When tadpoles matter: One-loop corrections for spectator Higgs in inflation

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ABSTRACT: We consider the classical attractor regime of the spectator Abelian Higgs model in power-law inflation, and compute the one-loop corrections to its evolution. For computations we utilize dimensional regularization and the propagators in the unitary gauge. The corrections to both the scalar condensate and the energymomentum tensor exhibit secular ultraviolet contributions, that tend to slow down the rolling of the scalar down its potential, and drive it away from the classical attractor. These corrections need not be suppressed if the U(1) charge is much larger than the scalar self-coupling, which is seen already in flat space. In addition, at late times the secular corrections necessarily invalidate the perturbative loop expansion. We find the late time secular corrections to be captured by the renormalization group, which opens up the possibility to resum them past the breakdown of perturbativity.

1 Introduction

The question of whether inflationary observables — expressed in terms of scalar and tensor cosmological perturbations — can be significantly affected by quantum loop corrections is still very much an open one. Quantum corrections to cosmological perturbations can arise in different manners: (i) as indirect corrections descending from the quantum corrections to the background fields, and (ii) as direct corrections arising from interactions between evolving perturbations. On a technical level, the indirect corrections descend from the corrections to the one-point functions (condensates), while the direct corrections are corrections to the connected two-point functions. The computation of former type is simpler as typically the loop consists of a single propagator.

While the understanding of the direct type of corrections to the cosmological perturbations is still marred by questions of gauge dependence [1], considerable progress has been made in understanding the indirect type of corrections, to which this work is devoted to as well. When it comes to corrections to condensates in cosmological expanding and accelerating spaces distinctions should be made between ultraviolet and infrared corrections. The former are in a sense universal as they depend on the ultraviolet structure of the theory captured by modes that are not strongly coupled to the background. That is why ultraviolet corrections in principle can be computed in any curved space [2–4]. These have found applications in inflation, investigating quantum corrections to the inflaton potential [5, 6], particularly in Higgs inflation [7–9] in connection to the quantum corrections to the Higgs effective potential [10–15], and in studies of how curvature corrections influence the Higgs potential stability [16–21] in inflation and reheating (for a review see [22]).

Infrared corrections derive from the tree-level effect of gravitational particle production [23–25] for fields non-conformally coupled to gravity, such as scalars or gravitons. It should be emphasized that infrared corrections can be more important that ultraviolet corrections. These effects are innate to accelerating expanding spacetimes, and have no analogue in flat space, as they are in effect a consequence of the cosmological horizon. As opposed to ultraviolet corrections, infrared ones are not universal and depend on the details of the model and the expanding spacetime, and it is much more challenging to quantify them. That is the reason why most works in the literature make a simplifying assumption of approximating the expanding spacetime by exactly de Sitter space, characterized by the constant Hubble rate. Such works date back to [2, 26–28], and the investigations of curvature corrections to symmetry breaking potentials and the study of phase transitions. Early results on corrections to the inflaton evolution suggested that the corrections are tiny [29, 30], which is due to the smallness of the couplings and the assumed perturbative regime. However, strong infrared effects for spectator fields can lead to symmetry restoration of the scalar models with symmetry-breaking potentials [31, 32], but also to dynamical

symmetry breaking in massless scalar electrodynamics [33–35] and the generation of a non-perturbatively large mass for the vector field [36–38]. When infrared effects are large typically one needs to describe them using non-perturbative methods, the Starobinsky's stochastic formalism [31, 39] being perhaps the most prominent one.

The de Sitter space is a very often good approximation for the realistic slowroll inflationary background. However, primordial cosmological observables crucially depend on the parameters measuring the deviation of primordial inflation from the exact de Sitter space. This deviation is usually encoded in the slow-roll parameters, where the principal one measures the rate of change of the Hubble rate, $\epsilon = -\dot{H}/H^2$, and the higher order ones encode the higher time derivatives. It was argued in Ref. [40] that quantum corrections introduce an additional fine-tuning problem into single scalar potential models of inflation, and presumably the same is true for spectator fields. This is why it is paramount to understand how quantum corrections depend on the slow-roll parameters. Efforts in that direction have been undertaken in inflation for spectator scalars [41–44] and for the inflaton [45–50], and even for the period of reheating [51, 52]. Particularly insightful are studies of quantum effects in power-law inflation, characterized by constant principal slow-roll parameter ϵ , and subsequently promoted to an adiabatic function of time. This is because analytically tractable computations are still feasible, both for the ultraviolet and for the infrared, and one can get a clear picture of the effects that corrections can engender. A particularly important reference for this work is [41] which considered a symmetrybreaking scalar potential model in power-law inflation with the assumption of the scaling solution, and it found that inclusion of quantum effects can significantly affect the scalar potential by dynamically restoring the symmetry of the potential in the small field regime, more rapidly than in de Sitter.

In this work we extend the analysis of Ref. [41] and compute one-loop corrections to the condensate of the Abelian Higgs model — scalar electrodynamics with a symmetry breaking potential. For the sake of clarity we adopt several simplifying assumptions. We assume that the Abelian Higgs model is a spectator, in the sense that it does not source the evolution of the expanding background. For the background we assume it is exact power-law inflation, characterized by the constant principal slow-roll parameter ϵ . This still allows for an analytically tractable analysis of the problem, at least at one-loop order. We consider the scalar modulus of the non-minimally coupled Abelian Higgs model to be in the attractor regime in power-law inflation, where it tracks the evolution of the decreasing background Hubble rate, $\overline{\phi} \propto H$. This scaling attractor can be seen as a dynamical generalization of the symmetry breaking minimum in equilibrium theories, as it is formed by the competition between the non-minimal coupling and the quartic self-coupling.

The main focus of this work are quantum corrections to the evolution of the scaling attractor. We account for loop corrections to the attractor descending from both the charged scalar and the vector. These are quantified by computing one-loop

corrections to the scalar one-point function, and to the graviton one-point function (that is sourced by the energy-momentum tensor. Our analysis utilizes propagators for power-law inflation, namely the scalar one thas was worked out in [53], and the vector one that was constructed in [54] in the unitary gauge. An important difference between the analysis performed in this work and that in [41] is in that the only source of breaking of scaling we encounter descends from contributions connected to the ultraviolet scale μ introduced by the counterterms. The non-vanishing scalar condensate generates masses for scalar and vector perturbations, which regulates their infrared sector and prevents large infrared effects from developing, except in the small condensate limit which is beyond the scope of this work. Our finding show that there is a secular ultraviolet effect that typically slows down the rolling of the spectator scalar down its potential, and eventually drives the condensate away from the classical attractor. Interestingly, this effect is completely absent in the de Sitter limit, which again points to the importance of considering quantum effects in more realistic inflationary backgrounds.

We perform the loop computations using dimensional regularization and we adopt a rather conservative approach to renormalization. Namely we forbid counterterm coefficients from depending on the principal slow roll parameter ϵ , that we consider to be a geometric quantity, that describes a particular realization of the system, and does not characterize the theory. This severely restricts which terms can be subtracted by finite parts of the counterterms, thus making the results more robust to renormalization prescription dependencies. Furthermore, we shy away from using any initial state subtractions, as these introduce renormalization scheme dependence to the initial state, and generate corrections which anyway decay at late times we are interested in.

This work is organized as follows. In section 2 we present the model and set the stage for the problem. Sections 3 and 4 are devoted to the calculation of the one-loop scalar condensate and energy-momentum tensor, where we discuss in detail the countertems needed to renormalize the results in dimensional regularization. Section 5 is devoted to studying various limits and comparing them to the literature. Particular attention is paid to the analysis of secular effects and their growth, and to the conditions under which pertubation theory breaks. In section 6 we present a preliminary analysis of how to emply renormalization group tools to restore perturbativity of the results. In particular, we show there that the RG resummation allows one to write the complete one-loop results in the tree-level form, up to an additional term in the energy-momentum tensor, which is due to the perturbative nonrenormalizability of quantum gravity. In section 7 we discuss our main results.

2 Preliminaries

The Abelian Higgs model is a commonly utilized as a toy model for the electroweak sector of the standard model. In this work we consider its nonminimally coupled variant, and the quantum corrections to its dynamics in power-law inflation. In this section we summarize the properties of the background, and recount the main properties of the model.

2.1 Power-law inflation

Spatially flat expanding cosmological spaces are well described by the Friedmann-Lemaître-Robertson-Walker (FLRW) invariant line element,

$$ds^{2} = -dt^{2} + a^{2}(t)d\vec{x}^{2} = a^{2}(\eta)\left[-d\eta^{2} + d\vec{x}^{2}\right], \qquad (2.1)$$

where time is conveniently given either in physical time coordinate t or the conformal time coordinate η , the two being related via the scale factor, $dt = ad\eta$, and where equal time spatial hypersurfaces are Euclidean spaces covered by Cartesian coordinates x^i . The expansion of the space is encoded by the scale factor a, and its first several derivatives. The first derivative is usually given as the physical Hubble rate H, or the conformal Hubble rate \mathcal{H} ,

$$H = \frac{1}{a}\frac{da}{dt}, \qquad \qquad \mathcal{H} = \frac{1}{a}\frac{da}{d\eta}, \qquad \qquad \mathcal{H} = aH, \qquad (2.2)$$

while in inflation the second derivative is encoded by the principal slow-roll parameter,

$$\epsilon = -\frac{1}{H^2} \frac{dH}{dt} = 1 - \frac{1}{\mathcal{H}^2} \frac{d\mathcal{H}}{d\eta} \,. \tag{2.3}$$

In this work we consider power-law inflation [55, 56] that is defined by the constant slow-roll parameter,

$$\epsilon = \text{const.}\,, \qquad \qquad 0 < \epsilon < 1\,. \tag{2.4}$$

Even though this model is not a realistic model of inflation, as it is excluded by observations [57] that favour an adiabatically evolving ϵ , it is more realistic than the de Sitter space, $\epsilon = 0$, often utilized to study quantum field theoretic effects in inflation. In addition to being almost as mathematically tractable as de Sitter, power law inflation incorporates the effect of the non-vanishing slow-roll parameter, and the evolving Hubble rate $H = H_0 a^{-\epsilon}$. The scale factor and the conformal Hubble rate in power-law inflation take the form,

$$a(\eta) = \left[1 - (1 - \epsilon)H_0(\eta - \eta_0)\right]^{-\frac{1}{1 - \epsilon}}, \qquad \mathcal{H}(\eta) = H_0 a^{1 - \epsilon}, \qquad (2.5)$$

where η_0 is the initial time at which $a(\eta_0) = 1$, and $H_0 = \mathcal{H}(\eta_0)$. The curvature tensors in power-law inflation are,

$$R_{\mu\nu\rho\sigma} = 2H^2 g_{\mu[\rho}g_{\sigma]\nu} + 4\epsilon \left(a^2 \delta^0_{[\mu}g_{\nu][\sigma}\delta^0_{\rho]}\right), \qquad (2.6)$$

$$R_{\mu\nu} = (D - 1 - \epsilon)H^2 g_{\mu\nu} + (D - 2)\epsilon H^2 \left(a^2 \delta^0_{\mu} \delta^0_{\nu}\right), \qquad (2.7)$$

$$R = (D-1)(D-2\epsilon)H^2.$$
 (2.8)

In addition, it is useful to note that the Weyl tensor,

$$C_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} - \frac{4}{(D-2)}g_{\mu][\rho}R_{\sigma][\nu} + \frac{2R}{(D-1)(D-2)}g_{\mu[\rho}g_{\sigma]\nu}.$$
 (2.9)

2.2 Nonminimally coupled Abelian Higgs model

The Abelian Higgs model consists of an Abelian gauge field A_{μ} interacting with a charged complex scalar Φ . Its action in *D*-dimensional curved space is,

$$S[A_{\mu}, \Phi, \Phi^{*}] = \int d^{D}x \sqrt{-g} \left[-\frac{Z_{A}^{0}}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - Z_{\phi}^{0} g^{\mu\nu} (D_{\mu} \Phi)^{*} (D_{\nu} \Phi) -\lambda_{0} (\Phi^{*} \Phi)^{2} - \xi_{0} R \Phi^{*} \Phi \right], \qquad (2.10)$$

where $g_{\mu\nu}$ denotes the metric tensor $[g^{\mu\rho}g_{\mu\nu} = \delta^{\rho}_{\nu}, g = \det(g_{\mu\nu})]$, R is the Ricci scalar, and the bare couplings (denoted by sub/superscripts 0) in (2.10) are split into their renormalized values plus counterterms (denoted by prefix δ),

$$Z_A^0 = 1 + \delta Z_A, \quad Z_\phi^0 = 1 + \delta Z_\phi, \quad \lambda_0 = \lambda + \delta \lambda, \quad \xi_0 = \xi + \delta \xi, \quad q_0 = q + \delta q. \quad (2.11)$$

The counterterms are necessary to absorb ultraviolet divergences of quantum loops, and are organized as a power series in \hbar , the dependence on which is henceforth suppressed by adopting the natural units $\hbar = c = 1$. In the unitary gauge, defined by the condition Im(Φ) = 0, the action (2.10) reads [54],

$$S[A_{\mu},\phi] = \int d^{D}x \sqrt{-g} \left[-\frac{Z_{A}^{0}}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{(q_{0}\phi)^{2}}{2} g^{\mu\nu} A_{\mu} A_{\nu} - \frac{Z_{\phi}^{0}}{2} g^{\mu\nu} (\partial_{\mu}\phi) (\partial_{\nu}\phi) - \frac{\lambda_{0}}{4} \phi^{4} - \frac{\xi_{0}}{2} R \phi^{2} \right].$$
(2.12)

where $\phi \equiv \operatorname{Re}(\Phi)/\sqrt{2}$ is the canonically normalized real scalar field. Note that in the limit $q \to 0$ the unitary gauge action reduces to the one for the self-interacting real scalar field. This is the sense in which all the results presented here can be seen in the limit of vanishing charge.

Following [54], we consider the Abelian Higgs model in rigid power-law inflation. Only the scalar is supposed to have a homogeneous and isotropic condensate, which satisfies the equation of motion,

$$\left[\Box -\xi R - \lambda \overline{\phi}^2\right] \overline{\phi} = 0, \qquad (2.13)$$

that admits an attractor solution,

$$\overline{\phi} = \frac{\overline{\phi}_0}{H_0} H \,, \tag{2.14}$$

with the amplitude set by the parameters of the model and the slow-roll parameter of the background,

$$\frac{\overline{\phi}_0}{H_0} = \pm \sqrt{\frac{1}{\lambda} \Big[\epsilon (D-1-2\epsilon) - \xi (D-1)(D-2\epsilon) \Big]} \xrightarrow{D \to 4} \pm \sqrt{\frac{1}{\lambda} \Big[(1-6\xi)(2-\epsilon) - 2(1-\epsilon)^2 \Big]}$$
(2.15)

For this attractor solution to exist the amplitude above should be real, which puts some bounds on the non-minimal coupling,

$$\xi < \frac{\epsilon(3-2\epsilon)}{6(2-\epsilon)} \equiv \xi_{\rm cr} \,, \tag{2.16}$$

dependent on the slow-roll parameter. We study scalar and vector fluctuations around this background solution. The vector field A_{μ} is assumed not to have a classical condensate and is treated as a fluctuation, while the full scalar field, $\phi = \overline{\phi} + \varphi$, is expanded in fluctuations φ around the classical attractor condensate defined in (2.14) and (2.15). We capture the nonlinear evolution of quantized fluctuations $\hat{\varphi}$ and \hat{A}_{μ} perturbatively, by computing corrections to the evolution of the linearized (noninteracting) fluctuations $\hat{\varphi}^0$ and \hat{A}^0_{μ} . We organize the corrections in powers of the linearized fields, which generates the usual loop expansion from the interaction picture. Each power of the linearized field counts as one power of $\sqrt{\hbar}$, which we henceforth suppress by adopting natural units $\hbar = 1$.

Linearized perturbations around the attractor condensate (2.14) and their twopoint functions have been worked out in the unitary gauge in [54]. The linearized scalar perturbations,

$$S_{S}^{(2)}[A_{\mu},\varphi] = \int d^{D}x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} (\partial_{\mu}\varphi)(\partial_{\nu}\varphi) - \frac{3\lambda}{2} \overline{\phi}^{2} \varphi^{2} - \frac{\xi}{2} R \varphi^{2} \right].$$
(2.17)

correspond to a spectator scalar with an effective mass $3\lambda \overline{\phi}^2$ and the nonminimal coupling to gravity ξ . Since the tree-level condensate (2.14) scales as the Hubble rate, for the purposes of dynamics we can consider the linearized scalar perturbations to possess just an effective mass,

$$M_{s}^{2} = 3\lambda\overline{\phi}^{2} + \xi R = \left[3\epsilon(D-1-2\epsilon)-2\xi(D-1)(D-2\epsilon)\right]H^{2} \xrightarrow{D\to4} 3\left[-4\xi(2-\epsilon)+\epsilon(3-2\epsilon)\right]H^{2}, \tag{2.18}$$

or an effective non-minimal coupling,

$$\xi_s = \frac{M_s^2}{R} \xrightarrow{D \to 4} -2\xi + \frac{\epsilon(3-2\epsilon)}{2(2-\epsilon)}.$$
(2.19)

Nevertheless, when computing the energy-momentum tensor, it is important to make the distinction between the two commensurate contributions in (2.17). The latter of the two quantities is bound by $\xi_s > (3-2\epsilon)\epsilon/[6(2-\epsilon)]$ due to (2.16), which ensures there are no infrared divergences for CTBD states [58, 59].

Linearized vector perturbations,

$$S_{V}^{(2)}[A_{\mu}] = \int d^{D}x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{(q\overline{\phi})^{2}}{2} g^{\mu\nu} A_{\mu} A_{\nu} \right]$$
(2.20)

behave as though massive, with the effective mass induced by the scalar condensate,

$$M_{\nu}^{2} = \left(q\overline{\phi}\right)^{2} = \frac{q^{2}}{\lambda} \left[\epsilon(D-1-2\epsilon) - \xi(D-1)(D-2\epsilon)\right] H^{2} \xrightarrow{D \to 4} \frac{q^{2}}{\lambda} \left[-6\xi(2-\epsilon) + \epsilon(3-2\epsilon)\right] H^{2}, \tag{2.21}$$

or as effectively non-minimally coupled,

$$\xi_{V} = \frac{M_{V}^{2}}{R} \xrightarrow{D \to 4} \frac{q^{2}}{\lambda} \left[-\xi + \frac{\epsilon(3 - 2\epsilon)}{6(2 - \epsilon)} \right].$$
(2.22)

Eq. (2.16) implies $\xi_V > 0$ and $M_V^2 > 0$, which ensures stability and infrared finiteness of vector perturbations, and distinguishes our model from vector curvaton models [60].

In this work we compute the one-loop effects that interacting perturbations impart on the evolution of the scalar condensate in the attractor (2.14), and on the energy-momentum tensor; the latter accounts for the backreaction onto the powerlaw inflation. This computation requires the two-point functions of scalar and vector perturbations, and in particular their coincidence limits collected in the following subsection.

2.3 Two-point functions

The two-point functions for linearized fluctuations of the model in (2.12) in the unitary gauge in power-law inflation have been worked out in [54]. They are expectation values of linearized fields (denoted by superscript 0). In this work we compute the one-loop corrections to the scalar condensate, and to the energy-momentum tensor. These are simplest one-loop corrections, where the loop is formed by a single propagator only. That is why we require only the dimensionally regulated coincidence limits of the propagators from [54]. For the scalar this is,

$$\left\langle \hat{\varphi}^{0}(x)\hat{\varphi}^{0}(x)\right\rangle = \mathbf{\Gamma}(\nu_{s})(1-\epsilon)^{2}H^{2}\left[\left(\frac{D-3}{2}\right)^{2}-\nu_{s}^{2}\right],$$
(2.23)

and for the vector,

$$\begin{split} \left\langle \hat{A}^{0}_{\mu}(x)\hat{A}^{0}_{\nu}(x)\right\rangle &= \mathbf{\Gamma}(\nu_{\nu})(1-\epsilon)^{2}H^{2}\left[\left(\frac{D-3}{2}\right)^{2}-\nu_{\nu}^{2}\right] \\ &\times \left\{\frac{1}{D}\left[(D-1)+\left(D-1+\frac{(D-4)\epsilon}{2(1-\epsilon)}\right)\left(2+\frac{(D-4)(2-\epsilon)}{2(1-\epsilon)}\right)\frac{(1-\epsilon)^{2}H_{0}^{2}}{(q\overline{\phi}_{0})^{2}}\right]g_{\mu\nu} \right. \\ &\left.+\frac{(D-4)\epsilon}{2(1-\epsilon)}\left(2+\frac{(D-4)(2-\epsilon)}{2(1-\epsilon)}\right)\frac{(1-\epsilon)^{2}H_{0}^{2}}{(q\overline{\phi}_{0})^{2}}\left(a^{2}\delta^{0}_{\mu}\delta^{0}_{\nu}\right)\right\}, \end{split}$$
(2.24)

where we have defined a coefficient,

$$\Gamma(\nu) \equiv \frac{\Gamma(\frac{2-D}{2}) \left[(1-\epsilon)H \right]^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D-3}{2}+\nu) \Gamma(\frac{D-3}{2}-\nu)}{\Gamma(\frac{1}{2}+\nu) \Gamma(\frac{1}{2}-\nu)}$$
(2.25)
$$\overset{D \to 4}{\sim} \frac{\Gamma(\frac{2-D}{2}) \left[(1-\epsilon)H \right]^{D-4}}{(4\pi)^{\frac{D}{2}}} \left\{ 1 + \frac{(D-4)}{2} \left[\psi(\frac{1}{2}+\nu) + \psi(\frac{1}{2}-\nu) \right] + \mathcal{O}[(D-4)^2] \right\},$$

divergent in D=4 on the account of $\Gamma\left(\frac{2-D}{2}\right) \stackrel{D\to4}{\sim} 2/(D-4)$, that depends on indices of scalar and vector perturbations, respectively,

$$\nu_s^2 = \left(\frac{D-1-\epsilon}{2(1-\epsilon)}\right)^2 - \frac{\xi_s(D-1)(D-2\epsilon)}{(1-\epsilon)^2} \xrightarrow{D\to4} \frac{25}{4} - \frac{2(1-6\xi)(2-\epsilon)}{(1-\epsilon)^2}, \qquad (2.26)$$

$$\nu_{V}^{2} = \left(\frac{D-3-\epsilon}{2(1-\epsilon)}\right)^{2} - \frac{\xi_{V}(D-1)(D-2\epsilon)}{(1-\epsilon)^{2}} \xrightarrow{D\to4} \frac{1}{4} + \frac{2q^{2}}{\lambda} \left[1 - \frac{(1-6\xi)(2-\epsilon)}{2(1-\epsilon)^{2}}\right]. \quad (2.27)$$

In four dimensions the range of these indices is limited by (2.16),

$$\nu_s^2 \in \left(-\infty, \frac{9}{4}\right), \qquad \nu_v^2 \in \left(-\infty, \frac{1}{4}\right).$$
(2.28)

We also need a dimensionally regulated scalar kinetic term, which is easily computed by first taking derivatives of the scalar Wightman two-point function and then using expressions from Sec. 7.5.3 of [54], 1

$$\left\langle \partial_{\mu} \hat{\varphi}^{0}(x) \, \partial_{\nu} \hat{\varphi}^{0}(x) \right\rangle = \mathbf{\Gamma}(\nu_{s}) (1-\epsilon)^{4} H^{4} \left[\left(\frac{D-3}{2} \right)^{2} - \nu_{s}^{2} \right] \\ \times \left\{ -\frac{1}{D} \left[\left(\frac{D-1}{2} \right)^{2} - \nu_{s}^{2} \right] g_{\mu\nu} + \left[\frac{(D-2)\epsilon}{2(1-\epsilon)} \right]^{2} \left(a^{2} \delta_{\mu}^{0} \delta_{\nu}^{0} \right) \right\}.$$
(2.29)

¹If the coincidence limit is defined in terms of the Feynman propagator instead care needs to be taken to discard possible local terms resulting from derivatives acting on the time-ordeing. In addition, in the unitary gauge attention must be paid to the fact that the Feynman propagator is not the Green's function of the theory [54], and to the difference between \mathcal{T} and \mathcal{T}^* ordered products, similarly to what was pointed out in perturbative quantum gravity by Donoghue [61].

Likewise we need the coincidence limit of the vector field strength correlator, obtained by taking the coincident limit of the correlator computed in Sec. 8 of [54],

$$\left\langle \hat{F}^{0}_{\mu\nu}(x)\hat{F}^{0}_{\rho\sigma}(x) \right\rangle = \mathbf{\Gamma}(\nu_{\nu})(1-\epsilon)^{4}H^{4} \left[\left(\frac{D-3}{2}\right)^{2} - \nu_{\nu}^{2} \right]$$

$$\times \left\{ -\frac{4}{D} \left[\left(\frac{D-1}{2}\right)^{2} - \nu_{\nu}^{2} \right] g_{\mu[\rho}g_{\sigma]\nu} - \frac{2(D-4)\epsilon}{(1-\epsilon)} \left[1 - \frac{(D-4)\epsilon}{2(1-\epsilon)} \right] \left(a^{2}\delta^{0}_{[\mu}g_{\nu][\sigma}\delta^{0}_{\rho]} \right) \right\}.$$

$$(2.30)$$

Even though for convenience we have introduced a number of parameters in this section, it should be emphasized that there are only two independent coupling constants in the problem, λ and q, and two background parameters, ϵ and H_0 . Nevertheless, we use the parameters introduced here interchangeably, to make the expressions more compact. Six particularly useful combinations when presenting final results are are,

$$(1-\epsilon)^2 H^2 \xrightarrow{D \to 4} -\frac{\lambda \overline{\phi}^2}{2} + \frac{1}{2} \left(\frac{1}{6} - \xi\right) R, \qquad (2.31a)$$

$$(1-\epsilon)^2 \left(\frac{1}{4} - \nu_s^2\right) H^2 \xrightarrow{D \to 4} 3\lambda \overline{\phi}^2 - \left(\frac{1}{6} - \xi\right) R, \qquad (2.31b)$$

$$(1-\epsilon)^2 \left(\frac{9}{4} - \nu_s^2\right) H^2 \xrightarrow{D \to 4} 2\lambda \overline{\phi}^2, \qquad (2.31c)$$

$$(1-\epsilon)^2 \left(\frac{25}{4} - \nu_s^2\right) H^2 \xrightarrow{D \to 4} 2\left(\frac{1}{6} - \xi\right) R, \qquad (2.31d)$$

$$(1-\epsilon)^2 \left(\frac{1}{4} - \nu_V^2\right) H^2 \xrightarrow{D \to 4} (q\overline{\phi})^2, \qquad (2.31e)$$

$$(1-\epsilon)^2 \left(\frac{9}{4} - \nu_v^2\right) H^2 \xrightarrow{D \to 4} (q\overline{\phi})^2 - \lambda\overline{\phi}^2 + \left(\frac{1}{6} - \xi\right) R.$$
(2.31f)

3 Scalar tadpole

Expanding the unitary gauge action (2.12) to cubic order in perturbations,

$$S^{(3)}[A_{\mu},\varphi] = \int d^{D}x \sqrt{-g} \left[-\overline{\phi} \left(q^{2}\varphi A^{\mu}A_{\mu} - \lambda\varphi^{3} \right) - \varphi \left(\delta\lambda\overline{\phi}^{3} + \delta\xi R\overline{\phi} - \delta Z_{\phi} \Box\overline{\phi} \right) \right], \quad (3.1)$$

defines the vertices and counterterms that generate the one-loop corrections to the tadpole, whose diagramatic representation is given in Fig. 1. Note that the divergent counterterm coefficients count as two powers of pertrubations.

The equation of motion for the one-loop correction to the scalar one-point function takes the form,

$$\left[\Box -\xi R - 3\lambda \overline{\phi}^{2}\right] \left\langle \hat{\varphi} \right\rangle = \mathcal{S}_{S} + \mathcal{S}_{V} + \delta \mathcal{S}, \qquad (3.2)$$



Figure 1. Diagrams depicting the one-loop corrections to the one-point function from the scalar perturbation. Dashed line correspond to the scalar propagator, and wavy ones to the vector propagator. The first two diagrams descend from the vertices defined in the cubic action (3.1), which also defines the counterterms represented by the last diagram.

Where the scalar loop and the vector loop sources, corresponding to the first and the second diagram in Fig. 1 respectively, are

$$\mathcal{S}_{S} = 3\lambda \overline{\phi} \left\langle \hat{\varphi}^{0} \hat{\varphi}^{0} \right\rangle, \qquad \qquad \mathcal{S}_{V} = q^{2} \overline{\phi} g^{\mu\nu} \left\langle \hat{A}^{0}_{\mu} \hat{A}^{0}_{\nu} \right\rangle, \qquad (3.3)$$

and the counterterms, corresponding to the last amputated diagram in Fig. 1 contribute as,

$$\delta \mathcal{S} = \left(\delta \xi - \xi \delta Z_{\phi}\right) R \overline{\phi} + \left(\delta \lambda - \lambda \delta Z_{\phi}\right) \overline{\phi}^{3} \\ = \left[\left(\delta \xi - \xi \delta Z_{\phi}\right) (D - 1) (D - 2\epsilon) H^{2} + \left(\delta \lambda - \lambda \delta Z_{\phi}\right) \overline{\phi}^{2} \right] \overline{\phi}, \qquad (3.4)$$

where we have used the tree-level equation of motion (2.13) to write them in this form. The counterterm coefficients will be chosen so as to absorb the divergences from the scalar and the vector source (3.3). These divergences are absorbed independently for each source, up to arbitrary finite parts. Accordingly, we split the counterterm contribution to the source into three parts,

$$\delta \mathcal{S} = \left[\delta \mathcal{S}\right]_{S}^{\text{div.}} + \left[\delta \mathcal{S}\right]_{V}^{\text{div.}} + \left[\delta \mathcal{S}\right]^{\text{fin.}}, \qquad (3.5)$$

and this split translates onto the analogous split of all the counterterm coefficients. Since the renormalizability of the theory should not depend on the background spacetime, the counterterm coefficients abould be independent of the packground parameters such as ϵ . This breaks some redundancy between the coefficients.

3.1 Scalar loop source

This is the simplest part to compute. Scalar tadpole source can be written as, using the coincident scalar two-point function (2.23) and expressions from Sec. 2.2,

$$S_{s} = \overline{\phi} \, \mathbf{\Gamma}(\nu_{s}) \left\{ \frac{3D(D-2)(D-6)\lambda}{32(D-1)} \left[1 - \frac{4(D-1)\xi}{(D-2)} \right] (D-1)(D-2\epsilon) H^{2} + \frac{3(D+2)(8-D)\lambda^{2}}{8} \overline{\phi}^{2} + \mathcal{O}\left[(D-4)^{2} \right] \right\}.$$
(3.6)

Comparing with the counterterms (3.4), the simplest ϵ -independent choices are,

$$\left[\delta\xi - \xi\delta Z_{\phi}\right]_{S}^{\text{div.}} = \frac{\mu^{D-4}\,\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{3D(D-2)(6-D)\lambda}{32(D-1)} \left[1 - \frac{4(D-1)\xi}{(D-2)}\right], \quad (3.7a)$$

$$\left[\delta\lambda - \lambda\delta Z_{\phi}\right]_{S}^{\text{div.}} = \frac{\mu^{D-4}\,\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{3(D+2)(D-8)\lambda^{2}}{8}\,. \tag{3.7b}$$

Taking the $D \to 4$ limit of the scalar loop tadpole source produces a fully renormalized result,

$$\mathcal{S}_{S} + \left[\delta \mathcal{S}\right]_{S}^{\text{div.}} \xrightarrow{D \to 4} \frac{3\lambda \overline{\phi} H^{2}}{16\pi^{2}} (1-\epsilon)^{2} \left(\frac{1}{4} - \nu_{S}^{2}\right) \left[2\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \Psi(\nu_{S})\right], \qquad (3.8)$$

where we defined,

$$\Psi(\nu) \equiv \psi\left(\frac{1}{2} + \nu\right) + \psi\left(\frac{1}{2} - \nu\right),\tag{3.9}$$

where $\psi(z) = \frac{d}{dz} \ln[\Gamma(z)]$ denotes the digamma function.

3.2 Vector loop source

For the vector tadpole source we only need the fully contracted coincident vector propagator from (2.24). Using some relations between parameters from Sec. (2.2) we write it in the form,

$$S_{V} = \overline{\phi} \, \Gamma(\nu_{V}) \left\{ \frac{(D-1)q^{2}}{2} \Big[2q^{2} - (D-2)\lambda \Big] \overline{\phi}^{2} + 3(D-4)q^{2} \Big[\frac{3}{4} - \frac{(1-\epsilon)^{3}H_{0}^{2}}{(q\overline{\phi}_{0})^{2}} \Big] \epsilon H^{2} + \frac{(D-2)(8-D)q^{2}}{16} \Big[1 - \frac{8(D-1)\xi}{(8-D)} \Big] (D-1)(D-2\epsilon)H^{2} + \mathcal{O}\big[(D-4)^{2} \big] \right\}.$$
(3.10)

Comparing this source with the form of the counterterms it is clear that the two divergent terms have the same structure. Therefore, the simplest ϵ -independent choice for counterterm coefficients seems to be,

$$\left[\delta\xi - \xi\delta Z_{\phi}\right]_{V}^{\text{div.}} = \frac{\mu^{D-4}\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{(D-2)(D-8)q^{2}}{16} \left[1 - \frac{8(D-1)\xi}{(8-D)}\right], \quad (3.11)$$

$$\left[\delta\lambda - \lambda\delta Z_{\phi}\right]_{V}^{\text{div.}} = \frac{\mu^{D-4}\,\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{(D-1)q^{2}}{2} \Big[(D-2)\lambda - 2q^{2} \Big] \,. \tag{3.12}$$

This leads to the renormalized vector tadpole source,

$$\mathcal{S}_{V} + \left[\delta \mathcal{S}\right]_{V}^{\text{div.}} \xrightarrow{D \to 4} \frac{3q^{2}\overline{\phi}H^{2}}{16\pi^{2}} \left\{ (1-\epsilon)^{2} \left(\frac{9}{4} - \nu_{V}^{2}\right) \left[2\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \Psi(\nu_{V})\right] + \left[\frac{3}{2} - \frac{2(1-\epsilon)^{3}H_{0}^{2}}{\left(q\overline{\phi}_{0}\right)^{2}}\right]\epsilon \right\}.$$
(3.13)

3.3 Solving tadpole equation

Here we solve equation (3.2) for the condensate correction $\langle \hat{\varphi} \rangle$, with sources computed in (3.8) and (3.13). We can write the equation as,

$$-\left[\frac{1}{a^2}\partial_0^2 + \frac{2H}{a}\partial_0 + 6\xi_s(2-\epsilon)H^2\right]\left\langle\hat{\varphi}\right\rangle = \frac{3\overline{\phi}H^2}{16\pi^2}\left[2\mathcal{A}\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \mathcal{B}\right],\qquad(3.14)$$

where the coefficients are read off from the sources (3.8) and (3.13) computed in the preceding sections. The first coefficient, $\mathcal{A} = \mathcal{A}_S + \mathcal{A}_V$, receives contributions from both the scalar and vector loop source, which are, respectively, succinctly written as,

$$\mathcal{A}_{s} = \lambda (1-\epsilon)^{2} \left(\frac{1}{4} - \nu_{s}^{2}\right), \qquad \qquad \mathcal{A}_{v} = q^{2} (1-\epsilon)^{2} \left(\frac{9}{4} - \nu_{v}^{2}\right). \qquad (3.15)$$

Analogous is true for the second coefficient, $\mathcal{B} = \mathcal{B}_{s} + \mathcal{B}_{v} + \delta \mathcal{B}$, where scalar and vector contributions are, respectively,

$$\mathcal{B}_{s} = \lambda (1-\epsilon)^{2} \left(\frac{1}{4} - \nu_{s}^{2}\right) \Psi(\nu_{s}), \qquad (3.16a)$$

$$\mathcal{B}_{V} = q^{2}(1-\epsilon)^{2} \left(\frac{9}{4} - \nu_{V}^{2}\right) \Psi(\nu_{V}) + q^{2} \epsilon \left[\frac{3}{2} - \frac{2(1-\epsilon)^{3}H_{0}^{2}}{\left(q\overline{\phi}_{0}\right)^{2}}\right], \qquad (3.16b)$$

but in addition it receives a contribution from the finite parts of the counterterms,

$$\frac{3\,\delta\mathcal{B}}{16\pi^2} = 6(2-\epsilon) \left[\delta\xi - \xi\delta Z_\phi\right]^{\text{fin.}} + \frac{\overline{\phi}_0^2}{H_0^2} \left[\delta\lambda - \lambda\delta Z_\phi\right]^{\text{fin.}}.$$
(3.17)

The solution of equation (3.14) is not complicated,

$$\left\langle \hat{\varphi} \right\rangle = \frac{3\overline{\phi}}{16\pi^2} \left[2\mathscr{A} \ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \mathscr{B} \right] + \frac{3\overline{\phi}}{16\pi^2} \left[\mathscr{C}_+ a^{p_+} + \mathscr{C}_- a^{p_-} \right], \tag{3.18}$$

where the left term is a dynamical contribution, with the two coefficients fixed by the sources of the equation of motion (3.14),

$$\mathscr{A} = -\frac{H_0^2}{2\lambda\overline{\phi}_0^2}\mathcal{A}, \qquad \qquad \mathscr{B} = -\frac{H_0^2}{2\lambda\overline{\phi}_0^2}\Big[\mathcal{B} - 6\epsilon(1-\epsilon)\mathscr{A}\Big], \qquad (3.19)$$

while the right term contains two free constants of integration corresponding to homogeneous solutions that redshift away, as their powers,

$$p_{\pm} = -(1-\epsilon) \left(\frac{3}{2} \pm \nu_s\right), \qquad (3.20)$$

both have a negative real part (note that ν_s can be real or imaginary). Imposing initial conditions on $\langle \hat{\varphi} \rangle$ and its derivative at the initial time η_0 fixes the free integration

constants \mathscr{C}_+ and \mathscr{C}_- in (3.18). Requiring that the condensate corrections are minimized at the initial moment η_0 , in the sense that both $\langle \hat{\varphi} \rangle (\eta_0) = 0$ and $\partial_0 \langle \hat{\varphi} \rangle (\eta_0) = 0$, fixes the two constants, ²

$$\mathscr{C}_{+} = \frac{p_{-}}{(p_{+} - p_{-})} \left[2\mathscr{A} \ln \left[\frac{(1 - \epsilon)H_{0}}{\mu} \right] + \mathscr{B} + \frac{2\epsilon\mathscr{A}}{p_{-}} \right], \qquad (3.21a)$$

$$\mathscr{C}_{-} = \frac{p_{+}}{(p_{-} - p_{+})} \left[2\mathscr{A} \ln\left[\frac{(1-\epsilon)H_{0}}{\mu}\right] + \mathscr{B} + \frac{2\epsilon\mathscr{A}}{p_{+}} \right].$$
(3.21b)

The choice of such coefficients amounts to a finite renormalization of the initial state, and is important when discussing the fine-tuning issues in inflation [40]. However, here we are primarily interested in corrections descending from dynamics. Given that the state-dependent contributions decay in time in comparison, we shall henceforth not consider them, and set $\mathscr{C}_{+} = \mathscr{C}_{-} = 0$. Thus, the one-loop correction to the condensate receives additive contributions, $\langle \hat{\varphi} \rangle = \langle \hat{\varphi} \rangle_{s} + \langle \hat{\varphi} \rangle_{v}$, from the scalar and the vector loop, respectively,

$$\left\langle \hat{\varphi} \right\rangle_{s} = -\frac{3\lambda\overline{\phi}}{16\pi^{2}} \left[\frac{3\lambda\overline{\phi}^{2} - \left(\frac{1}{6} - \xi\right)R}{2\lambda\overline{\phi}^{2}} \right] \left[2\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \Psi(\nu_{s}) + \frac{3\epsilon(1-\epsilon)H^{2}}{\lambda\overline{\phi}^{2}} \right], \quad (3.22)$$

$$\left\langle \hat{\varphi} \right\rangle_{v} = -\frac{3q^{2}\overline{\phi}}{16\pi^{2}} \left\{ \left[\frac{\left(q\overline{\phi}\right)^{2} - \lambda\overline{\phi}^{2} + \left(\frac{1}{6} - \xi\right)R}{2\lambda\overline{\phi}^{2}} \right] \left[2\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \Psi(\nu_{v}) + \frac{3\epsilon(1-\epsilon)H^{2}}{\lambda\overline{\phi}^{2}} \right] + \frac{3\epsilon H^{2}}{4\lambda\overline{\phi}^{2}} \right\} + \frac{3\lambda\overline{\phi}}{16\pi^{2}} \times \frac{\epsilon(1-\epsilon)^{3}H^{4}}{\left(\lambda\overline{\phi}^{2}\right)^{2}}, \quad (3.23)$$

where the simplest choice for the finite parts of the counterterms in (3.17) is $\delta \mathcal{B} = 0$, which implies,

$$\left[\delta\xi - \xi\delta Z_{\phi}\right]^{\text{fin.}} = 0, \qquad \left[\delta\lambda - \lambda\delta Z_{\phi}\right]^{\text{fin.}} = 0, \qquad (3.24)$$

that we henceforth assume. Note that in expressions above the argument of the logarithms, and the mode function indices are expressed in terms of the scalar field and curvature scalar using (2.31).

4 Energy-momentum tensor

In order to have a well defined one-loop energy-momentum tensor, in addition to the action for the model in (2.12), we need to consider purely geometrical higherderivative counterterms [62],

$$S_{\text{ctm.}}[g_{\mu\nu}] = \int d^D x \sqrt{-g} \left[\alpha_R R^2 + \alpha_C \mathcal{C}^2 + \alpha_G \mathcal{G} \right], \qquad (4.1)$$

²The choice of \mathscr{C}_{\pm} in (3.21) is valid both when p_{\pm} are real or a pair of complex conjugate numbers. In the latter case, $(\mathscr{C}_{+}a^{p_{+}})^{*} = \mathscr{C}_{-}a^{p_{-}}$ such that $\langle \hat{\varphi} \rangle$ in (3.18) remains real.

where the first counterterm is the square of the Ricci tensor, the second is the square of the Weyl tensor (2.9), $C^2 = C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}$, and the last one is the Gauss-Bonnet invariant $\mathcal{G} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$. Only the first of these counterterms contributes to renormalization on FLRW background. The variation of the C^2 term is proportional to the Weyl tensor which vanishes for conformally flat spacetimes (2.9), and the Gauss-Bonnet part is only a surface term that does not contribute in the bulk. Thus, the full energy-momentum tensor operator is defined as

$$\hat{T}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_*}{\delta g_{\mu\nu}} \Big|_{\substack{\phi \to \hat{\phi} \\ A \to \hat{A}}} = Z_A \left[\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} - \frac{1}{4} g_{\mu\nu} g^{\rho\sigma} \right] g^{\alpha\beta} \hat{F}_{\rho\alpha} \hat{F}_{\sigma\beta}$$

$$+ \left(q_0 \hat{\phi} \right)^2 \left[\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \right] \hat{A}_{\rho} \hat{A}_{\sigma} + Z_{\phi} \left[\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \right] \left(\partial_{\rho} \hat{\phi} \right) \left(\partial_{\sigma} \hat{\phi} \right)$$

$$- \frac{\lambda_0}{4} \hat{\phi}^4 g_{\mu\nu} + \xi_0 \left[G_{\mu\nu} + g_{\mu\nu} \Box - \nabla_{\mu} \nabla_{\nu} \right] \hat{\phi}^2 + \alpha_R H^R_{\mu\nu} ,$$

$$(4.2)$$

where the last term is,

$$H^{R}_{\mu\nu} = 4\nabla_{\mu}\nabla_{\nu}R - 4g_{\mu\nu} \Box R - 4RR_{\mu\nu} + g_{\mu\nu}R^{2}$$

$$\xrightarrow{\epsilon = \text{const.}} (D-1)(D-2\epsilon)(D-4-6\epsilon)H^{4}\Big[(D-1-4\epsilon)g_{\mu\nu} - 4\epsilon(a^{2}\delta^{0}_{\mu}\delta^{0}_{\nu})\Big].$$
(4.3)

We expand the energy-momentum tensor operator (4.2) up to quadratic order in fluctuating fields,

$$\hat{T}_{\mu\nu} = \overline{T}_{\mu\nu} + \hat{t}^{(1)}_{\mu\nu} + \hat{t}^{(2)}_{\mu\nu} + \dots$$
(4.4)

where higher orders do not contribute at one-loop level. The three parts are defined and their expectation values computed in the following three subsections.

4.1 Tree-level part

The classical contribution to the energy-momentum tensor derives from the scalar condensate (2.14) only,

$$\overline{T}_{\mu\nu} = \left[\delta^{\rho}_{\mu}\delta^{\sigma}_{\nu} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\right] \left(\partial_{\rho}\overline{\phi}\right) \left(\partial_{\sigma}\overline{\phi}\right) - \frac{\lambda}{4}\overline{\phi}^{4}g_{\mu\nu} + \xi \left[G_{\mu\nu} + g_{\mu\nu}\Box - \nabla_{\mu}\nabla_{\nu}\right]\overline{\phi}^{2}.$$
 (4.5)

Evaluated on the attractor solution (2.14) it reads,

$$\overline{T}_{\mu\nu} = \frac{(1-6\xi)\epsilon + (D-4)\xi}{4} \left[-(D-1-4\epsilon)g_{\mu\nu} + 4\epsilon \left(a^2\delta^0_{\mu}\delta^0_{\nu}\right) \right] \left(\overline{\phi}H\right)^2$$
$$\xrightarrow{D\to4} \frac{(1-6\xi)\epsilon}{4} \left[-(3-4\epsilon)g_{\mu\nu} + 4\epsilon \left(a^2\delta^0_{\mu}\delta^0_{\nu}\right) \right] \left(\overline{\phi}H\right)^2, \tag{4.6}$$

and it is indeed covariantly conserved, $\nabla^{\mu} \overline{T}_{\mu\nu} = 0$. It describes an effective ideal fluid with an equation of state dependent on the background,

$$\overline{w} = -1 + \frac{4\epsilon}{3} \,. \tag{4.7}$$

The energy-momentum tensor of the condensate, which is a spectator from the perspective of the expansion, redshifts away faster $(\rho \propto H^4 \propto a^{-4\epsilon})$ than some fluid driving the expansion of the power-law inflation $(\rho \propto H^2 \propto a^{-2\epsilon})$

4.2 Part linear in fluctuations

Graviton couples to linearized fluctuations too. This would be the contribution to the energy-momentum tensor coming from the correction to the scalar tadpole. However, this split should only go so far on the account of it not being conserved by itself! Therefore, contributions cannot be split in a physical way.



Figure 2. Diagrams depicting the one-loop corrections to the graviton one-point function descending from the one-loop corrections to the scalar one-point function in Fig. 1. Curly lines correspond to the graviton propagator, dashed line to the scalar propagator, and wavy lines to the vector propagator, while encircled crosses stand for the classical condensate insertions. The last diagram stands for counterterms. Amputating the graviton propagator leaves the contribution to the one-loop energy-momentum tensor.

$$\left\langle \hat{t}_{\mu\nu}^{(1)} \right\rangle = \left[2\delta^{\rho}_{(\mu}\delta^{\sigma}_{\nu)} - g_{\mu\nu}g^{\rho\sigma} \right] \left(\partial_{\rho}\overline{\phi} \right) \partial_{\sigma} \left\langle \hat{\varphi} \right\rangle - \lambda\overline{\phi}^{3} \left\langle \hat{\varphi} \right\rangle g_{\mu\nu} + 2\xi \left[G_{\mu\nu} + g_{\mu\nu}\Box - \nabla_{\mu}\nabla_{\nu} \right] \overline{\phi} \left\langle \hat{\varphi} \right\rangle.$$

$$\tag{4.8}$$

This contribution is not covariantly conserved by itself. In fact, from the tree-level equation of motion (2.13), and the one-loop tadpole equation (3.2) it follows,

$$\nabla^{\mu} \left\langle \hat{t}_{\mu\nu}^{(1)} \right\rangle = \left(\partial_{\nu} \overline{\phi} \right) \left[\mathcal{S}_{S} + \mathcal{S}_{V} + \delta \mathcal{S} \right].$$
(4.9)

This non-conservation has nothing to do with quantum anomalies, or quantum physics for that matter. Rather, it is a consequence of the part quadratic in fluctuations, discussed in the following section, contributing to the energy-momentum tensor at the same order as the part linear in fluctuations discussed in this section.

Plugging in the tree-level attractor solution for the condensate (2.14), and the one-loop condensate correction (3.18) into (4.8) gives the tadpole contribution to the one-loop energy-momentum tensor. Just like the condensate correction it receives dynamically generated contributions, and the initial state dependent contributions,

$$\left\langle \hat{t}_{\mu\nu}^{(1)} \right\rangle = \left\langle \hat{t}_{\mu\nu}^{(1)} \right\rangle_{\text{dyn.}} + \mathcal{T}_{\mu\nu}^{+} + \mathcal{T}_{\mu\nu}^{-}.$$
 (4.10)

where the former contribution is,

$$\left\langle \hat{t}_{\mu\nu}^{(1)} \right\rangle_{\text{dyn.}} = \frac{3}{8\pi^2} \left[2\mathscr{A} \ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \mathscr{B} \right] \left[\overline{T}_{\mu\nu} - \frac{\lambda}{4} \overline{\phi}^4 g_{\mu\nu} \right]$$

$$- \frac{\left(\overline{\phi}H\right)^2 \epsilon \mathscr{A}}{8\pi^2} \left\{ \left[(1-6\xi)(2-5\epsilon) - 2(1-\epsilon) \right] g_{\mu\nu} - \left[(1-6\xi)(1+5\epsilon) - (1-\epsilon) \right] \left(a^2 \delta_{\mu}^0 \delta_{\nu}^0 \right) \right\},$$

$$(4.11)$$

while the latter contributions are,

$$\mathcal{T}_{\mu\nu}^{\pm} = \frac{3\mathscr{C}_{\pm}a^{p_{\pm}}(\overline{\phi}H)^{2}}{16\pi^{2}} \left\{ g_{\mu\nu} \times (3-3\epsilon+p_{\pm}) \left[2\xi (1+2\epsilon-p_{\pm})-\epsilon \right] + (a^{2}\delta_{\mu}^{0}\delta_{\nu}^{0}) \times 2 \left[\epsilon(\epsilon-p_{\pm}) + \xi p_{\pm} - \xi (3\epsilon-p_{\pm}) (2\epsilon-p_{\pm}) \right] \right\}.$$
 (4.12)

These initial state dependent contributions are conserved by themselves, $\nabla^{\mu} \mathcal{T}^{\pm}_{\mu\nu} = 0$. They redshift away compared to the dynamical contribution and we do not consider them further.

4.3 Part quadratic in fluctuations



Figure 3. Diagrams depicting the one-loop corrections to the graviton one-point function descending from vacuum fluctuations. Curly lines correspond to the graviton propagator, dashed line to the scalar propagator, and wavy lines to the vector propagator. The last diagram stands for counterterms. Amputating the graviton propagator leaves the contribution to the one-loop energy-momentum tensor.

The renormalized expectation value of the quadratic part of the energy-momentum tensor is a sum of all three contributions from Fig. 3,

$$\left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle = \left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{S} + \left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{S} + \delta T_{\mu\nu} , \qquad (4.13)$$

where the first contribution comes from the leftmost digram with a scalar loop,

$$\left\langle \hat{t}^{(2)}_{\mu\nu} \right\rangle_{s} = \left[\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \right] \left\langle \partial_{\rho} \hat{\varphi}^{0} \partial_{\sigma} \hat{\varphi}^{0} \right\rangle - \frac{3\lambda}{2} \overline{\phi}^{2} \left\langle \hat{\varphi}^{0} \hat{\varphi}^{0} \right\rangle g_{\mu\nu} + \xi \Big[G_{\mu\nu} + g_{\mu\nu} \,\Box - \nabla_{\mu} \nabla_{\nu} \Big] \left\langle \hat{\varphi}^{0} \hat{\varphi}^{0} \right\rangle,$$

$$(4.14)$$

the second contribution comes from the middle diagram with the vector loop,

$$\left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{V} = \left[\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} g^{\alpha\beta} - \frac{1}{4} g_{\mu\nu} g^{\rho\sigma} g^{\alpha\beta} \right] \left\langle \hat{F}^{0}_{\rho\alpha} \hat{F}^{0}_{\sigma\beta} \right\rangle + \left(q \overline{\phi} \right)^{2} \left[\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \right] \left\langle \hat{A}^{0}_{\rho} \hat{A}^{0}_{\sigma} \right\rangle, \quad (4.15)$$

and the remaining contribution comes from counterterms, denoted by the rightmost diagram,

$$\delta T_{\mu\nu} = \delta Z_{\phi} \left[\delta^{\rho}_{(\mu} \delta^{\sigma}_{\nu)} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \right] \left(\partial_{\rho} \overline{\phi} \right) \left(\partial_{\sigma} \overline{\phi} \right) - \frac{\delta \lambda}{4} \overline{\phi}^{4} g_{\mu\nu} + \delta \xi \left[G_{\mu\nu} + g_{\mu\nu} \Box - \nabla_{\mu} \nabla_{\nu} \right] \overline{\phi}^{2} + \delta \alpha_{R} H^{R}_{\mu\nu} \,.$$
(4.16)

This contribution is also not conserved on its own,

$$\nabla^{\mu} \left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle = -\left(\partial_{\nu} \overline{\phi} \right) \left[\mathcal{S}_{S} + \mathcal{S}_{V} + \delta \mathcal{S} \right], \qquad (4.17)$$

but rather its non-conservation precisely cancels the one in (4.9), so that the entire one-loop energy momentum tensor is covariantly conserved, as it should be,

$$\nabla^{\mu} \left(\left\langle \hat{t}_{\mu\nu}^{(1)} \right\rangle + \left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle \right) = 0.$$
(4.18)

The counterterm part, evaluated for power-law inflation reads,

$$\delta T_{\mu\nu} = g_{\mu\nu} \times \left\{ -\frac{\delta\lambda}{4} \overline{\phi}^4 + \frac{\epsilon^2}{2} \delta Z_{\phi} (\overline{\phi}H)^2 + \delta\alpha_R (D-1)(D-2\epsilon)(D-1-4\epsilon)(D-4-6\epsilon)H^4 - \frac{\delta\xi}{2} \left[(D-1)(D-2) - 6(D-2)\epsilon + 12\epsilon^2 \right] H^2 \overline{\phi}^2 \right\} + \left(a^2 \delta^0_\mu \delta^0_\nu\right) \times \left\{ \left(\delta Z_\phi - 6\delta\xi\right) \epsilon^2 (\overline{\phi}H)^2 + 24\delta\alpha_R (D-1)(D-2\epsilon)\epsilon^2 H^4 + (D-4)\delta\xi\epsilon (\overline{\phi}H)^2 - 4(D-4)\delta\alpha_R (D-1)(D-2\epsilon)\epsilon H^4 \right\}.$$
(4.19)

Just as for the tadpole, the scalar and the vector loops are renormalized independently in the energy-monetum tensor, and we split the contributions,

$$\delta T_{\mu\nu} = \left[\delta T_{\mu\nu}\right]_{S}^{\text{div.}} + \left[\delta T_{\mu\nu}\right]_{V}^{\text{div.}} + \left[\delta T_{\mu\nu}\right]^{\text{fin.}}.$$
(4.20)

4.3.1 Scalar loop contribution

For the scalar part the scalar parts of counterterms from before are enough. This is because the vector parts of counterterms vanish for $q \rightarrow 0$.

$$\left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{s} = g_{\mu\nu} \times \mathbf{\Gamma}(\nu_{s})(1-\epsilon)^{2}H^{2} \left[\left(\frac{D-3}{2} \right)^{2} - \nu_{s}^{2} \right]$$

$$\times \frac{1}{D} \left\{ -\frac{(D-2)^{2}}{4} \left[1 - \frac{4(D-1)\xi}{(D-2)} \right] (D-1-D\epsilon)\epsilon H^{2} - 3\lambda \overline{\phi}^{2} \right\}$$

$$+ \left(a^{2} \delta_{\mu}^{0} \delta_{\nu}^{0} \right) \times \mathbf{\Gamma}(\nu_{s}) \left[1 - \frac{4(D-1)\xi}{(D-2)} \right] \frac{(D-2)^{2} \epsilon^{2} H^{2}}{32} \times \left\{ (D+2)(8-D)\lambda \overline{\phi}^{2} - \frac{D(D-2)(6-D)}{4} \left[1 - \frac{4(D-1)\xi}{(D-2)} \right] (D-2\epsilon) H^{2} + \mathcal{O} \left[(D-4)^{2} \right] \right\}.$$

$$(4.21)$$

Since the factor $\Gamma(\nu_s)$ defined in (2.25) is divergent in D = 4, so are most of the contributions to the naive expectation value above. These need to be absorbed by the counterterms in (4.19), by judiciously choosing four counterterm coefficients. Note that we already have two conditions on these coefficients (3.7) from renormalizing the scalar contribution to the tadpole. Therefore, there are only two independent conditions that we can require here. They are most conveniently found by requiring that the divergences of the $(a^2 \delta^0_{\mu} \delta^0_{\nu})$ part are absorbed, since that is accomplished by a simple comparison with (4.19), and the simplest choice is

$$\left[\delta Z_{\phi} - 6\delta\xi\right]_{S}^{\text{div.}} = \frac{\mu^{D-4} \Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{(D-2)^{2}(D+2)(D-8)\lambda}{32} \left[1 - \frac{4(D-1)\xi}{(D-2)}\right], \quad (4.22)$$

$$\left[\delta\alpha_{R}\right]_{s}^{\text{div.}} = \frac{\mu^{D-4}\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{D(D-2)^{2}}{768(D-1)} \left[1 - \frac{4(D-1)\xi}{(D-2)}\right]^{2}.$$
(4.23)

In addition to fixing $\delta \alpha_R$, this fixes the three counterterms,

$$\left[\delta\xi\right]_{S}^{\text{div.}} = \frac{\mu^{D-4}\,\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{3D\lambda\left[1-(D-1)(D-2)\xi\right]}{8(D-1)}\,,\tag{4.24a}$$

$$\left[\delta\lambda\right]_{s}^{\text{div.}} = \frac{\mu^{D-4}\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \left[-\frac{9D\lambda^{2}}{4}\right],\tag{4.24b}$$

$$\left[\delta Z_{\phi}\right]_{S}^{\text{div.}} = \frac{\mu^{D-4} \Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \left[-3(D-4)\lambda\right], \qquad (4.24c)$$

which agrees (up to finite contributions) with the counterterms found in [41]. Note that the wavefunction renormalization is finite, which agrees with what was also found in Ref. [63] in a different setting. Now we have for the renormalized scalar part of the energy-momentum tensor,

$$\left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{S} + \left[\delta T_{\mu\nu} \right]_{S}^{\text{div.}} \xrightarrow{D \to 4} -g_{\mu\nu} \times \frac{3\lambda\overline{\phi}^{2}}{64\pi^{2}} (1-\epsilon)^{2} H^{2} \left(\frac{1}{4} - \nu_{S}^{2} \right) \left[2\ln\left[\frac{(1-\epsilon)H}{\mu} \right] + \Psi(\nu_{S}) - \frac{1}{2} \right]$$
$$- \frac{(1-6\xi)H^{4}}{64\pi^{2}} (1-\epsilon)^{2} \left(\frac{1}{4} - \nu_{S}^{2} \right) \left\{ g_{\mu\nu} \times \left[(3-4\epsilon)\epsilon \left(2\ln\left[\frac{(1-\epsilon)H}{\mu} \right] + \Psi(\nu_{S}) \right) + 1 - \frac{17\epsilon}{6} \right]$$
$$- \left(a^{2} \delta_{\mu}^{0} \delta_{\nu}^{0} \right) \times 4\epsilon \left[\epsilon \left(2\ln\left[\frac{(1-\epsilon)H}{\mu} \right] + \Psi(\nu_{S}) \right) + \frac{1}{3} \right] \right\}.$$
(4.25)

The covariant contribution in the first line accounts for the non-conservation, while the remainder is conserved by itself.

4.3.2 Vector loop contribution

$$\begin{split} \left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{V} &= g_{\mu\nu} \times \mathbf{\Gamma}(\nu_{V}) (1-\epsilon)^{4} H^{4} \left[\left(\frac{D-3}{2} \right)^{2} - \nu_{V}^{2} \right] \\ & \times \left\{ -\frac{(D-1)}{D} \left(\frac{9}{4} - \nu_{V}^{2} \right) - \frac{3(D-4)(3-\epsilon)}{8(1-\epsilon)} + \mathcal{O} \left[(D-4)^{2} \right] \right\}. \end{split}$$
(4.26)

As before, the two remaining conditions for the counterterm coefficients are most easily read off from the $(a^2 \delta^0_{\mu} \delta^0_{\nu})$ part, and the most convenient choice seems to be,

$$\left[\delta Z_{\phi} - 6\delta\xi\right]_{V}^{\text{div.}} = \mathcal{O}\left[(D-4)^{0}\right], \qquad \left[\delta\alpha_{R}\right]_{V}^{\text{div.}} = 0, \qquad (4.27)$$

which then uniquely fixes all the divergent parts of the counterterm coefficients tied to vector loops,

$$\left[\delta\xi\right]_{V}^{\text{div.}} = \frac{\mu^{D-4}\,\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \frac{(D-2)(D-8)q^2}{16}\,,\tag{4.28a}$$

$$\left[\delta\lambda\right]_{V}^{\text{div.}} = \frac{\mu^{D-4}\,\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \left[-(D-1)q^{4}\right],\tag{4.28b}$$

$$\left[\delta Z_{\phi}\right]_{V}^{\text{div.}} = \frac{\mu^{D-4} \Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times \left[-\frac{(D-1)(D-2)q^{2}}{2}\right], \qquad (4.28c)$$

$$\left[\delta\alpha_{R}\right]_{V}^{\text{div.}} = \frac{\mu^{D-4}\Gamma\left(\frac{2-D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \times 0, \qquad (4.28d)$$

such that $[\delta Z_{\phi} - 6\delta \xi]^{\text{div.}} = -7/(32\pi^2)$, in agreement with (4.27). Inserting the values for these coefficients in (4.19) and adding it to (4.26) produces a finite renormalized result upon taking the four-dimensional limit,

$$\left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{V} + \left[\delta T_{\mu\nu} \right]_{V}^{\text{div.}} \xrightarrow{D \to 4} - \frac{3(q\overline{\phi})^{2}}{64\pi^{2}} H^{2} \left\{ \left[(1-\epsilon)^{2} \left(\frac{9}{4} - \nu_{V}^{2}\right) \left(2\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \Psi(\nu_{V}) - \frac{1}{2} \right) \right. \\ \left. - 2\epsilon \left(\frac{(1-\epsilon)^{3}H^{2}}{\left(q\overline{\phi}\right)^{2}} - \frac{3}{4} \right) - \frac{(3-4\epsilon)(2+7\epsilon)}{6} \right] \times g_{\mu\nu} + \frac{2(2+7\epsilon)\epsilon}{3} \times \left(a^{2}\delta_{\mu}^{0}\delta_{\nu}^{0}\right) \right\}.$$
(4.29)

4.3.3 Full one-loop results

Adding up the scalar loop contribution to the linear (4.11) and quadratic (4.25) parts of the energy-momentum tensor, and writing them in terms of the scalar perturbation mass (2.18) gives,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{s} = -\frac{\lambda}{4\pi^{2}} \left[\frac{3\lambda \overline{\phi}^{2} - \left(\frac{1}{6} - \xi\right) R}{2\lambda \overline{\phi}^{2}} \right] \left\{ \left[2\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \Psi(\nu_{s}) + \frac{2+15\epsilon}{6\epsilon} - \frac{(1-\epsilon)}{2\epsilon(1-6\xi)} + \frac{9\epsilon(1-\epsilon)H^{2}}{2\lambda \overline{\phi}^{2}} \right] \overline{T}_{\mu\nu} - \frac{g^{\rho\sigma} \overline{T}_{\rho\sigma}}{8(1-\epsilon)} g_{\mu\nu} \right\}.$$
(4.30)

and adding up the vector loop contributions from (4.11) and (4.29), and writing them in terms of scalar and vector masses (2.18) and (2.21) gives,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{\nu} = -\frac{3q^2}{8\pi^2} \left\{ \left[\frac{\left(q\overline{\phi}\right)^2 - \lambda\overline{\phi}^2 + \left(\frac{1}{6} - \xi\right)R}{2\lambda\overline{\phi}^2} \right] \left[2\ln\left[\frac{(1-\epsilon)H}{\mu}\right] + \Psi(\nu_{\nu}) + \frac{1+5\epsilon}{3\epsilon} - \frac{(1-\epsilon)}{3\epsilon(1-\epsilon)} + \frac{3\epsilon(1-\epsilon)H^2}{\lambda\overline{\phi}^2} \right] \overline{T}_{\mu\nu} + \left[\frac{(2+7\epsilon)}{12\epsilon(1-6\xi)} + \frac{3\epsilon H^2}{4\lambda\overline{\phi}^2} \right] \overline{T}_{\mu\nu} - \left[\frac{\left(q\overline{\phi}\right)^2 - \lambda\overline{\phi}^2 + \left(\frac{1}{6} - \xi\right)R}{2\lambda\overline{\phi}^2} \right] \frac{g^{\rho\sigma}\overline{T}_{\rho\sigma}}{8(1-\epsilon)} g_{\mu\nu} \right\} + \frac{3\lambda}{8\pi^2} \times \frac{\epsilon(1-\epsilon)^3 H^4}{\left(\lambda\overline{\phi}^2\right)^2} \overline{T}_{\mu\nu}, \quad (4.31)$$

where $\overline{T}_{\mu\nu}$ is the tree level result given in (4.6). Recalling that $\nabla^{\mu}\overline{T}_{\mu\nu} = 0$, one can easily check that both (4.30) and (4.31) are separately covariantly conserved, ³ as they should be. We emphasize this is true only after linear and quadratic contributions are added up. The final expressions (4.30) and (4.31) are rather complicated, and one may be tempted to subtract some of the terms by a judicious choice of finite counterterms. A closer look at the finite part of the energy-momentum tensor counterterms in (4.19),

$$\left[\delta T_{\mu\nu}\right]^{\text{fn.}} = \left\{ \left[\delta Z_{\phi}\right]^{\text{fn.}} + \frac{144\lambda(2-\epsilon)}{(1-6\xi)} \frac{H^2}{\lambda \overline{\phi}^2} \left[\delta \alpha_R\right]^{\text{fn.}} \right\} \times \overline{T}_{\mu\nu} \,. \tag{4.32}$$

where we took account of the simple choice for the condensate correction (3.24), reveals that further simplifications are rather limited if we restrict ourselves to counterterm coefficients being independent of ϵ . Therefore, we shall not pursue this, and instead we shall focus our analysis on the role of secular effects, and perturbativity of the one-loop results (4.30)–(4.31).

5 Various limits

In this section we discuss various limits of the one-loop corrections to the condensate (3.22) and (3.23), and to the energy momentum tensor (4.30), and (4.31), and compare them with the results from the literature when possible.

5.1 De Sitter limit

The de Sitter limit is defined by the constant physical Hubble rate, $H = H_0$, obtained in the limit of vanishing principal slow-roll parameter, $\epsilon \rightarrow 0$, where the Ricci scalar is $R_0 = 12H_0^2$. At tree level the condensate (2.15) is constant, while the energymomentum tensor (4.6) vanishes,

$$\overline{\phi} \xrightarrow{\epsilon \to 0} \overline{\phi}_0 = \pm H_0 \sqrt{\frac{-12\xi}{\lambda}}, \qquad \overline{T}_{\mu\nu} \xrightarrow{\epsilon \to 0} 0.$$
 (5.1)

³Useful relations are $\nabla_{\nu} \left(g^{\rho\sigma} \overline{T}_{\rho\sigma} \right) = -4\epsilon a H \delta^0_{\nu} \left(g^{\rho\sigma} \overline{T}_{\rho\sigma} \right)$, and $\nabla^{\mu} \left[\ln(a) \overline{T}_{\mu\nu} \right] = -(H/a) \overline{T}_{0\nu}$.

The effective masses of fluctuations are also constant in de Sitter,

$$M_s^2 \xrightarrow{\epsilon \to 0} m_s^2 = -24\xi H_0^2, \qquad M_v^2 \xrightarrow{\epsilon \to 0} m_v^2 = -\frac{12\xi q^2}{\lambda} H_0^2 = \frac{q^2 m_s^2}{2\lambda}. \tag{5.2}$$

Consequently, the condensate one-loop corrections (3.22) and (3.23) are also constant, and reduce to,

$$\frac{\left\langle \hat{\varphi} \right\rangle_{s}}{\overline{\phi}_{0}} = -\frac{3\lambda}{16\pi^{2}} \left[\frac{3\lambda\overline{\phi}_{0}^{2} - \left(\frac{1}{6} - \xi\right)R_{0}}{2\lambda\overline{\phi}_{0}^{2}} \right] \left[2\ln\left(\frac{H_{0}}{\mu}\right) + \Psi\left(\nu_{s}^{0}\right) \right], \tag{5.3}$$

$$\frac{\left\langle \hat{\varphi} \right\rangle_{V}}{\overline{\phi}_{0}} = -\frac{3q^{2}}{16\pi^{2}} \left[\frac{\left(q\overline{\phi}_{0}\right)^{2} - \lambda\overline{\phi}_{0}^{2} + \left(\frac{1}{6} - \xi\right)R_{0}}{2\lambda\overline{\phi}_{0}^{2}} \right] \left[2\ln\left(\frac{H_{0}}{\mu}\right) + \Psi\left(\nu_{V}^{0}\right) \right], \quad (5.4)$$

where the mode function indices are,

$$\nu_{S}^{2} \xrightarrow{\epsilon \to 0} \left(\nu_{S}^{0}\right)^{2} = \frac{9}{4} - 24\xi = \frac{9}{4} - \frac{m_{S}^{2}}{H_{0}^{2}}, \qquad \nu_{V}^{2} \xrightarrow{\epsilon \to 0} \left(\nu_{V}^{0}\right)^{2} = \frac{1}{4} - \frac{12q^{2}\xi}{\lambda} = \frac{1}{4} - \frac{m_{V}^{2}}{H_{0}^{2}}.$$
(5.5)

For some purposes it is more convenient to express the de Sitter limit in terms of the effective masses of fluctuations,

$$\frac{\left\langle \hat{\varphi} \right\rangle_{s}}{\overline{\phi}_{0}} \xrightarrow{\epsilon \to 0} - \frac{3\lambda}{16\pi^{2}} \left[\frac{m_{s}^{2} - 2H_{0}^{2}}{m_{s}^{2}} \right] \left[2\ln\left(\frac{H_{0}}{\mu}\right) + \Psi\left(\nu_{s}^{0}\right) \right], \tag{5.6}$$

$$\frac{\left\langle \hat{\varphi} \right\rangle_{V}}{\overline{\phi}_{0}} \xrightarrow{\epsilon \to 0} - \frac{3q^{2}}{16\pi^{2}} \left[\frac{m_{V}^{2} + 2H_{0}^{2}}{m_{S}^{2}} \right] \left[2\ln\left(\frac{H_{0}}{\mu}\right) + \Psi\left(\nu_{V}^{0}\right) \right].$$
(5.7)

Note that no secular corrections remain, as they are proportional to ϵ and vanish in the exact de Sitter limit. For the energy-momentum tensor we first observe that, even though the classical tree-level contribution vanishes in the de Sitter limit, the the singular limit,

$$\frac{1}{\epsilon}\overline{T}_{\mu\nu} \xrightarrow{\epsilon \to 0} -\frac{3(1-6\xi)}{4} \left(\overline{\phi}_0 H_0\right)^2 g_{\mu\nu} , \qquad (5.8)$$

is finite, so that de Sitter limits of one-loop corrections (4.30) and (4.31) are,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{S} \xrightarrow{\epsilon \to 0} \frac{1}{128\pi^{2}} \left(m_{S}^{2} - 2H_{0}^{2} \right)^{2} g_{\mu\nu} , \qquad (5.9)$$

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{V} \xrightarrow{\epsilon \to 0} \frac{3}{128\pi^2} m_{V}^2 \left(m_{V}^2 + 4H_0^2 \right) g_{\mu\nu} \,.$$
 (5.10)

Curiously, the de Sitter limit of the one-loop energy-momentum tensor exhibits no dependence on the renormalization scale μ , which implies the absence of UV divergences. This is a peculiarity of the nonminimally coupled Abelian Higgs model (2.1) that we consider here, where it is the typically negative non-minimal coupling, satisfying Eq. (2.16), that allows for the existence of the symmetry-breaking condensate in the attractor regime.

One-loop energy momentum tensors of spectator scalars and vectors have been computed in the literature, but only in the limit of vanishing condensate. Therefore they cannot be compared directly to our result (5.9) and (5.9), that comprise of the tadpole contribution from Sec. 4.2 and the quadratic perturbation contribution of Sec. 4.3. Each of the contributions are separately conserved in the de Sitter limit, as opposed to the general case when only their sum is conserved. We can only compare the quadratic parts to the existing results in the literature.

Expression (2.17), from which the one-loop energy-momentum tensor descends from, suggests that the proper comparison for the scalar loop contribution is comparing the de Sitter limit of (4.25),

$$\left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{S} + \left[\delta T_{\mu\nu} \right]_{S}^{\text{div.}} \xrightarrow{D \to 4} \frac{(1+12\xi)H_{0}^{4}}{32\pi^{2}} \left\{ -36\xi \left[2\ln\left(\frac{H_{0}}{\mu}\right) + \Psi\left(\nu_{S}^{0}\right) \right] + (1+12\xi) \right\} g_{\mu\nu} \,. \tag{5.11}$$

to the result for the spectator scalar field with non-minimal coupling ξ and mass $m^2 = 3\lambda \overline{\phi}_0^2 = -36\xi H_0^2$. This was first computed in [64], and the reported results agree with our, up to renormalization scheme ambiguities, and the conformal anomaly. The de Sitter limit of the vector loop contribution to the quadratic part in (4.15) suggests we should compare the de Sitter limit of the expectation value (4.29),

$$\left\langle \hat{t}_{\mu\nu}^{(2)} \right\rangle_{V} + \left[\delta T_{\mu\nu} \right]_{V}^{\text{div.}} \xrightarrow{D \to 4} -\frac{3m_{V}^{2}}{64\pi^{2}} \left\{ \left(m_{V}^{2} + 2H_{0}^{2} \right) \left[2\ln\left(\frac{H_{0}}{\mu}\right) + \Psi\left(\nu_{V}^{0}\right) \right] - \frac{m_{V}^{2}}{2} - 2H_{0}^{2} \right\} g_{\mu\nu} .$$
(5.12)

to the Stueckelberg model with mass $m_V^2 = (q\overline{\phi}_0)^2$ computed in [65, 66], where the same result was reported, up to renormalization scheme ambiguities, and the conformal anomaly.

5.2 Flat space limit

The Minkowski limit is strictly speaking not accessible naively. However, we can take it by expressing the results in terms of effective scalar masses first, and then take $H_0 \rightarrow 0$ with keeping m_s constant. It is most convenient to do this from the de Sitter limit of the preceding subsection. Taking into account that digamma functions in reduce to the following forms in the flat space limit, ⁴

$$\Psi\left(\nu_{s}^{0}\right) \stackrel{H_{0}\to0}{\sim} 2\ln\left(\frac{m_{s}}{H_{0}}\right), \qquad \Psi\left(\nu_{v}^{0}\right) \stackrel{H_{0}\to0}{\sim} 2\ln\left(\frac{m_{v}}{H_{0}}\right), \qquad (5.13)$$

⁴This follows from $\nu_s^0 \overset{H_0 \to 0}{\sim} im_s/H_0$ and $\nu_v^0 \overset{H_0 \to 0}{\sim} im_v/H_0$, and the limit,

$$\Psi(i|z|) = \psi\Big(\frac{1}{2} + i|z|\Big) + \psi\Big(\frac{1}{2} - i|z|\Big) \stackrel{z \to \infty}{\sim} 2\ln(|z|) \,.$$

the one-loop tadpole corrections are,

$$\frac{\left\langle \hat{\varphi} \right\rangle_{S}}{\overline{\phi}_{0}} \xrightarrow{H_{0} \to 0} -\frac{3\lambda}{8\pi^{2}} \ln\left(\frac{m_{S}}{\mu}\right), \qquad \frac{\left\langle \hat{\varphi} \right\rangle_{V}}{\overline{\phi}_{0}} \xrightarrow{H_{0} \to 0} -\frac{3q^{2}}{8\pi^{2}} \frac{m_{V}^{2}}{m_{S}^{2}} \ln\left(\frac{m_{V}}{\mu}\right). \quad (5.14)$$

These correspond to the results of Minkowski space computations [67], up to renormalization scheme dependent terms.

While we are able to infer the flat space limit from the condensate correction in de Sitter space, we are not able to do so for the energy-momentum tensor. The reason is that the mass term and the non-minimal coupling term are not distinguishable in de Sitter for the evolution of the scalar. However, they are distinguished by the energy-momentum tensor which involves the variational derivative with respect to the metric, which treats the mass term and the non-minimal coupling terms differently.

5.3 Large field limit/large non-minimal coupling/large masses

The large field limit $\overline{\phi}/H \gg 1$ in our model corresponds to the limit of large negative nonminimal coupling $\xi \ll -1.^5$ Here we have for the tree-level condensate (2.15) and the energy-momentum tensor (4.6),

$$\frac{\overline{\phi}}{H} \stackrel{\xi \ll -1}{\sim} \pm \sqrt{\frac{-6\xi(2-\epsilon)}{\lambda}}, \qquad \overline{T}_{\mu\nu} \stackrel{\xi \ll -1}{\sim} \frac{-3\xi\epsilon}{2} \left[-(3-4\epsilon)g_{\mu\nu} + 4\epsilon \left(a^2\delta^0_{\mu}\delta^0_{\nu}\right) \right] \left(\overline{\phi}H\right)^2.$$
(5.15)

For fluctuations this is essentially the limit of large effective masses,

$$\frac{M_s^2}{H^2} \stackrel{\xi \ll -1}{\sim} -12\xi(2-\epsilon) \gg 1, \qquad \qquad \frac{M_v^2}{H^2} \stackrel{\xi \ll -1}{\sim} -\frac{6\xi q^2(2-\epsilon)}{\lambda} \gg 1, \qquad (5.16)$$

provided the ratio of couplings q^2/λ is not hierarchical. Given that the mode function indices in this limit reduce to,

$$\nu_{s}^{2} \stackrel{\xi \ll -1}{\sim} \frac{12\xi(2-\epsilon)}{(1-\epsilon)^{2}} \ll -1, \qquad \nu_{v}^{2} \stackrel{\xi \ll -1}{\sim} \frac{q^{2}}{\lambda} \times \frac{6\xi(2-\epsilon)}{(1-\epsilon)^{2}} \ll -1, \qquad (5.17)$$

which implies that,

$$\Psi(\nu_S) \stackrel{\xi \ll -1}{\sim} 2\ln\left[\frac{M_S}{(1-\epsilon)H}\right], \qquad \Psi(\nu_V) \stackrel{\xi \ll -1}{\sim} 2\ln\left[\frac{M_V}{(1-\epsilon)H}\right], \tag{5.18}$$

and produces the result for the one-loop condensate corrections,

$$\frac{\left\langle \hat{\varphi} \right\rangle_{S}}{\overline{\phi}} \stackrel{\xi \ll -1}{\longrightarrow} -\frac{3\lambda}{8\pi^{2}} \ln\left(\frac{M_{S}}{\mu}\right), \qquad \qquad \frac{\left\langle \hat{\varphi} \right\rangle_{V}}{\overline{\phi}} \stackrel{\xi \ll -1}{\longrightarrow} -\frac{3q^{2}}{8\pi^{2}} \frac{M_{V}^{2}}{M_{S}^{2}} \ln\left(\frac{M_{V}}{\mu}\right). \tag{5.19}$$

 $^{^5 \}rm We$ still assume the non-minimal coupling is small enough not to induce significant backreaction on the expansion rate.

that are a direct non-equilibrium generalization of the Coleman-Weinberg result [68] with time-dependent effective masses.

For the one-loop energy-momentum tensor the large field limit is,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{S} \stackrel{\xi \ll -1}{\sim} - \frac{\lambda}{2\pi^{2}} \ln\left(\frac{M_{S}}{\mu}\right) \times \overline{T}_{\mu\nu}, \qquad \left\langle \hat{t}_{\mu\nu} \right\rangle_{V} \stackrel{\xi \ll -1}{\sim} - \frac{3q^{2}}{4\pi^{2}} \frac{M_{V}^{2}}{M_{S}^{2}} \ln\left(\frac{M_{V}}{\mu}\right) \times \overline{T}_{\mu\nu}, \quad (5.20)$$

where we neglected constant corrections to the logs (that can nonetheless be enhanced by a relative factor $1/\epsilon$ in comparison when $\epsilon \ll 1$.) The expressions above are not conserved on the account of time dependence of masses of fluctuations. However, the non-conservation is only at the subleading limit that is countered by the subleading terms we neglected.

5.4 Small field limit

The small field limit $\overline{\phi}/H \ll 1$ is implemented by taking the critical value limit for the nonminimal coupling,

$$\xi = \xi_{\rm cr} - \Delta \xi = \frac{\epsilon (3 - 2\epsilon)}{6(2 - \epsilon)} - \Delta \xi, \qquad 0 < \Delta \xi \ll 1, \qquad (5.21)$$

such that the condition (2.16) is close to saturation, where the tree-level condensate field value is vanishing small,

$$\frac{\lambda \overline{\phi}^2}{H^2} \stackrel{\Delta \xi \to 0}{\sim} 6(2 - \epsilon) \Delta \xi \,. \tag{5.22}$$

The mode function indices in the small field limit are,

$$\nu_s^2 = \frac{9}{4} - \frac{12(2-\epsilon)\Delta\xi}{(1-\epsilon)^2}, \qquad \nu_v^2 = \frac{1}{4} - \frac{q^2}{\lambda} \frac{6(2-\epsilon)\Delta\xi}{(1-\epsilon)^2}, \qquad (5.23)$$

so that the digamma functions contribute as,

$$\Psi(\nu_s) \stackrel{\Delta \xi \to 0}{\sim} -\frac{(1-\epsilon)^2}{4(2-\epsilon)\Delta\xi} + \frac{7}{3} - 2\gamma_E, \qquad \Psi(\nu_v) \stackrel{\Delta \xi \to 0}{\sim} -\frac{\lambda}{q^2} \frac{(1-\epsilon)^2}{6(2-\epsilon)\Delta\xi} + 1 - 2\gamma_E.$$
(5.24)

This sufficies to evaluate the limiting behaviour of our results. The condensate corrections (3.22) and (3.22) diverge in this limit,

$$\frac{\left\langle \hat{\varphi} \right\rangle_{s}}{\overline{\phi}} \stackrel{\Delta \xi \to 0}{\sim} - \frac{\lambda (1 - 3\epsilon) (1 - \epsilon)^{3}}{128\pi^{2} (2 - \epsilon)^{2} \Delta \xi^{2}} + \mathcal{O}(1/\Delta \xi), \qquad (5.25)$$

$$\frac{\left\langle \hat{\varphi} \right\rangle_{V}}{\overline{\phi}} \stackrel{\Delta \xi \to 0}{\sim} \frac{(\lambda - 3\epsilon q^{2})(1 - \epsilon)^{3}}{192\pi^{2}(2 - \epsilon)^{2}\Delta\xi^{2}} + \mathcal{O}(1/\Delta\xi), \qquad (5.26)$$

which is actually a consequence of the singular definition of the variable. In the unitary gauge we emply here the scalar field is actually the modulus of the complex scalar field, which is not defined at the origin. In fact, a closer look at the leading relative condensate correction above reveals it to be negative, meaning that the meaningful description breaks down before the singular point. Similar behaviour is observed for the energy-momentum tensor corrections,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{s} \overset{\Delta\xi \to 0}{\simeq} - \frac{\epsilon (1-\epsilon)^{5} (1-4\epsilon)}{32\pi^{2} (2-\epsilon)^{2} \Delta\xi} H^{4} \left[-(3-4\epsilon) g_{\mu\nu} + 4\epsilon \left(a^{2} \delta_{\mu}^{0} \delta_{\nu}^{0} \right) \right] + \mathcal{O} \left(\Delta\xi^{0} \right), \quad (5.27)$$

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{v} \overset{\Delta\xi \to 0}{\simeq} \frac{\epsilon (1-\epsilon)^{5}}{32\pi^{2} (2-\epsilon)^{2} \Delta\xi} \left[1 - \frac{3\epsilon q^{2}}{\lambda} \right] H^{4} \left[-(3-4\epsilon) g_{\mu\nu} + 4\epsilon \left(a^{2} \delta_{\mu}^{0} \delta_{\nu}^{0} \right) \right] + \mathcal{O} \left(\Delta\xi^{0} \right), \quad (5.28)$$

that are finite only in the de Sitter limit,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{S} \xrightarrow[\epsilon \to 0]{\Delta \xi \to 0} \frac{H^{4}}{32\pi^{2}} g_{\mu\nu}, \qquad \left\langle \hat{t}_{\mu\nu} \right\rangle_{V} \xrightarrow[\epsilon \to 0]{\Delta \xi \to 0} 0, \qquad (5.29)$$

where the scalar loop contribution reduces to the contribution of the massless, minimally coupled scalar, supplemented by the finite contribution obtained from taking the singular limit $\xi \to 0$ of the nonminimally coupled case (consistent with [69– 71]), while the vector loop contribution reduces to the vanishing one of a massless photon [72–74]. However, behaviour of the results for nonvanishing ϵ points to the need of examining this regime in a different gauge, that is adapted to the vanishing condensate limit, unlike the unitary gauge we use here.

5.5 Hierarchal couplings

Interesting limit is provided by a hierarchy between the scalar self-coupling constant and the U(1) charge,

$$q^2/\lambda \gg 1\,,\tag{5.30}$$

when the vector loop provides a parametrically larger contribution than the scalar one. Furthermore, given that in this limit,

$$\Psi(\nu_V) \stackrel{q^2 \gg \lambda}{\sim} 2\ln\left[\frac{q\overline{\phi}}{(1-\epsilon)H}\right],\tag{5.31}$$

the dominant contribution to the condensate correction,

$$\frac{\left\langle \hat{\varphi} \right\rangle_{V}}{\overline{\phi}} \stackrel{q^{2} \gg \lambda}{\sim} - \frac{3q^{4}}{16\pi^{2}\lambda} \ln\left(\frac{q\overline{\phi}}{\mu}\right), \qquad (5.32)$$

and to the energy-momentum tensor correction,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{V} = -\frac{3q^{4}}{8\pi^{2}\lambda} \ln\left(\frac{q\overline{\phi}}{\mu}\right) \times \overline{T}_{\mu\nu} \,.$$
 (5.33)

derive from the vector loop. We see that the hierarchy (5.30) of coupling constants can lead to a big enhancement of the loop corrections. This is a feature of the model that is present already in flat space. However, the secular logarithm multiplying this correction is innate to curved spacetimes, which is discussed in detail in Sec. 6.

5.6 Vanishing U(1) charge

Even though the limit $q \to 0$ is a singular limit of the unitary gauge [54], the scalar field ϕ in the unitary gauge is a gauge-independent observable, and it is not sensitive to the singularity of the this limit. Therefore, we may infer the result in the limit of vanishing U(1) charge. Interestingly, the vector loop contributions do not vanish, neither for the condensate,

$$\frac{\left\langle \hat{\varphi} \right\rangle_{V}}{\overline{\phi}} \xrightarrow{q \to 0} \frac{3\lambda}{16\pi^{2}} \times \frac{\epsilon (1-\epsilon)^{3} H^{4}}{\left(\lambda \overline{\phi}^{2}\right)^{2}}, \qquad (5.34)$$

nor for the energy-momentum tensor,

$$\left\langle \hat{t}_{\mu\nu} \right\rangle_{V} \xrightarrow{q \to 0} \frac{3\lambda}{8\pi^{2}} \times \frac{\epsilon (1-\epsilon)^{3} H^{4}}{\left(\lambda \overline{\phi}^{2}\right)^{2}} \times T^{(0)}_{\mu\nu} \,.$$
 (5.35)

Nevertheless, inorder to be sure of this limit the computation should be redone in a gauge regular in the vanishing U(1) charge limit.

6 Late time limit and an RG explanation

The time dependence of the Hubble rate in the logarithms of the one-loop corrections implies the secular breakdown of perturbation theory after long enough time, when,

$$\ln\left[\frac{(1-\epsilon)H}{\mu}\right] \stackrel{a\to\infty}{\sim} -\epsilon \ln(a) \gg 1, \qquad (6.1)$$

(6.4)

becomes large enough. This term dominates when $a \gg e^{1/\epsilon}$. In inflation this number is very big, but our results are valid for finite ϵ . Therefore, the late time limit of the one-loop corrections are,

$$\left\langle \hat{\phi} \right\rangle = \overline{\phi} \times \left[1 + \frac{3\lambda}{16\pi^2} C_{\phi} \times \epsilon \ln(a) \right], \qquad \left\langle \hat{T}_{\mu\nu} \right\rangle = \overline{T}_{\mu\nu} \times \left[1 + \frac{3\lambda}{8\pi^2} C_T \times \epsilon \ln(a) \right], \quad (6.2)$$

where the scalar and vector loops contribute differently to each constant factor,

$$C_{\phi} = \frac{3\lambda\overline{\phi}^{2} - \left(\frac{1}{6} - \xi\right)R}{\lambda\overline{\phi}^{2}} + \frac{q^{2}}{\lambda} \left[\frac{\left(q\overline{\phi}\right)^{2} - \lambda\overline{\phi}^{2} + \left(\frac{1}{6} - \xi\right)R}{\lambda\overline{\phi}^{2}}\right] = \frac{q^{4}}{\lambda^{2}} + \frac{1}{\Xi - 1}\left(\frac{q^{2}}{\lambda} - 1\right) + 2,$$

$$(6.3)$$

$$C_{T} = \frac{2}{3} \left[\frac{3\lambda\overline{\phi}^{2} - \left(\frac{1}{6} - \xi\right)R}{\lambda\overline{\phi}^{2}}\right] + \frac{q^{2}}{\lambda} \left[\frac{\left(q\overline{\phi}\right)^{2} - \lambda\overline{\phi}^{2} + \left(\frac{1}{6} - \xi\right)R}{\lambda\overline{\phi}^{2}}\right] = \frac{q^{4}}{\lambda^{2}} + \frac{1}{\Xi - 1}\left(\frac{q^{2}}{\lambda} - \frac{2}{3}\right) + \frac{4}{3}$$

where $\Xi = (1-6\xi)(2-\epsilon)/[2(1-\epsilon)^2]$. It is worth noting that had we applied the effective potential approximation for our computation, and was utilized in [50], we would have

missed the terms of order $\mathcal{O}(\epsilon)$ in the coefficients above that appear in the dominant late-time contribution. This can be tracked down to the classical equation (2.13) that the tree-level condensate is assumed to satisfy. The effective potential approximation in our case would correspond to neglecting the d'Alembertian term in that equation. Even though this drops the time derivatives, the condensate is still time-dependent because of the non-minimal coupling, and retains the same scaling, but its amplitude does not account correctly for the $\mathcal{O}(\epsilon)$ terms.

The factors (6.3) and (6.4) that multiply the secular corrections in (6.2) depend only on the two independent combinations of model parameters, q^2/λ and Ξ , and this dependence is depicted in Fig. 4. The case of hierarchal couplings $q^2 \gg \lambda$



Figure 4. Density plots of the coefficient (6.3) multiplying the late time correction of the condensate (*left*), and of the coefficient (6.4) multiplying the late time correction of the energy-momentum tensor (*right*). Thin black curves denote points of equal value of the coefficient; curves are equidistant in values of the coefficients. The value increases from the dark shading to the light shading. The thick, black, nearly straight lines denote where the coefficient vanishes, and the values the coefficients to the left of these lines are negative, while values to the right are positive.

from Sec. (5.5) is particularly interesting. It is captured by a simple expression independent of the non-minimal coupling,

$$\left\langle \hat{\phi} \right\rangle \stackrel{a \gg 1}{\sim} \overline{\phi} \left[1 + \frac{q^4}{\lambda} \frac{3\epsilon}{16\pi^2} \ln(a) \right], \qquad \left\langle \hat{T}_{\mu\nu} \right\rangle \stackrel{a \gg 1}{\sim} \overline{T}_{\mu\nu} \left[1 + \frac{q^4}{\lambda} \frac{3\epsilon}{8\pi^2} \ln(a) \right], \quad (6.5)$$

where the hierarchy of coupling constants can make up for the ϵ -suppression in inflation.

Looking at the late time-expressions (6.3), in particular the condensate correction on the left, it might be tempting to resum the secular correction, $\langle \hat{\varphi} \rangle \rightarrow \overline{\phi}_0 a^{-\epsilon(1-\delta)}$ into a renormalized slow-roll parameter $\epsilon \rightarrow \epsilon(1-\delta)$, where $\delta = 3\lambda C_{\phi}/(16\pi^2)$. This resummation would be correct provided that the equation of motion for $\langle \hat{\varphi} \rangle$ is linear and autonomous, and that the secular logarithm is an artifact of the time-dependent perturbation theory. This is very often what is implicitly or explicitly assumed when the so-called Dynamical renormalization group is invoked to resum the secular corrections (see e.g. [75, 76]). However, the original construction of the Dynamical renormalization group [77] is essentially a reformulation of multiple scale analysis, and consequently necessitates knowing the dynamical equation before anything can be resummed, and not only the first few perturbative corrections. The fact that this resummation does not work here is obviated by the energy-momentum correction that cannot be resummed by the same slow-roll parameter correction on the account of the coefficients in (6.3) and (6.4) being different. In fact, the secular correction in (6.2) is an effect of ultraviolet physics in a nonequilibrium setting. The appropriate framework to describe (and eventually resum) them is the standard renormalization group formalism, implemented in a nonequilibrium setting. The following section is devoted to discussing some aspects of it.

6.1 Renormalization group explanation

The secular corrections in (6.2) at late times dominate over the tree-level result, and thus invalidate the perturbative approach. The first step towards extending the results past this point is to understand which equation governs these secular corrections. It is that equation that ultimately provides the resummation scheme. Here we demonstrate that, at the perturbative level, the secular corrections are governed by the standard renormalization group equations applied to a nonequilibrium setting.

The utility of the renormalization group hinges on our ability to recognize the relevant reference scale μ_0 that appears together with the arbitrary renormalization scale μ , as a logarithm of a dimensionless ratio, $\ln(\mu_0/\mu)$. The appropriate choice of the reference scale ultimately allows the renormalization group formalism to effectively resum all such logarithms, thus extending the validity of perturbative results. For systems with multiple physical scales, such as the one we consider here, different regimes will have different reference scales [4]. Moreover, for dynamical systems the reference scale will in general be *time-dependent*. Looking at the final one-loop corrections (3.22)–(3.23) for the condensate, and (4.30)–(4.31) for the energy-momentum tensor, the natural choice for the reference scale seems to be $\mu_0 = (1-\epsilon)H$. However, as various limits considered in Sec. 5 demonstrate, the relevant reference scale depends on the parameter regime. It is the digamma functions, that are inextricably tied to the logarithms they appear with, that set the correct reference scale, without having to choose it by hand. This is perhaps most evident in the hierarchal limit of Sec. 5.5, where the reference scale is recognized as $\mu_0 = q\overline{\phi}$.

The system at hand is a multiscale problem, for which in general it is not possible to choose a single reference scale which captures the large logarithms in all regimes. Here this is essentially due to corrections descending from two independent loops — the scalar and the vector one. Potentially large logarithms are captured by the reference scale if it accounts for the digamma function accompanying the logarithm. In our case we can make two choices which capture either the full large logarithms descending from the scalar loop, or ones descending from the vector loop, respectively,

$$\ln\left[\frac{(1-\epsilon)^2 H^2}{\mu^2}\right] + \Psi(\nu_s) \longrightarrow \ln\left(\frac{\mu_0^2}{\mu^2}\right), \quad \text{or} \quad \ln\left[\frac{(1-\epsilon)^2 H^2}{\mu}\right] + \Psi(\nu_v) \longrightarrow \ln\left(\frac{\mu_0^2}{\mu^2}\right), \tag{6.6}$$

where $(1-\epsilon)^2 H^2 = \frac{1}{2} \left[\left(\frac{1}{6} - \xi \right) R - \lambda \overline{\phi}^2 \right] > 0$. However, we cannot do both simultaneously. That would require utilizing the full machinery of multiscale renormalization group [78–81]. Nevertheless, this is not necessary for our purposes, as the late time behaviour is not marred by such subtleties. In fact, there is universality in the late-time secular corrections (6.2). This is because the tree-level quantities that make up different reference scales all scale like the Hubble rate. Therefore, whatever μ_0 precisely is, at late times we always have,

$$\ln\left(\frac{\mu_0}{\mu}\right) \stackrel{a \to \infty}{\sim} -\epsilon \ln(a) \ll -1, \qquad (6.7)$$

even if $m_s \gg H$ or $m_V \gg H$.

The observations above can be formalized using the renormalization group machinery. Our results depend on the arbitrary renormalization scale μ ,

$$\left\langle \hat{\phi} \right\rangle = \left\langle \hat{\phi} \right\rangle \left(H, \left\{ \lambda_n \right\}, \ln\left[\frac{\mu_0}{\mu}\right] \right), \qquad \left\langle \hat{T}_{\mu\nu} \right\rangle = \left\langle \hat{T}_{\mu\nu} \right\rangle \left(H, \left\{ \lambda_n \right\}, \ln\left[\frac{\mu_0}{\mu}\right] \right), \qquad (6.8)$$

where $\{\lambda_n\} = \{\lambda, \xi, Z_\phi, \alpha_R\}$ stands for all the coupling constants. Note that we have chosen to treat the scalar wavefunction renormalization as a coupling constant with its associated β -function for convenience, instead of treating it as the anomalous dimension γ of the field; this distinction is essentially immaterial [68]. The renormalization scale μ does not have a physical meaning by itself. What does have physical sense is performing several measurements at some scale μ_* . This process determines the couplings as functions of the ratio μ/μ_*

$$\lambda_n \to \lambda_n \left(\ln \left[\frac{\mu_*}{\mu} \right] \right), \tag{6.9}$$

such that the dependence on μ completely disappears from the result,

$$\mu \frac{d}{d\mu} \left\langle \hat{\phi} \right\rangle \left(H, \left\{ \lambda_n \left(\ln \left[\frac{\mu_*}{\mu} \right] \right) \right\}, \ln \left[\frac{\mu_0}{\mu} \right] \right) = 0, \qquad (6.10)$$

$$\mu \frac{d}{d\mu} \langle \hat{T}_{\mu\nu} \rangle \Big(H, \left\{ \lambda_n \left(\ln \left[\frac{\mu_*}{\mu} \right] \right) \right\}, \ln \left[\frac{\mu_0}{\mu} \right] \Big) = 0, \qquad (6.11)$$

which is guaranteed by the running couplings satisfying the running equations,

$$\mu \frac{d}{d\mu} \lambda_n = \beta_n \big(\{ \lambda_m \} \big) \,, \tag{6.12}$$

where β_n are the β -functions associated to couplings λ_n , that are determined by the structure of divergences. The one-loop β -functions are obtained by acting $-\mu \frac{\partial}{\partial \mu}$ on

the counterterm coefficients given in sections 4.3.1 and 4.3.2, and taking the limit $D \rightarrow 4$,

$$\beta_{\xi} = -\frac{\lambda(1-6\xi)}{16\pi^2} + \frac{q^2}{16\pi^2}, \qquad \beta_{\lambda} = \frac{9\lambda^2}{8\pi^2} + \frac{3q^4}{8\pi^2}, \qquad \beta_{Z_{\phi}} = \frac{3q^2}{8\pi^2}, \qquad \beta_{\alpha_R} = -\frac{(1-6\xi)^2}{1152\pi^2}.$$
(6.13)

It is straightforward to check that our one-loop corrections satisfy the Callan-Symanzik equations to the given order,

$$\mu \frac{\partial}{\partial \mu} \left\langle \hat{\varphi} \right\rangle = -\beta_n \frac{\partial}{\partial \lambda_n} \overline{\phi} \,, \qquad \qquad \mu \frac{\partial}{\partial \mu} \left\langle \hat{t}_{\mu\nu} \right\rangle = -\beta_n \frac{\partial}{\partial \lambda_n} \overline{T}_{\mu\nu} \,, \qquad (6.14)$$

where we have reintroduced an arbitrary Z_{ϕ} and $\alpha_{\scriptscriptstyle R}$ dependence in the tree-level expressions,

$$\overline{\phi}(H,\xi,\lambda,Z_{\phi}) = \pm H\sqrt{\frac{1}{\lambda} \left[Z_{\phi}\epsilon(3-2\epsilon) - 6\xi(2-\epsilon) \right]}, \qquad (6.15)$$

$$\overline{T}_{\mu\nu}(H,\xi,\lambda,Z_{\phi},\alpha_{R}) = \left[-\frac{1}{4}(3-4\epsilon)\epsilon g_{\mu\nu} + \epsilon^{2}\left(a^{2}\delta_{\mu}^{0}\delta_{\nu}^{0}\right)\right] \times \left[(Z_{\phi}-6\xi)\left(\overline{\phi}H\right)^{2} + 144\alpha_{R}(2-\epsilon)H^{4}\right].$$
 (6.16)

Then, having correctly identified the relevant late-time reference scale, as described at the beginning of this section, we immediately infer the late-time limit,

$$\left\langle \hat{\varphi} \right\rangle \stackrel{a \to \infty}{\sim} \epsilon \ln(a) \times \beta_n \frac{\partial}{\partial \lambda_n} \overline{\phi}, \qquad \left\langle \hat{t}_{\mu\nu} \right\rangle \stackrel{a \to \infty}{\sim} \epsilon \ln(a) \times \beta_n \frac{\partial}{\partial \lambda_n} \overline{T}_{\mu\nu}.$$
 (6.17)

Apart from efficiently reproducing the late-time correction obtained from the loop computation, the real power of the renormalization group is in its ability to resum the large logarithms. This is accomplished by first making a variable substitution, and adopting the dimensionless ratio that includes the reference scale μ_0 as the new variable,

$$s = \ln\left(\frac{\mu_0}{\mu}\right),\tag{6.18}$$

such that the Callan-Symanzik equations read,

$$\frac{d}{ds}\langle\hat{\phi}\rangle\Big(H,\lambda_n\big(\ln\big[\frac{\mu_*}{\mu_0}\big]+s\big),s\Big)=0\,,\qquad \frac{d}{ds}\langle\hat{T}_{\mu\nu}\rangle\Big(H,\lambda_n\big(\ln\big[\frac{\mu_*}{\mu_0}\big]+s\big),s\Big)=0\,,\quad(6.19)$$

and the equations for the running couplings,

$$\frac{d}{ds}\lambda_n = \beta_n(\{\lambda_m\}). \tag{6.20}$$

Once we have solved for the running couplings as functions of s, the Callan-Symanzik equations allow us to express the results choosing any s. Particularly useful is the

choice that removes as much of the explicit *s*-dependence from the results, and translates it to the *s*-dependence of the couplings. This choice corresponds to the following,

$$\left\langle \hat{\phi} \right\rangle = \left\langle \hat{\phi} \right\rangle \left(H, \lambda_n \left(\ln \left[\frac{\mu_*}{\mu_0} \right] \right), 0 \right), \qquad \left\langle \hat{T}_{\mu\nu} \right\rangle = \left\langle \hat{T}_{\mu\nu} \right\rangle \left(H, \lambda_n \left(\ln \left[\frac{\mu_*}{\mu_0} \right] \right), 0 \right). \tag{6.21}$$

provided we had identified the relevant reference scale μ_0 . What (6.21) accomplishes when compared with (6.19) is that the explicit dependence on the Hubble rate *via s* is moved to the coupling constants. Working at the one-loop level corresponds to solving Eqs. (6.20) to linear order in *s*,

$$\xi(s) = \xi(0) - \left[\frac{\lambda(0)\left[1 - 6\xi(0)\right]}{16\pi^2} - \frac{q^2(0)}{16\pi^2}\right]s, \qquad (6.22a)$$

$$\lambda(s) = \lambda(0) + \left[\frac{9\lambda^2(0)}{8\pi^2} + \frac{3q^4(0)}{8\pi^2}\right]s, \qquad (6.22b)$$

$$Z_{\phi}(s) = Z_{\phi}(0) + \frac{3 q^2(0)}{8\pi^2} s, \qquad (6.22c)$$

$$\alpha_R(s) = \alpha_R(0) - \frac{\left[1 - 6\xi(0)\right]^2}{1152\pi^2} s.$$
(6.22d)

It is straightforward to see that when these solutions are plugged into the tree-level result they reproduce exactly the late-time behaviour,

$$\left\langle \hat{\phi} \right\rangle = \overline{\phi} + \left\langle \hat{\varphi} \right\rangle \equiv \overline{\phi} \left(H, \xi(s), \lambda(s), Z_{\phi}(s) \right),$$
(6.23)

$$\left\langle \hat{T}_{\mu\nu} \right\rangle = \overline{T}_{\mu\nu} + \left\langle \hat{t}_{\mu\nu} \right\rangle \equiv \overline{T}_{\mu\nu} \left(H, \xi(s), \lambda(s), Z_{\phi}(s), \alpha_{R}(s) \right).$$
(6.24)

Note that, even though all of the time-dependence has been absorbed into effective running of the coupling constants, this does not imply that the effect is not observable. On the contrary, quantum corrections will cause the ratio $\langle \hat{\phi} \rangle / H$ to acquire time dependence, which is a physical effect. We emphasize that the classical form in (6.23)–(6.24) is in general achieved only at asymptotically late times. At intermediate times, other logarithmically enhanced contributions can in general occur. However, since we are here primarily interested in the question of restoring perturbativity at late times, this analysis suffices.

The analysis of this section opens up an exciting possibility of resumming the corrections past the breakdown of the perturbative expansion, and obtaining reliable late-time behaviour utilizing the machinery of the renormalization group. Accomplishing this requires supplementing the equations for the running couplings (6.20) with an additional two for the U(1) charge q, and for the vector wavefunction renormalization Z_A , and then solving the entire system exactly. For this we first need the one-loop β -functions for q and Z_A in the unitary gauge, which do not appear at the level of one-loop analysis we performed here, but are important for resumming the late-time behaviour correctly. This is left for future work. Finally, we emphasize that

the renormalization group can explain the large ultraviolet logarithms, but not the infrared logarithms [82], which in inflation are captured by a variant of Starobinsky's stochastic formalism [31, 39], as was accomplished in [83, 84].

7 Discussion

Quantum effects in inflation are widely studied utilizing de Sitter space as an idealized model space for inflation, and quantum loop corrections have been calculated in many models of interest. On the other hand, comparatively little is known about quantum effects in more realistic models of inflation, where the expansion rate is an adiabatic function of time. Of particular interest are models which exhibit novel secular effects that do not occur in de Sitter limit, and this work is devoted to one such system. We consider power-law inflation, characterized by a constant principal slow-roll parameter $\epsilon = -\dot{H}/H^2$, as a mathematically tractable model of inflating background, on which we studied the behaviour of a spectator non-minimally coupled Abelian Higgs model. The Abelian Higgs model is considered to be in a classical attractor regime, characterized by the scaling behaviour, $\overline{\phi}/H = \text{const.}$, in which the condensate tracks the evolution of the Hubble rate. This attractor can be seen as a dynamical generalization of a symmetry-breaking minimum. We computed dimensionally regulated one-loop corrections to the condensate depicted in Fig. 1, and to the energy-momentum tensor depicted in Figs. 2 and 3. The tadpole contributions to the energy-momentum tensor in Fig. 2 are often not considered, but are of essential importance because, only when they are added to the contributions from vacuum fluctuations in Fig. 3, is the one-loop energy-momentum tensor conserved. The computations are performed in the unitary gauge,⁶ in which both quantities of our interest are manifestly gauge-independent observables. Our results (3.22)-(3.23) and (4.30)–(4.31) capture both the infrared and ultraviolet effects, and they go beyond the effective potential approximation, in that they descend from the full one-loop effective action corrections, which include kinetic term corrections that need not be small for finite ϵ backgrounds.

Particularly interesting are the secular corrections to both the condensate and the energy-momentum tensor given in (6.2)–(6.4), which dominate at late times. The corrected late time result takes the form of the tree-level result multiplied by a timedependent amplitude. Since this amplitude grows in time, the corrections can be seen as driving the evolution away from the classical attractor given in (2.14) and (2.15), pointing to the quantum instability of the attractor. Classically the condensate rolls

⁶We find no obstacle in computing dimensionally regulated loop correction in the unitary gauge, conforming with the experience from flat space equilibrium computations [67]. However, Ref. [10] reports problems with the unitary gauge for dynamical scalar condensates, when compared to the covariant gauge computations. This issue should be resolved by repeating the explicit computations of this paper in some analogue of the R_{ξ} gauge.

down the potential tracking the evolution of the Hubble rate. The late time one-loop correction is enhanced by a logarithm of the scale factor (the number of e-foldings) multiplied by the coefficient that takes positive values for the most range of the model parameters, as seen from Fig. 4. Thus the one-loop correction tends to slow down the rolling of the spectator scalar down its potential.

It is interesting that the secular corrections we find are multiplied by the principal slow-roll parameter ϵ , and thus are suppressed in inflation, and vanish in the exact de Sitter limit $\epsilon = 0$, that is often employed as a tractable model of inflation. This points to the fact that power-law inflation, and other classes of inflating spacetimes closer to more realistic slow-roll inflation can harbour *qualitatively* different, and potentially important effects. While in inflation the slow-roll parameter acts as a suppression factor, our results are valid for finite constant (or adiabatically evolving) ϵ , including instances where it does not act as a suppression. Nonetheless, given enough time the secular correction will necessarily overcome the possible suppressions coming from the slow-roll parameter or the coupling constants, leading to the breakdown of perturbation theory. Similar types of corections have also been noticed ⁷ in [41, 50, 85, 86]. This serves as a very good motivation for further studies of quantum corrections to observables in power-law inflation, using both perturbative [41, 87], and non-perturbative [42–44] methods.

Another observation to be made is that the possible hierarchy between the coupling constants, $q^2 \gg \lambda$, discussed in Sec. 5.5, leads to an unsuppressed loop correction. This correction is furthermore made bigger at late times by the secular logarithm, exacerbating the problem of applying perturbation theory. Unlike the secular correction, the coupling hierarchy enhancement survives in the de Sitter limit. In fact it is already seen in flat space [68], where the vector loop moves the minimum of the effective potential by an amount larger than tree-level. Despite that one expects perturbativity to be respected at two- and higher loops [88, 89]. In cosmological expanding spaces, on the other hand, this perturbativity will necessarily be spoiled at late times due to secular corrections we report here.

The secular corrections we find are an ultraviolet effect, and can ultimately be accounted for by the renormalization group, as we show in Sec. 6.1. This is because the secular logarithm always appears together with the logarithm of the renormalization scale μ in the form $\ln[(1-\epsilon)H/\mu]$ given in (6.1). Consequently, the secular correction can be captured by an effective time-dependent running of the coupling constants given by the renormalization group equations. Note that

⁷It is interesting that for quantum-gravitational corrections found in [85, 86] the one-loop correction is actually decaying with respect to the tree-level result. Even though there is a $\ln(a)$ enhancement factor descending from the UV corrections, just like the one we find here, the correction decays as a power law as it is additionally multiplied by a decaying H^2 that has to be there to form a dimensionless product with the dimensionful loop-counting parameter $\kappa^2 = 16\pi G_N$, where G_N is the Newton's constant.

this does not imply that all the physical scales are simply rescaled differently at different points in time. It is the fact that the quantum-corrected condensate no longer scales as the Hubble rate, and it is their ratio that has a physical meaning. It would be interesting to investigate the full power of the renormalization group to resum the late time behaviour, and provide reliable results beyond the breakdown of perturbation theory. For the case at hand, that would first require computing the beta functions for the U(1) charge and the vector field wavefunction renormalization, which do not appear in the strictly perturbative computation at hand. Early efforts in this direction are undertaken in Refs. [83, 84], but a lot more is needed to fully develop the formalism, especially for its higher loop implementation.

It would, furthermore, be interesting to examine quantum corrections to the condensate when the scalar field is the inflaton [90] as well as the quantum backreaction from cosmological perturbations [91–95]. A notable model is Higgs inflation [7, 8], and stability of the quantum model is of particular interest, which can be addressed without [9], or with taking into account the background curvature corrections [17–19] and additional fine tuning problems [40, 45–50]. Important questions to be addressed are how the secular corrections discussed in this work affect stability of Higgs inflationary models, as well as the amplitude of the scalar and tensor spectra. Secular corrections to the reheating period following Higgs inflation is also of great interest [18, 19, 51, 52].

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