two opposite cases by which groundwater is exploited. The first is competition, where each user is myopic and strives to maximize their profit at every time, and the second when either there is one single owner of the groundwater resource or when all farmers cooperate to maximize the net present value of total profits over time (Burt, 1965;1967). The "optimal control" depletion entails finding a withdrawal strategy $q(t)$ that maximizes:

$$\Pi = \int_0^\infty \{R(q(t)) - C(q(t), h(t))\} e^{-it} dt \quad (18)$$

subject to Equation (9) that describes how $q(t)$ and $h(t)$ are related. Here, $R(q)$ is the revenue from irrigated agriculture, $C(q,h)$ the withdrawal costs and i the discount rate. Appendices C and D provide the derivations of the withdrawal $q(t)$ and depletion $h(t)$ trajectories for respectively the competition depletion case and the optimal control depletion case, i.e. by maximizing Equation (18). These have the general form of:

$$q(t) = B - n\kappa_i(d - A_i)e^{\kappa_i t} \quad (19)$$

$$h(t) = A_i + (d - A_i)e^{\kappa_i t} \; i = 1, 2 \quad (20)$$

With the following values for the coefficients:

$$B = r + \frac{(Q_i + q_sA)}{WvC + A} \quad (21)$$

Competition ($i = 1$):

$$A_1 = \frac{a^2}{kp_p}\left\{r + \frac{(Q_i + q_sA)}{WvC + A}\right\} - \frac{ap_0}{p_p} + \frac{rC + \alpha}{1-\beta} \quad (22)$$

$$\kappa_1 = -\frac{kp_p}{na^2} \quad (23)$$

Optimal control ($i = 2$):

$$A_2 = \left(\frac{1}{i} + \frac{a^2}{kp_p}\right)\left\{r + \frac{(Q_i + q_sA)}{WvC + A}\right\} - \frac{ap_0}{p_p} + \frac{rC + \alpha}{1-\beta} \quad (24)$$

$$\kappa_2 = \frac{i - \sqrt{i^2 + 4\frac{kip_p}{na^2}}}{2} \quad (25)$$

Like Gisser and Sánchez (1980), Appendix D also shows under which circumstances the results for competition resemble those of optimal control. This is the case if: (a) porosity n is large (productive aquifers), (b) the water productivity a is high; (c) pumping costs p_p are low; (d) the price elasticity k is small; (e) the discount rate i is high.

As with the equilibrium regime, we can include the environmental externalities in the case of the depletion under optimal control. We keep

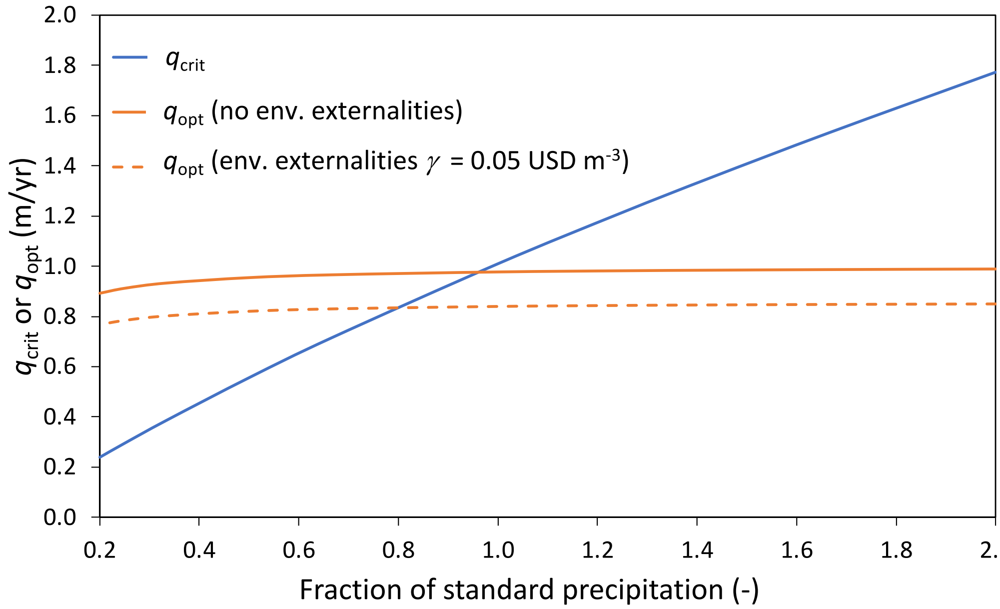

Fig. 3. Example of dependence of critical withdrawal rate q_{crit} (blue line) and economically optimal withdrawal rate under presumed equilibrium q_{opt} (solid orange line) on climate wetness/dryness, with a drier climate than the standard case on the left and a wetter climate on the right; the value 1 at the x-axis is the standard case with parameters in Table 1; a value of 0.2 means one fifth of the standard precipitation, a value of 2.0 twice the standard precipitation; profit maximization leads to an equilibrium regime as long as $q_{opt} < q_{crit}$; the dashed orange line shows the impact of including environmental externalities. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the additional environmental costs equal to γq considering that, even if streamflow is no longer affected above the value belonging to $q = q_{crit}$, additional environmental externalities are likely to occur through falling groundwater levels that are not accounted for in the simple representation of Fig. 1, such as the desiccation of phreatophytes, remote wetlands and springs and land subsidence that may also lead to irreversible aquifer storage reduction. This is very similar to way environmental externalities were included by Esteban and Albiac (2011). Including the environmental costs to γq into account in Equation (D1) yields the same result as equations (19)-(21), (24), (25), but with the substitution (with $\gamma < ap_0$)

$$p_o \rightarrow p_{o,e} = \frac{ap_o - \gamma}{a} \tag{26}$$

As expected, including the environmental externalities leads to decreased head decline and pumping rate under optimal control, as follows from the smaller value of A_2 (Equation (24)).

When evaluating the occurrence of the Gisser-Sánchez effect we can now compare three cases: depletion under competition $q_{cmp}(t)$, depletion under optimal control $q_{ctr}(t)$, and depletion under optimal control including environmental externalities $q_{ctr,e}(t)$. Apart from comparing the NPV of the farmer's profit over time (Equation D1) of these cases, i.e. Π_{cmp}, Π_{ctr} and $\Pi_{ctr,e}$, adding environmental externalities also requires comparing the NPV of social welfare of the three cases:

$$\Pi_{cmp(sw)} = \int_0^\infty \left\{R\left(q_{cmp}(t)\right) - C\left(q_{cmp}(t), h_{cmp}(t)\right) - \gamma q_{cmp}(t)\right\} e^{-it} dt \tag{27a}$$

$$\Pi_{ctr(sw)} = \int_0^\infty \left\{R(q_{ctr}(t)) - C(q_{ctr}(t), h_{ctr}(t)) - \gamma q_{ctr}(t)\right\} e^{-it} dt \tag{27b}$$

$$\Pi_{ctr,e(sw)} = \Pi_{ctr,e} = \int_0^\infty \left\{R\left(q_{ctr,e}(t)\right) - C\left(q_{ctr,e}(t), h_{ctr,e}(t)\right)\right\} e^{-it} dt \tag{27c}$$

For illustration, we show a case (parameters in SI units in Table 2), which represents a semi-arid area over an aquifer of 1000 km^2, without

Table 2
Hydrogeological and hydroeconomic parameters used in the examples shown in Fig. 4. See the text for an explanation of the symbols.

Hydrogeological parameters			Hydroeconomic parameters (Wheat)		
A	1.000 10^9	m^2	p_O	0.18	USD kg^{-1}
q_s	0	m yr^{-1}	k	11	kg^2 USD^{-1} m^2 yr^{-1}
Q_i	1.578 10^9	m^3 yr^{-1}	a	2	kg m^{-3}
d	295	m	p_p	0.0015	USD m^{-1} m^{-3}
W	20	m	i	0.03	yr^{-1}
v	3.078 10^7	m yr^{-1}			
C	13.7	yr	γ	0.05	USD m^{-3}
n	0.3	–			
r	0.183	m yr^{-1}			

surface runoff, a recharge of approximately 180 mm yr^{-1}, an upstream discharge of the stream of 50 m^3/s, an effective river-bed width of 20 m, a river flow velocity of 1 m s^{-1}, a specific yield of 0.3 and a C parameter of 5000 days. The high C parameter entails a limited density of perennial streams and/or a medium to small transmissivity. Under this hydrogeological setting we again assume hydroeconomic parameter values thought to be typical for wheat production in the U.S. (FAO, 2021a, b). We assume a discount rate of 3% per year and also include a case with environmental externalities with a unit cost of $\gamma = 0.05$ USD m^{-3} groundwater pumped. From Equation (6) we find that the critical withdrawal rate q_{crit} for this case is 0.35 m yr^{-1}, while the optimal groundwater withdrawal under presumed equilibrium conditions (from Equation (14) q_{opt} is 0.89 m yr^{-1} (0.76 m yr^{-1} if we tax the environmental externalities). If q tends toward q_{opt} or exceeds q_{crit}, we expect a depletion regime.

Fig. 4a and Fig. 4b provide the withdrawal trajectories over time (as m^3 m^{-2} yr^{-1}) and the groundwater depletion, i.e., head, over time (in m) for the competition, optimal control and optimal control including externalities. Fig. 4a shows that in case of competition, early-time withdrawal rates are higher than in case of optimal control depletion, leading to a larger final groundwater depth (Fig. 4b). This is even more the case if environmental externalities are included. Following Esteban and Albiac (2011), Fig. 4b also shows that by penalizing environmental

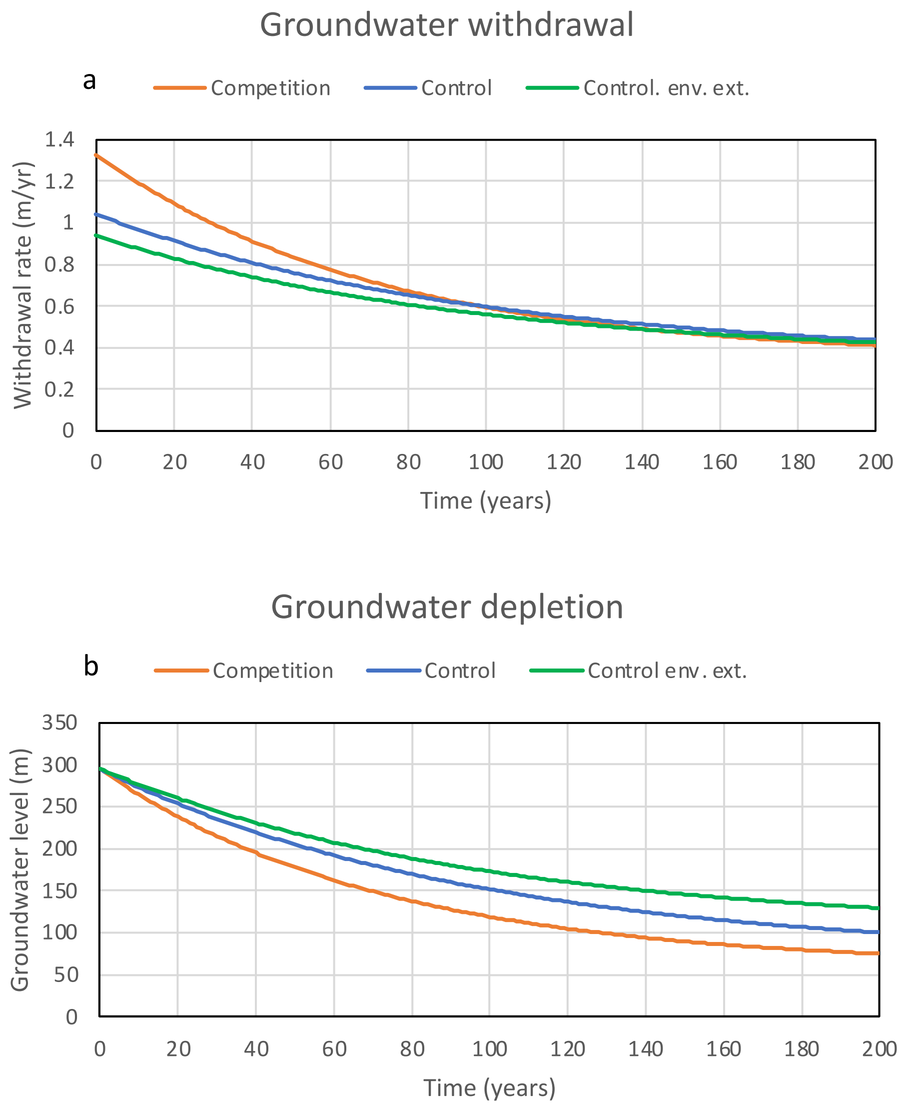

Fig. 4. Withdrawal trajectories (a) and groundwater depletion (head decline) (b) under competition (red lines) and optimal control (blue lines) and optimal control with environmental externalities (green lines). Hydrogeological and hydroeconomic parameters are given in Table 2. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

externalities the differences in depletion between the competition and optimal control cases become more pronounced. The net present value of all future profits (Equation (18) for the 1000 km^2 area overlying this aquifer equals 2.32 billion USD for competition, 3.47 billion USD for optimal control and 2.49 billion USD for optimal control with environmental externalities. It shows that even if environmental externalities are included, slightly larger profits are achieved under full cooperation. In case we calculate social welfare by reducing the farmers' profit with the cost of the environmental externalities (Equations 27), we find the highest value for optimal control with environmental externalities (2.49 billion USD), followed by optimal control (2.00 billion USD) and competition (0.59 billion USD).

3.4. Four quadrants of groundwater withdrawal strategies

Based on the analyses and results in sections 3.2 and 3.3 it becomes clear that one can distinguish four quadrants of groundwater withdrawal strategies, depending on the values of q_{crit} (Equation (6), q_{opt} (under a presumed equilibrium regime; Equation (14)) and the actual value of groundwater withdrawal q. In the following, we will denote these quadrants by a combination of two letters: E for equilibrium regime (q < q_{crit}) versus D for depleting regime ($q > q_{crit}$) and P for the situation where an optimal withdrawal rate is possible ($q_{opt} < q_{crit}$) versus N for the situation where an optimal withdrawal rate is not possible ($q_{opt} > q_{crit}$). This leads to the following codes for the four quadrants: EP, EN, DN and DP (see Fig. 5).

In the following we describe the four quadrants in more detail and how withdrawal strategies differ between quadrants. Note that these descriptions are based on the limited notion of optimality, i.e. profit maximization. It is always possible to also include environmental externalities, which will have impact on the boundaries between the quadrants, i.e. replacing q_{opt} by $q_{opt,e}$, and also changes the meaning of "optimal withdrawal rates" from those maximizing profit to those that maximize social welfare.

1. EP: $q < q_{crit}$ and $q_{opt} < q_{crit}$. In case both the actual and optimal withdrawal rate are smaller than the critical rate (maximum capture), the optimal withdrawal rate can be achieved under equilibrium conditions, which leads to $q = q_{opt}$ (Equation 14) when maximizing profit.
2. EN: $q < q_{crit}$ and $q_{opt} > q_{crit}$. If the optimal withdrawal rate exceeds the critical rate (maximum capture), no optimal rate is possible under equilibrium conditions. Even though the actual withdrawal rate is smaller than the critical rate, maximization of profit will

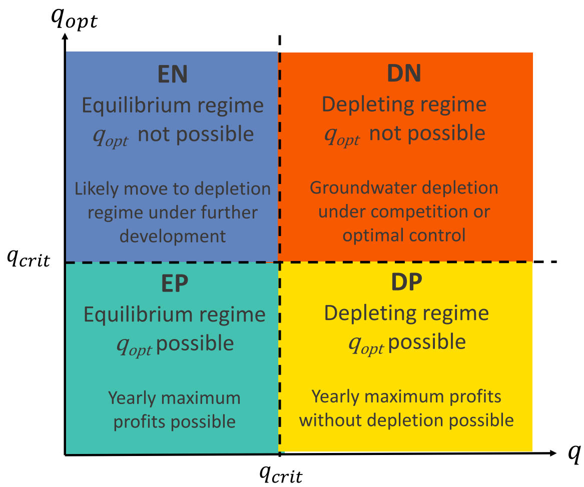

Fig. 5. Four quadrants of groundwater withdrawal strategies; EP (lower left; green; $q < q_{crit}$ and $q_{opt} < q_{crit}$); EN (upper left; blue; $q < q_{crit}$ and $q_{opt} > q_{crit}$), DN (upper right; red; $q > q_{crit}$ and $q_{opt} > q_{crit}$) and DP DN (lower right; yellow; $q > q_{crit}$ and $q_{opt} < q_{crit}$). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

increase it until it exceeds q_{crit}. Unless there is some form of control, this will eventually lead to persistent depletion (quadrant DN) where withdrawal rates are given by Equations 19–25 depending on competition or control.

3. DN: $q > q_{crit}$ and $q_{opt} > q_{crit}$. In this case withdrawal is already above the critical limit and depletion occurs. Also, an optimal rate under equilibrium conditions is not possible. Thus, withdrawal rates are given by Equations 19–25 depending on competition or optimal control.
4. DP: $q > q_{crit}$ and $q_{opt} < q_{crit}$. This is the peculiar situation that even though some optimal withdrawal rate under equilibrium conditions is possible, actual withdrawal rates are too high. This defies rationality, which may be explained by several factors, such as subsidized energy for pumping leading to small values of unit pumping costs p_p, lack of information about the hydrogeological situation on the relationship between water use and yield, or non-exclusive access by exporting pumped groundwater or groundwater use by other sectors. Under these circumstances, there is the possibility to revert to a physically stable withdrawal regime and at the same time maximize profit: $q = q_{opt}$ (Equation 14).

4. Global application

We demonstrate the developed hydroeconomic model for optimal groundwater withdrawal in a global case study covering all areas where crops are irrigated with groundwater. The model is used to provide first order estimates of: 1) the occurrence of the four quadrants of withdrawal strategies; 2) optimal withdrawal rates for the areas with equilibrium regimes (with and without environmental externalities); and 3) the global occurrence of the Gisser-Sánchez effect for regions with depletion regimes by comparing depletion under competition, under optimal control and under optimal control with environmental externalities. Since, this is a global case study that is necessarily subject to large parameter uncertainties, we investigate the robustness of the results by performing a sensitivity study. Additionally, we present the results in the main text as relative changes between non-optimal and optimal withdrawal strategies.

4.1. Global datasets and parameters

Bierkens et al (2021) present a global application of the simple conceptual model of groundwater withdrawal with groundwater-surface water interaction (Fig. 2). We use this application as a basis for a global application of the hydroeconomic model of optimal withdrawal rates developed in section 3. For its global application, Bierkens et al. (2021) in turn used the output and parameters of the global hydrology and groundwater resources model PCR-GLOBWB 2 (Sutanudjaja et al., 2018). PCR-GLOBWB 2 simulates the flow and storage of water at the land surface, including river discharge, for 5 arcminute cells (~10 km at the equator) at daily time step and for the global landmass except Greenland an Antarctica. Instead of applying the simple conceptual model for an aquifer of size A, Bierkens et al. (2021) applied it to each PCR-GLOBWB grid cell separately, where the analysis is limited to cells with groundwater withdrawal for irrigation (Portmann et al., 2010; Siebert and Döll, 2010). The upstream inflow Q_i for each cell is then given by the incoming river discharge as simulated by PCR-GLOBWB averaged over the period 2000–2015. Similarly, the values of densities q, q_s, r and the streamflow velocity v (Fig. 2; Appendix A) are average values over the period 2000–2015 obtained from PCR-GLOBWB. Note that the groundwater withdrawal value q provided by PCR-GLOBWB 2 which is used in the global analysis here also includes groundwater withdrawal from domestic and industrial water use (which are relatively small in agricultural areas analyzed here). Also, note that by using as inflow Q_i the upstream discharge from a PCR-GLOBWB simulation we account for the average upstream withdrawals from surface water and groundwater by all sectors in the period 2000–2015. The groundwater-surface water interaction parameter C, the specific yield parameter n and the dimensions of the rivers W and d are parameters that can also be directly obtained from the PCR-GLOBWB 2 parameterization. Table 3 provides an overview of the hydrogeological inputs used along with the hydroeconomic parameters. We refer to the Supplement of Bierkens et al (2021) for maps of the hydrogeological parameters, as well as an elaborate evaluation of the accuracy of this model.

Following the conceptual groundwater-surface water model of Bierkens et al. (2021), the hydroeconomic model is applied to the PCR-GLOBWB 2 cells (5 arcminute) restricted to those cells where groundwater is withdrawn for irrigation purposes. The land cover model of PCR-GLOBWB 2 distinguishes between 26 different crop types following

Table 3
Parameter and input values used in global-scale analyses at 5 arc-minute cells (~10 km at the equator). All hydrogeological inputs obtained from PCR-GLOBWB 2 with input variables averaged over the period 2000–2015.

Parameter	Value
Hydrogeological parameters (from Sutanudjaja et al., 2018)	
A	Cell area 5 arc-minute cells (m^2)
q_s	Sum of surface runoff and interflow (m yr^{-1}) of a cell
Q_i	Upstream discharge of a cell (m^3 yr^{-1})
d	Stream bottom elevation (m)
W	Stream width (m)
v	Streamflow velocity (m yr^{-1})
C	C = J/n (years), with J the characteristic response time of the groundwater reservoir
n	Specific yield (–) from the groundwater reservoir in PCR-GLOBWB.
r	Net recharge (recharge minus capillary rise) (m yr^{-1}).
q	Withdrawal rate (m/yr).
Hydroeconomic parameters	
p_o	Intercept of crop-specific and country specific demand curve (USD kg^{-1}) based on maximum crop specific producer's price (FAO, 2021a) minus non-water related productions costs which result in a reduction factor of 0.32 (Vocke and Ali, 2013)
k	Price elasticity of demand (kg^2 USD^{-1} m^{-2} yr^{-1}). Calculated per country from maximum and minimum prices and maximum and minimum yields of wheat and citrus (FAO, 2021a) as representative for staple and cash crops
a	Water productivity per crop (kg m^{-3}) (FAO, 2021b)
p_p	Unit pumping costs (USD m^{-1} m^{-3}) (Bierkens et al., 2021)
i	Discount rate per country (yr^{-1}) (World Bank, 2020); reference year 2016).
γ	Unit costs of externalities (USD m^{-3}) we assume $\gamma = 0.05$ USD m^{-3}

the MIRCA dataset (Portmann et al., 2010). We however performed the global analysis for the dominant crop type per cell only. The parameters of the demand curve were obtained as follows: The intercept p_0 (USD kg^{-1}) was based on the maximum crop price (producer price) per country over the period 1991–2015 as reported in FAOSTAT (FAO, 2021a). This price was multiplied by a factor 0.32 to account for non-water related production costs (machinery, labour, fertilizer etc.). This reduction factor was based on an analysis of farm-level costs, yields and revenues of wheat production at a large number of U.S. farms (Vocke and Ali, 2013), showing the non-water related costs on farms with irrigated crops to be 68 % of the revenues on average. For the price elasticity k (kg^2 USD^{-1} m^{-2} yr^{-1}) the maximum and minimum price p_{max}, p_{min} and maximum and minimum yield Y_{max}, Y_{min} over the period 1991–2015 (FAO, 2021a) were used and the price elasticity estimated as $k = (Y_{max} - Y_{min})/ (p_{max}\text{-} p_{min})$. This was done separately for each country, but to reduce the effort of analyses, for two crops: for wheat, taken as representative for all staple crops and for citrus as representative for all cash crops. The water productivity a per crop type was obtained by multiplying water use efficiency data from FAO (2021b) (per crop the same for all countries) with country-specific irrigation efficiencies as used in PCR-GLOBWB (Sutanudjaja et al., 2018). Country-specific discount rates were obtained from World Bank (2020). The most challenging parameter to obtain is the unit pumping costs p_p (USD m^{-1} m^{-3}), i.e. the costs of withdrawal of one m^3 groundwater per meter depth of the water table. These have been computed following a procedure described in more detail in the Supplementary Information. These costs include material costs and labour costs to dig and construct the well, the costs of the pumps and the irrigation infrastructure, the energy costs to lift the water to the surface and bring it to the crops and the interest payments on loans needed to invest in withdrawal and irrigation infrastructure. This cost model thus includes both fixed and variable costs. For each grid and its most abundant crop type we have combined the average fixed costs with the average variable costs (across all the different well depths and withdrawal volumes) and divided these by the average well depths and withdrawal rates to arrive at an average unit cost (USD $m^{-1}m^{-3}$) per cell. Well depths were estimated from the long-term average groundwater depths calculated with the global groundwater model of De Graaf et al. (2017) as minimum depth and the depth to economic depletion (total costs of water exceeding revenues) as maximum depth. The supplementary Figs. S1–S5 provide global maps at 5 arcminute resolution for the hydroeconomic parameters used.

The unit costs of the externalities γ is difficult, if not impossible, to estimate at the global scale. Its value depends on the proximity of individual wells to groundwater-dependent ecosystems and the unique flora and fauna in those systems. Thus, in this paper we use a fixed value of γ as an example to show how optimal pumping rates change when social welfare rather than farmer profits is maximized. In the examples in Section 3 we use a value of 0.05 USD m^{-3} pumped, which is the same as used by Esteban and Albiac (2011) for Spain. This, however, renders agriculture non-profitable for a few percent of the area under irrigation. These are areas where the maximum crop price per unit groundwater pumped ap_0 is smaller than $\gamma = 0.05$ USD m^{-3} (See Supplementary Fig. S6). For these areas we capped the value of γ to $0.5ap_0$. This and the values in Supplementary Fig. S6 point to the fact that the unit price of irrigation water is quite low for many parts of the world, confirming the econometric analysis of Bierkens et al (2019).

4.2. Sensitivity analyses

Apart from the assumptions that make for a tractable hydroeconomic model (Section 3.1), additional assumptions are made to support a global application. These are the country-specific values of the hydroeconomic parameters p_o, k, a, p_p, i the single prefactor for all non-water related costs of 0.32, the imposed environmental $\gamma = 0.05$ USD m^{-3} and the fact that for each cell we apply the model to only one crop, i.e. the most dominant one. These values are obviously subject to considerable uncertainty and may even vary within a country. A full uncertainty analysis would require information about these parameter uncertainties in the form of a joint a priori probability distribution, which we do not have. To obtain some idea about the robustness of the results shown, we perform a sensitivity analysis instead. To do this we change the parameters p_o, k, a, p_p, i and the factors 0.32 by +/- 20 %, adding two values of environmental externalities ($\gamma = 0.05$ and $\gamma = 0.1$ USD m^{-3}) and additionally replace the most dominant crop by the second dominant crop per cell. We redo the calculations of the occurrence of the four quarters and the outputs of the hydroeconomic model: q_{opt}, π_{max} and the discharge capture $dQ(q_{opt})$ for the equilibrium regimes (EP and EN) and $s_{cmp}(\infty)$, $s_{ctr}(\infty)$, Π_{cmp} and Π_{ctr} for the depleting regimes (DP and DN). We show the results as tables and plots denoting spatial distributions of these outputs in the Supplementary Information.

4.3. Global results

Global occurrence of four quadrants of optimal withdrawal strategies

Based on the maps of parameters mentioned in Table 3 (See the maps of hydrogeological parameters in Bierkens et al. (2021) and the maps of hydroeconomic parameters in the Supplementary Figs. S1–S5) we calculated global maps of q_{crit} and q_{opt} (Supplementary Figs. S7 and S8 respectively). By comparing these with the map of global groundwater withdrawal q, we calculated a global map (Fig. 6) with the occurrence of the four quadrants as defined in section 3.3 (Fig. 5).

Fig. 6 shows that most cells with groundwater withdrawal have current withdrawal rates that are situated in the equilibrium regime (EP and EN). However, most of this area has an optimal equilibrium withdrawal rate that exceeds the critical limit (EN). This entails that upon further groundwater development without some control mechanism, a tendency towards profit maximization will eventually lead to an increase of groundwater withdrawal that moves the system to a regime with groundwater depletion. The cells that are currently in a depleting regime (DN and DP) can be found in the regions that are well known groundwater depletion hotspots (Wada et al., 2010; Gleeson et al., 2012; Döll et al., 2014; Rodell et al., 2018). About 14 %. of these cells are in regions where in fact the optimal groundwater withdrawal rate is lower than the critical rate. Some of these areas are situated in North Africa and Arabian Peninsula. A possible explanation is that the wish for some countries to become self-sufficient in food production drives the use of groundwater beyond what is economically optimal and that this is made possible from other sources of national income (e.g. subsidizing crops or energy use). The sensitivity analysis (Supplementary Table S1) shows that the percentages of global occurrence of the four quadrants are not very sensitive to changes in the hydroeconomic parameters. Any change that decreases profit decreases q_{opt} resulting in a slightly larger percentage of the occurrence of the classes EP and DP. Particularly the inclusion of environmental externalities has a noticeable impact on the occurrence of the regimes.

Results for the equilibrium regime $q < q_{crit}$ (EP and EN)

For the equilibrium regime (EP and EN) we calculated the optimal withdrawal rates without q_{opt} (Equation 14) and with environmental externalities $q_{opt,e}$ (Equation 16). Also, we calculated the yearly maximum profit π_{max} (Equation 15) and the profit when including environmental externalities $\pi_{max,e}$ (Equation 17). For the cells where $q_{opt} > q_{crit}$ (EN) we have set the profit equal to the profit that can be attained at the critical value $\pi_{max} = \pi(q_{crit})$, which is the maximum yearly profit (USD ha^{-1} yr^{-1}) that can be attained without groundwater depletion. Additionally, using the equation of Table A1 (Bierkens et al., 2021) we also calculated the impact of withdrawal on streamflow reduction under current and optimal (with and without externalities) groundwater withdrawal rates $dQ(q)$, $dQ(q_{opt})$ and $dQ(q_{opt,e})$. The absolute values of q_{opt}, $q_{opt,e}$, $\pi(q)$, π_{max}, $\pi_{max,e}$, $dQ(q)$, $dQ(q_{opt})$ and $dQ(q_{opt,e})$ are given in Supplementary Figs. S8 to S15 respectively for reference. However, we caution against taking such absolute values at face value, since they

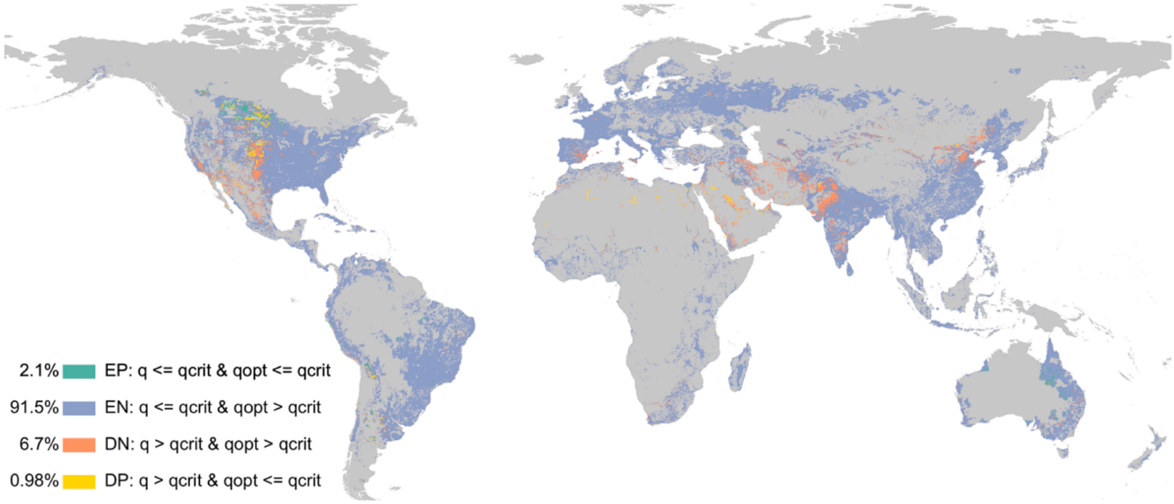

Fig. 6. Global occurrence of the four quadrants of groundwater withdrawal strategies (EP, EN, DN, DP) based on the values of actual groundwater withdrawal rates and the critical and optimal withdrawal rates q_{crit} and q_{opt}; light grey areas are without groundwater irrigation.

depend on global datasets and generalizations, which are inevitable at the spatial scale of this study. Therefore, we opt to present relative changes, which are a measure of global sensitivities, instead.

Fig. 7 presents for the case without environmental externalities the relative change in withdrawal rates when moving from q to q_{opt} (Fig. 7a), the associated relative change (increase) in profit between $\pi(q)$ and π_{max} (Fig. 7b) and the associated relative change in streamflow between $dQ(q)$ and $dQ(q_{opt})$ (Fig. 7c). Fig. 7a shows that for the EN quadrant areas ($q < q_{crit}$; $q_{opt} > q_{crit}$), the withdrawal rates obviously increase (towards q_{crit}), while for the areas in the EP quadrant ($q < q_{crit}$; $q_{opt} < q_{crit}$), both increases as well as decreases in withdrawal rates lead to maximum profits. The increase in profit can be quite substantial, being over 100 % or a factor two for many regions. However, this comes at the expense of a large decrease in streamflow, especially for the areas in the EN quadrant. In a small fraction of the area in the EP quadrant the optimal groundwater withdrawal is reached by decreasing the withdrawal rate, with positive impacts on streamflow.

Fig. 8 shows that the impact of including environmental externalities is very small and does not lead to more social welfare (Fig. 7a) and less streamflow reduction (Fig. 7b). This results from the fact that in most cases both q_{opt} and $q_{opt,e}$ are larger than q_{crit} and would lead to a depleting regime under economic development without further control; we have set q_{opt} and $q_{opt,e}$ equal to q_{crit} in this case. In less than a few percent of the cases either q_{opt} and $q_{opt,e}$ are smaller than q_{crit} (the EP regime) or at least $q_{opt,e}$ is smaller than q_{crit} (the EN regime). Here we see that including environmental externalities does lead to substantial higher value of social welfare (Fig. 7a) and to a reduction in streamflow capture up to 50 % (i.e. not including externalities leads to a doubling of capture – Fig. 7b). Thus, although internalizing environmental externalities will lead to increased social welfare and reduced environmental damage in principle, this does not hold for most areas with groundwater withdrawal since even with externalities, socially optimal withdrawal rates would surpass the critical withdrawal rates and lead to a depleting regime where optimal pumping rates are no longer physically sustainable.

The results of the sensitivity analysis, i.e. by evaluating the impact of changing the hydroeconomic parameters on the spatial distribution of q_{opt}, π_{max} and $dQ(q_{opt})$, are shown in Supplementary Fig. S16 in the form of quantile plots of the spatial variation of values compared to the reference situation (the maps in Supplementary Figs. S8, S11 and S15). As can be seen, any change that decreases unit costs leads to higher optimal withdrawal rates, larger profits and larger capture of streamflow. Results are most sensitive to unit cost parameters such as other production costs, pumping costs p_p and externalities γ and less to parameters that indirectly impact profits such as water productivity and price elasticity. The results shown in Figs. 7 and 8, even if they are already shown as relative differences, will change subject to uncertain hydroeconomic parameters. However, the sensitivities shown are limited and will not change the conclusions that can be drawn from the results.

Results for the depleting regime $q > q_{crit}$ (DN and DP)

For the depleting regime we calculated the following variables: final groundwater level decline under competition $s_{cmp}(t\to\infty)$, optimal control $s_{ctr}(t\to\infty)$ and optimal control including environmental externalities $s_{ctr,e}(t\to\infty)$ using Equations 19–24 and the associated NPV of cumulative profits over time Π_{cmp}, Π_{ctr} and $\Pi_{ctr,e}$ We also calculated the social welfare, here defined as the NPV of cumulative profits minus the costs of environmental externalities (Equations 27), for the cases: $\Pi_{cmp(sw)}$, $\Pi_{ctr(sw)}$ and $\Pi_{ctr,e(sw)}$, where the latter is equal to $\Pi_{ctr,e}$. Maps of the absolute values of are given in Supplementary Figs. S17–S24.

Here, we show the relative differences of these variables between depletion under competition and optimal control with and without externalities (Figs. 9 and 10). The relative differences in the final groundwater decline $s(t\to\infty)$ between the competition and control cases are mostly small in all areas where currently groundwater depletion occurs, confirming the Gisser-Sánchez effect (Fig. 9a). However, including environmental externalities reduces the final groundwater level decline in various parts of the world such as North America, Mexico, the northern Indus Valley and the North-China plane. Thus, the Gisser-Sánchez effect in terms of groundwater decline is less conspicuous when environmental externalities are included. Fig. 10a shows that the difference in the NPV of cumulative profits Π mirror that of the groundwater decline confirming the Gisser-Sánchez effect. However, the differences in social welfare between competition and optimal control with environmental externalities are considerable, with significantly larger values for the control case with externalities compared to competition, as can be seen from Fig. 10b.

Supplementary Fig. S25 and Table S2 (for the discount rate) show the results of the sensitivity analysis applied to $s_{cmp}(\infty)$, $s_{ctr}(\infty)$, Π_{cmp} and Π_{ctr}. Clearly, changing the hydroeconomic parameters has a larger impact on the results than for the equilibrium case. We observe similar tendencies, where parameters that directly reduce unit costs lead to larger groundwater level decline and increased profits, however sensitivities are larger, which is due to the cumulative nature of groundwater level decline and accumulation of profits over time. Comparison of the competition and control spatial distributions show that they are very

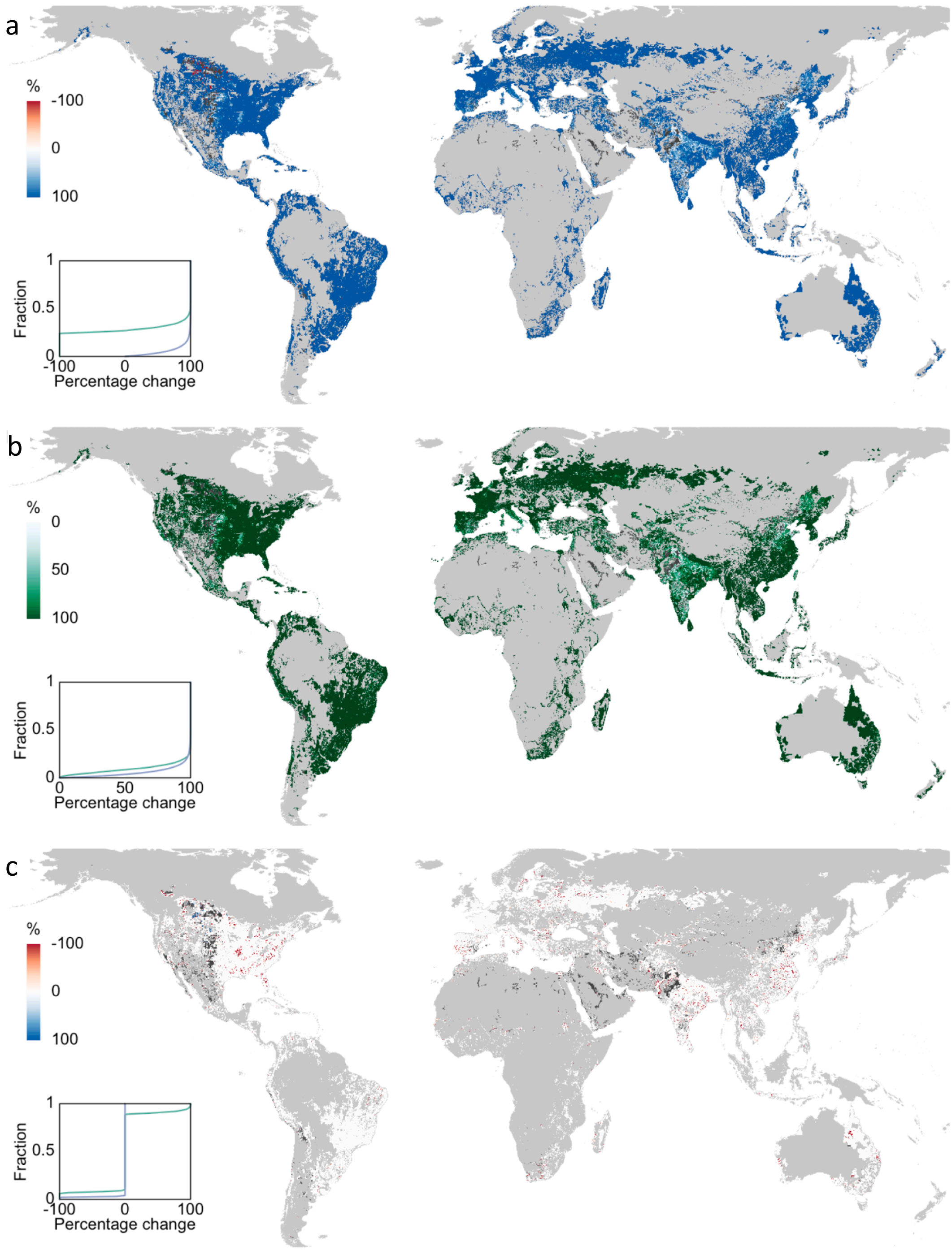

Fig. 7. Impacts of changing to optimal withdrawal rates for the equilibrium regime (EP,EN) ($q < q_{crit}$); note that if $q_{opt} > q_{crit}$ (EN) we set $q_{opt} = q_{crit}$; (a) relative change in withdrawal rate (q_{opt}–q)/q (%); (b) relative change in profit $(\pi_{max} - \pi(q))/\pi(q)$ (%); (c) relative change in impact on streamflow and (dQ(q)- dQ(q_{opt}))/ dQ(q) (%); insets show the cumulative frequency distributions of relative change, with the green colour the EP quadrant and the blue colour the EN quadrant; black pixels are areas with groundwater withdrawal falling in the depleting regimes ($q > q_{crit}$) (EN and EP); grey background colour in the maps identify areas without groundwater irrigation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

similar (Gisser-Sánchez). Also, the sensitivities are similar, which entails that the relative changes in Figs. 9 and 10 are not very sensitive to parameter uncertainty.

5. Discussion

The combined hydrogeological-hydroeconomic model here presented provides a quick analytical tool to estimate, at first order, optimal groundwater withdrawal strategies at large scales. The lumped-conceptual nature of the model that ensures tractability and its global parameterization come at the price of ignoring many factors. Site-specific outcomes may be different from our global maps, but they highlight regional differences between the four quadrants combining groundwater withdrawal regimes and optimal strategies (Figs. 5 and 6). To account for uncertainties in model hydroeconomic parameters we present the results in terms of relative differences. We also added a sensitivity analysis which indicated that parameter uncertainty will change results. However, as sensitivities are limited for the equilibrium

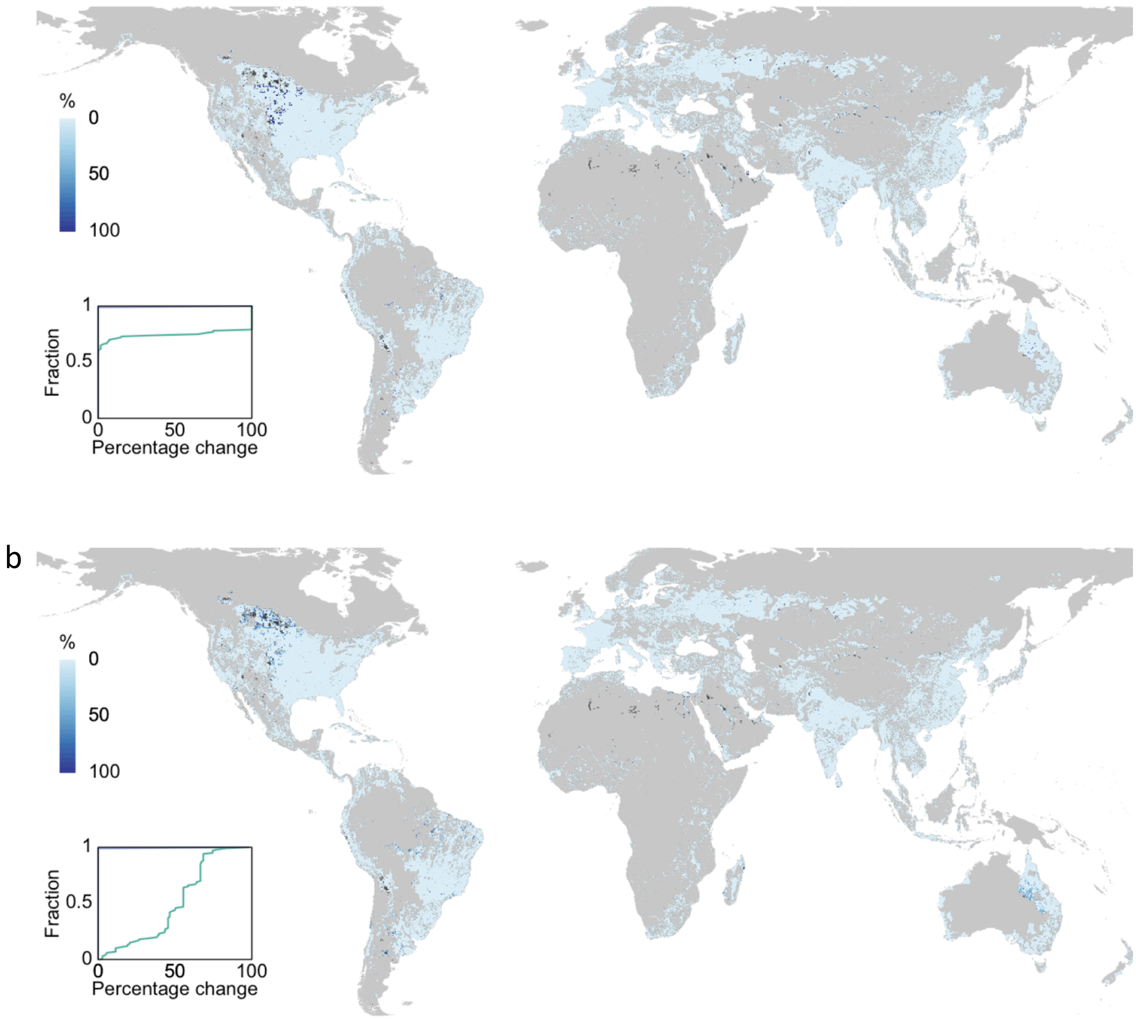

Fig. 8. The impact of including environmental externalities when calculating the optimal withdrawal rates for the equilibrium regime (EP, EN); (a) relative change in social welfare $(\pi_{max,e} - (\pi_{max} - \gamma q_{opt}))/(\pi_{max,e} - \gamma q_{opt})$ (%); (b) relative change in impact on streamflow ($dQ(q_{opt})$- $dQ(q_{opt,e})$)/ $dQ(q_{opt})$ (%) when environmental externalities are not included; insets show the cumulative frequency distributions of relative change, with the green colour the EP quadrant and the blue colour the EN quadrant; black pixels are areas with groundwater withdrawal falling in the depleting regimes($q > q_{crit}$) (EN and EP); grey background colour in the maps identify areas without groundwater irrigation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

regime and very similar for withdrawal under competition and control for the depleting regime, we argue that parameter uncertainty will not change the conclusions that can be drawn from the results. The discount rate requires some additional discussion, however. Although not very uncertain at a given time and readily reported, discount rates for a given country can vary over time, at least in the order of several percent points. Obviously, changes of these magnitudes over time will have a notable impact on the size of the Gisser-Sánchez effect and such changes are not accounted for in the analyses presented. Thus, conclusions drawn are always conditional to the current discount rates.

Apart from the uncertainty in model parameters, the model also relies on some additional simplifying assumptions. First, our analysis focuses on the economic limits to groundwater depletion. In reality, physical limits such as the occurrence of low permeability (Gleeson et al., 2015) or saline groundwater (van Weert et al., 2009) may limit to what depth groundwater levels can be exploited. Also, in many parts of the world, groundwater is extracted from deeper confined aquifers with considerable overpressure. This means that extraction costs are not necessarily proportional to the depth of extraction, as assumed here. Second, just as the hydrogeology of the subsurface is very much simplified in our model, so is the hydrology. For instance, surface water systems of different order have bottom elevations of varying heights. Thus, instead of a single threshold between equilibrium and depleting regimes as shown in Fig. 2 and Equation (2), a more gradual change between the two regimes can be expected based on multiple drainage levels (cf. Bierkens and te Stroet, 2007). Also, the local impact of groundwater withdrawal on surface water levels and the degree of capture depends on the location of the wells with respect to the streams. Third, we do not consider that farmers will likely adapt to the higher costs occurring from groundwater level decline, such as changing crop types, e.g. changing the type of irrigation infrastructure, fallowing or using deficit irrigation (Döll et al., 2014). Fourth, our analyses are steady state, ignoring the impacts of temporal variability where multiple consecutive dry years can spur the establishment of additional groundwater wells (Scanlon et al., 2012) that will be subsequently used in wetter years as well. At the same time, if the infrastructure is in place, farmers may deal with temporal variability by temporarily depleting groundwater reserves in drier years in conjunction with maintaining a strategic drought reserve during wetter years (Provencer and Burt, 1993). Fifth, our analysis considers profit per unit area, without consideration of farm size. The size of a farm, however, determines whether optimal withdrawal rates or optimal groundwater depletion trajectories result in a sufficiently large farmer's income to support the farmer and his family, which is needed for economic sustainability. Also,

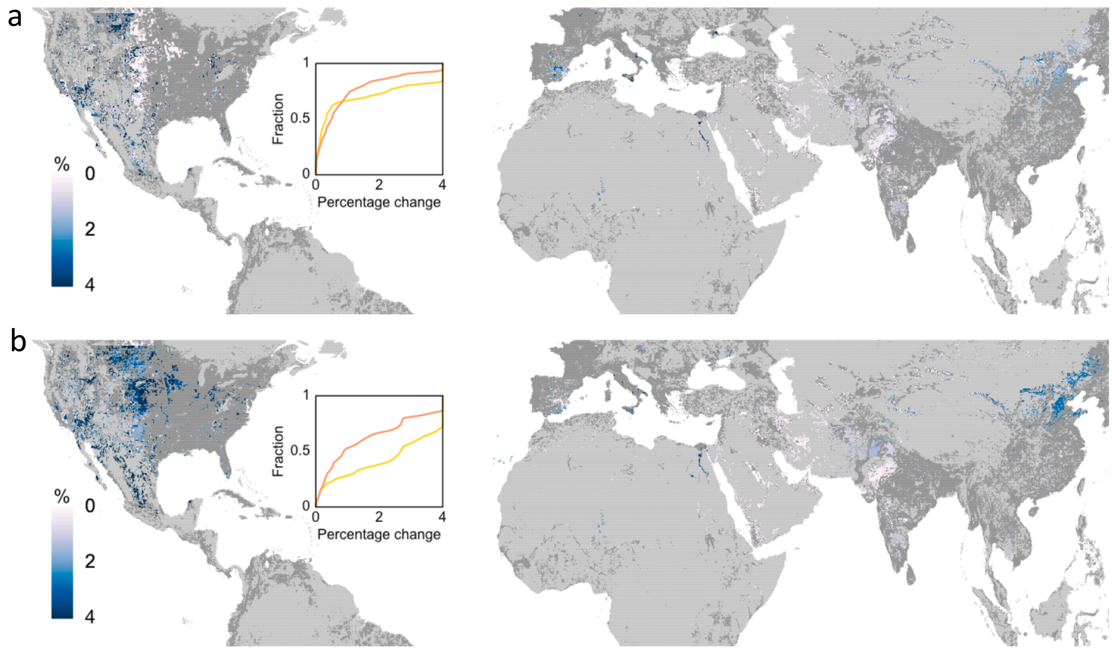

Fig. 9. Differences in groundwater level decline between competition and optimal control (intertemporal efficiency) for the depleting regime (DN and DP); a) relative increase of groundwater level decline under competition compared to control $(s_{cmp}(\infty) - s_{ctr}(\infty))/s_{ctr}(\infty)$ (%); (b) relative increase of groundwater level decline under competition compared to optimal control including environmental externalities $(s_{cmp}(\infty) - s_{ctr,e}(\infty))/s_{ctr,e}(\infty)$ (%). dark grey pixels are areas with groundwater withdrawal falling in the equilibrium regimes (EP, EN; $q < q_{crit}$); insets show the cumulative frequency distributions of relative change, with the red colour the DN and the yellow colour the DP quadrant; light grey areas are without groundwater irrigation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

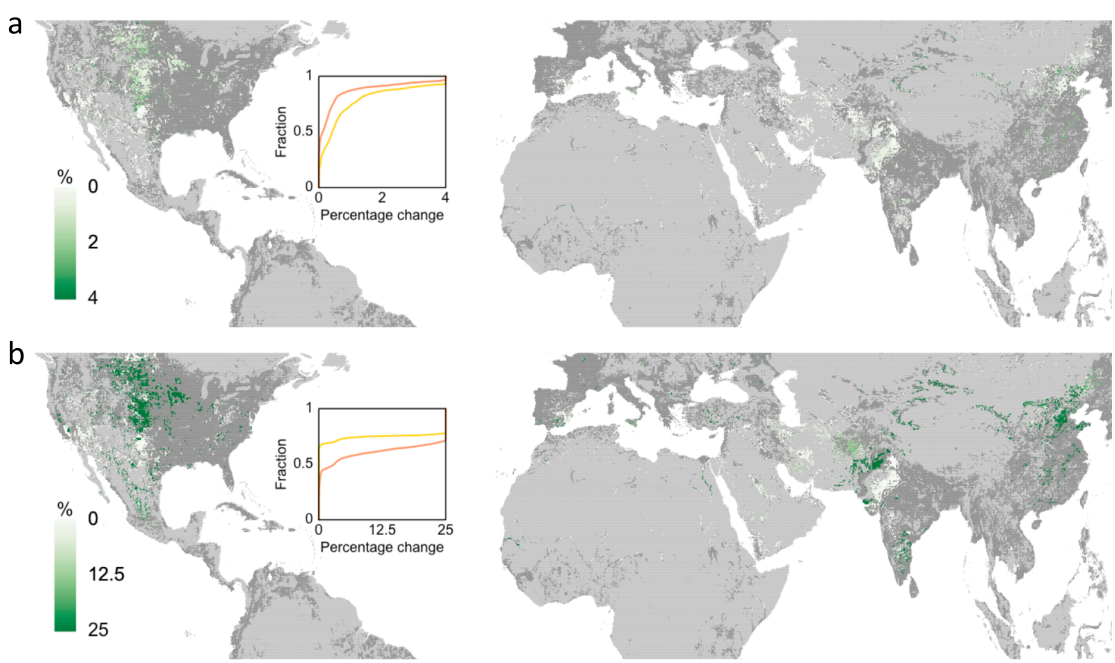

Fig. 10. Differences in profits and social welfare between competition and optimal control (intertemporal efficiency) for the depleting regime (DN and DP); a) relative increase in NPV of cumulative profits of optimal control versus competition $(\Pi_{ctr} - \Pi_{cmp})/\Pi_{cmp}$; (b) relative increase in NPV of social welfare of optimal control including environmental externalities versus competition $(\Pi_{ctr,e(sw)} - \Pi_{cmp(sw)})/\Pi_{cmp(sw)}$ (5); dark grey pixels are areas with groundwater withdrawal falling in the equilibrium regimes (EP, EN; $q < q_{crit}$); insets show the cumulative frequency distributions of relative change, with the red colour the DN and the yellow colour the DE quadrant; light grey areas are without groundwater irrigation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the farm size determines whether a farmer can raise the capital needed to deepen a well or increase the pump capacity under falling groundwater levels. Sixth, the model and the associated withdrawal strategies are based on the assumption that farmers' behaviour can approximated by global hydroeconomic optimization with farmers that are either myopic or fully cooperative, while many other social and cultural factors may be more important. Some of the simplifications and assumptions described above can be relaxed and added to the framework presented. However, most of these require the use of physically-deterministic groundwater flow and transport models coupled with hydro-economic optimization or simulation routines (Harou et al., 2009) or socio-hydrological agency (Sivapalan et al., 2012), which are difficult to apply at the continental to global scales on account of a heavy computational burden and large parameterization challenges.

As a final remark, we note that "optimal groundwater withdrawal" could be defined in a much broader sense than maximizing profit. Even when including environmental externalities, the notion of "social welfare" as used in this paper is limited. A broader definition of social welfare would e.g. have to include the opportunity costs of alternative water use and equity among beneficiaries (Moench, 1992). Following the notion of "strong sustainability" (Ayres et al. 2001), optimal groundwater management would additionally include the intrinsic value of groundwater dependent ecosystems, the importance of groundwater in places of cultural significance and its role in Earth system functions, such as advocated by the groundwater sustainability research community (e.g., Elshall et al., 2020; Gleeson et al., 2020, Zwarteveen et al., 2021; Huggins et al., 2023).

6. Conclusions

We combined a lumped-conceptual model of large-scale non-linear groundwater-surface water interaction under groundwater withdrawal with a hydroeconomic model to derive analytical solutions of optimal groundwater withdrawal. The inclusion of non-linear groundwater-surface water interaction allowed for incorporating the impact of capture and analysing, at first order, hydroeconomic optimality for physically stable (equilibrium) and physically non-stable (depleting) groundwater withdrawal regimes. For both regimes, we included the possibility to internalize environmental externalities. Based on the relative value of the actual withdrawal rate q with respect to the critical withdrawal rate q_{crit} that distinguishes an equilibrium (physically stable) from a groundwater depleting regime and the optimal equilibrium withdrawal rate q_{opt}, four quadrants in terms of withdrawal strategies could be distinguished. We used the combined hydrogeological-hydroeconomic model to map the global occurrence of these four quadrants and to globally estimate optimal groundwater withdrawal rates and trajectories with and without including externalities.

Global results show that most of the area with global groundwater withdrawal for irrigation are still in the equilibrium regime ($q < q_{crit}$), but also that these regions would tend towards a depleting regime if groundwater withdrawal for irrigation would be uncontrolled and tend towards an optimal withdrawal rate, even with environmental externalities included. For the quadrants with groundwater depletion ($q > q_{crit}$) we derived and compared depletion trajectories under competition, optimal control and optimal control including environmental externalities and assessed globally where the differences between these modes were small, which is known as the Gisser-Sánchez (1980) effect. We found that the Gisser-Sánchez effect is globally ubiquitous, but only if environmental externalities are ignored. The inclusion of environmental externalities in optimal control withdrawal resulted in notably reduced groundwater decline and larger values of social welfare in many of the major depletion areas.

7. Contributions

MB developed the theory and derived the equations. MB and NW designed the global case study. NW performed the numerical calculations with parameterizations from RvB. MB wrote the manuscript with inputs from all authors.

CRediT authorship contribution statement

Marc F.P. Bierkens: Conceptualization, Data curation, Formal analysis, Software, Visualization, Writing – original draft. **L.P.H. Rens van Beek:** Data curation, Formal analysis, Writing - review & editing. **Niko Wanders:** Data curation, Investigation, Software, Visualization, Writing – review & editing.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Marc F.P. Bierkens reports financial support was provided by European Research Council. Niko Wanders reports financial support was provided by Dutch Research Council.

Data availability

Data will be made available on request.

Acknowledgements

MB acknowledges support from the ERC Advanced Grant scheme (Grant no. 101019185 – GEOWAT). NW received funding from the Dutch Research Council (NWO) (Grant no. 016.Veni.181.049).

Appendix A. . Conceptual model of large-scale groundwater withdrawal with groundwater–surface water interaction: equations summary

Table A1
Overview of expressions of groundwater and surface water with time and related properties for the equilibrium and depleting regimes.

(*continued on next page*)

Table A1 (*continued*)

$\alpha = \frac{Q_i C + q_s AC + WvdC}{WvC + A}$ $\quad \beta = \frac{A}{WvC + A}$ $\quad q_{\text{crit}} = r + \frac{Q_i + q_s A}{WvC + A}$		
$q \le q_{\text{crit}}$	$q > q_{\text{crit}}$	
	$t_{\text{crit}} = \frac{nC}{1-\beta} \ln\left(\frac{qC}{qC - (rC + \alpha) + d(1-\beta)}\right)$	
	$t \le t_{\text{crit}}\ (h \ge d)$	$t > t_{\text{crit}}\ (h < d)$
$h(t) = \frac{rC + \alpha}{1-\beta} - \left(\frac{qC}{1-\beta}\right)\left[1 - e^{-\left(\frac{1-\beta}{nC}\right)t}\right]$ $h(\infty) = \frac{rC + \alpha - qC}{1-\beta}$	$h(t) = \frac{rC + \alpha}{1-\beta} - \left(\frac{qC}{1-\beta}\right)\left[1 - e^{-\left(\frac{1-\beta}{nC}\right)t}\right]$	$h(t) = d + \left[\frac{r-q}{n} + \frac{(Q_i + q_s A)}{n(WvC + A)}\right](t - t_{\text{crit}})$
$h_s(t) = \alpha + \beta h(t)$ $h_s(\infty) = \alpha + \frac{\beta(rC + \alpha - qC)}{1-\beta}$	$h_s(t) = \alpha + \beta h(t)$	$h_s = d + \frac{(Q_i + q_s A)C}{WvC + A}$
$Q(t) = Q_i + q_s A - \frac{A\alpha}{C} + \frac{A(1-\beta)}{C} h(t)$ $Q(\infty) = Q_i + (q_s + r - q)A$	$Q(t) = Q_i + q_s A - \frac{A\alpha}{C} + \frac{A(1-\beta)}{C} h(t)$	$Q = \frac{(Q_i + q_s A)WvC}{WvC + A}$
$q_{stor} = qe^{-\left(\frac{1-\beta}{nC}\right)t}$ $q_{cap} = q\left(1 - e^{-\left(\frac{1-\beta}{nC}\right)t}\right)$	$q_{stor} = qe^{-\left(\frac{1-\beta}{nC}\right)t}$ $q_{cap} = q\left(1 - e^{-\left(\frac{1-\beta}{nC}\right)t}\right)$	$q_{stor} = q - \left(r + \frac{(Q_i + q_s A)}{(WvC + A)}\right)$ $q_{cap} = r + \frac{(Q_i + q_s A)}{(WvC + A)}$

The following variables are portrayed (for parameters q, A, q_s, Q_i, n, W, v, C, d: see Fig. 2 and its explanation in the main text):

q_{crit} Critical withdrawal rate (or maximum capture) ($m^3\ m^{-2}\ yr^{-1}$) above which the groundwater level becomes disconnected from the stream.
t_{crit} Critical time (years after start of withdrawal) at which the groundwater level becomes disconnected from the stream, i.e. $h < h_s$.
$h(t)$ Groundwater head (m) over time.
$h(\infty)$ Equilibrium groundwater head (m) at $t=\infty$ that only occurs in case $q \le q_{crit}$.
$h_s(t)$ Surface water level (m) over time.
$h_s(\infty)$ Equilibrium surface water level (m), which is different when $q \le q_{crit}$ than when $q > q_{crit}$.
$Q(t)$ Surface water discharge ($m^3\ yr^{-1}$) over time.
$Q(\infty)$ Equilibrium surface water discharge ($m^3\ yr^{-1}$), which is different when $q \le q_{crit}$ than when $q > q_{crit}$.
$q_{stor}(t)$ Part of the pumped groundwater that comes out of storage, which is different when $q \le q_{crit}$ than when $q > q_{crit}$.
$q_{cap}(t)$ Part of the pumped groundwater that comes from capture (reduction in streamflow), which is different when $q \le q_{crit}$ than when $q > q_{crit}$.

Table A1 provides an overview of the mathematical expressions derived for each of these properties. The left column shows the stable regime where upon commencement of withdrawal after some time an equilibrium is reached with equilibrium groundwater levels $h(\infty)$, streamflow $Q(\infty)$ and surface water level h_s. The middle and right columns show the results of unstable groundwater withdrawal. The behavior of $h(t)$, $Q(t)$ $h_s(t)$ follows that of the stable regime until time $t = t_{crit}$ when the groundwater level drops below the bottom of the surface water. After this time the groundwater level $h(t)$ shows a persistent decline and surface water level $h_s(t)$, streamflow $Q(t)$ and the fraction of water pumped from capture become constant.

Appendix B. . Optimum groundwater withdrawal under an equilibrium withdrawal regime ($q_{opt} < q_{crit}$)

The revenue R (USD $m^{-3}\ yr^{-1}$) from crop irrigation is based on an inverse demand function in order to include (endogenous) price effects:

$$R(Y) = \int_0^Y p(Y)dY \tag{B1}$$

with Y crop yield (kg m^{-2} yr^{-1}) and p (USD kg^{-1}) the price at the gate minus the sum of all unit production costs except water (e.g., land, labour, fertilizer etc.). Furthermore, we assume that yield is related to irrigation water withdrawn q (m^3 m^{-2} yr^{-1}) (and applied) following a Cobb-Douglas type production function:

$$Y(q) = aq^b \tag{B2}$$

with $0 \leq b \leq 1$. We further assume that $b = 1$, which entails that prefactor a (kg m^{-3}) can be interpreted as the water productivity for the given crop. Assuming $p(Y)$ is based on the inverse of a linear demand function with demand elasticity k (kg^2 USD^{-1} m^{-2} yr^{-1}) we have from (B1) and (B2):

$$R(Y(q)) = \int_0^Y \left(p_0 - \frac{y}{k}\right)dy = \int_0^q \left(p_0 - \frac{aq}{k}\right)adq = ap_0q - \frac{a^2q^2}{2k} \tag{B3}$$

This is a quadratic function which is zero at $q = 0$ and $q = \frac{2kp_0}{a}$ with a maximum revenue at $q = \frac{kp_0}{a}$ with $R_{\max} = \frac{kp_0^2}{2}$.

The pumping costs $C_q(D,q)$ (USD m^{-2} yr^{-1}) are assumed to be proportional to the head or water table decline s (m) due to withdrawal and the withdrawal rate q (m^3 m^{-2} yr^{-1}) as follows:

$$C_q(s,q) = p_p sq \tag{B4}$$

with p_p (USD m^{-1} m^{-3}) the (unit) pumping costs per m^3 water per m. Note that (B4) assumes that under natural circumstances the groundwater is close to the surface and initial pumping costs at $s = 0$ are therefore negligible.

For the stable withdrawal regime ($q \leq q_{crit}$) we have the following (large-scale) equilibrium (See Table A1 and explained symbols therein):

$$s = h(0) - h(\infty) = \frac{Cq}{1-\beta} \tag{B5}$$

so that resulting pumping costs then become

$$C_q(h,q) \equiv C_q(q) = \frac{p_pCq^2}{1-\beta} \tag{B6}$$

Denoting $\pi(q) = R(q) - C_q(q)$ as profit (revenue minus costs), we find maximum withdrawal rate that still yields a profit from $\pi(q) = 0$:

$$\pi(q) = ap_0q - \frac{a^2q^2}{2k} - \frac{p_pCq^2}{1-\beta} = 0 \tag{B7}$$

From which follows the maximum withdrawal rate that still provides a profit

$$q_{max} = \frac{2kap_0(1-\beta)}{a^2(1-\beta) + 2kp_pC} \tag{B8}$$

In case access to the aquifer is not controlled, the tragedy of the commons would push the aquifer exploitation to q_{max}. For a restricted access aquifer, e.g., only the land owners sitting on top of the aquifer have access, competition would yield an optimal withdrawal rate q_{opt} for which the marginal revenue and costs are equal: $\frac{d\pi}{dq} = 0 \rightarrow \frac{dR}{dq} = \frac{dC_{hq}}{dq}$. This gives:

$$\frac{d\pi}{dq} = ap_0 - \frac{2a^2q}{2k} - \frac{2p_pCq}{1-\beta} = 0 \tag{B9}$$

from which follows:

$$q_{opt} = \frac{kap_0(1-\beta)}{a^2(1-\beta) + 2kp_pC} \tag{B10}$$

Inserting (B10) into (B7) then results in a maximum profit of:

$$\pi_{\max} = \frac{ka^2p_o^2(1-\beta)}{2a^2(1-\beta) + 4kp_pC} \tag{B11}$$

Fig. A1 shows the revenue/costs functions (left panel) and profit function (right panel).

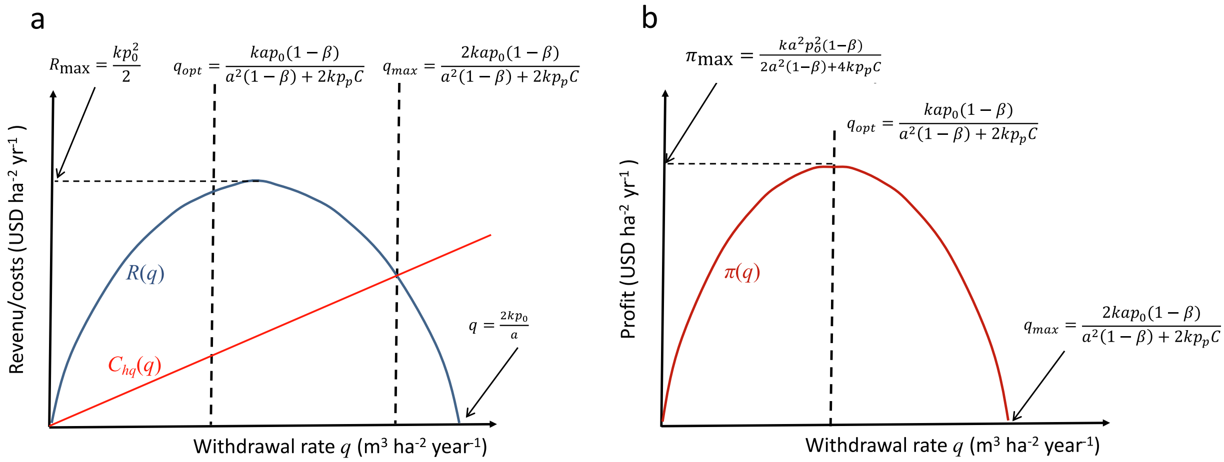

Fig. A1. Revenue and cost functions (a) and profit (b) for a stable groundwater withdrawal regime in case revenue from groundwater withdrawal is given by the integral of a linear inverse demand function.

The cost of environmental externalities can be readily included in the optimization by adding the cost C_{dQ} of streamflow loss $Q(q) - Q(q = 0)$. Since in at equilibrium all pumped water comes out of capture, these costs are proportional to the withdrawal rate q: $C_{dQ} = \gamma q$ (with the unit cost γ in USD m^{-3} pumped). Including these costs in (B7) then results in:

$$q_{opt,e} = \frac{k(ap_0 - \gamma)(1-\beta)}{a^2(1-\beta) + 2kp_pC} \tag{B12}$$

$$\pi_{max,e} = \frac{k(ap_o - \gamma)^2(1-\beta)}{2a^2(1-\beta) + 4kp_pC} \tag{B13}$$

These results can be obtained under the condition that the costs of environmental externalities are limited to $\gamma < ap_0$, since otherwise withdrawal becomes zero.

Equation (B12) now provides a measure of social welfare. To compare the difference in social welfare $\Delta\pi$ (in USD m^{-2} yr^{-1} including environmental costs) with and without taking account of environmental externalities we calculate:

$$\Delta\pi = \pi_{max,e} - (\pi_{max} - \gamma q_{opt}) \tag{B14}$$

From (B13), B(10) and (B11) then obtain:

$$\Delta\pi = \frac{k(ap_o - \gamma)^2(1-\beta)}{2a^2(1-\beta) + 4kp_pC} - \frac{ka^2p_o^2(1-\beta)}{2a^2(1-\beta) + 4kp_pC} + \gamma\frac{kap_0(1-\beta)}{a^2(1-\beta) + 2kp_pC} \tag{B15}$$

$$= \frac{k(ap_o - \gamma)^2(1-\beta) - ka^2p_o^2(1-\beta) + 2\gamma kap_0(1-\beta)}{2a^2(1-\beta) + 4kp_pC} \tag{B16}$$

This difference is positive (increase of social welfare) if

$$(ap_o - \gamma)^2 - a^2p_o^2 + 2\gamma ap_0 > 0 \tag{B17}$$

which is always the case:

$$a^2p_o^2 - 2\gamma ap_0 + \gamma^2 - a^2p_o^2 + 2\gamma ap_0 = \gamma^2 > 0 \tag{B18}$$

From (B12) it is also possible to assess what the value of γ (in USD per m^{-3} pumped) should be to assure that a certain environmental flow limit Q_{env} of streamflow is not exceeded, i.e. as a form of taxation to prevent ecological damage. From Table A1 we find the relationship between the maximum environmental pumping rate and Q_{env}:

$$Q_{env} = Q_i + (q_s + r - q_{env})A \tag{B19}$$

From which follows

$$q_{env} = \frac{Q_i + Aq_s + Ar - Q_{env}}{A} \tag{B20}$$

The value of γ needed to make sure that $q_{opt} \le q_{env}$ can be readily derived from (B12) as:

$$\frac{k(ap_0 - \gamma)(1-\beta)}{a^2(1-\beta) + 2kp_pC} \le q_{env} \tag{B21}$$

$$\gamma \ge ap_0 - \left(\frac{a^2(1-\beta) + 2kp_pC}{k(1-\beta)}\right) q_{env} \tag{B22}$$

We can use the same equation as B22 to find the additional taxation of water that is needed make sure that the optimal withdrawal rate remains within the equilibrium regime $q_{opt} \le q_{crit}$, by replacing q_{env} in (B22) with $q_{crit} = r + (Q_i + q_sA)/(WvC + A)$.

Appendix C. . Groundwater withdrawal for a depletion withdrawal regime (q and $q_{opt} > q_{crit}$, $h < d$) under competition for water

We assume an aquifer of finite extent where access is limited to the users living on top of it. If multiple users are in full competition for water, they will not be able to forego on maximizing the current profits from using the groundwater, as any water left in the ground will be used by other users, incurring more pumping costs later (Negri, 1989). So, in full competition each land owner will maximize current profits by equating marginal revenue to marginal costs.

Marginal revenue is again given by the inverse of the demand function following (B3) and the production function (B2) with $b = 1$:

$$\frac{dR}{dq} = ap_0 - \frac{a^2 q}{k} \tag{C1}$$

The marginal cost of withdrawal follows from (B4), with s the decline of groundwater due to withdrawal and h the actual head:

$$\frac{dC_q}{dq} = p_p s = p_p \left\{ \frac{rC + \alpha}{1 - \beta} - h \right\} \tag{C2}$$

Equating marginal cost to marginal revenue gives:

$$ap_0 - \frac{a^2 q}{k} = p_p \left\{ \frac{rC + \alpha}{1 - \beta} \right\} - p_p h \tag{C3}$$

from which follows a relationship between optimal withdrawal rate q and groundwater head h:

$$q = \frac{kp_0}{a} - \frac{kp_p}{a^2} \left\{ \frac{rC + \alpha}{1 - \beta} \right\} + \frac{kp_p}{a^2} h \tag{C4}$$

For $q > q_{crit}$ and $h < d$ we can write the derivative $h'(t) = dh/dt$ as (see Table 1):

$$h' = \left[\frac{r - q}{n} + \frac{(Q_i + q_s A)}{n(WvC + A)} \right] \tag{C5}$$

with $\beta = \frac{A}{WvC+A}$ (C5) can be written as:

$$h' = \left[\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right] - \frac{q}{n} \tag{C6}$$

Substitution of (C4) in (C6) for q then gives:

$$h' = \left[\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right] - \frac{kp_0}{na} + \frac{kp_p}{na^2} \left\{ \frac{rC + \alpha}{1 - \beta} \right\} - \frac{kp_p}{na^2} h \tag{C7}$$

Equation is (C7) is a first order ordinary differential equation of the form $h' = b_0 - b_1 h$ with b_0 including the first three right-hand terms of (C7) and $b_1 = kp_p/na^2$, which has the general solution:

$$h(t) = \frac{b_0}{b_1} + C_1 e^{-\frac{kp_p}{na^2}t} \tag{C8}$$

with $h(0) = d$, we have that $C_1 = d - b_0/b_1$ and renaming $A = b_0/b_1$ we have for $h(t)$:

$$h(t) = A + (d - A) e^{-\frac{kp_p}{na^2}t} \tag{C9}$$

with

$$A = \frac{a^2}{kp_p} \left\{ r + \frac{(Q_i + q_s A)}{WvC + A} \right\} - \frac{ap_0}{p_p} + \frac{rC + \alpha}{1 - \beta} \tag{C10}$$

The evolution of the withdrawal rate under competition is obtained by substitution of (C9) into (C4) which gives after some manipulation:

$$q(t) = B + \frac{kp_p}{a^2}(d - A) e^{-\frac{kp_p}{na^2}t} \tag{C11}$$

with

$$B = r + \frac{(Q_i + q_s A)}{WvC + A} \tag{C12}$$

Which shows that withdrawal rates exponentially decrease to the critical withdrawal rate for large times (compare Table A1), which is also the maximum capture when streams are disconnected from the water table. Equations (C10) and (C11) further shows that by increasing the pumping costs by e.g., taxation $p_p' = p_p + p_T$, the final water table decline is limited, but at the expense of reaching the final reduced withdrawal rate earlier in time.

The net return over time is obtained from combining revenue (C3) and cost (integral of C2):

$$\pi(t) = ap_0 q(t) - \frac{a^2 q(t)^2}{2k} - p_p \left[\left\{ \frac{rC + \alpha}{1 - \beta} \right\} - h(t) \right] q(t) \tag{C13}$$

with $h(t)$ and $q(t)$ given by (C9) and (C11) respectively. The total economic net present value (NPV) is then given by (with i the discount rate):

$$\Pi = \int_0^\infty \left\{ ap_0 q(t) - \frac{a^2 q(t)^2}{2k} - p_p \left[\left\{ \frac{rC + \alpha}{1 - \beta} \right\} - h(t) \right] q(t) \right\} e^{-it} dt \tag{C14}$$

Appendix D. . Groundwater withdrawal for a depletion withdrawal regime (q and $q_{opt} > q_{crit}$, $h < d$) under managed aquifer depletion (optimal control)

Equation (C14) provides the net present value of groundwater use for irrigation under the assumption of competition between users resulting in withdrawal rates and associated groundwater levels according to (C9) and (C11) respectively. In case some managing agent is able to force all the users to strive for inter-temporal efficiency, a withdrawal trajectory $q(t)$ is sought that maximizes (C14)

$$\Pi = \int_0^\infty \left\{ ap_0 q(t) - \frac{a^2 q(t)^2}{2k} - p_p \left[\left\{ \frac{rC + \alpha}{1-\beta} \right\} - h(t) \right] q(t) \right\} e^{-it} dt \tag{D1}$$

subject to

$$h' = \left[\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right] - \frac{q}{n} \qquad h(0) = d \tag{D2}$$

To find the solution to this optimal control problem we closely follow the derivation given by Gisser and Sánchez (1980) for a simpler case without groundwater and surface water interaction. They use the Pontryagan principle by writing the Hamiltonian $\mathscr{H}$ as:

$$\mathscr{H} = -e^{-it} \left\{ ap_0 q(t) - \frac{a^2 q(t)^2}{2k} - p_p \left[\left\{ \frac{rC + \alpha}{1-\beta} \right\} - h(t) \right] q(t) \right\} + \lambda(t) \left\{ \left[\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right] - \frac{q}{n} \right\} \tag{D3}$$

and differentiate to obtain the system (leaving the variable t out of $h(t)$, $q(t)$ and $\lambda(t)$ for convenience):

$$\frac{\partial \mathscr{H}}{\partial q} = -e^{-it} \left\{ ap_0 - \frac{a^2 q}{k} - p_p \left[\left\{ \frac{rC + \alpha}{1-\beta} \right\} - h \right] \right\} - \frac{\lambda}{n} = 0 \tag{D4}$$

$$\lambda' = -\frac{\partial \mathscr{H}}{\partial h} = e^{-it} p_p q \tag{D5}$$

From (D4) we find for λ:

$$\lambda = \frac{na^2}{k} q e^{-it} - np_p h e^{-it} - \left[ap_0 - p_p \left\{ \frac{rC + \alpha}{1-\beta} \right\} \right] ne^{-it} \tag{D6}$$

Taking the derivative with respect to time t of (D6) and equating this to (D5) gives:

$$\lambda' = \frac{-na^2 i}{k} q e^{-it} + \frac{na^2}{k} q' e^{-it} + nip_p h e^{-it} - np_p h' e^{-it} + ni \left[ap_0 - p_p \left\{ \frac{rC + \alpha}{1-\beta} \right\} \right] ne^{-it} = e^{-it} p_p q \tag{D7}$$

Dividing by ne^{-it} and collecting the time derivatives on the left side:

$$-\frac{a^2}{k} q' - p_p h' = \frac{-a^2 i}{k} q + + ip_p h - p_p \frac{q}{n} + i \left[ap_0 - p_p \left\{ \frac{rC + \alpha}{1-\beta} \right\} \right] \tag{D8}$$

Substitution of (D2) for h' in (D8) then gives:

$$-\frac{a^2}{k} q' - p_p \left\{ \left[\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right] - \frac{q}{n} \right\} = \frac{-a^2 i}{k} q + + ip_p h - p_p \frac{q}{n} + i \left[ap_0 - p_p \left\{ \frac{rC + \alpha}{1-\beta} \right\} \right] \tag{D9}$$

which results in the following first order ordinary differential equation for q':

$$q' = iq - \frac{kip_p}{a^2} h + \frac{kp_p}{a^2} \left\{ \left(\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right) + i \left(\frac{rC + \alpha}{1-\beta} \right) \right\} - \frac{kip_0}{a} \tag{D10}$$

Equation (D2) and (D10) then yield a system of coupled linear ordinary differential equations describing the evolution of $q(t)$ and $h(t)$ that maximizes net present value across time. In order to solve these, we introduce the following intermediate constants:

$$a_2 = -\frac{1}{n} \tag{D11a}$$

$$a_3 = \left(\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right) \tag{D11b}$$

$$b_1 = -\frac{kip_p}{a^2} \tag{D11c}$$

$$b_2 = i \tag{D11d}$$

$$b_3 = \frac{kp_p}{a^2} \left\{ \left(\frac{Ar + Q_i\beta + Aq_s\beta}{nA} \right) + i \left(\frac{rC + \alpha}{1-\beta} \right) \right\} - \frac{kip_0}{a} \tag{D11e}$$

The dynamic system is then written as:

$$h' = a_2 q + a_3 \tag{D12}$$

$$q' = b_1 h + b_2 q + b_3 \tag{D13}$$

Starting with the homogenous solution ($a_3 = 0, b_3 = 0$), differentiating (D13)

$$q'' = b_1 h' + b_2 q' \tag{D14}$$

and inserting the homogenous version of (D12) for *h'* yields:

$$q'' = b_2 q' + b_1 a_2 q \tag{D15}$$

Equation (D15) is a second order ordinary differential equation whose general solution is given by:

$$q(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{D16}$$

with C_1, C_2 constants depending on initial and final conditions and λ_1, λ_2 the roots of the characteristic equation $\lambda^2 - b_2\lambda - b_1 a_2 = 0$. This results in the following values for λ_1, λ_2 :

$$\lambda_{1,2} = \frac{b_2 \pm \sqrt{b_2^2 - 4b_1 a_2}}{2} = \frac{i \pm \sqrt{i^2 + 4\frac{kip_p}{na^2}}}{2} \tag{D17}$$

From $h' = a_2 q = -\frac{1}{n} q$ we then obtain the homogeneous solution for $h(t)$:

$$h(t) = \int h' dt = -\frac{C_1}{n\lambda_1} e^{\lambda_1 t} - \frac{C_2}{n\lambda_2} e^{\lambda_2 t} \tag{D18}$$

The non-homogeneous equations can be obtained by considering that for $t \to \infty, q', h' \to 0$. Equating (D12) and (D13) for $t \to \infty$ then yields:

$$q(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} - \frac{a_3}{a_2} \tag{D19}$$

$$h(t) = -\frac{C_1}{n\lambda_1} e^{\lambda_1 t} - \frac{C_2}{n\lambda_2} e^{\lambda_2 t} + \frac{b_3 - b_2\left(\frac{a_3}{a_2}\right)}{-b_3} \tag{D20}$$

Finally, the transversality condition states that for $t \to \infty, \lambda(t) \to 0$ (Beavis and Dobbs, 1990). From (D17) it then follows that C_1 should be zero. Taking $C_1 = 0$, inserting (D11a-e) into (D19) and (D20) and taking the initial condition $h(0) = d$, and taking $\beta = \frac{A}{WvC+A}$ then yields the following solutions to the optimal control problem:

$$h(t) = A + (d - A)e^{\lambda_2 t} \tag{D21}$$

$$q(t) = B - n\lambda_2 (d - A)e^{\lambda_2 t} \tag{D22}$$

With *A* and *B* given by

$$A = \left(\frac{1}{i} + \frac{a^2}{kp_p}\right)\left\{r + \frac{(Q_i + q_s A)}{WvC + A}\right\} - \frac{ap_0}{p_p} + \frac{rC + \alpha}{1 - \beta} \tag{D23}$$

$$B = r + \frac{(Q_i + q_s A)}{WvC + A} \tag{D24}$$

Note the resemblance of these solutions to (C9)-(C12) for free competition. Also, like in free competition, the withdrawal rate eventually reduces to the critical withdrawal rate or maximum capture (from recharge and streamflow leakage).

Just as done by Gisser and Sánchez (1980), we can further analyse under which conditions (D21)-(D24) revert to (C9)-(C12.). This is done by writing the part underneath the square-root part of λ_2 (D17) as:

$$i^2 + 4\frac{kip_p}{na^2} = \left(i + 2\frac{kp_p}{na^2}\right)^2 - \left(2\frac{kp_p}{na^2}\right)^2 \tag{D25}$$

If the third term $\left(2kp_p/na^2\right)^2$ in (D25) can be neglected then we have

$$\lambda_2 \approx -\frac{kp_p}{na^2} \tag{D26}$$

and the optimal control solutions resembles those of competition. This is the case if: (a) porosity *n* is large (productive aquifers), (b) the water productivity *a* is high; (c) pumping costs p_p are low; (d) the price elasticity *k* is small; (e) the discount rate *i* is high. It also shows that in that case the constant *A* (D23) starts to resemble that defined in (C10).

Including environmental externalities in D1as an additional cost proportional pumping rate γq (Esteban and Albiac, 2011) with γ the unit environmental costs of pumping (USD m^{-3})

$$\Pi = \int_0^\infty \left\{ap_0 q(t) - \frac{a^2 q(t)^2}{2k} - p_p\left[\left\{\frac{rC + \alpha}{1 - \beta}\right\} - h(t)\right]q(t) - \gamma q(t)\right\}e^{-it}dt \tag{D27}$$

yields the same results as (D21) to (D24), if we replace $ap_0 q - \gamma q = (ap_0 - \gamma)q$ with $ap_{0,e}q$ with:

$$p_o \to p_{o,e} = \frac{ap_o - \gamma}{a} \tag{26}$$

Note that the value of A (Equation D23) then becomes smaller, which results in less depletion and smaller overall pumping rates. Just as in the

Appendix E. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jhydrol.2024.131145.

References

Alley, W.M., Reilly, T.E. and Franke, O.L. (1999). *Sustainability of groundwater resources*. United States Geol. Survey, Circ. 1186 (Denver, CO: USGS).

Ayres, R., et al., 2001. Strong versus Weak Sustainability. Environ. Ethics. https://doi.org/10.5840/enviroethics200123225.

Beavis, B., Dobbs, I., 1990. Optimisation and Stability Theory for Economic Analysis. Cambridge University Press, New York.

Bierkens, M.F.P., te Stroet, C.B.M., 2007. Modelling non-linear water table dynamics and specific discharge through landscape analysis. J. Hydrol. 332, 412–426.

Bierkens, M.F.P., Wada, Y., 2019. Non-renewable groundwater use and groundwater depletion: a review. Environ. Res. Lett. 14, 063002.

Bierkens, M.F.P., Sutanudjaja, E.H., Wanders, N., 2021. Large-scale sensitivities of groundwater and surface water to groundwater withdrawal. Hydrol. Earth Syst. Sci. 25, 5859–5878.

Bredehoeft, J.D., 2002. The water budget myth revisited: why hydrogeologists model. Ground Water 40, 340–345.

Brown, G., Deacon, R., 1972. Economic optimization of a single cell aquifer. Water Resour. Res. 8, 557–564.

Brunner, P., Cook, P.G., Simmons, C.T., 2009. Hydrogeologic controls on disconnection between surface water and groundwater. Water Resour. Res. 45, W01422.

Brunner, P., Cook, P.G., Simmons, C.T., 2011. Disconnected surface water and groundwater: From theory to practice. Groundwater 49, 460–467.

Burt, O.R., 1964. Optimal resource use over time with an application to ground water. Manage. Sci. 11, 80–93.

Burt, O.R., 1967. Temporal allocation of groundwater. Water Resour. Res. 3, 45–56.

Döll, P., Mueller Schmied, H., Schuh, C., Portmann, F.T., Eicker, A., 2014. Global-scale assessment of groundwater depletion and related groundwater abstractions: Combining hydrological modeling with information from well observations and GRACE satellites. Water Resour. Res. 50, 5698–5720.

Domenico, P.A., Anderson, V., Case, C.M., 1968. Optimal groundwater mining. Water Resour. Res. 4, 247–255.

Elshall, A.S., Arik, A.D., El-Kadi, A.I., Pierce, S., Ye, M., Burnett, K.M., Wada, C.A., Bremer, L.L., Chun, G., 2020. Groundwater sustainability: a review of the interactions between science and policy. Environ. Res. Let. 15, 93004.

Esteban, E., Albiac, J., 2011. Groundwater and ecosystems damages: questioning the Gisser-Sánchez effect. Ecol. Econ. 70, 2062–2069.

Food and Agriculture Organization of the United Nations (FAO) (2021a). *FAOSTAT producer prices*. http://www.fao.org/faostat/en/#data/PP.

Food and Agriculture Organization of the United Nations (FAO) (2021b). *Crop water information*. http://www.fao.org/land-water/databases-and-software/crop-information/en/.

Foster, T., Brozovic, N., Butler, A.P., 2015. Analysis of the impacts of well yield and groundwater depth on irrigated agriculture. J. Hydrol. 523, 86–96.

Gisser, M., Sánchez, D., 1980a. Competition versus optimal control in groundwater withdrawal. Water Resour. Res. 16, 638–642.

Gisser, M., Sánchez, D.A., 1980b. Some additional economic aspects of ground water resources and replacement flows in semi-arid agricultural areas. Int. J. Contr. 31, 331–341.

Gleeson, T., Befus, K.M., Jasechko, S., Luijendijk, E., Cardenas, M.B., 2015. The global volume and distribution of modern groundwater. Nat. Geosci. 9, 161–167.

Gleeson, T., Cuthbert, M.O., Perrone, D., 2020. Global groundwater sustainability, resources, and systems in the Anthropocene. An. Rev. Earth Plan. Sci. 48, 431–463.

Gleeson, T., Wada, Y., Bierkens, M. F.P. and van Beek, L.P.H. (2012). Water balance of global aquifers revealed by groundwater footprint. *Nature* 488, 197-200.

Godfray, H.C.J., Beddington, J.R., Crute, I.R., Haddad, L., Lawrence, D., Muir, J.F., et al., 2010. Food security: The challenge of feeding 9 billion people. Science 327, 812–818.

Halvorsen, R. (Ed.), 2018. The Economics of Nonrenewable Resources. Edward Elgar Publishing, Cheltenham.

Halvorsen, R., Layton, F.L. (Eds.), 2015. Handbook on the Economics of Natural Resources. Edward Elgar Publishing, Cheltenham.

Harou, J.J., Pulido-Velazquez, M., Rosenberg, D.E., Medellín-Azuara, J., Lund, J.R., Howitt, R.E., 2009. Hydro-economic models: concepts, design, applications, and future prospects. J. Hydrol. 375, 627–643.

Hotelling, H., 1931. The economics of exhaustible resources. J. Political Econ. 39, 137–175.

Huggins, X., Gleeson, T., Castilla-Rho, J., Holley, C., Re, V., Famiglietti, J.S., 2023. Groundwater connections and sustainability in social-ecological systems. Groundwater 61, 463–478.

Jasechko, S., Perrone, D., 2021. Global groundwater wells at risk of running dry. Science 372, 418–421.

Konikow, L.F., Kendy, E., 2005. Groundwater depletion: A global problem. Hydrogeol. J. 13, 317–320.

Konikow, L.F., Leake, S.A., 2014. Depletion and capture: revisiting the source of water derived from wells. Groundwater 52, 100–111.

Koundouri, P., 2000. Three approaches to measuring natural resource scarcity: theory and application to groundwater. PhD Thesis, Department of Economics, Faculty of Economics and Politics. University of Cambridge, Cambridge, U.K.

Koundouri, P., 2004. Current issues in the economics of groundwater resource management. J. Econ. Surv. 5, 703–740.

MacEwan, D., Cayar, M., Taghavi, A., Mitchell, D., Hatchett, S., Howitt, R., 2017. Hydroeconomic modeling of sustainable groundwater management. Water Resour. Res. 53, 2384–2403.

Moench, M.H. (1992). Chasing the watertable: equity and sustainability in groundwater management. *Econ. Polit. Weekly* 27, A171–A177.

Negri, D.H., 1989. The common property aquifer as a differential game. Water Resour. Res. 25 (1), 9–15. https://doi.org/10.1029/WR025i001p00009.

Portmann, F.T., Siebert, S., Döll, P., 2010. MIRCA2000-Global monthly irrigated and rainfed crop areas around the year 2000: A new high-resolution data set for agricultural and hydrological modelling. Global Biogeochem. Cy. 24, GB1011.

Provencer, B., Burt, O.R., 1993. The externalities associated with the common property exploitation of groundwater. J. Environ. Econ. Manage. 24, 139–158.

Rodell, M., Famiglietti, J.S., Wiese, D.N., Reager, J.T., Beaudoing, H.K., Landerer, F.W., Lo, M.-H., 2018. Emerging trends in global freshwater availability. Nature 557, 651–659.

Scanlon, B.R., Longuevergne, L., Long, D., 2012. Ground referencing GRACE satellite estimates of groundwater storage changes in the California Central Valley USA. Water Resour. Res. 48, W04520.

Siebert, S., Döll, P., 2010. Quantifying blue and green virtual water contents in global crop production as well as potential production losses without irrigation. J. Hydrol. 384, 198–217.

Siebert, S., Kummu, M., Porkka, M., Döll, P., Ramankutty, N., Scanlon, B.R., 2015. A global data set of the extent of irrigated land from 1900 to 2005. Hydrol. Earth Syst. Sci 19, 1521–1545.

Sivapalan, M., Savenije, H.H.G., Blöschl, G., 2012. Sociohydrology: a new science of people and water. Hydrol. Process. 26, 1270–1276.

Sutanudjaja, E.H., van Beek, L.P.H., Wanders, N., Wada, Y., Bosmans, J.H.C., Drost, N., van der Ent, R.J., de Graaf, I.E.M., Hoch, J.M., de Jong, K., Karssenberg, D., López López, P., Peßenteiner, S., Schmitz, O., Straatsma, M.W., Vannametee, E., Wisser, D., Bierkens, M.F.P., 2018. PCR-GLOBWB 2: a 5 arcmin global hydrological and water resources model. Geosci. Model Dev. 11, 2429–2453.

Theis, C.V., 1940. The source of water derived from wells: Essential factors controlling the response of an aquifer to development. Civil Eng. 10, 277–280.

Van Weert, F.J., Van der Gun, J. and Reckman, J. (2009). *Global overview of saline groundwater occurrence and genesis*. Utrecht: IGRAC Report GP-2009-1.

Vocke, G., and Ali, M., 2012. *U.S. Wheat Production Practices, Costs, and Yields: Variations Across Regions.* Report EIB-116. U.S. Department of Agriculture, Economic Research Service.

Wada, Y., 2016. Modeling Groundwater Depletion at Regional and Global Scales: Present State and Future Prospects. Surv. Geophys. 37, 419–451.

Wada, Y., van Beek, L.P.H., van Kempen, C.M., Reckman, J.W.T., Vasak, S., Bierkens, M.F.P., 2010. Global depletion of groundwater resources. Geoph. Res. Lett. 37, L20402.

Wada, Y., van Beek, L.P.H., Bierkens, M.F.P., 2012. Nonsustainable groundwater sustaining irrigation: A global assessment. Water Resourc. Res. 48, W00L06, 1–18.

Wang, W., Dai, Z., Zhao, Y., Li, J., Duan, L., Wang, Z., Zhu, L., 2016. A quantitative analysis of hydraulic interaction processes in stream-aquifer systems. Sci. Rep. 6, 19876.

World Bank (2020). *Lending interest rate* (%). https://data.worldbank.org/indicator/FR.INR.LEND?view=map.

Worthington, V., Burt, O.R., Brustkern, R.L., 1985. Optimal management of a confined groundwater system. J. Environ. Econ. Manage. 12, 229–245.

Zwarteveen, M., Kuper, M., Olmos-Herrera, C., Dajani, M., Kemerink-Seyoum, J., et al., 2021. Transformations to groundwater sustainability: from individuals and pumps to communities and aquifers. Curr. Opin. Environ. Sustain. 49, 88–97.