



# Even the photon propagator must break de Sitter symmetry

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## ABSTRACT

The propagator for the massless vector field in de Sitter space cannot maintain de Sitter invariance in the general covariant gauge, except in the exactly transverse limit. This is due to a previously overlooked Ward-Takahashi identity that the propagator must satisfy. We construct the propagator that satisfies all the necessary conditions, and that preserves cosmological symmetries and dilation, but breaks spatial special conformal transformations. Our propagator vanishes in the infrared, contrary to previously reported de Sitter invariant results, which is relevant for loop computations.

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## 1. Introduction

Understanding interacting quantum field theory in de Sitter space is of paramount importance for apprehending the physics of the primordial inflationary phase of the Universe. Computations of quantum loop corrections in realistic slow-roll inflation are still prohibitively difficult, so simplifications are necessary. The de Sitter space is often taken as an appropriate idealization for two reasons: (i) it is close enough to slow-roll inflation that is phenomenologically relevant, and (ii) it is a maximally symmetric space. It is the latter that is very often useful when describing physical systems – more symmetric they are the simpler the description. Such is our experience in Minkowski space, where Poincaré invariance provides an efficient organizational principle for computations. It is often assumed that de Sitter symmetries provide the same level of simplifications and organize the computations in an economical manner. Even though adhering to symmetries is the right approach in many circumstances, it must not be taken for granted in de Sitter space.

Two-point functions of free fields are essential ingredients for perturbative computations in quantum field theory. In maximally symmetric spaces it is natural to assume that they respect symmetries of the background spacetime. However, it has long been known that issues with this approach arise already for arguably the simplest system of minimally coupled, massless scalar (MMCS), whose propagator satisfies the equation of motion,

$$\sqrt{-g} \square i \Delta(x; x') = i \delta^D(x - x'), \quad (1)$$

where  $\square = g^{\mu\nu} \nabla_\mu \nabla_\nu$  denotes the d'Alembertian (wave) operator. Even though this equation is invariant under de Sitter symmetries, it does not admit a de Sitter invariant solution with the appropriate singularity structure [1,2] in any number  $D$  of space-time dimensions. This letter is devoted to pointing out a similar, but more subtle, obstruction to maintaining de Sitter symmetry for the massless vector propagator in the general covariant gauge. Even though the equation of motion does allow a de Sitter invariant solution, it is the Ward-Takahashi identity, overlooked thus far for propagators of massless vector fields, that prevents a de Sitter invariant solution. Here we present a solution for the photon propagator in  $D$ -dimensional spacetime, appropriate for dimensionally regulated quantum loop computations, that accounts for both the equation of motion and the Ward-Takahashi identity.

## 2. Photon in the general covariant gauge

The physical photon is conformally coupled to gravity in four dimensions, which is no longer true in  $D$ -dimensional spacetime,

$$S[A_\mu] = \int d^D x \sqrt{-g} \left[ -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right], \quad (2)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the field strength tensor of the vector potential  $A_\mu$ . We consider the spacetime to be the expanding Poincaré patch of de Sitter where the metric,  $g_{\mu\nu} = a^2(\eta) \eta_{\mu\nu}$ , is conformal to the Minkowski space metric,  $\eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$ ,  $g = \det(g_{\mu\nu})$ , and  $a(\eta) = 1/[1 - H(\eta - \eta_0)]$  is the scale factor expressed in terms of conformal time  $\eta$  and a constant Hubble parameter  $H$ . We shall not consider symmetry breaking theories, in which the vector field can acquire a mass, nor shall we consider spatially compact global coordinates on de Sitter where the problem of linearization instability arises [3].

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Quantization of the theory requires fixing a gauge. Choosing the general covariant gauge,

$$S_{\text{gf}}[A_\mu] = \int d^D x \sqrt{-g} \left[ -\frac{1}{2\xi} (\nabla^\mu A_\mu)^2 \right], \quad (3)$$

allows to maintain de Sitter symmetries of the dynamics. However, it breaks conformal coupling of the photon even in four spacetime dimensions, and for any choice of the gauge-fixing parameter  $\xi$ .

Central objects in perturbative nonequilibrium quantum field theory are the two-point functions determined from the free theory. The relevant ones are the Feynman propagator and the positive frequency Wightman function,

$$i[\Delta_\mu]_\nu(x; x') = \langle \Omega | \mathcal{T} [\hat{A}_\mu(x) \hat{A}_\nu(x')] | \Omega \rangle, \quad (4)$$

$$i[\Delta_\mu^\pm]_\nu(x; x') = \langle \Omega | \hat{A}_\mu(x) \hat{A}_\nu(x') | \Omega \rangle, \quad (5)$$

where  $|\Omega\rangle$  is the state,  $\mathcal{T}$  stands for time ordering, and  $\hat{A}_\mu(x)$  is the free photon field operator. The propagator equation of motion in this gauge is,

$$\sqrt{-g} \mathcal{D}^{\mu\nu} i[\Delta_\alpha]_\nu(x; x') = \delta_\alpha^\mu i\delta^D(x-x'), \quad (6)$$

where we set  $\hbar = 1$ , and the kinetic operator is,

$$\mathcal{D}^{\mu\nu} = g^{\mu\nu} \square - \left(1 - \frac{1}{\xi}\right) \nabla^\mu \nabla^\nu - R^{\mu\nu}, \quad (7)$$

where  $R^{\mu\nu} = (D-1)H^2 g^{\mu\nu}$  is the Ricci tensor in de Sitter. Due to the exchange symmetry  $(\mu, x) \leftrightarrow (\nu, x')$ , the photon propagator (4) obeys an equation analogous to (6) on the other leg  $(\nu, x')$ . The Wightman function obeys the same equation of motion (6), but without the local term on the right-hand-side.

The photon two-point functions in covariant gauges (3) have been considered in several works over the last decades, starting from the seminal work of Allen and Jacobson [4], who reported many results, among which the de Sitter space covariant gauge propagator for  $\xi=1$  in  $D$  spacetime dimensions. Subsequent works have extended and generalized this result. Tsamis and Woodard [5] reported the transverse massive vector propagator, whose massless limit reduces to the photon propagator in the Landau gauge ( $\xi \rightarrow 0$ ) in  $D$  dimensions; Youssef [6] computed the propagator for arbitrary  $\xi$  in  $D=4$  spacetime dimensions; and Fröb and Higuchi [7] reported the result for a massive vector propagator, with the massless limit producing the photon propagator for arbitrary  $\xi$  and arbitrary  $D$ . The last of these encompasses all the previously reported results as special cases. Only propagators from [4] and [5] were used for loop computations in de Sitter. The former was re-derived and used to study scalar electrodynamics [8], while the latter was utilized for both scalar electrodynamics [10,11,9,12], and for quantum gravity interacting with electromagnetism [13,14].

In this letter we are interested only in the massless vector (photon) propagators. All of the photon propagators reported in previous works satisfy the equation of motion (6). However, it was pointed out recently in [15] that photon propagators should satisfy additional subsidiary conditions dictated by the consistent canonical quantization in average/multiplier gauges. Among them is the condition that the double divergence of both the Feynman propagator and the Wightman function should vanish off-coincidence. But the reported results violate this condition,<sup>1</sup>

$$\nabla^\mu \nabla'^\nu i[\Delta_\mu]_\nu(x; x') \stackrel{!}{=} -\xi \frac{i\delta^D(x-x')}{\sqrt{-g}} - \xi \frac{H^D \Gamma(D)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})}, \quad (8)$$

<sup>1</sup> The Wightman function exhibits the same problem as in (8), except that the local term on the right-hand-side is absent.

except in the limit  $\xi \rightarrow 0$  when the offending term vanishes, suggesting that only the Tsamis-Woodard result is consistent. This is a problem that needs to be addressed. In the companion paper [16] to this letter we consider the problem from the first principles of canonical quantization and construct the photon propagator as a sum over modes. This letter is devoted to resolving the problem in (8) in an elegant manner by considering the Becchi-Rouet-Stora-Tyutin (BRST) quantization [17]. Our approach is similar to the one employed in [18] for studying retarded and advanced Green's functions for massive vector fields. However, the problem in (8) is not encountered for these Green's functions, as for them the problematic term is absent.

### 3. BRST quantization

In addition to introducing the gauge-fixing term (3), the BRST formalism requires the inclusion of the Faddeev-Popov ghost action for Grassmann fields  $c$  and  $\bar{c}$ ,

$$S_{\text{gh}}[c, \bar{c}] = \int d^D x \sqrt{-g} g^{\mu\nu} (\nabla_\mu \bar{c}) (\nabla_\nu c), \quad (9)$$

to the complete gauge-fixed action,  $S_* = S + S_{\text{gf}} + S_{\text{gh}}$ , which is invariant under infinitesimal BRST transformations,

$$A_\mu \rightarrow A_\mu + \theta \xi \partial_\mu c, \quad \bar{c} \rightarrow \bar{c} - \theta \nabla^\mu A_\mu, \quad c \rightarrow c, \quad (10)$$

parametrized by  $\theta$ , that are generated by the associated conserved BRST charge,

$$Q = \int d^{D-1} x a^{D-4} \left[ a^2 (\nabla^\mu A_\mu) \partial_0 c + \xi F_{0i} \partial_i c \right]. \quad (11)$$

The ghost propagator,  $i\Delta_c(x; x') = \langle \Omega | \mathcal{T} [\hat{c}(x) \hat{c}(x')] | \Omega \rangle$ , satisfies,

$$\sqrt{-g} \square i\Delta_c(x; x') = i\delta^D(x-x'). \quad (12)$$

This equation implies that the ghost propagator equals the MMCS propagator,  $i\Delta_c(x; x') = i\Delta(x; x')$ . Thus, the ghost propagator must break de Sitter symmetry. The natural choice for the MMCS propagator in the Poincaré patch of de Sitter is the one preserving cosmological symmetries, but breaking dilations and special spatial conformal transformations [19]. For our purposes it is best to write it as a limit [20],

$$i\Delta(x; x') = \lim_{\lambda \rightarrow \nu+1} i\Delta_\lambda(x; x'), \quad \nu = \frac{D-3}{2}, \quad (13)$$

of a propagator with an effectively slightly tachyonic mass  $M^2 = [(D-1)^2/4 - \lambda^2]H^2 < 0$ ,

$$i\Delta_\lambda(x; x') = \mathcal{F}_\lambda(y) + \mathcal{W}_\lambda(u), \quad (14)$$

which consists of a de Sitter invariant part,

$$\begin{aligned} \mathcal{F}_\lambda(y) &= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D-1}{2} + \lambda) \Gamma(\frac{D-1}{2} - \lambda)}{\Gamma(\frac{D}{2})} \\ &\quad \times {}_2F_1\left(\frac{D-1}{2} + \lambda, \frac{D-1}{2} - \lambda, \frac{D}{2}, 1 - \frac{y}{4}\right), \end{aligned} \quad (15)$$

dependent on a de Sitter invariant distance,

$$y = aa' H^2 \left[ \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\epsilon)^2 \right], \quad (16)$$

and the de Sitter breaking part [20],

$$\mathcal{W}_\lambda = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(2\lambda) \Gamma(\lambda)}{\Gamma(\frac{D-1}{2}) \Gamma(\frac{1}{2} + \lambda)} \frac{e^{(\lambda - \frac{D-1}{2})u}}{\lambda - \frac{D-1}{2}} \left(\frac{k_0}{H}\right)^{D-1-2\lambda}, \quad (17)$$

dependent on  $u = \ln(aa')$ , where  $k_0 < H$  is some infrared scale.

#### 4. Subsidiary condition

We derive the subsidiary condition for the photon propagator [21] by considering the expectation value of an anticommutator of the BRST charge operator  $\hat{Q}$  in Eq. (11), with a judiciously chosen product of the vector potential and anti-ghost operators,

$$\{\hat{Q}, \hat{c}(x)\hat{A}_\nu(x')\} = i\left[\nabla^\mu \hat{A}_\mu(x)\hat{A}_\nu(x') + \xi \hat{c}(x)\partial'_\nu \hat{c}(x')\right]. \quad (18)$$

Since the BRST charge operator annihilates physical states,  $\hat{Q}|\Omega\rangle = 0$ , the expectation value of this anticommutator must vanish. This produces the desired subsidiary condition,

$$\nabla^\mu i[\mu\Delta_\nu](x; x') = -\xi \partial'_\nu i\Delta(x; x'), \quad (19)$$

relating the photon propagator to the Faddeev-Popov ghost propagator. The photon propagator must therefore satisfy both the equation of motion (6) and the subsidiary condition (19). The latter has seemingly gone unnoticed thus far, apart from the exact transverse limit  $\xi \rightarrow 0$  [5]. This subsidiary condition is the Ward-Takahashi identity of the free theory. It is the massless limit of the previously derived identity for massive vector fields [18] adapted to the Feynman propagator and Wightman functions. The crucial observation is that this condition does not admit a de Sitter invariant solution for the photon propagator, and the reason behind it is the MMCS propagator appearing on the left hand side. After the derivative is acted on it,

$$\nabla^\mu i[\mu\Delta_\nu](x; x') = -\xi (\partial'_\nu y) \frac{\partial \mathcal{F}_{v+1}}{\partial y} - \xi (\partial'_\nu u) \frac{H^{D-2} \Gamma(D-1)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})}, \quad (20)$$

the right hand side of the subsidiary condition still breaks de Sitter symmetry. It preserves homogeneity, isotropy, and dilations, but it breaks spatial special conformal transformations.

#### 5. Solving for the propagator

Consider first the equation of motion (6), which simplifies upon plugging in the Ward-Takahashi identity (19) for the middle term,

$$\begin{aligned} & [\square - (D-1)H^2] i[\mu\Delta_\nu](x; x') \\ &= g_{\mu\nu} \frac{i\delta^D(x-x')}{\sqrt{-g}} + (1-\xi) \partial_\mu \partial'_\nu i\Delta(x; x'). \end{aligned} \quad (21)$$

Upon decomposing the propagator,<sup>2</sup>

$$i[\mu\Delta_\nu](x; x') = i[\mu\Delta_\nu^T](x; x') + i[\mu\Delta_\nu^L](x; x'), \quad (22)$$

into a transverse part,

$$\nabla^\mu i[\mu\Delta_\nu^T](x; x') = \nabla'^\nu i[\mu\Delta_\nu^T](x; x') = 0, \quad (23)$$

and a longitudinal part,

$$i[\mu\Delta_\nu^L](x; x') = \partial_\mu \partial'_\nu L(x; x'), \quad (24)$$

the equation of motion (21) breaks up into the transverse,

$$\begin{aligned} & [\square - (D-1)H^2] i[\mu\Delta_\nu^T](x; x') \\ &= g_{\mu\nu} \frac{i\delta^D(x-x')}{\sqrt{-g}} + \partial_\mu \partial'_\nu i\Delta(x; x'), \end{aligned} \quad (25)$$

and the longitudinal equation,

$$\partial_\mu \partial'_\nu \square L(x; x') = -\xi \partial_\mu \partial'_\nu i\Delta(x; x'). \quad (26)$$

The Ward-Takahashi identity (19) constrains only the longitudinal part,

$$\partial'_\nu \square L(x; x') = -\xi \partial'_\nu i\Delta(x; x'). \quad (27)$$

The transverse equation (25) is (the massless limit of) the equation solved by Tsamis and Woodard [5], and corresponds to the propagator in the  $\xi \rightarrow 0$  limit. Even though the MMCS propagator in the source on the right-hand side of (25) breaks de Sitter symmetry, the two derivatives acting on it annihilate the de Sitter breaking part.

The longitudinal equation is, interestingly, a derivative of the subsidiary condition (19). Thus, any solution satisfying the subsidiary condition will automatically satisfy the longitudinal equation of motion (27). But, importantly, not all solutions of the equation of motion will satisfy the subsidiary condition! It is the second derivative in the longitudinal equation that enables a de Sitter-invariant solution for the longitudinal part. However, as can be seen from (20), the Ward-Takahashi identity (19) necessitates breaking of de Sitter symmetry.

One solves Eq. (27) by requiring,

$$\square L(x; x') = -\xi i\Delta(x; x'), \quad (28)$$

which is the equation for the so-called integrated propagator, that is solved by [22],

$$L(x; x') = \frac{\xi}{2\lambda} \frac{\partial}{\partial \lambda} i\Delta_\lambda(x; x') \Big|_{\lambda \rightarrow v+1}. \quad (29)$$

Combining the transverse part worked out in [5] with the longitudinal part worked out here, the propagator in the general covariant gauge can be written in a convenient covariant basis that emphasizes our main point,

$$i[\mu\Delta_\nu](x; x') = (\partial_\mu \partial'_\nu y) C_1 + (\partial_\mu y)(\partial'_\nu y) C_2 + (\partial_\mu u)(\partial'_\nu u) C_4. \quad (30)$$

The first two terms are composed out of de Sitter invariant tensor structures multiplied by de Sitter invariant scalar structure functions, which depend on  $y$  only,

$$C_1 = \frac{-1}{2vH^2} \left[ \left(v + \frac{1}{2}\right) \mathcal{F}_v + \left(1 - \frac{\xi}{\xi_s}\right) \frac{\partial}{\partial y} \frac{\partial}{\partial v} \mathcal{F}_{v+1} \right], \quad (31)$$

$$C_2 = \frac{-1}{2vH^2} \left[ \frac{1}{2} \frac{\partial}{\partial y} \mathcal{F}_v + \left(1 - \frac{\xi}{\xi_s}\right) \frac{\partial^2}{\partial y^2} \frac{\partial}{\partial v} \mathcal{F}_{v+1} \right], \quad (32)$$

where  $v$  is defined in (13), and  $\xi_s = (D-1)/(D-3)$  is what we refer to as the simple covariant gauge. The last term in (30) consists of a de Sitter breaking tensor structure multiplied by a constant,

$$C_4 = \xi \times \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{(D-1) \Gamma(\frac{D}{2})}, \quad (33)$$

implying that our propagator preserves dilations, but breaks special spatial conformal transformations.

All the de Sitter invariant photon propagator results reported in the literature [4–7] are captured by the  $C_1$  and  $C_2$  parts of the solution. The nonvanishing constant  $C_4$  in (33) however, has been overlooked thus far. But it is this part that guarantees that the Ward-Takahashi identity and the equation of motion are simultaneously satisfied, now producing the correct expression,

$$\nabla^\mu \nabla'^\nu i[\mu\Delta_\nu](x; x') = -\xi \frac{i\delta^D(x-x')}{\sqrt{-g}}, \quad (34)$$

<sup>2</sup> The Ward-Takahashi identity (19) forbids contributions in (22) that are transverse on one leg and longitudinal on the other.

instead of (8). Thus, expressions (30)–(33) constitute a complete solution for the Feynman propagator for the massless vector field, that satisfies both the equation of motion (6) and the subsidiary condition (19).

The positive frequency Wightman function is now easily inferred from the solution for the Feynman propagator by simply changing the  $i\varepsilon$  prescription for  $y$  in (30)–(32) to the appropriate one,  $y_{-+} = aa'[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' - i\varepsilon)^2]$ . This ensures that the Wightman function satisfies a homogeneous equation of motion,

$$\sqrt{-g} \mathcal{D}^{\mu\nu} i[\Delta_{\alpha}^+](x; x') = 0, \quad (35)$$

and an appropriate subsidiary condition,

$$\nabla^{\mu} i[\Delta_{\mu}^+](x; x') = -\xi \partial_{\nu}' i[\Delta^+](x; x'), \quad (36)$$

containing the positive-frequency Wightman function for the ghost, obtained from (13) and (14) by the same substitution  $y \rightarrow y_{-+}$ . Thus, the double divergence vanishes,

$$\nabla^{\mu} \nabla'^{\nu} i[\Delta_{\mu}^+](x; x') = 0, \quad (37)$$

resolving the problems reported in [15].

This result agrees with the independent mode sum analysis in our companion paper [16]. Apart from correctly accounting for the subsidiary condition, the missing de Sitter breaking term resolves some issues that we outline in the remainder of the letter.

## 6. Infrared behavior

The missing term in the photon two-point function we report here does not only solve the issue of non-vanishing double divergence, which has gone unnoticed for a long time, but also addresses the concern regarding the infrared behavior of the photon two-point function. It was reported by Youssef [6] (in  $D=4$ ) and by Rendell [23] (in  $D$  dimensions) that the two-point function does not vanish in the deep infrared,

$$i[\Delta_{\mu\nu}](x; x') \underset{!}{\overset{|y| \rightarrow \infty}{\sim}} -\xi \frac{H^{D-2} aa' \delta_{\mu}^0 \delta_{\nu}^0 \Gamma(D-1)}{(4\pi)^{\frac{D}{2}} (D-1) \Gamma(\frac{D}{2})}. \quad (38)$$

Even though this is a gauge dependent statement [6], since  $\xi$  ranges on the entire real line, it nonetheless is not innocuous, as it is a consequence of failing to account for the Ward-Takahashi identity (19). This behavior is precisely removed by the missing term (33) that we report, so that in the deep infrared the photon two-point function vanishes,

$$i[\Delta_{\mu\nu}](x; x') \xrightarrow{|y| \rightarrow \infty} 0. \quad (39)$$

While this is immaterial at the linear level, it does influence the loops.

## 7. Energy-momentum tensor

Not accounting for the de Sitter breaking part (33) of the photon propagator can lead to inconsistencies in how photons source gravity. There are two definitions possible for the photon energy-momentum tensor, that have to coincide on-shell. We can either define it as a variation of the gauge-invariant action,

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = (\delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} - \frac{1}{4} g_{\mu\nu} g^{\rho\sigma}) g^{\alpha\beta} F_{\rho\alpha} F_{\sigma\beta}, \quad (40)$$

or as a variation of the gauge-fixed action,

$$\begin{aligned} T_{\mu\nu}^{\star} &= \frac{-2}{\sqrt{-g}} \frac{\delta S_{\star}}{\delta g^{\mu\nu}} = T_{\mu\nu} - \frac{2}{\xi} A_{(\mu} \nabla_{\nu)} \nabla^{\rho} A_{\rho} \\ &\quad + \frac{g_{\mu\nu}}{\xi} \left[ A_{\rho} \nabla^{\rho} \nabla^{\sigma} A_{\sigma} + \frac{1}{2} (\nabla^{\rho} A_{\rho})^2 \right] \\ &\quad - 2(\partial_{(\mu} \bar{c})(\partial_{\nu)} c) + g_{\mu\nu} g^{\rho\sigma} (\partial_{\rho} \bar{c})(\partial_{\sigma} c). \end{aligned} \quad (41)$$

Classically the two give the same answer as they differ by BRST-exact terms only, which all vanish on-shell.<sup>3</sup> The quantized theory has to maintain this property at the level of expectation values. Operators associated to the two definitions of the energy-momentum tensor are defined by Weyl ordered (symmetrized) products of field operators, and the difference between their expectation values is best expressed in terms of the Wightman function and derivatives acting on it,

$$\begin{aligned} &\langle \hat{T}_{\mu\nu}^{\star}(x) \rangle - \langle \hat{T}_{\mu\nu}(x) \rangle \\ &= \left\{ \frac{g_{\mu\nu}}{2\xi} \nabla^{\rho} \nabla'^{\sigma} i[\Delta_{\rho\sigma}^+](x; x') \right. \\ &\quad - \frac{1}{\xi} \left[ \delta_{(\mu}^{\rho} \nabla_{\nu)}' \nabla'^{\sigma} + \delta_{(\mu}^{\sigma} \nabla_{\nu)} \nabla^{\rho} \right] i[\Delta_{\rho\sigma}^+](x; x') \\ &\quad + \frac{g_{\mu\nu}}{2\xi} \left[ \nabla'^{\rho} \nabla'^{\sigma} + \nabla^{\sigma} \nabla^{\rho} \right] i[\Delta_{\rho\sigma}^+](x; x') \\ &\quad \left. - \left[ 2\nabla_{\mu} \nabla_{\nu}' - g_{\mu\nu} \nabla^{\rho} \nabla_{\rho}' \right] i[\Delta^+](x; x') \right\} \Big|_{x' \rightarrow x}. \end{aligned} \quad (42)$$

The terms in the last three lines above cancel on the account of a derivative of the Ward-Takahashi identity,

$$\nabla_{\rho} \nabla^{\mu} i[\Delta_{\mu\nu}^+](x; x') = -\xi \partial_{\rho} \partial_{\nu}' i[\Delta^+](x; x'), \quad (43)$$

which is insensitive to the de Sitter breaking part that drops out because of the second derivative. The remaining term from the first line,

$$\langle \hat{T}_{\mu\nu}^{\star}(x) \rangle - \langle \hat{T}_{\mu\nu}(x) \rangle = \frac{g_{\mu\nu}}{2\xi} \nabla^{\rho} \nabla'^{\sigma} i[\Delta_{\rho\sigma}^+](x; x') \Big|_{x' \rightarrow x}, \quad (44)$$

has to vanish on its own. This is precisely in the form of the problematic expression (8) we started with. If we were to disregard the de Sitter breaking part (33) of the photon propagator, this difference would not vanish,

$$\langle \hat{T}_{\mu\nu}^{\star}(x) \rangle - \langle \hat{T}_{\mu\nu}(x) \rangle \stackrel{!}{=} -\frac{g_{\mu\nu}}{2} \frac{H^D \Gamma(D)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})},$$

but would have a cosmological constant form. Since this contribution is independent of the gauge-fixing parameter  $\xi$ , it would be difficult to recognize it as an unphysical answer, even though it clearly originates from not accounting for the gauge sector constraints. In fact, when one uses the de Sitter breaking two-point function, inserting (37) into (44) shows that the two definitions give the same answer,

$$\langle \hat{T}_{\mu\nu}^{\star}(x) \rangle - \langle \hat{T}_{\mu\nu}(x) \rangle = 0. \quad (45)$$

When computed in dimensional regularization the expectation value of the energy-momentum tensor in  $D=4$  vanishes,  $\langle \hat{T}_{\mu\nu}(x) \rangle = 0$ , as shown in [16].

<sup>3</sup> BRST-exact objects are the ones that can be written as (anti)commutators of other objects with the BRST charge. They vanish on-shell by the definition of the formalism.

## 8. Discussion

The photon propagator in de Sitter in the general covariant gauge takes the form (30) with the three structure functions given in (31)–(33). The de Sitter invariant parts in (31)–(32) have been derived in previous works [4–7]. It is the nonvanishing constant  $C_4$  in (33), overlooked thus far, that is our main contribution. This contribution *breaks* de Sitter symmetry. In particular, it breaks special spatial conformal transformations, while preserving dilations, spatial homogeneity and isotropy. Even though this term is a homogeneous solution of the propagator equation of motion (6), it cannot be discarded. It is the Ward-Takahashi identity (19), necessary for the construction of a consistent propagator, that requires it. The de Sitter breaking can be traced back to the Faddeev-Popov ghost propagator, which satisfies the equation for the massless, minimally coupled scalar (12), which does not admit a de Sitter invariant solution. This ghost propagator appears in the Ward-Takahashi identity (19), whose solution allows a dilation-preserving photon propagator. At the linear level, this necessary modification of the photon propagator is of little physical significance, as it is confined to the pure gauge sector. However, it can be of paramount importance when interactions are considered, as failure to implement it, in general leads to incorrect results.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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