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Photon propagator in de Sitter space in the general covariant gauge

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Abstract: We consider a free photon field in D-dimensional de Sitter space, and construct its propagator in the general covariant gauge. Canonical quantization is employed to define the system starting from the classical theory. This guarantees that the propagator satisfies both the equation of motion *and* subsidiary conditions descending from gauge invariance and gauge fixing. We first construct the propagator as a sum-over-modes in momentum space, carefully accounting for symmetry properties of the state. We then derive the position space propagator in a covariant representation, that is our main result. Our conclusions disagree with previous results as we find that the position space photon propagator *necessarily breaks de Sitter symmetry*, except in the exact transverse gauge limit.

Keywords: de Sitter space, Early Universe Particle Physics, Gauge Symmetry, Non-Equilibrium Field Theory

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Contents

1 Introduction

Linear dynamics of a massless vector field — the photon — in expanding cosmological spaces is considered to be particularly simple and well understood. The photon couples conformally to gravity and thus effectively does not sense the expansion. This makes the dynamics of its two physical polarizations no more complicated than in flat space. However, the conformal coupling of the photon can be broken via couplings to non-trivial field condensates or other non-conformally coupled fields such as scalars or gravitons. Primordial inflation is where these effects can be particularly important because of the huge scale of the expansion rate. There are a number of cases where vector fields play an important role in inflation, some of which are vector inflation [1–6], preheating after inflation [7, 8], axion inflation [9–12], inflationary magnetogenesis from tree level dynamics [13–15] (for a review see [16]) and from dynamically generated condensates during inflation [17, 18], the Schwinger effect in inflation [19–26], and corrections to the standard model and inflaton effective potential [27–34]. Particularly interesting are cases where conformal coupling is broken by quantum loop effects in inflation. This happens for interactions with light spectator scalar fields [35–51] or with inflationary gravitons [52–59]. Both cases point to large non-perturbative effects. A useful idealization of the slow-roll inflationary spacetime is a rigid de Sitter space, which is one of the three maximally symmetric spacetimes, and maximal symmetry implies considerable conceptual and computational simplifications. In this work we consider D-dimensional photon two-point functions in the expanding patch of de Sitter space, appropriate for dimensionally regulated, nonequilibrium perturbative computations in the Schwinger-Keldysh (also known as *in-in* or closed-time-path) formalism.

While at the linear level we have the luxury of explicitly isolating the physical degrees of freedom — the two transverse polarizations — this is no longer as straightforward at the interacting level. Instead, it is preferable and considerably simpler to perform computations in a particular gauge with gauge-dependent quantities, and only later project out the physical information. Fixing the gauge can be done in a number of ways, but the choice preferred for computations is the one where no field components are explicitly eliminated. These are referred to as average gauges (or multiplier gauges), and are characterized by adding a gauge-fixing term to the original gauge-invariant action. Arguably the most natural choice for the gauge-fixing term is the *general covariant gauge,*

$$S_{\rm gf}[A_\mu] = \int d^Dx\, \sqrt{-g} \left[-\frac{1}{2\xi} \left(g^{\mu\nu} \nabla_\mu A_\nu\right)^2 \right] , \tag{1.1}$$

that comes with one free real parameter ξ, defining a one-parameter family of gauges. Any physical quantity must be independent on ξ, which makes these gauges particularly useful. It is commonly held that photon propagators in the general covariant gauge are de Sitter invariant, which is supported by the existing literature. It his work we challenge this belief.

Covariant gauge propagators for different values of the gauge-fixing parameters have been derived in several works, starting with the seminal work by Allen and Jacobson [60]. Among several different cases and spacetimes, they derived the photon propagator in de Sitter space of arbitrary spacetime dimension D, in $\xi = 1$ covariant gauge. To this end they made a de Sitter invariant ansatz for the propagator, solved the resulting simplified equations of motion, and fixed the ambiguities by considering the singularity structure. This method was used in several subsequent works to derive propagators in different covariant gauges. In [39] the propagator from [60] was rederived in a somewhat different form. Tsamis and Woodard [61] used the method when considering a massive vector propagator whose massless limit corresponds to the photon in Landau gauge $\xi \to 0$ in D-dimensional de Sitter. To obtain the propagator they required their *Ansatz* to be transverse, in addition to being de Sitter invariant. The methods from [60] was also used by Youssef [62] to derive the photon propagator for arbitrary gauge-fixing parameter in four space-time dimensions. The most general photon propagator, valid for arbitrary ξ and D, was reported by Fröb and Higuchi [63]. Unlike the preceding works, they utilized canonical quantization of the gauge-fixed vector sector of the Stueckelberg model, containing a massive vector field. The resulting propagator they report was derived as a sum-over-modes, and its de Sitter invariant massless limit encompasses all previous results as special cases.[1]

It would seem that little more could be said about covariant gauge photon propagators in de Sitter. Nevertheless, it was pointed out recently [65] that photon propagators in so-called average (or multiplier) gauges, in addition to solving the equations of motion, must satisfy subsidiary conditions that are a consequence of gauge symmetry. These subsidiary conditions derive from the quantization of the first-class constraints of the classical theory, and amount to the condition that correlators of first-class constraints have to vanish in the quantized theory. It was found that the photon propagators reported in the literature do not satisfy all the subsidiary conditions [65], except in the gauge $\xi \to 0$. This is a problem that needs to be addressed, and it motivates us here to consider the construction of the photon propagator in covariant gauges from the first principles of canonical quantization.

The canonical quantization is based on the canonical structure only, and is conceptually divorced from symmetries of the background spacetime. Even though making this distinction is often not necessary, here we find it important when trying to understand where the problem with the propagators comes from and how to resolve it. We pay special attention to how particular symmetry properties of quantum states are imposed, by considering conserved charges from both the gauge-invariant and gauge-fixed formulations, that serve as generators of de Sitter symmetry transformations. Our main result is rather surprising — the physically de Sitter invariant quantum state of the photon does not admit a de Sitter

[1] In ref. [64] the method of making the de Sitter invariant *Ansatz* was considered for the case of general ξ and D. The analysis there would have reproduced the de Sitter invariant result of [63], had the integrals in the reported result been evaluated.

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invariant propagator in the general covariant gauge (except in the limit $\xi\!\to\!0$). We derive this result by representing the propagator as a sum-over-modes in momentum space, which we subsequently solve to find the propagator in position space. The resulting expression consists of a de Sitter invariant part, that corresponds to the massless limit of [63], and a previously missed de Sitter breaking part which ensures that the subsidiary conditions are respected. In a companion letter [66] we show how one can obtain the same result in position space, starting from BRST quantization and utilizing Ward-Takahashi identities.

This paper is organized in eight sections, the first of which is concluding. The following section collects definitions and results on propagators and scalar mode functions used throughout the paper. The third section recounts the canonical formulation of the photon in multiplier gauges and canonical quantization. The dynamics of field operators is solved for in the fourth section, and the fifth section is devoted to the construction of the quantum state and discussion of its symmetries. The main results are derived in the sixth section, that gives the solution for the propagator satisfying all the required subsidiary conditions. Checks of the propagator solution are performed in the seventh section by considering two simple observables, while the concluding eighth section contains a discussion of the main results. More technical details are relegated to two appendices.

2 Preliminaries

This section collects definitions and results frequently used in subsequent sections. First the Poincaré patch of de Sitter space is defined, and then some useful results on scalar two-point functions and scalar mode functions in de Sitter are recalled.

2.1 De Sitter space

The invariant line element of the D-dimensional Friedman-Lemaître-Robertson-Walker (FLRW) spacetime,

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^{\,2} = a^2(\eta)\big[-d\eta^2 + d\vec{x}^{\,2}\big]\,, \tag{2.1}$$

defines the associated conformally flat metric $g_{\mu\nu}\!=\!a^2(\eta)\text{diag}(-1,1,\dots,1)$, where the speed of light is taken to be unity, $c\!=\!1$. Here Cartesian coordinates $\vec{x}$ span $(D\!-\!1)-$dimensional flat spatial slices, while time is parametrized either by the physical time t, or by conformal time η. The two times are related by $dt\!=\!a(\eta)d\eta$, where a is the scale factor that encodes the dynamics of the expansion, which is usually expressed either in terms of the physical Hubble rate, $H\!=\!(da/dt)/a$, or the conformal Hubble rate, $\mathcal{H}\!=\!(da/d\eta)/a$, the two being related by $\mathcal{H}\!=\!aH$.

The expanding Poincaré patch of de Sitter space is defined as the FLRW spacetime with a constant physical Hubble rate, $H = \texttt{const.}$, and the conformal Hubble rate and the scale factor take the following functional form,

$$\mathcal{H} = \frac{H}{1\!-\!H(\eta\!-\!\eta_0)}\,, \qquad\qquad a(\eta) = \frac{\mathcal{H}}{H}\,, \tag{2.2}$$

where η_0 is the initial time, for which $a(\eta_0) = 1$. The conformal time ranges on the interval $\eta\!\in\!(-\infty,\eta_0\!+\!1/H)$.

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2.2 Scalar two-point functions

Two-point functions of scalar fields often appear as building blocks of two-point functions for higher spin fields in de Sitter. This will be true for the photon propagator we construct in this work, so here we summarize and recall some of the properties of scalar field mode functions and the two-point functions constructed out of them.

The positive-frequency Wightman function[2] can be taken as the elementary two-point function. It satisfies a homogeneous equation of motion,

$$(\Box - M_\lambda^2)\, i\big[{}^{-}\Delta^{+}\big]_\lambda(x;x') = 0\,, \tag{2.3}$$

where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the d'Alembertian, and where the effective mass, conveniently parametrized by λ,

$$M_\lambda^2 = \bigg[\Big(\frac{D-1}{2}\Big)^2 - \lambda^2\bigg] H^2\,, \tag{2.4}$$

receives contributions from both the scalar field mass and its non-minimal coupling to the Ricci scalar. The negative frequency Wightman function is just a complex conjugate of the positive-frequency one, $i\big[{}^{+}\Delta^{-}\big]_\lambda(x;x') = \big\{ i\big[{}^{-}\Delta^{+}\big]_\lambda(x;x')\big\}^*$, and satisfies the same homogeneous equation of motion. The Feynman propagator involves time-ordering, and is expressed in terms of the Heaviside step function as,

$$i\big[{}^{+}\Delta^{+}\big](x;x') = \theta(\eta-\eta')\, i\big[{}^{-}\Delta^{+}\big](x;x') + \theta(\eta'-\eta)\, i\big[{}^{+}\Delta^{-}\big](x;x')\,. \tag{2.5}$$

and satisfies a sourced equation of motion,

$$(\Box - M_\lambda^2)\, i\big[{}^{+}\Delta^{+}\big]_\lambda(x;x') = \frac{i\delta^D(x-x')}{\sqrt{-g}}\,. \tag{2.6}$$

The Dyson propagator is its complex conjugate, $i\big[{}^{-}\Delta^{-}\big](x;x') = \big\{ i\big[{}^{+}\Delta^{+}\big](x;x')\big\}^*$, and satisfies a conjugate of eq. (2.6).

The scalar two-point functions admit a sum-over-modes representation,

$$i\big[{}^{-}\Delta^{+}\big]_\lambda(x;x') = (aa')^{-\frac{D-2}{2}} \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\, \mathscr{U}_\lambda(\eta,\vec{k})\big[\mathscr{U}_\lambda(\eta',\vec{k})\big]^*\,, \tag{2.7}$$

in terms of the conformally rescaled scalar field mode function $\mathscr{U}_\lambda(\eta,k)$. We summarize the properties of the scalar mode functions in the following subsection. The integral in eq. (2.7) is generally divergent for real coordinates, and analytic continuation is called for. The prescription $\eta \to \eta - i\varepsilon/2$ and $\eta' \to \eta' - i\varepsilon/2$ preserves its naive properties under complex conjugation, and defines it as a distributional limit $\varepsilon \to 0_+$. Depending on the value of parameter λ solutions for two-point functions are qualitatively different. For $\lambda < (D-1)/2$ there are de Sitter invariant solutions, while for $\lambda \geq (D-1)/2$ they do not exist, see e.g. refs. [67, 68]. We summarize these two cases for position space scalar two-point functions in the two concluding subsections.

[2] Our naming for two-point functions follows the Keldysh polarity conventions.

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2.2.1 Scalar mode functions

The equation of motion for scalar mode functions in de Sitter is,

$$\Big[\partial_0^2 + k^2 - \Big(\lambda^2 - \frac{1}{4}\Big)\mathcal{H}^2\Big]\mathscr{U}_\lambda(\eta,\vec{k}) = 0\,, \tag{2.8}$$

where λ is a constant defined in (2.4). Its general solution in de Sitter space,

$$\mathscr{U}(\eta,\vec{k}) = \alpha(\vec{k})\,U_\lambda(\eta,k) + \beta(\vec{k})\,U_\lambda^*(\eta,k)\,, \tag{2.9}$$

is given by the positive-frequency Chernikov-Tagirov-Bunch-Davies (CTBD) mode function [69, 70],

$$U_\lambda(\eta,k) = e^{\frac{i\pi}{4}(2\lambda+1)} e^{\frac{-ik}{H}} \sqrt{\frac{\pi}{4\mathcal{H}}}\, H_\lambda^{(1)}\Big(\frac{k}{\mathcal{H}}\Big)\,. \tag{2.10}$$

where $H_\lambda^{(1)}$ is the Hankel function of the first kind, and $\alpha(\vec{k})$ and $\beta(\vec{k})$ are complex Bogolybov coefficients satisfying, $|\alpha(\vec{k})|^2 - |\beta(\vec{k})|^2 = 1$. The flat space limit of the this mode function is

$$U_\lambda(\eta,k) \overset{H\to 0}{\sim} \frac{e^{-ik(\eta-\eta_0)}}{\sqrt{2k}}\bigg\{1 + \Big(\lambda^2 - \frac{1}{4}\Big)\Big[\frac{iH}{2k} + \Big(\lambda^2 - \frac{9}{4} - 4ik(\eta-\eta_0)\Big)\frac{H^2}{8k^2} + \mathcal{O}(H^3)\Big]\bigg\}\,, \tag{2.11}$$

where η_0 is the initial time at which $a(\eta_0) = 1$. We make frequent use of recurrence relations between contiguous scalar mode functions,

$$\Big[\partial_0 + \Big(\lambda + \frac{1}{2}\Big)\mathcal{H}\Big]U_\lambda = -ikU_{\lambda+1}\,, \qquad \Big[\partial_0 - \Big(\lambda + \frac{1}{2}\Big)\mathcal{H}\Big]U_{\lambda+1} = -ikU_\lambda\,, \tag{2.12}$$

which follow from the recurrence relations for Hankel functions (cf. (10.6.2) in [71, 72]). Using these the Wronskian is conveniently written as,

$$\mathrm{Re}\Big[U_\lambda(\eta,k)U_{\lambda+1}^*(\eta,k)\Big] = \frac{1}{2k}\,. \tag{2.13}$$

We also use two identities that follow from the equation of motion (2.8),

$$\Big[\partial_0^2 + k^2 - \Big(\lambda^2 - \frac{1}{4}\Big)\mathcal{H}^2\Big]\Big(\mathcal{H}U_{\lambda+1}\Big) = 2\mathcal{H}^3\Big[2(\lambda+1)U_{\lambda+1} - \frac{ik}{\mathcal{H}}U_\lambda\Big]\,, \tag{2.14}$$

$$\Big[\partial_0^2 + k^2 - \Big(\lambda^2 - \frac{1}{4}\Big)\mathcal{H}^2\Big]\frac{\partial U_\lambda}{\partial\lambda} = 2\lambda\mathcal{H}^2 U_\lambda\,, \tag{2.15}$$

the former one by applying the recurrence relations (2.12), and the later by taking a parametric derivative of (2.8).

2.2.2 De Sitter invariant scalar two-point functions

For $\lambda < (D-1)/2$, the positive-frequency Wightman function of the real scalar field (2.3) in power-law inflation has a sum-over-modes representation (2.7),

$$i\big[{}^-\Delta^+\big]_\lambda(x;x') = (aa')^{-\frac{D-2}{2}} \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\, U_\lambda(\eta,k)U_\lambda^*(\eta',k). \tag{2.16}$$

This integral representation evaluates to,

$$i\big[{}^{-}\!\Delta^{+}\big]_{\lambda}(x;x') = \mathcal{F}_{\lambda}(y_{-+})\,, \tag{2.17}$$

where y_{-+} is the $i\varepsilon$-regulated distance function appropriate for the positive-frequency Wightman function,

$$y_{-+}(x;x') = \mathcal{H}\mathcal{H}'(\Delta x^2_{-+}) = \mathcal{H}\mathcal{H}'\Big[\|\vec{x}-\vec{x}\,'\|^2 - (\eta-\eta'-i\varepsilon)^2\Big]\,, \tag{2.18}$$

and the propagator function is expressed in terms of a hypergeometric function,

$$\begin{aligned}\mathcal{F}_{\lambda}(y) = {} & \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma\big(\frac{D-1}{2}+\lambda\big)\,\Gamma\big(\frac{D-1}{2}-\lambda\big)}{\Gamma\big(\frac{D}{2}\big)} \\ & \times {}_2F_1\bigg(\Big\{\frac{D-1}{2}+\lambda, \frac{D-1}{2}-\lambda\Big\}, \Big\{\frac{D}{2}\Big\}, 1-\frac{y}{4}\bigg)\,,\end{aligned} \tag{2.19}$$

that satisfies the hypergeometric equation in a different guise,

$$\bigg[(4y-y^2)\frac{\partial^2}{\partial y^2} + D(2-y)\frac{\partial}{\partial y} + \lambda^2 - \Big(\frac{D-1}{2}\Big)^2\bigg]\mathcal{F}_{\lambda}(y) = 0\,. \tag{2.20}$$

The scalar Feynman propagator (2.5) — known as the Chernikov-Tagirov propagator [69] — takes the same form as the Wightman function,

$$i\big[{}^{+}\!\Delta^{+}\big]_{\lambda}(x;x') = \mathcal{F}_{\lambda}(y_{++})\,, \tag{2.21}$$

where the argument of the propagator function is substituted by the appropriate one according to (2.5),[3]

$$y_{++} = \mathcal{H}\mathcal{H}'(\Delta x^2_{++}) = \mathcal{H}\mathcal{H}'\Big[\|\vec{x}-\vec{x}\,'\|^2 - (|\eta-\eta'|-i\varepsilon)^2\Big]\,. \tag{2.22}$$

Henceforth we suppress the polarity indices denoting different $i\varepsilon$-prescriptions for distance functions y that distinguish between two-point functions, as they should be clear from the context. A useful representation of the propagator function in (2.19) is a power series around $y\!=\!0$,

$$\begin{aligned}\mathcal{F}_{\lambda}(y) = {} & \frac{H^{D-2}\,\Gamma\big(\frac{D-2}{2}\big)}{(4\pi)^{\frac{D}{2}}} \bigg\{ \Big(\frac{y}{4}\Big)^{-\frac{D-2}{2}} + \frac{\Gamma\big(\frac{4-D}{2}\big)}{\Gamma\big(\frac{1}{2}+\lambda\big)\,\Gamma\big(\frac{1}{2}-\lambda\big)} \sum_{n=0}^{\infty} \\ & \times \bigg[\frac{\Gamma\big(\frac{3}{2}+\lambda+n\big)\,\Gamma\big(\frac{3}{2}-\lambda+n\big)}{\Gamma\big(\frac{6-D}{2}+n\big)\,(n+1)!} \Big(\frac{y}{4}\Big)^{n-\frac{D-4}{2}} - \frac{\Gamma\big(\frac{D-1}{2}+\lambda+n\big)\,\Gamma\big(\frac{D-1}{2}-\lambda+n\big)}{\Gamma\big(\frac{D}{2}+n\big)\,n!} \Big(\frac{y}{4}\Big)^{n} \bigg] \bigg\}\,.\end{aligned} \tag{2.23}$$

Furthermore, Gauss' relations between hypergeometric functions (cf. (9.137) of [73]) allow us to derive recurrence relations between contiguous propagator functions,

$$2\frac{\partial \mathcal{F}_{\lambda}}{\partial y} = (2-y)\frac{\partial \mathcal{F}_{\lambda+1}}{\partial y} + \Big(\lambda-\frac{D-3}{2}\Big)\mathcal{F}_{\lambda+1}\,, \tag{2.24}$$

$$2\frac{\partial \mathcal{F}_{\lambda+1}}{\partial y} = (2-y)\frac{\partial \mathcal{F}_{\lambda}}{\partial y} - \Big(\lambda+\frac{D-1}{2}\Big)\mathcal{F}_{\lambda}\,, \tag{2.25}$$

that we utilize in section 6.

[3] The $i\varepsilon$ prescription for the Feynman propagator in (2.22) follows from the prescription for the Wightman function in (2.18) and the definition (2.5), upon using the properties of the step function: $\theta(\Delta\eta)+\theta(-\Delta\eta)=1$, $[\theta(\Delta\eta)]^2=\theta(\Delta\eta)$, and $\theta(\Delta\eta)\times\theta(-\Delta\eta)=0$, where $\Delta\eta=\eta-\eta'$.

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2.2.3 De Sitter breaking scalar two-point functions

When the index of the scalar mode function is $\lambda > (D-1)/2$, the CTBD mode function leads to an unphysical infrared divergent Wightman function. In such cases the physical mode function must be modified in the infrared by introducing Bogolyubov coefficients in (2.9) that suppress the singular behaviour, and lead to a well defined sum-over-modes representation (2.16). Choosing them to preserve cosmological symmetries leads to the following Wightman function [67],

$$i\big[{}^{-}\Delta^{+}\big]_{\lambda}(x;x') = \mathcal{F}_{\lambda}(y) + \mathcal{W}_{\lambda}(y,u,v)\,, \tag{2.26}$$

composed of the de Sitter invariant part (2.19), and the de Sitter breaking part $\mathcal{W}_{\lambda}$, that depends on y and two other bi-local variables respecting spatial homogeneity and isotropy,

$$u = \ln(aa')\,, \qquad v = \ln(a/a')\,. \tag{2.27}$$

The general expression for the de Sitter breaking part can be found in [68], but here we only need it for a restricted range of λ,

$$\mathcal{W}_{\lambda}(u) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(2\lambda)\,\Gamma(\lambda)}{\Gamma\big(\frac{D-1}{2}\big)\,\Gamma\big(\frac{1}{2}+\lambda\big)} \frac{e^{(\lambda-\frac{D-1}{2})u}}{\lambda-\frac{D-1}{2}} \Big(\frac{k_0}{H}\Big)^{D-1-2\lambda}\,, \qquad \frac{D-1}{2} < \lambda < \frac{D+1}{2}\,, \tag{2.28}$$

where it depends on u only, and where $k_0 < H$ is some infrared scale. The Feynman propagator is then inferred from the Wightman function in the same way as described in section 2.2.2, by changing the implicit $i\varepsilon$-prescription of y.

For our purposes the limiting case of the massless, minimally coupled scalar, $\lambda \to (D-1)/2$, is particularly important. In this limit the two-point function (2.26) reproduces the finite Onemli-Woodard two-point function [74], as the divergence in the de Sitter breaking part cancels the one from the de Sitter invariant part, which is divergent in any number of dimensions. Nevertheless, the quantity $\partial\mathcal{F}_{\lambda}/\partial y$ is finite if first the derivative is performed, and then the limit $\lambda \to (D-1)/2$ is taken. Another important expression that we encounter is,

$$\Big(\frac{D-1}{2}-\lambda\Big)\mathcal{F}_{\lambda}(y) \xrightarrow{\lambda\to\frac{D-1}{2}} \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma\big(\frac{D}{2}\big)}\,, \tag{2.29}$$

which is valid for arbitrary y, and which allows us to use recurrence relations (2.24) and (2.25) in this limit.

3 Photon in FLRW

The free photon in D-dimensional curved space is defined by the covariant action,

$$S[A_{\mu}] = \int d^Dx\,\sqrt{-g}\,\Big[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\Big]\,, \tag{3.1}$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the vector field strength. The action is invariant under U(1) gauge transformations, $A_{\mu}(x) \to A_{\mu}(x) + \partial_{\mu}\Lambda(x)$, for some arbitrary function $\Lambda(x)$. Quantization of this free theory allows one to work out the two-point functions necessary for perturbative loop computations in interacting theories containing massless vector fields.

Canonical quantization of the photon, where all the components of the vector potential are treated on equal footing, is based on the canonical formulation in the so-called *multiplier gauges* (also known as *average gauges* and sometimes referred to as covariant gauges). We begin by consider how the generally covariant gauge (1.1) is implemented in the classical theory. This serves to transparently define canonical quantization, which is summarized in the concluding part of this section.

3.1 Gauge-invariant photon

Our starting point is the canonical formulation of the gauge-invariant system (3.1), in which we decompose the indices into spatial and temporal ones, and plug in the de Sitter metric,

$$S[A_\mu] = \int d^D x \, a^{D-4} \biggl[\frac{1}{2} F_{0i} F_{0i} - \frac{1}{4} F_{ij} F_{ij} \biggr] . \tag{3.2}$$

Henceforth in all expressions with decomposed indices we write them lowered, and adopt the convention that all the repeated spatial indices are summed over. There is little advantage in trying to maintain some sense of manifest covariance in the canonical formulation when the time direction plays a preferred role.

The promotion of the first time derivatives of vector field components to independent fields, $F_{0i} \to V_i$, $\partial_0 A_0 \to V_0$, and the introduction of the accompanying Lagrange multipliers Π_0 and Π_i that ensure on-shell equivalence, defines an intermediate first-order action,

$$\begin{aligned} \mathcal{S}[A_0, V_0, \Pi_0, A_i, V_i, \Pi_i] = \int d^D x \, \biggl\{ a^{D-4} \biggl[\frac{1}{2} V_i V_i - \frac{1}{4} F_{ij} F_{ij} \biggr] + \Pi_i \bigl(F_{0i} - V_i \bigr) \\ + \Pi_0 \bigl(\partial_0 A_0 - V_0 \bigr) \biggr\} , \end{aligned} \tag{3.3}$$

sometimes referred to as the extended action. Solving for as many velocity fields as possible on-shell, which in this case means the spatial components,[4]

$$V_i \approx \overline{V}_i = a^{4-D} \Pi_i \, , \tag{3.4}$$

and plugging the solutions back into the action (3.3) then defines the canonical action,

$$\begin{aligned} \mathscr{S}[A_0, \Pi_0, A_i, \Pi_i, \ell] &\equiv \mathcal{S}[A_0, V_0 \to \ell, \Pi_0, A_i, \overline{V}_i, \Pi_i] \\ &= \int d^D x \, \Bigl[\Pi_0 \partial_0 A_0 + \Pi_i \partial_0 A_i - \mathscr{H} - \ell \, \Psi_1 \Bigr] , \end{aligned} \tag{3.5}$$

where,

$$\mathscr{H} = \frac{a^{4-D}}{2} \Pi_i \Pi_i + \Pi_i \partial_i A_0 + \frac{a^{D-4}}{4} F_{ij} F_{ij} \, , \tag{3.6}$$

is the canonical Hamiltonian density, and where we have relabeled $V_0 \to \ell$ to emphasize that in the canonical action ℓ is a Lagrange multiplier responsible for generating the primary constraint,

$$\Psi_1 = \Pi_0 \approx 0 \, , \tag{3.7}$$

[4]We use the Dirac notation $\approx$ to denote weak (on-shell) equalities that are valid at the level of equations of motion, as opposed to $=$ denoting strong (off-shell) equalities that are valid at the level of the action.

which in turn generates a secondary constraint on-shell,

$$\partial_0 \Psi_1 \approx \partial_i \Pi_i \equiv \Psi_2 \approx 0\,. \tag{3.8}$$

These two form a complete set of first-class constraints, $\{\Psi_1, \Psi_2\} = 0$. The Poisson brackets of the canonical variables[5] follow from the symplectic part of the canonical action (3.5),

$$\{A_0(\eta,\vec{x}), \Pi_0(\eta,\vec{x}\,')\} = \delta^{D-1}(\vec{x}-\vec{x}\,')\,, \qquad \{A_i(\eta,\vec{x}), \Pi_j(\eta,\vec{x}\,')\} = \delta_{ij}\delta^{D-1}(\vec{x}-\vec{x}\,')\,. \tag{3.9}$$

The dynamical equations descending from the canonical action do not fix the Lagrange multiplier ℓ, which can be chosen arbitrarily. This property is the canonical formulation equivalent of the more familiar invariance under local transformations that the configuration space action (3.1) possesses.

3.2 Gauge-fixed photon

One particularly convenient way to fix the gauge is the multiplier gauge, where we by hand fix the Lagrange multiplier in the canonical action (3.5) to be a function of canonical variables, which here we choose as,

$$\ell \to \overline{\ell} = -\frac{\xi a^{4-D}}{2}\Pi_0 + \partial_i A_i - (D-2)\mathcal{H}A_0\,, \tag{3.10}$$

with ξ an arbitrary real parameter. This leads to the gauge-fixed action,

$$\mathscr{S}_\star[A_0, \Pi_0, A_i, \Pi_i] \equiv \mathscr{S}[A_0, \Pi_0, A_i, \Pi_i, \overline{\ell}\,] = \int d^Dx \left[\Pi_0 \partial_0 A_0 + \Pi_i \partial_0 A_i - \mathscr{H}_\star\right], \tag{3.11}$$

where the gauge-fixed Hamiltonian is,

$$\mathscr{H}_\star = \frac{a^{4-D}}{2}\Pi_i\Pi_i - \frac{a^{4-D}\xi}{2}\Pi_0\Pi_0 + \Pi_i\partial_i A_0 + \Pi_0 \partial_i A_i - (D-2)\mathcal{H}\Pi_0 A_0 + \frac{a^{D-4}}{4}F_{ij}F_{ij}\,. \tag{3.12}$$

This gauge fixed canonical action now uniquely defines the gauge fixed dynamics, but it no longer encodes the first-class constraints (3.7) and (3.8). We have to require them as *subsidiary conditions,*

$$\Psi_1 = \Pi_0 \approx 0\,, \qquad \Psi_2 = \partial_i \Pi_i \approx 0\,, \tag{3.13}$$

in addition to the gauge fixed action. These are preserved if they are demanded on the initial time hypersurface.[6] Thus we split the description of the system into the dynamics described by the gauge-fixed action (3.11), and kinematics given by the subsidiary conditions (3.13). This structure is crucial when quantizing the system.

[5]Sometimes the canonical momenta are defined with upper indices so as to have Poisson brackets in a seemingly covariant form. We find little use for notation, which clearly does not reintroduce manifest covariance in the canonical formulation, and can be misleading at times.

[6]This property is guaranteed by the equations of motion the constraints satisfy,

$$\partial_0 \Psi_1 = \Psi_2 + (D-2)\mathcal{H}\Psi_1\,, \qquad \partial_0\Psi_2 = \nabla^2 \Psi_1\,.$$

The utility of the particular choice (3.10) for the multiplier is revealed once we derive the gauge fixed Lagrangian action, that takes the form,

$$S_\star[A_\mu] = S[A_\mu] + S_{\rm gf}[A_\mu] \,, \tag{3.14}$$

where the gauge-fixing term,

$$S_{\rm gf}[A_\mu] = \int d^Dx \, \sqrt{-g} \left[-\frac{1}{2\xi} \big(g^{\mu\nu} \nabla_\mu A_\nu \big)^2 \right] , \tag{3.15}$$

is precisely the general covariant gauge-fixing term 1.1. The two subsidiary conditions substituting for first-class constraints in the Lagrangian formalism take the form,

$$\nabla^\mu A_\mu \approx 0 \,, \qquad\qquad \partial_i F_{0i} \approx 0 \,. \tag{3.16}$$

Implementing these at the initial value surface guarantees they are conserved for all times.

3.3 Quantized photon

The gauge fixed photon of the preceding section is readily quantized in the Heisenberg picture. The dynamics is quantized by applying standard rules of canonical quantization to the gauge fixed canonical action. Canonical variables are promoted to field operators, and their Poisson brackets (3.9) are promoted to commutators,

$$\big[\hat{A}_0(\eta,\vec{x}), \hat{\Pi}_0(\eta,\vec{x}^{\,\prime}) \big] = i\delta^{D-1}(\vec{x} - \vec{x}^{\,\prime}), \quad \big[\hat{A}_i(\eta,\vec{x}), \hat{\Pi}_j(\eta,\vec{x}^{\,\prime}) \big] = \delta_{ij} \, i\delta^{D-1}(\vec{x} - \vec{x}^{\,\prime}), \tag{3.17}$$

where we set $\hbar = 1$, and the canonical operator equations of motion,

$$\partial_0 \hat{A}_0 = -\xi a^{4-D} \hat{\Pi}_0 + \partial_i \hat{A}_i - (D-2)\mathcal{H} \hat{A}_0 \,, \tag{3.18}$$

$$\partial_0 \hat{\Pi}_0 = \partial_i \hat{\Pi}_i + (D-2)\mathcal{H} \hat{\Pi}_0 \,, \tag{3.19}$$

$$\partial_0 \hat{A}_i = a^{4-D} \hat{\Pi}_i + \partial_i \hat{A}_0 \,, \tag{3.20}$$

$$\partial_0 \hat{\Pi}_i = \partial_i \hat{\Pi}_0 + a^{D-4} \partial_j \hat{F}_{ji} \,, \tag{3.21}$$

take the same form as in the classical theory.

The constraints (3.13) of the classical theory require a more careful quantization. It is straightforward to associate Hermitian operators associated to classical first-class constraints,

$$\hat{\Psi}_1 = \hat{\Pi}_0 \,, \qquad\qquad \hat{\Psi}_2 = \partial_i \hat{\Pi}_i \,, \tag{3.22}$$

but the constraints clearly cannot be implemented at the operator level, as they would contradict canonical commutation relations (3.17). This is of no consequence, since the actual property that needs to be satisfied in the quantized theory is that all correlators of the Hermitian constraint operators (3.22) have to vanish. In particular, for Gaussian states we consider in this paper, this means that all the two-point functions of Hermitian constraints must vanish,

$$\langle \Omega | \hat{\Pi}_0(x) \hat{\Pi}_0(x') | \Omega \rangle = 0 \,, \quad \langle \Omega | \partial_i \hat{\Pi}_i(x) \hat{\Pi}_0(x') | \Omega \rangle = 0 \,, \quad \langle \Omega | \partial_i \hat{\Pi}_i(x) \partial_j' \hat{\Pi}_j(x') | \Omega \rangle = 0 \,. \tag{3.23}$$

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This implies that it is appropriate to implement the constraints as conditions on states. The form that such quantum *subsidiary conditions* take is not immediately obvious, since requiring Hermitian constraint operators (3.22) to annihilate the physical state would again contradict canonical commutation relations (3.17). The consistent way of implementing the constraints as subsidiary conditions on states is to require a non-Hermitian *subsidiary constraint operator* $\hat{K}(\vec{x})$ to annihilate the *ket* state vector, and its conjugate to annihilate the *bra* state vector,

$$\hat{K}(\vec{x})|\Omega\rangle = 0\,, \qquad\qquad \langle\Omega|\hat{K}^\dagger(\vec{x}) = 0\,. \tag{3.24}$$

This non-Hermitian constraint operator is constructed as an invertible linear combination of Hermitian constraints,

$$\hat{K}(\vec{x}) = \int d^{D-1}x' \Big[f_1(\eta, \vec{x}-\vec{x}\,')\hat{\Psi}_1(\eta,\vec{x}\,') + f_2(\eta,\vec{x}-\vec{x}\,')\hat{\Psi}_2(\eta,\vec{x}\,') \Big] \,. \tag{3.25}$$

This guarantees that physical conditions (3.23) are always satisfied in a way consistent with canonical commutation relations (3.17). The structure of the subsidiary constraint operator becomes clearer when considered in momentum space in section 4.4.

Since definition (3.25) is invertible, it implies that we can decompose the two Hermitian constraints in terms of the non-Hermitian subsidiary constraint operator and its conjugate. This will be used in section 5 to simplify expressions. It suffices here to define this inverse symbolically as,

$$\hat{\Psi}_1(x) = \hat{K}_1(x) + \hat{K}_1^\dagger(x)\,, \qquad\qquad \hat{\Psi}_2(x) = \hat{K}_2(x) + \hat{K}_2^\dagger(x)\,, \tag{3.26}$$

where $\hat{K}_1(x)$ and $\hat{K}_2(x)$ are linear in $\hat{K}(\vec{x})$, and their conjugates are linear in $\hat{K}^\dagger(\vec{x})$.

4 Field operator dynamics

The free theory defines the propagators for perturbative computations in the interacting theory. The dynamics of the free theory is determined by solving the equations of motion of the field operators, which we do in this section. The most convenient way to solve for field operators is in spatial comoving momentum space. After decomposing field operators into transverse and scalar sectors, we solve for the mode functions in momentum space, and determine the accompanying commutation relations. The mode functions are expressed in terms of CTBD scalar mode functions (2.10) and their parametric derivatives. Our solutions are consistent with previously obtained solutions for photon mode functions in $D\!=\!4$ [75, 76], and for massive Stueckelberg vector field mode functions [63]. The section concludes with a discussion of momentum space non-Hermitian constraint operators defining the subsidiary condition on the space of states.

4.1 Decomposition of field operators

It is convenient to split the spatial components of field operators,

$$\hat{A}_i = \hat{A}_i^{\scriptscriptstyle T} + \hat{A}_i^{\scriptscriptstyle L}\,, \qquad\qquad \hat{\Pi}_i = \hat{\Pi}_i^{\scriptscriptstyle T} + \hat{\Pi}_i^{\scriptscriptstyle L}\,, \tag{4.1}$$

into transverse parts, $\hat{A}^T_i = \mathbb{P}^T_{ij}\hat{A}_j$ and $\hat{\Pi}^T_i = \mathbb{P}^T_{ij}\hat{\Pi}_j$, and longitudinal parts, $\hat{A}^L_i = \mathbb{P}^L_{ij}\hat{A}_j$ and $\hat{\Pi}^L_i = \mathbb{P}^L_{ij}\hat{\Pi}_j$, defined via the projection operators,

$$\mathbb{P}^T_{ij} = \delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}\,, \qquad\qquad \mathbb{P}^L_{ij} = \frac{\partial_i\partial_j}{\nabla^2}\,, \tag{4.2}$$

where $\nabla^2 = \partial_i\partial_i$ is the Laplace operator. The projectors are both idempotent, $\mathbb{P}^T_{ij}\mathbb{P}^T_{jk} = \mathbb{P}^T_{ik}$ and $\mathbb{P}^L_{ij}\mathbb{P}^L_{jk} = \mathbb{P}^L_{ik}$, and mutually orthogonal, $\mathbb{P}^T_{ij}\mathbb{P}^L_{jk} = \mathbb{P}^L_{ij}\mathbb{P}^T_{jk} = 0$. Thus $\partial_i A^T_i = \partial_i \Pi^T_i = 0$, and $\partial_i A_i = \partial_i A^L_i$, $\partial_i \Pi_i = \partial_i \Pi^L_i$. Given the isotropy of spatially flat cosmological spaces, it is most convenient to examine operator dynamics in the comoving momentum space,

$$\hat{A}_0(\eta,\vec{x}) = a^{\frac{2-D}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}}\, \hat{\mathcal{A}}_0(\eta,\vec{k})\,, \tag{4.3a}$$

$$\hat{\Pi}_0(\eta,\vec{x}) = a^{\frac{D-2}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}}\, \hat{\pi}_0(\eta,\vec{k})\,, \tag{4.3b}$$

$$\hat{A}^L_i(\eta,\vec{x}) = a^{\frac{2-D}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}} \frac{(-i)k_i}{k}\, \hat{\mathcal{A}}_L(\eta,\vec{k})\,, \tag{4.3c}$$

$$\hat{\Pi}^L_i(\eta,\vec{x}) = a^{\frac{D-2}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}} \frac{(-i)k_i}{k}\, \hat{\pi}_L(\eta,\vec{k})\,, \tag{4.3d}$$

$$\hat{A}^T_i(\eta,\vec{x}) = a^{\frac{4-D}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}} \sum_{\sigma=1}^{D-2} \varepsilon_i(\sigma,\vec{k})\, \hat{\mathcal{A}}_{T,\sigma}(\eta,\vec{k})\,, \tag{4.3e}$$

$$\hat{\Pi}^T_i(\eta,\vec{x}) = a^{\frac{D-4}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}} \sum_{\sigma=1}^{D-2} \varepsilon_i(\sigma,\vec{k})\, \hat{\pi}_{T,\sigma}(\eta,\vec{k})\,, \tag{4.3f}$$

where momentum space Hermitian operators behave as $\hat{\mathcal{O}}^\dagger(\vec{k}) = \hat{\mathcal{O}}(-\vec{k})$. Here we have introduced transverse polarization tensors with the following properties,

$$k_i\, \varepsilon_i(\sigma,\vec{k}) = 0\,, \qquad\qquad \big[\varepsilon_i(\sigma,\vec{k})\big]^* = \varepsilon_i(\sigma,-\vec{k})\,, \tag{4.4a}$$

$$\varepsilon^*_i(\sigma,\vec{k})\, \varepsilon_i(\sigma',\vec{k}) = \delta_{\sigma\sigma'}\,, \qquad \sum_{\sigma=1}^{D-2} \varepsilon^*_i(\sigma,\vec{k})\varepsilon_j(\sigma,\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2}\,, \tag{4.4b}$$

where $k = \|\vec{k}\|$. The nonvanishing canonical commutators of the momentum space field operators are,

$$\big[\hat{\mathcal{A}}_0(\eta,\vec{k}), \hat{\pi}_0(\eta,\vec{k}')\big] = \big[\hat{\mathcal{A}}_L(\eta,\vec{k}), \hat{\pi}_L(\eta,\vec{k}')\big] = i\delta^{D-1}(\vec{k}+\vec{k}')\,, \tag{4.5a}$$

$$\big[\hat{\mathcal{A}}_{T,\sigma}(\eta,\vec{k}), \hat{\pi}_{T,\sigma'}(\eta,\vec{k}')\big] = \delta_{\sigma\sigma'}\, i\delta^{D-1}(\vec{k}+\vec{k}')\,. \tag{4.5b}$$

The equations of motion for the transverse sector read,

$$\partial_0 \hat{\mathcal{A}}_{T,\sigma} = \hat{\pi}_{T,\sigma} + \frac{1}{2}(D-4)\mathcal{H}\hat{\mathcal{A}}_{T,\sigma}\,, \tag{4.6}$$

$$\partial_0 \hat{\pi}_{T,\sigma} = -k^2\hat{\mathcal{A}}_{T,\sigma} - \frac{1}{2}(D-4)\mathcal{H}\hat{\pi}_{T,\sigma}\,, \tag{4.7}$$

while the ones for the scalar sector are,

$$\partial_0\hat{\mathcal{A}}_0 = -\xi a^2\hat{\pi}_0 + k\hat{\mathcal{A}}_L - \frac{1}{2}(D-2)\mathcal{H}\hat{\mathcal{A}}_0\,, \tag{4.8}$$

$$\partial_0\hat{\pi}_0 = k\hat{\pi}_L + \frac{1}{2}(D-2)\mathcal{H}\hat{\pi}_0\,, \tag{4.9}$$

$$\partial_0\hat{\mathcal{A}}_L = a^2\hat{\pi}_L - k\hat{\mathcal{A}}_0 + \frac{1}{2}(D-2)\mathcal{H}\hat{\mathcal{A}}_L\,, \tag{4.10}$$

$$\partial_0\hat{\pi}_L = -k\hat{\pi}_0 - \frac{1}{2}(D-2)\mathcal{H}\hat{\pi}_L\,. \tag{4.11}$$

4.2 Transverse sector

Equations (4.6) and (4.7) combine into a single second order one,

$$\Big[\partial_0^2 + k^2 - \Big(\nu^2 - \frac{1}{4}\Big)\mathcal{H}^2\Big]\hat{\mathcal{A}}_{T,\sigma} = 0\,, \tag{4.12}$$

$$\hat{\pi}_{T,\sigma} = \Big[\partial_0 - \Big(\nu + \frac{1}{2}\Big)\mathcal{H}\Big]\hat{\mathcal{A}}_{T,\sigma}\,, \tag{4.13}$$

where we introduce and henceforth make frequent use of the parameter,

$$\nu = \frac{D-3}{2}\,. \tag{4.14}$$

It follows from comparing with the mode equation (2.8), and from recurrence relations (2.12) that the equations are solved by,

$$\hat{\mathcal{A}}_{T,\sigma}(\eta,k) = U_\nu(\eta,k)\,\hat{b}_T(\sigma,\vec{k}) + U_\nu^*(\eta,k)\,\hat{b}_T^\dagger(\sigma,-\vec{k})\,, \tag{4.15}$$

$$\hat{\pi}_{T,\sigma}(\eta,k) = -ikU_{\nu-1}(\eta,k)\,\hat{b}_T(\sigma,\vec{k}) + ikU_{\nu-1}^*(\eta,k)\,\hat{b}_T^\dagger(\sigma,-\vec{k})\,. \tag{4.16}$$

From the canonical commutation relations (4.5) it then follows that the initial condition operators are the creation and annihilation operators $\hat{b}_T^\dagger(\sigma,\vec{k})$ and $\hat{b}_T(\sigma,\vec{k})$, respectively, whose non-vanishing commutators are,

$$[\hat{b}_T(\sigma,\vec{k}),\hat{b}_T^\dagger(\sigma',\vec{k}')] = \delta_{\sigma\sigma'}\,\delta^{D-1}(\vec{k}-\vec{k}')\,. \tag{4.17}$$

4.3 Scalar sector

The two equations (4.9) and (4.11) of the scalar sector decouple, and combine into a single second order one,

$$\Big[\partial_0^2 + k^2 - \Big(\nu^2 - \frac{1}{4}\Big)\mathcal{H}^2\Big]\hat{\pi}_L = 0\,, \tag{4.18}$$

$$\hat{\pi}_0 = -\frac{1}{k}\Big[\partial_0 + \Big(\nu + \frac{1}{2}\Big)\mathcal{H}\Big]\hat{\pi}_L\,, \tag{4.19}$$

where ν is given in (4.14), so that solutions are readily read off from (2.8)–(2.10) and (2.12),

$$\hat{\pi}_L(\eta,\vec{k}) = kU_\nu(\eta,k)\hat{b}_P(\vec{k}) + kU_\nu^*(\eta,k)\hat{b}_P^\dagger(-\vec{k})\,, \tag{4.20}$$

$$\hat{\pi}_0(\eta,\vec{k}) = ikU_{\nu+1}(\eta,k)\hat{b}_P(\vec{k}) - ikU_{\nu+1}^*(\eta,k)\hat{b}_P^\dagger(-\vec{k})\,. \tag{4.21}$$

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The remaining equations (4.8) and (4.10) now combine into a second order one sourced by homogeneous solutions (4.20) and (4.21),

$$\Big[\partial_0^2 + k^2 - \Big(\nu^2 - \frac{1}{4}\Big)\mathcal{H}^2\Big]\hat{\mathcal{A}}_0 = (1-\xi)\frac{\mathcal{H}^2 k}{H^2}\hat{\pi}_L - 2\xi\frac{\mathcal{H}^3}{H^2}\hat{\pi}_0 \,, \tag{4.22}$$

$$\hat{\mathcal{A}}_L = \frac{1}{k}\Big[\partial_0 + \Big(\nu+\frac{1}{2}\Big)\mathcal{H}\Big]\hat{\mathcal{A}}_0 + \frac{\xi\mathcal{H}^2}{H^2 k}\hat{\pi}_0 \,. \tag{4.23}$$

The homogeneous parts of the solution are the same as before,

$$\begin{aligned}\hat{\mathcal{A}}_0(\eta,\vec{k}) &= U_\nu(\eta,k)\hat{b}_H(\vec{k}) + U_\nu^*(\eta,k)\hat{b}_H^\dagger(-\vec{k}) \\ &\quad + v_0(\eta,k)\hat{b}_P(\vec{k}) + v_0^*(\eta,k)\hat{b}_P^\dagger(-\vec{k}) \,,\end{aligned} \tag{4.24}$$

$$\begin{aligned}\hat{\mathcal{A}}_L(\eta,\vec{k}) &= -iU_{\nu+1}(\eta,k)\hat{b}_H(\vec{k}) + iU_{\nu+1}^*(\eta,k)\hat{b}_H^\dagger(-\vec{k}) \\ &\quad - iv_L(\eta,k)\hat{b}_P(\vec{k}) + iv_L^*(\eta,k)\hat{b}_P^\dagger(-\vec{k}) \,,\end{aligned} \tag{4.25}$$

while the particular mode functions v_0 and v_L satisfy,

$$\Big[\partial_0^2 + k^2 - \Big(\nu^2 - \frac{1}{4}\Big)\mathcal{H}^2\Big]v_0 = \frac{-i\xi\mathcal{H}^3 k}{(\nu+1)H^2}\Big[2(\nu+1)U_{\nu+1} - \frac{ik}{\mathcal{H}}U_\nu\Big] + \Big(1-\frac{\xi}{\xi_s}\Big)\frac{\mathcal{H}^2 k^2}{H^2}U_\nu \,, \tag{4.26}$$

$$v_L = \frac{i}{k}\Big[\partial_0 + \Big(\nu+\frac{1}{2}\Big)\mathcal{H}\Big]v_0 - \frac{\xi\mathcal{H}^2}{H^2}U_{\nu+1} \,, \tag{4.27}$$

where we introduced what we refer to as the *simple covariant gauge*,

$$\xi_s = \frac{\nu+1}{\nu} = \frac{D-1}{D-3} \,, \tag{4.28}$$

which in the flat space limit corresponds to the D-dimensional Fried-Yennie gauge [77, 78]. The photon two-point function takes the simplest form in this gauge, as already noted in [62]. The solutions for the particular mode functions are readily found from identities (2.14) and (2.15),

$$v_0 = \frac{-i\xi k}{2(\nu+1)H}\Big[\frac{\mathcal{H}}{H}U_{\nu+1} - U_\nu\Big] - \Big(1-\frac{\xi}{\xi_s}\Big)\frac{ik}{2H}\Big[\frac{ik}{\nu H}\frac{\partial U_\nu}{\partial\nu} + U_\nu\Big] \,, \tag{4.29}$$

$$v_L = \frac{-i\xi k}{2(\nu+1)H}\Big[\frac{\mathcal{H}}{H}U_\nu - U_{\nu+1}\Big] - \Big(1-\frac{\xi}{\xi_s}\Big)\frac{ik}{2H}\Big[\frac{ik}{\nu H}\frac{\partial U_{\nu+1}}{\partial\nu} + \frac{\mathcal{H}}{\nu H}U_\nu + U_{\nu+1}\Big] \,, \tag{4.30}$$

where the homogeneous parts were fixed by requiring the Wronskian-like relation,

$$\mathrm{Re}\Big(v_0 U_{\nu+1}^* + v_L U_\nu^*\Big) = 0 \,, \tag{4.31}$$

and a regular flat space limit,

$$v_0 \xrightarrow{H\to 0} \frac{1}{4}\Big[(1+\xi) + 2(1-\xi)ik(\eta-\eta_0)\Big]\frac{e^{-ik(\eta-\eta_0)}}{\sqrt{2k}} \,, \tag{4.32a}$$

$$v_L \xrightarrow{H\to 0} \frac{1}{4}\Big[-(1+\xi) + 2(1-\xi)ik(\eta-\eta_0)\Big]\frac{e^{-ik(\eta-\eta_0)}}{\sqrt{2k}} \,. \tag{4.32b}$$

Using the Wronskian-like relation above we can invert the momentum space field operators for the initial conditions operators, and infer their commutation relations from the canonical ones (4.5), with the non-vanishing ones being,

$$[\hat{b}_H(\vec{k}), \hat{b}_P^\dagger(\vec{k}\,')] = [\hat{b}_P(\vec{k}), \hat{b}_H^\dagger(\vec{k}\,')] = -\delta^{D-1}(\vec{k}-\vec{k}\,') \,. \tag{4.33}$$

These commutators are not canonical, in the sense that they are not the ones of creation/annihilation operators. It is actually advantageous to keep them in this form for the purpose of constructing the space of states, as it simplifies matters and makes the structure more transparent [65].

4.4 Subsidiary condition

Physical states have to satisfy the subsidiary condition (3.24) descending from the first-class constraints. For this purpose we need to construct the non-Hermitian constraint operator (3.25). It takes a considerably simpler, diagonal form in momentum space,

$$\hat{K}(\vec{x}) = a^{\frac{D-2}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}}\, \hat{\mathcal{K}}(\vec{k}) \,, \qquad \hat{K}^\dagger(\vec{x}) = a^{\frac{D-2}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}\, e^{i\vec{k}\cdot\vec{x}}\, \hat{\mathcal{K}}^\dagger(-\vec{k}) \,, \tag{4.34}$$

where it is a simple linear combination of the momentum space Hermitian constraints,

$$\hat{\mathcal{K}}(\vec{k}) = c_0(\eta,\vec{k})\hat{\pi}_0(\eta,\vec{k}) + c_L(\eta,\vec{k})\hat{\pi}_L(\eta,\vec{k}) \,, \tag{4.35}$$

so that the equivalent subsidiary conditions are,

$$\hat{\mathcal{K}}(\vec{k})|\Omega\rangle = 0 \,, \qquad \langle\Omega|\hat{\mathcal{K}}^\dagger(\vec{k}) = 0 \,, \qquad \forall\vec{k} \,. \tag{4.36}$$

The non-Hermitian constraint is time-independent, which implies the following first order equations of motion for the coefficient functions,

$$\partial_0 c_0 - k c_L + \frac{1}{2}(D-2)\mathcal{H}c_0 = 0 \,, \qquad \partial_0 c_L + k c_0 - \frac{1}{2}(D-2)\mathcal{H}c_L = 0 \,. \tag{4.37}$$

These equations combine into a second order one,

$$\left[\partial_0^2 + k^2 - \left(\nu^2 - \frac{1}{4}\right)\right] c_0 = 0 \,, \tag{4.38}$$

$$c_L = \frac{1}{k}\left[\partial_0 + \left(\nu + \frac{1}{2}\right)\mathcal{H}\right] c_0 \,, \tag{4.39}$$

whose general solution, according to (2.8)–(2.10), can be written as,

$$c_0(\eta,\vec{k}) = \alpha(\vec{k}) U_\nu(\eta,k) + \beta(\vec{k}) U_\nu^*(\eta,k) \,, \tag{4.40}$$

$$c_L(\eta,\vec{k}) = -i\alpha(\vec{k}) U_{\nu+1}(\eta,k) + i\beta(\vec{k}) U_{\nu+1}^*(\eta,k) \,, \tag{4.41}$$

where $\alpha(\vec{k})$ and $\beta(\vec{k})$ are some free functions of momenta with immaterial normalization. We parametrize these coefficient functions so that they are consistent with homogeneity and isotropy[7] of cosmological spacetimes [65], such that the non-Hermitian constraint reads,[8]

$$\hat{\mathcal{K}}(\vec{k}) = \mathrm{e}^{i\theta(\vec{k})}\Big(\mathrm{e}^{-i\varphi(k)}\mathrm{ch}[\rho(k)]\,\hat{b}_P(\vec{k}) + \mathrm{e}^{i\varphi(k)}\mathrm{sh}[\rho(k)]\,\hat{b}_P^\dagger(-\vec{k})\Big)\,, \tag{4.42}$$

where $\theta(\vec{k})$, $\varphi(k)$, and $\rho(k)$ are real functions of momentum. It is also convenient to define an accompanying operator,

$$\hat{\mathcal{B}}(\vec{k}) = e^{i\theta(\vec{k})}\Big(e^{-i\varphi(k)}\mathrm{ch}[\rho(k)]\,\hat{b}_H(\vec{k}) + e^{i\varphi(k)}\mathrm{sh}[\rho(k)]\,\hat{b}_H^\dagger(-\vec{k})\Big)\,, \tag{4.43}$$

that preserves commutation relations (4.33),

$$[\hat{\mathcal{K}}(\vec{k}), \hat{\mathcal{B}}^\dagger(\vec{k}')] = [\hat{\mathcal{B}}(\vec{k}), \hat{\mathcal{K}}^\dagger(\vec{k}')] = -\delta^{D-1}(\vec{k}-\vec{k}')\,, \tag{4.44}$$

so that in this sense these definitions can be seen as Bogolyubov transformations between $\hat{b}_P$ and $\hat{b}_H$, and $\hat{\mathcal{K}}$ and $\hat{\mathcal{B}}$. We have to construct the space of states on which the algebra in (4.44) is represented, such that it contains the kernel of $\hat{\mathcal{K}}$ as a subspace. The following section is devoted to this task.

5 Constructing the state

The quantization of the dynamics outlined in the preceding two sections needs to be supplemented by the construction of the space of states. Its construction in quantum field theory is necessarily intertwined with the possible existence of global symmetries that the vacuum is sought to respect. This section presents a detailed analysis of both physical and gauge-fixed de Sitter symmetries of the photon in the general covariant gauge. Physical symmetries are related to the gauge-invariant action (3.1) and are a property of physical polarizations only, while gauge-fixed symmetries are related to the gauge-fixed action (3.14) and are a property of the physical and gauge sectors alike. We define the vacuum state in momentum space mode-by-mode, by requiring the state (i) to produce vanishing expectation values of all the physical de Sitter generators, and (ii) to be an eigenstate of all the gauge-fixed de Sitter generators, with vanishing eigenvalues. This is a rather involved section and for readers not interested in the technical details of the procedure we provide a brief summary of the results in the concluding part 5.4 of this section.

5.1 De Sitter symmetries

There are $\frac{1}{2}D(D{+}1)$ isometries of the Poincaré patch of de Sitter space (see e.g. [79]). These are the coordinate transformations that leave the de Sitter line element invariant. There is

[7] A more general parametrization is considered in [65],

$$\hat{\mathcal{K}}(\vec{k}) = \mathcal{N}(\vec{k})\mathrm{e}^{i\theta(\vec{k})}\Big(\mathrm{e}^{-i\varphi(\vec{k})}\mathrm{ch}[\rho(\vec{k})]\,\hat{b}_P(\vec{k}) + \mathrm{e}^{i\varphi(-\vec{k})}\mathrm{sh}[\rho(-\vec{k})]\,\hat{b}_P^\dagger(-\vec{k})\Big)\,,$$

with $\mathcal{N}(\vec{k}) = \big(\mathrm{ch}[\rho(\vec{k})]\mathrm{ch}[\rho(-\vec{k})] - \mathrm{sh}[\rho(\vec{k})]\mathrm{sh}\rho[(-\vec{k})]\big)^{-\frac{1}{2}}$, that reduces to the one in (4.42) when homogeneity and isotropy are imposed.

[8] It is also possible to exchange the roles of $\hat{b}_P(\vec{k})$ and $\hat{b}_P^\dagger(\vec{k})$. The entire analysis can be repeated. However, that choice would not be consistent with the requirements imposed in section 5.

the same number of corresponding active vector field transformations that are symmetries of both the gauge invariant (3.1) and the gauge fixed action (3.14). The isometries and the associated infinitesimal active field transformations are:

- *Spatial translations* — $(D-1)$ transformations,

$$\eta \longrightarrow \eta\,, \qquad x_i \longrightarrow x_i + \alpha_i\,, \tag{5.1}$$

with the associated active field transformation,

$$A_\mu(x) \longrightarrow A_\mu(x) - \alpha_i \partial_i A_\mu(x)\,. \tag{5.2}$$

- *Spatial rotations* — $\frac{1}{2}(D-1)(D-2)$ transformations,

$$\eta \longrightarrow \eta\,, \qquad x_i \longrightarrow x_i + 2\omega_{ij}x_j\,, \qquad (\omega_{ij} = -\omega_{ji})\,, \tag{5.3}$$

with the associated active field transformation,

$$A_\mu(x) \longrightarrow A_\mu(x) + 2\omega_{ij}x_i\partial_j A_\mu(x) + 2\delta^i_\mu \omega_{ij} A_j(x)\,. \tag{5.4}$$

- *Dilation* — one transformation,

$$\eta \longrightarrow \eta - \frac{\delta}{a}\,, \qquad x_i \longrightarrow (1+H\delta)x_i\,, \tag{5.5}$$

with the associated active field transformation,

$$A_\mu(x) \longrightarrow A_\mu(x) + \frac{\delta}{a}\partial_0 A_\mu(x) - \delta H x_i \partial_i A_\mu(x) - \delta H A_\mu(x)\,. \tag{5.6}$$

- *Spatial special conformal transformations* — $(D-1)$ transformations,

$$\eta \longrightarrow \eta + \frac{\theta_j x_j}{a}\,, \qquad x_i \longrightarrow x_i - H\theta_j x_j x_i - \frac{H\theta_i}{2}\left[\frac{1}{H^2}\left(\frac{1}{a^2}-1\right) - x_j x_j\right], \tag{5.7}$$

with the associated active field transformation,

$$A_\mu(x) \longrightarrow A_\mu(x) - \frac{\theta_i x_i}{a} F_{0\mu}(x) + \left\{ H\theta_j x_j x_i + \frac{H\theta_i}{2}\left[\frac{1}{H^2}\left(\frac{1}{a^2}-1\right) - x_j x_j\right]\right\}\partial_i A_\mu(x)\,. \tag{5.8}$$

Note that the transformations above are written in such a way that in the Minkowski limit, $H \to 0$, they reduce to the Poincaré transformations. In particular, spatial translations and rotations do not change, while dilation (5.5) reduces to time translation, and spatial special conformal transformation (5.7) reduces to Lorentz boosts.

In gauge theories one should distinguish between two sets of global symmetry generators — one set descending from the gauge-invariant action (3.2) that accounts for the physical symmetries of the system, and another set descending from the gauge-fixed action (3.14) that accounts for the symmetries of dynamics of the gauge-fixed system. The two sets in general do not have to contain the same number of generators since gauge-fixing is allowed to break global symmetries of the gauge-invariant system. In the case at hand both the gauge-invariant action and the gauge-fixed action are invatiant under all the de Sitter symmetry transformations above. Correspondingly, the two sets of generators we compute next are of the same dimensionality.

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5.2 Physical symmetries

Physical symmetries of the photon in de Sitter are accounted for by the symmetry generators descending from the gauge-invariant action (3.1),

$$P_i = \int d^{D-1}x \left(-\Pi_j F_{ij} \right), \tag{5.9}$$

$$M_{ij} = \int d^{D-1}x \left(2x_{[i} F_{j]k} \Pi_k \right), \tag{5.10}$$

$$Q = \int d^{D-1}x \left(\frac{a^{3-D}}{2a} \Pi_i \Pi_i + \frac{a^{D-5}}{4} F_{ij} F_{ij} - H x_i F_{ij} \Pi_j \right), \tag{5.11}$$

$$\begin{aligned} K_i = \int d^{D-1}x \bigg[& -\frac{x_i}{2a} \left(a^{4-D} \Pi_j \Pi_j + \frac{a^{D-4}}{2} F_{jk} F_{jk} \right) + H x_i x_j F_{jk} \Pi_k \\ & + \frac{1}{2H} \left(\frac{1}{a^2} - 1 - H^2 x_k x_k \right) F_{ij} \Pi_j \bigg], \end{aligned} \tag{5.12}$$

and they satisfy the de Sitter algebra weakly (up to constraints),

$$\begin{aligned} &\{P_i, P_j\} \approx 0, && \{P_i, M_{ij}\} \approx 2P_{[i}\delta_{j]k}, && \{M_{ij}, M_{kl}\} \approx 4\delta_{i][k} M_{l][j}, \\ &\{Q, P_i\} \approx H P_i, && \{Q, M_{ij}\} \approx 0, && \{Q, Q\} \approx 0, \\ &\{K_i, P_j\} \approx -\delta_{ij} Q - H M_{ij}, && \{K_i, M_{jk}\} \approx 2\delta_{i[j} K_{k]}, && \\ &\{K_i, Q\} \approx -P_i + H K_i, && \{K_i, K_j\} \approx M_{ij}. && \end{aligned} \tag{5.13}$$

In the flat space limit, $H \to 0$, this algebra reduces to the Poincaré algebra. Quantizing the generators requires one to address operator ordering. Namely, every term containing a classical constraint should be ordered so that the non-Hermitian subsidiary constraint operator is on the right of the product, and that its conjugate is on the left. This is accomplished by writing them as (cf. eqs. (A.15a), (A.16a), (A.17a) and (A.18a) from appendix A.3),

$$\hat{P}_i = \hat{P}_i^T + \int d^{D-1}x \left(\hat{K}_2^\dagger \hat{A}_i^T + \hat{A}_i^T \hat{K}_2 \right), \tag{5.14}$$

$$\hat{M}_{ij} = \hat{M}_{ij}^T + \int d^{D-1}x \left(2\hat{K}_2^\dagger x_{[i} \hat{A}_{j]}^T + 2x_{[i} \hat{A}_{j]}^T \hat{K}_2 \right), \tag{5.15}$$

$$\hat{Q} = \hat{Q}^T + \int d^{D-1}x \left(-\frac{a^{3-D}}{2} \hat{\Psi}_2 \nabla^{-2} \hat{\Psi}_2 - H \hat{K}_2^\dagger x_i \hat{A}_i^T - H x_i \hat{A}_i^T \hat{K}_2 \right), \tag{5.16}$$

$$\begin{aligned} \hat{K}_i = \hat{K}_i^T + \int d^{D-1}x \bigg[& \frac{a^{D-3}}{2} x_i \hat{\Psi}_2 \frac{1}{\nabla^2} \hat{\Psi}_2 + a^{3-D} \left(\hat{K}_2^\dagger \frac{1}{\nabla^2} \Pi_i^T + \hat{\Pi}_i^T \frac{1}{\nabla^2} \hat{K}_2 \right) \\ & + (D-3) H \left(\hat{K}_2^\dagger \frac{1}{\nabla^2} \hat{A}_i^T + \hat{A}_i^T \frac{1}{\nabla^2} \hat{K}_2 \right) + H x_i x_j \left(\hat{K}_2^\dagger \hat{A}_j^T + \hat{A}_j^T \hat{K}_2 \right) \\ & + \frac{1}{2H} \left(\frac{1}{a^2} - 1 - H^2 x_j x_j \right) \left(\hat{K}_2^\dagger \hat{A}_i^T + \hat{A}_i^T \hat{K}_2 \right) \bigg], \end{aligned} \tag{5.17}$$

where the purely transverse parts are computed to be (cf. eqs. (A.15c), (A.16c), (A.17c) and (A.18c) from appendix A.3),

$$\hat{P}_i^T = \int d^{D-1}k \, k_i \, \hat{\mathcal{E}}_j^\dagger(\vec{k}) \hat{\mathcal{E}}_j(\vec{k}) = \int d^{D-1}k \, k_i \sum_{\sigma=1}^{D-2} \hat{b}_T^\dagger(\sigma, \vec{k}) \, \hat{b}_T(\sigma, \vec{k}), \tag{5.18}$$

$$\hat{M}_{ij}^T = \int d^{D-1}k \left[\hat{\mathcal{E}}_k^\dagger(\vec{k}) \left(i k_i \frac{\partial}{\partial k_j} - i k_j \frac{\partial}{\partial k_i} \right) \hat{\mathcal{E}}_k(\vec{k}) + 2\hat{\mathcal{E}}_{[i}^\dagger(\vec{k}) \hat{\mathcal{E}}_{j]}(\vec{k}) \right], \tag{5.19}$$

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$$\hat{Q}^{T} = \int d^{D-1}k \left[k\hat{\mathcal{E}}_i^\dagger(\vec{k})\hat{\mathcal{E}}_i(\vec{k}) - \frac{iHk_j}{2}\left(\frac{\partial \hat{\mathcal{E}}_i^\dagger(\vec{k})}{\partial k_j}\hat{\mathcal{E}}_i(\vec{k}) - \hat{\mathcal{E}}_i^\dagger(\vec{k})\frac{\partial \hat{\mathcal{E}}_i(\vec{k})}{\partial k_j}\right)\right], \tag{5.20}$$

$$\begin{aligned}\hat{K}_i^{T} = \int d^{D-1}k \Bigg[& \frac{ik}{2}\left(\frac{\partial \hat{\mathcal{E}}_j^\dagger(\vec{k})}{\partial k_i}\hat{\mathcal{E}}_j(\vec{k}) - \hat{\mathcal{E}}_j^\dagger(\vec{k})\frac{\partial \hat{\mathcal{E}}_j(\vec{k})}{\partial k_i}\right) - \frac{(D-3)^2 H k_i}{8k^2}\hat{\mathcal{E}}_j^\dagger(\vec{k})\hat{\mathcal{E}}_j(\vec{k}) \\ & + \frac{H}{2}\left(\frac{\partial \hat{\mathcal{E}}_i^\dagger(\vec{k})}{\partial k_j}\hat{\mathcal{E}}_j(\vec{k}) + \hat{\mathcal{E}}_j^\dagger(\vec{k})\frac{\partial \hat{\mathcal{E}}_i(\vec{k})}{\partial k_j} - \hat{\mathcal{E}}_i^\dagger(\vec{k})\frac{\partial \hat{\mathcal{E}}_j(\vec{k})}{\partial k_j} - \frac{\partial \hat{\mathcal{E}}_j^\dagger(\vec{k})}{\partial k_j}\hat{\mathcal{E}}_i(\vec{k})\right) \\ & + 2k_i \frac{\partial \hat{\mathcal{E}}_k^\dagger(\vec{k})}{\partial k_j}\frac{\partial \hat{\mathcal{E}}_k(\vec{k})}{\partial k_j} - 2k_j \frac{\partial \hat{\mathcal{E}}_k^\dagger(\vec{k})}{\partial k_j}\frac{\partial \hat{\mathcal{E}}_k(\vec{k})}{\partial k_i} - 2k_j \frac{\partial \hat{\mathcal{E}}_k^\dagger(\vec{k})}{\partial k_i}\frac{\partial \hat{\mathcal{E}}_k(\vec{k})}{\partial k_j}\Bigg], \end{aligned} \tag{5.21}$$

with the expressions written compactly using a short-hand notation,

$$\hat{\mathcal{E}}_i(\vec{k}) = \sum_{\sigma=1}^{D-2} \varepsilon_i(\sigma,\vec{k})\, \hat{b}_{T}(\sigma,\vec{k}), \tag{5.22}$$

and where products have been normal ordered in the usual manner. Since none of the generators (5.14)–(5.17) depends on either A_0 or A_i^{L}, they automatically qualify as observables since all of them commute with the non-Hermitian constraint,

$$[\hat{\mathcal{K}}(\vec{k}), \hat{G}_I] = 0, \qquad \hat{G}_I = \{\hat{P}_i, \hat{M}_{ij}, \hat{Q}, \hat{K}_i\}, \tag{5.23}$$

for any choice of the parameters in (4.42).

Firstly, we want to specify a *physically* de Sitter-invariant state. This is not implemented as in theories without constraints by considering eigenstates of the generators, but rather by computing expectation values of polynomials of generators and showing that the values factorize. In other words, this means,

$$\langle \Omega | \mathscr{P}(\hat{P}_i, \hat{M}_{ij}, \hat{Q}, \hat{K}_i) | \Omega \rangle = \mathscr{P}(\overline{P}_i, \overline{M}_{ij}, \overline{Q}, \overline{K}_i), \tag{5.24}$$

where the arguments on the right-hand-side are expectation values of generators,

$$\overline{P}_i = \langle \Omega | \hat{P}_i | \Omega \rangle, \quad \overline{M}_{ij} = \langle \Omega | \hat{M}_{ij} | \Omega \rangle, \quad \overline{Q} = \langle \Omega | \hat{Q} | \Omega \rangle, \quad \overline{K}_i = \langle \Omega | \hat{K}_i | \Omega \rangle. \tag{5.25}$$

Making this consistent with the generator algebra (5.13) then implies that all of the expectation values of generators (5.25) have to vanish,

$$\overline{P}_i = 0, \qquad \overline{M}_{ij} = 0, \qquad \overline{Q} = 0, \qquad \overline{K}_i = 0. \tag{5.26}$$

Furthermore, only the fully transverse parts of the generators in (5.24) participate, as any parts containing scalar sector operators annihilate either the *ket* or the *bra* state,

$$\langle \Omega | \mathscr{P}(\hat{P}_i, \hat{M}_{ij}, \hat{Q}, \hat{K}_i) | \Omega \rangle = \langle \Omega | \mathscr{P}(\hat{P}_i^{T}, \hat{M}_{ij}^{T}, \hat{Q}^{T}, \hat{K}_i^{T}) | \Omega \rangle. \tag{5.27}$$

This means that requiring the state to be physically de Sitter symmetric puts conditions on the transverse sector only. Clearly (5.26) is satisfied by the state defined as,

$$\hat{b}_{T}(\sigma,\vec{k})|\Omega\rangle = 0, \qquad \forall \vec{k}, \sigma. \tag{5.28}$$

The rest of the Hilbert space for the transverse sector is then defined in the usual manner as a Fock space, by acting on $|\Omega\rangle$ with the transverse creation operators. It should be noted that de Sitter symmetries do not uniquely fix the state to satisfy (5.28). In fact they allow for a two-parameter class of states analogous to α-vacua of the scalar [80, 81]. Nevertheless, we consider (5.28) to define our state, as it minimizes energy and, in addition, it is the unique state with a regular flat space limit.

It is important to note that the gauge sector of the space of states is not fixed by the requirement of the physical de Sitter invariance, and can in fact be chosen freely without interfering with it. However, it is advantageous to fix the gauge sector to also respect de Sitter symmetries. This is discussed and implemented in the following subsection.

5.3 Gauge-fixed symmetries

The dynamics of the gauge-fixed theory is determined by the gauge-fixed action (3.14). We have chosen the gauge-fixing term to respect general covariance, and hence it respects de Sitter symmetries of section 5.1. Therefore, the full gauge-fixed action respects de Sitter symmetries, and accordingly it engenders the associated symmetry generators,

$$P_i^\star = \int d^{D-1}x \left(-\Pi_j \partial_i A_j - \Pi_0 \partial_i A_0 \right) , \tag{5.29}$$

$$M_{ij}^\star = \int d^{D-1}x \left(2\Pi_0 x_{[i}\partial_{j]} A_0 + 2\Pi_k x_{[i}\partial_{j]} A_k + 2\Pi_{[i} A_{j]} \right) , \tag{5.30}$$

$$Q^\star = \int d^{D-1}x \left\{ \frac{1}{a} \left[\frac{a^{4-D}}{2} \Pi_i \Pi_i - \frac{a^{4-D}\xi}{2} \Pi_0^2 - A_0 \partial_i \Pi_i + \Pi_0 \partial_i A_i - (D{-}2)\mathcal{H} \Pi_0 A_0 \right.\right.$$
$$\left.\left. + \frac{a^{D-4}}{4} F_{ij} F_{ij} \right] - H \Big[\Pi_j (1{+}x_i\partial_i) A_j + \Pi_0 (1{+}x_i\partial_i) A_0 \Big] \right\} , \tag{5.31}$$

$$K_i^\star = \int d^{D-1}x \left\{ -\frac{x_i}{a} \left[\frac{a^{4-D}}{2} \Pi_j \Pi_j - \frac{a^{4-D}\xi}{2} \Pi_0^2 - A_j \partial_j \Pi_0 - A_0 \partial_j \Pi_j - (D{-}2)\mathcal{H} \Pi_0 A_0 \right.\right.$$
$$\left. + \frac{a^{D-4}}{4} F_{jk} F_{jk} \right] + H x_j A_j \Pi_i + H \left[x_i x_k \partial_k A_j + \frac{1}{2H^2} \left(\frac{1}{a^2} {-} 1 {-} H^2 x_k x_k \right) \partial_i A_j \right.$$
$$\left.\left. + x_i A_j - x_j A_i \right] \Pi_j + H \left[x_i x_j \partial_j A_0 + \frac{1}{2H^2} \left(\frac{1}{a^2} {-} 1 {-} H^2 x_j x_j \right) \partial_i A_0 + x_i A_0 \right] \Pi_0 \right\} . \tag{5.32}$$

These generators satisfy de Sitter algebra (5.13) strongly, since the gauge-fixed action does not possess local symmetries any more. Quantizing these generators requires us to address operator ordering. The purely transverse part will be the same as for the gauge-invariant generators, while the remainder warrants a closer look. We need to order operators so that all the non-Hermitian constraints are on the right in the products, and their conjugates on the left. This is a tedious task, and here we report the results (cf. eqs. (A.15b), (A.16b), (A.17b) and (A.18b) from appendix A.3),

$$\hat{P}_i^\star = \hat{P}_i^{\scriptscriptstyle T} + \int d^{D-1}x \left(\hat{K}_2^\dagger \hat{A}_i^{\scriptscriptstyle L} + \hat{A}_i^{\scriptscriptstyle L} \hat{K}_2 - \hat{K}_1^\dagger \partial_i \hat{A}_0 - \partial_i \hat{A}_0 \hat{K}_1 \right) , \tag{5.33}$$

$$\hat{M}_{ij}^\star = \hat{M}_{ij}^{\scriptscriptstyle T} + \int d^{D-1}x \left(2\hat{K}_1^\dagger x_{[i}\partial_{j]} \hat{A}_0 + 2x_{[i}\partial_{j]} \hat{A}_0 \hat{K}_1 - 2\hat{K}_2^\dagger x_{[i} \hat{A}_{j]}^{\scriptscriptstyle L} - 2x_{[i} \hat{A}_{j]}^{\scriptscriptstyle L} \hat{K}_2 \right) , \tag{5.34}$$

$$
\begin{aligned}
\hat{Q}^\star = \hat{Q}^T + \frac{1}{a}\int d^{D-1}x \Big[& -\frac{a^{4-D}}{2}\hat{\Psi}_2\nabla^{-2}\hat{\Psi}_2 - \frac{a^{4-D}}{2}\xi\hat{\Psi}_1\hat{\Psi}_1 - \hat{K}_2^\dagger\hat{A}_0 - \hat{A}_0\hat{K}_2 \\
& + \hat{K}_1^\dagger\partial_i\hat{A}_i^L + \partial_i\hat{A}_i^L\hat{K}_1 + \mathcal{H}x_i\Big(\hat{K}_2^\dagger\hat{A}_i^L + \hat{A}_i^L\hat{K}_2 + \partial_i\hat{K}_1^\dagger\hat{A}_0 + \hat{A}_0\partial_i\hat{K}_1\Big)\Big]\,,
\end{aligned} \tag{5.35}
$$

$$
\begin{aligned}
\hat{K}_i^\star = \hat{K}_i^T + \frac{1}{a}\int d^{D-1}x \Big[& a^{4-D}\hat{K}_2^\dagger\nabla^{-2}\hat{\Pi}_i^T + a^{4-D}\hat{\Pi}_i^T\nabla^{-2}\hat{K}_2 - \hat{K}_1^\dagger\hat{A}_i^T - \hat{A}_i^T\hat{K}_1 \\
& + (D-3)\mathcal{H}\Big(\hat{K}_2^\dagger\nabla^{-2}\hat{A}_i^T + \hat{A}_i^T\nabla^{-2}\hat{K}_2\Big) + \frac{a^{4-D}}{2}x_i\Big(\hat{\Psi}_2\nabla^{-2}\hat{\Psi}_2 + \xi\hat{\Psi}_1\hat{\Psi}_1\Big) \\
& + x_i\Big(\partial_j\hat{K}_1^\dagger\hat{A}_j^L + \hat{A}_j^L\partial_j\hat{K}_1 + \hat{K}_2^\dagger\hat{A}_0 + \hat{A}_0\hat{K}_2 + (D-1)\mathcal{H}\Big(\hat{K}_1^\dagger\hat{A}_0 + \hat{A}_0\hat{K}_1\Big)\Big) \\
& + \frac{a}{2H}\Big(\frac{1}{a^2} - 1 - H^2x_jx_j\Big)\Big(\hat{K}_1^\dagger\partial_i\hat{A}_0 + \partial_i\hat{A}_0\hat{K}_1 - \hat{K}_2^\dagger\hat{A}_i^L - \hat{A}_i^L\hat{K}_2\Big) \\
& + \mathcal{H}x_ix_j\Big(\hat{K}_1^\dagger\partial_j\hat{A}_0 + \partial_j\hat{A}_0\hat{K}_1 - \hat{K}_2^\dagger\hat{A}_j^L - \hat{A}_j^L\hat{K}_2\Big)\Big]\,,
\end{aligned} \tag{5.36}
$$

where the transverse parts are given in (5.18)–(5.21). The next step is requiring that all the generators commute with $\hat{\mathcal{K}}$ (modulo $\hat{\mathcal{K}}$ itself). This requirement has two interpretations. The first is that it makes the gauge-fixed generators into observables; the second is that it puts a condition on the non-Hermitian constraint operator to be consistent with de Sitter symmetries. For spatial translations and spatial rotations this is accomplished by taking coefficients as in (4.42),

$$
[\hat{\mathcal{K}}(\vec{k}), \hat{P}_i^\star] = k_i\hat{\mathcal{K}}(\vec{k})\,, \qquad [\hat{\mathcal{K}}(\vec{k}), \hat{M}_{ij}^\star] = ie^{i\theta(\vec{k})}\Big(k_i\frac{\partial}{\partial k_j} - k_j\frac{\partial}{\partial k_i}\Big)\Big(e^{-i\theta(\vec{k})}\hat{\mathcal{K}}(\vec{k})\Big)\,. \tag{5.37}
$$

For dilations this is no longer the case,

$$
\begin{aligned}
[\hat{\mathcal{K}}(\vec{k}), \hat{Q}^\star] = & \Big(1 + 2\,\mathrm{sh}^2[\rho(k)]\Big)\Big(k - Hk\frac{\partial\varphi(k)}{\partial k}\Big)\hat{\mathcal{K}}(\vec{k}) + e^{i\theta(\vec{k})}iHk_i\frac{\partial}{\partial k_i}\Big(e^{-i\theta(\vec{k})}\hat{\mathcal{K}}(\vec{k})\Big) \\
& + \frac{(D+1)iH}{2}\hat{\mathcal{K}}(\vec{k}) - e^{i\theta(\vec{k}) + i\theta(-\vec{k})}\Big[\mathrm{sh}[2\rho(k)]\Big(k - Hk\frac{\partial\varphi(k)}{\partial k}\Big) + iHk\frac{\partial\rho(k)}{\partial k}\Big]\hat{\mathcal{K}}^\dagger(-\vec{k}),
\end{aligned} \tag{5.38}
$$

as the conjugate of the non-Hermitian constraint operator appears on the right hand side. Requiring that its coefficient vanishes selects two options:

$$
\texttt{option 1:} \qquad \rho(k) = 0\,, \tag{5.39}
$$

$$
\texttt{option 2:} \qquad \rho(k) = \rho = \text{const.}\,, \qquad \varphi(k) = \frac{k}{H}\,. \tag{5.40}
$$

The second option does not have a well defined flat space limit, as the Hubble rate appears in the denominator of the phase. This option would lead to the α-vacuum-equivalent for photons. We do not consider this option in the remainder of the paper. Rather, we consider only the first option (5.39), which has a regular flat space limit, consistent with the choice (5.28) for the transverse sector. Taking $\rho = 0$ allows one to also absorb phase $\varphi(k)$ into $\theta(\vec{k})$ since it becomes redundant; effectively we are taking $\varphi(k) = 0$. Moreover, we can dispense with the irrelevant phase altogether,[9] and simply take,

$$\hat{\mathcal{K}}(\vec{k}) = \hat{b}_{P}(\vec{k}) \,, \qquad\qquad \hat{\mathcal{B}}(\vec{k}) = \hat{b}_{H}(\vec{k}) \,. \tag{5.41}$$

Computing the remaining commutator with the generator of spatial special conformal transformations,

$$\begin{aligned}[\hat{\mathcal{K}}(\vec{k}), \hat{K}_i^\star] = {}& Hk_j \frac{\partial^2 \hat{\mathcal{K}}(\vec{k})}{\partial k_i \partial k_j} - \frac{Hk_i}{2} \frac{\partial^2 \hat{\mathcal{K}}(\vec{k})}{\partial k_j \partial k_j} - ik \biggl(1 + \frac{(D\!+\!1)iH}{2k} \biggr) \frac{\partial \hat{\mathcal{K}}(\vec{k})}{\partial k_i} \\ & - \frac{ik_i}{2k} \biggl(3 - \frac{(D\!-\!3)(D\!+\!1)iH}{4k} \biggr) \hat{\mathcal{K}}(\vec{k}) \,, \end{aligned} \tag{5.42}$$

reveals it to be consistent with that requirement. The generators take the form,

$$\hat{P}_i^\star = \hat{P}_i^T + \int d^{D-1}k \, k_i \Bigl(\hat{\mathcal{K}}^\dagger(\vec{k}) \hat{\mathcal{B}}(\vec{k}) + \hat{\mathcal{B}}^\dagger(\vec{k}) \hat{\mathcal{K}}(\vec{k}) \Bigr) \,, \tag{5.43}$$

$$\begin{aligned}\hat{M}_{ij}^\star = \hat{M}_{ij}^T - \int d^{D-1}k \, \biggl[& \hat{\mathcal{B}}^\dagger(\vec{k}) \biggl(ik_i \frac{\partial}{\partial k_j} - ik_j \frac{\partial}{\partial k_i} \biggr) \hat{\mathcal{K}}(\vec{k}) \\ & + \hat{\mathcal{K}}^\dagger(\vec{k}) \biggl(ik_i \frac{\partial}{\partial k_j} - ik_j \frac{\partial}{\partial k_i} \biggr) \hat{\mathcal{B}}(\vec{k}) \biggr] \,, \end{aligned} \tag{5.44}$$

$$\begin{aligned}\hat{Q}^\star = \hat{Q}^T - \int d^{D-1}k \, \biggl[& k \biggl(1 \!+\! \frac{(D\!+\!1)iH}{2k} \biggr) \hat{\mathcal{B}}^\dagger(\vec{k}) \hat{\mathcal{K}}(\vec{k}) + k \biggl(1 \!-\! \frac{(D\!+\!1)iH}{2k} \biggr) \hat{\mathcal{K}}^\dagger(\vec{k}) \hat{\mathcal{B}}(\vec{k}) \\ & - \frac{(1\!-\!\xi)k}{2} \hat{\mathcal{K}}^\dagger(\vec{k}) \hat{\mathcal{K}}(\vec{k}) + iHk_i \biggl(\hat{\mathcal{B}}^\dagger(\vec{k}) \frac{\partial \hat{\mathcal{K}}(\vec{k})}{\partial k_i} - \frac{\partial \hat{\mathcal{K}}^\dagger(\vec{k})}{\partial k_i} \hat{\mathcal{B}}(\vec{k}) \biggr) \biggr] \,, \end{aligned} \tag{5.45}$$

$$\begin{aligned}\hat{K}_i^\star = \hat{K}_i^T + \int d^{D-1}k \, \biggl[& i\hat{\mathcal{K}}^\dagger(\vec{k}) \hat{\mathcal{E}}_i(\vec{k}) - i\hat{\mathcal{E}}_i^\dagger(\vec{k}) \hat{\mathcal{K}}(\vec{k}) \\ & + \frac{i(1\!-\!\xi)}{4} k \biggl(\frac{\partial \hat{\mathcal{K}}^\dagger(\vec{k})}{\partial k_i} \hat{\mathcal{K}}(\vec{k}) - \hat{\mathcal{K}}^\dagger(\vec{k}) \frac{\partial \hat{\mathcal{K}}(\vec{k})}{\partial k_i} \biggr) + ik \biggl(1 \!+\! \frac{(D\!+\!1)iH}{2k} \biggr) \hat{\mathcal{B}}^\dagger(\vec{k}) \frac{\partial \hat{\mathcal{K}}(\vec{k})}{\partial k_i} \\ & - ik \biggl(1 \!-\! \frac{(D\!+\!1)iH}{2k} \biggr) \frac{\partial \hat{\mathcal{K}}^\dagger(\vec{k})}{\partial k_i} \hat{\mathcal{B}}(\vec{k}) + \frac{ik_i}{2k} \biggl(3 \!-\! \frac{i(D\!-\!3)(D\!+\!1)H}{4k} \biggr) \hat{\mathcal{B}}^\dagger(\vec{k}) \hat{\mathcal{K}}(\vec{k}) \\ & - \frac{ik_i}{2k} \biggl(3 \!+\! \frac{i(D\!-\!3)(D\!+\!1)H}{4k} \biggr) \hat{\mathcal{K}}^\dagger(\vec{k}) \hat{\mathcal{B}}(\vec{k}) - Hk_j \biggl(\frac{\partial^2 \hat{\mathcal{K}}^\dagger(\vec{k})}{\partial k_i \partial k_j} \hat{\mathcal{B}}(\vec{k}) + \hat{\mathcal{B}}^\dagger(\vec{k}) \frac{\partial^2 \hat{\mathcal{K}}(\vec{k})}{\partial k_i \partial k_j} \biggr) \\ & + \frac{Hk_i}{2} \biggl(\frac{\partial^2 \hat{\mathcal{K}}^\dagger(\vec{k})}{\partial k_j \partial k_j} \hat{\mathcal{B}}(\vec{k}) + \hat{\mathcal{B}}^\dagger(\vec{k}) \frac{\partial^2 \hat{\mathcal{K}}(\vec{k})}{\partial k_j \partial k_j} \biggr) \biggr] \,. \end{aligned} \tag{5.46}$$

Having ensured that the subsidiary non-Hermitian constraint operator is consistent with de Sitter invariant dynamics, we may finally define a de Sitter invariant state. This is implemented by requiring the state to be annihilated by all the gauge-fixed de Sitter symmetry generators (5.43)–(5.46). Given the subsidiary condition (4.36), this is possible only if the state satisfies,

$$\hat{\mathcal{B}}(\vec{k}) |\Omega\rangle = 0 \,, \qquad \langle \Omega | \hat{\mathcal{B}}^\dagger(\vec{k}) = 0 \,, \qquad \forall \vec{k} \,, \tag{5.47}$$

[9] The phase $\theta(\vec{k})$ does not affect the propagator. An interested reader can easily reintroduce it to expressions (5.41)–(5.46) by taking $\hat{\mathcal{K}}(\vec{k}) \!\to\! e^{-i\theta(\vec{k})} \hat{\mathcal{K}}(\vec{k})$ and $\hat{\mathcal{B}}(\vec{k}) \!\to\! e^{-i\theta(\vec{k})} \hat{\mathcal{B}}(\vec{k})$.

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Together with (5.39) these completely define the state, which serves as the vacuum for constructing the indefinite inner product space of states [65].

5.4 Summary: momentum space de Sitter invariant state

The most important result of this rather technical section is the momentum space construction of a de Sitter invariant quantum state. Such construction is considerably simpler than in position space since the spatial momentum modes do not couple at the linear level. Here we briefly summarize the results relevant for subsequent sections.

The physical properties of the system are described by the gauge-invariant action (3.1), which is invariant under de Sitter transformations from section 5.1. The associated generators (5.9)–(5.12) of these transformations are the conserved Noether charges that characterize the state. Requiring that all expectation values of these charges and their products vanish defines a class of physically de Sitter invariant states, and puts conditions on the transverse sector of the state only. The minimum energy state in this class is defined by,

$$\hat{b}_{T}(\sigma,\vec{k})|\Omega\rangle = 0\,, \qquad\qquad \forall\vec{k},\sigma\,, \tag{5.48}$$

which is the state we consider as the vacuum. The remainder of the transverse space of states is then constructed as a Fock space. The gauge fixed dynamics is given by the gauge fixed action (3.14), which is chosen to preserve all the de Sitter symmetries. Accordingly, there are conserved Noether charges (5.29)–(5.32) that generate the gauge fixed symmetries. Requiring that the full state, that includes both the transverse and scalar sectors, is de Sitter symmetric requires (i) that the non-Hermitian constraint operator (4.42) commutes with all the gauge fixed generators, which fixes $\hat{\mathcal{K}}(\vec{k})\!=\!\hat{b}_{P}(\vec{k})$, and (ii) that the state is an eigenstate of all the gauge fixed de Sitter generators mode-by-mode, which fixes,[10]

$$\hat{b}_{P}(\vec{k})|\Omega\rangle = 0\,, \qquad \hat{b}_{H}(\vec{k})|\Omega\rangle = 0\,, \qquad \forall\vec{k}\,. \tag{5.49}$$

The scalar sector space of states is then necessarily an indefinite inner product space, and is constructed by acting repeatedly with $\hat{b}_{P}^{\dagger}$ and $\hat{b}_{H}^{\dagger}$ on $|\Omega\rangle$. Conditions (5.48) and (5.49) fully define the state whose two-point functions we compute in the following section. The position space two-point function ultimately decides whether the state we had defined in momentum space makes physical sense. This attitude is informed by the case of the massless, minimally coupled scalar, for which the CTBD mode function (2.10) defines a de Sitter invariant state in momentum space, but whose respective two-point function in position space diverges in the infrared [81, 82], failing to describe a physically sensible state.

6 Two-point function

The basic building blocks of nonequilibrium perturbation theory for interacting vector fields in de Sitter are the various two-point functions determined in the free theory. In this section we first briefly introduce these two-point functions and the properties they must satisfy. We then proceed to compute the two-point functions by evaluating the integrals

[10] This is true up to the freedom in (5.40), defining the α-vacuum states of the scalar sector.

over the mode functions obtained in section 4. Rewriting the results in a covariant form reveals the existence of *one de Sitter breaking structure function*, which is the principal result of this work. The section concludes by examining particular limits of the two-point functions and comparing them with known results.

6.1 Generalities

The positive-frequency Wightman two-point function of the photon is defined as an expectation value of a product of two vector field operators,

$$i\big[{}_\mu^-\Delta_\nu^+\big](x;x') = \langle \Omega | \hat{A}_\mu(x) \hat{A}_\nu(x') | \Omega \rangle \,. \tag{6.1}$$

We compute this two-point function using the mode functions obtained in section 4. Its complex conjugate defines the negative-frequency Wightman function, $i\big[{}_\mu^+\Delta_\nu^-\big](x;x') = \big\{ i\big[{}_\mu^-\Delta_\mu^+\big](x;x') \big\}^*$, and the two together define the Feynman propagator,

$$i\big[{}_\mu^+\Delta_\nu^+\big](x;x') = \theta(\eta\!-\!\eta')\, i\big[{}_\mu^-\Delta_\nu^+\big](x;x') + \theta(\eta'\!-\!\eta)\, i\big[{}_\mu^+\Delta_\nu^-\big](x;x') \,. \tag{6.2}$$

and its conjugate, $i\big[{}_\mu^-\Delta_\nu^-\big](x;x') = \big\{ i\big[{}_\mu^+\Delta_\nu^+\big](x;x') \big\}^*$, known as the Dyson propagator.

The field operators in the definitions of the two-point functions above satisfy equations of motion (3.18)–(3.21), that can be written in a more familiar covariant form,

$$\mathcal{D}_\mu{}^\nu \, \hat{A}_\nu(x) = 0 \,, \qquad \mathcal{D}_{\mu\nu} = \nabla_\rho \nabla^\rho g_{\mu\nu} - \Big(1 - \frac{1}{\xi} \Big) \nabla_\mu \nabla_\nu - R_{\mu\nu} \,. \tag{6.3}$$

These equations of motion are inherited by the Wightman function (6.1),

$$\mathcal{D}_\mu{}^\rho \, i\big[{}_\rho^-\Delta_\nu^+\big](x;x') = 0 \,, \qquad \mathcal{D}'_\nu{}^\sigma \, i\big[{}_\mu^-\Delta_\sigma^+\big](x;x') = 0 \,, \tag{6.4}$$

while the Feynman propagator (6.2) picks up a delta function source on the account of the time-ordering in its definition and the canonical commutation relations (3.17),

$$\mathcal{D}_\mu{}^\rho \, i\big[{}_\rho^+\Delta_\nu^+\big](x;x') = g_{\mu\nu} \frac{i\delta^D(x\!-\!x')}{\sqrt{-g}} \,, \qquad \mathcal{D}'_\nu{}^\sigma \, i\big[{}_\mu^+\Delta_\sigma^+\big](x;x') = g_{\mu\nu} \frac{i\delta^D(x\!-\!x')}{\sqrt{-g}} \,, \tag{6.5}$$

The two-point functions we construct via the sum-over-modes ultimately satisfy these equations of motion.

In addition to satisfying the equations of motion, the photon two-point functions have to satisfy identities stemming from gauge invariance of the theory. These can be understood as the two-point functions of Hermitian constraints (3.23), which have to vanish according to the principles of canonical quantization. We can express them directly in terms of the two-point functions of vector potential fields since the two Hermitian constraints are given by $\nabla^\mu \hat{A}_\mu = \xi a^{2-D} \hat{\Pi}_0$ and $\partial_i \hat{F}_{0i} = a^{4-D} \partial_i \hat{\Pi}_i$. The subsidiary conditions thus take the following form for the Wightman function,

$$\nabla^\mu \nabla'^\nu i\big[{}_\mu^-\Delta_\nu^+\big](x;x') = 0 \,, \tag{6.6a}$$

$$\big(2 g^{ij} \delta^\mu_{[i} \partial_{0]} \partial_j \big) \nabla'^\nu i\big[{}_\mu^-\Delta_\nu^+\big](x;x') = 0 \,, \tag{6.6b}$$

$$\nabla^{\mu}\big(2g'^{kl}\delta^{\nu}_{[k}\partial'_{0]}\partial'_{l}\big)i\big[{}^{-}_{\mu}\Delta^{+}_{\nu}\big](x;x') = 0\,, \tag{6.6c}$$
$$\big(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_{j}\big)\big(2g'^{kl}\delta^{\nu}_{[k}\partial'_{0]}\partial'_{l}\big)i\big[{}^{-}_{\mu}\Delta^{+}_{\nu}\big](x;x') = 0\,. \tag{6.6d}$$

while for the Feynman propagator they also receive a local contribution due to the time-ordered product of operators in its definition,

$$\nabla^{\mu}\nabla'^{\nu}i\big[{}^{+}_{\mu}\Delta^{+}_{\nu}\big](x;x') = -\xi\,\frac{i\delta^{D}(x\!-\!x')}{\sqrt{-g}}\,, \tag{6.7a}$$
$$\big(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_{j}\big)\nabla'^{\nu}i\big[{}^{+}_{\mu}\Delta^{+}_{\nu}\big](x;x') = 0\,, \tag{6.7b}$$
$$\nabla^{\mu}\big(2g'^{kl}\delta^{\nu}_{[k}\partial'_{0]}\partial'_{l}\big)i\big[{}^{+}_{\mu}\Delta^{+}_{\nu}\big](x;x') = 0\,, \tag{6.7c}$$
$$\big(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_{j}\big)\big(2g'^{kl}\delta^{\nu}_{[k}\partial'_{0]}\partial'_{l}\big)i\big[{}^{+}_{\mu}\Delta^{+}_{\nu}\big](x;x') = -\nabla^{2}\frac{i\delta^{D}(x\!-\!x')}{\sqrt{-g}}\,. \tag{6.7d}$$

6.2 Sum over modes

Given the mode decompositions (4.3), and the solutions for the momentum space field operators (4.15), (4.24), and (4.25), and the definition of the physical state (5.28) and (4.36), components of the photon two-point function can be expressed as the following integrals over the mode functions,

$$\begin{aligned} &i\big[{}^{-}_{0}\Delta^{+}_{0}\big](x;x') \\ &\quad = (aa')^{-\frac{D-2}{2}}\!\int\!\frac{d^{D-1}k}{(2\pi)^{D-1}}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\Big[-U_{\nu}(\eta,k)v_{0}^{*}(\eta',k) - v_{0}(\eta,k)U_{\nu}^{*}(\eta',k)\Big]\,, \end{aligned} \tag{6.8}$$

$$\begin{aligned} &i\big[{}^{-}_{0}\Delta^{+}_{i}\big](x;x') \\ &\quad = (aa')^{-\frac{D-2}{2}}\!\int\!\frac{d^{D-1}k}{(2\pi)^{D-1}}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\,\frac{k_i}{k}\Big[U_{\nu}(\eta,k)v_{L}^{*}(\eta',k) + v_{0}(\eta,k)U_{\nu+1}^{*}(\eta',k)\Big]\,, \end{aligned} \tag{6.9}$$

$$\begin{aligned} i\big[{}^{-}_{i}\Delta^{+}_{j}\big](x;x') &= (aa')^{-\frac{D-4}{2}}\!\int\!\frac{d^{D-1}k}{(2\pi)^{D-1}}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)U_{\nu}(\eta,k)U_{\nu}(\eta',k) \\ &\quad -(aa')^{-\frac{D-2}{2}}\!\int\!\frac{d^{D-1}k}{(2\pi)^{D-1}}\,e^{i\vec{k}\cdot(\vec{x}-\vec{x}\,')}\,\frac{k_ik_j}{k^2}\Big[U_{\nu+1}(\eta,k)v_{L}^{*}(\eta',k) + v_{L}(\eta,k)U_{\nu+1}^{*}(\eta',k)\Big]\,, \end{aligned} \tag{6.10}$$

where we made use of eqs. (2.13) and (4.31), and where the $i\varepsilon$-prescriptions are implicit in the same way as for the scalar propagator (2.16). In the following two sections we solve these integrals using the solutions for the mode functions found in section 4. Plugging in the particular mode functions (4.29) and (4.30) into the sum over modes expressions (6.8)–(6.10), using recurrence relations for mode functions (2.12), and recognizing the scalar two-point functions (2.16) produces expressions,[11]

$$\begin{aligned} i\big[{}^{-}_{0}\Delta^{+}_{0}\big](x;x') &= \frac{-\xi}{2(\nu\!+\!1)H^2}\Big[\mathcal{H}\partial_0 + \mathcal{H}'\partial'_0 + (D\!-\!2)\big(\mathcal{H}^2\!+\!\mathcal{H}'^2\big)\Big]\mathcal{F}_{\nu}(y) \\ &\quad + \left(1\!-\!\frac{\xi}{\xi_s}\right)\frac{\nabla^2}{2\nu H^2}\,\frac{\partial}{\partial\nu}\mathcal{F}_{\nu}(y)\,, \end{aligned} \tag{6.11}$$

[11]Function $\mathcal{F}_{\nu+1}(y)$ appearing in expressions (6.11)–(6.13) is divergent in any dimension, but it is implied that all the derivatives acting on it, including the parametric one, are to be taken first, and only then the index set to $\nu\!=\!(D\!-\!3)/2$, which produces a well defined finite result.

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$$i\big[{}_0^{-}\Delta_i^{+}\big](x;x') = \frac{\partial_i'}{2\nu H^2}\Big[\mathcal{H}\mathcal{F}_{\nu+1}(y) - \mathcal{H}'\mathcal{F}_{\nu}(y)\Big] - \bigg(1-\frac{\xi}{\xi_s}\bigg)\frac{\partial_i'\partial_0}{2\nu H^2}\frac{\partial}{\partial\nu}\mathcal{F}_{\nu+1}(y)\,, \tag{6.12}$$

$$\begin{aligned} i\big[{}_i^{-}\Delta_j^{+}\big](x;x') = {}& \frac{\delta_{ij}\mathcal{H}\mathcal{H}'}{H^2}\mathcal{F}_{\nu}(y) - \bigg(1-\frac{\xi}{\xi_s}\bigg)\frac{\partial_i\partial_j'}{2\nu H^2}\frac{\partial}{\partial\nu}\mathcal{F}_{\nu+1}(y) \\ & - \frac{1}{2\nu H^2}\frac{\partial_i\partial_j'}{\nabla^2}\Big[\mathcal{H}\partial_0' + \mathcal{H}'\partial_0 + (D-1)\mathcal{H}\mathcal{H}'\Big]\mathcal{F}_{\nu}(y)\,. \end{aligned} \tag{6.13}$$

It is advantageous to rewrite these expressions by using the identity [83],

$$\Big[\mathcal{H}'\partial_0 + \mathcal{H}\partial_0' + (D-1)\mathcal{H}\mathcal{H}'\Big]f(y) = \frac{1}{2}\nabla^2 I[f](y)\,, \tag{6.14}$$

to eliminate the inverse Laplacian in the last component, and by acting some derivatives explicitly to obtain,

$$\begin{aligned} i\big[{}_0^{-}\Delta_0^{+}\big](x;x') = {}& \frac{1}{2\nu H^2}\frac{\xi}{\xi_s}\bigg\{(\mathcal{H}^2+\mathcal{H}'^2)\bigg[(2-y)\frac{\partial}{\partial y}-(D-2)\bigg] - 4\mathcal{H}\mathcal{H}'\frac{\partial}{\partial y}\bigg\}\mathcal{F}_{\nu}(y) \\ & + \frac{1}{2\nu H^2}\bigg(1-\frac{\xi}{\xi_s}\bigg)\bigg\{4(\mathcal{H}^2+\mathcal{H}'^2)\frac{\partial}{\partial y} - 2\mathcal{H}\mathcal{H}'\bigg[2(2-y)\frac{\partial}{\partial y}-(D-1)\bigg]\bigg\}\frac{\partial}{\partial y}\frac{\partial}{\partial\nu}\mathcal{F}_{\nu}(y)\,, \end{aligned} \tag{6.15}$$

$$\begin{aligned} i\big[{}_0^{-}\Delta_i^{+}\big](x;x') = {}& \frac{(\partial_i'y)}{2\nu H^2}\bigg[\mathcal{H}\frac{\partial}{\partial y}\mathcal{F}_{\nu+1}(y) - \mathcal{H}'\frac{\partial}{\partial y}\mathcal{F}_{\nu}(y)\bigg] \\ & - \frac{(\partial_i'y)}{2\nu H^2}\bigg(1-\frac{\xi}{\xi_s}\bigg)\bigg\{2\mathcal{H}'\frac{\partial}{\partial y} + \mathcal{H}\bigg[1-(2-y)\frac{\partial}{\partial y}\bigg]\bigg\}\frac{\partial}{\partial y}\frac{\partial}{\partial\nu}\mathcal{F}_{\nu+1}(y)\,, \end{aligned} \tag{6.16}$$

$$i\big[{}_i^{-}\Delta_j^{+}\big](x;x') = \frac{\delta_{ij}\mathcal{H}\mathcal{H}'}{H^2}\mathcal{F}_{\nu}(y) - \frac{\partial_i\partial_j'}{2\nu H^2}\bigg[\frac{1}{2}I[\mathcal{F}_{\nu}](y) + \bigg(1-\frac{\xi}{\xi_s}\bigg)\frac{\partial}{\partial\nu}\mathcal{F}_{\nu+1}(y)\bigg]\,. \tag{6.17}$$

In this way we have accomplished expressing the photon two-point function in terms of scalar two-point functions and their derivatives. However, this form of the two-point function is not practical, as it is not covariant, and moreover its properties are not obvious. In what follows we derive a more systematic, covariant representation.

6.3 Covariantization: de Sitter invariant ansatz

At this point there is nothing that motivates us to consider anything else than a de Sitter invariant ansatz,

$$i\big[{}_\mu^{-}\Delta_\nu^{+}\big]^{\rm dS}(x;x') = (\partial_\mu\partial_\nu'y)\,\mathcal{C}_1^{\rm dS}(y) + (\partial_\mu y)(\partial_\nu'y)\,\mathcal{C}_2^{\rm dS}(y)\,, \tag{6.18}$$

where $\mathcal{C}_1$ and $\mathcal{C}_2$ are the two scalar structure functions. We have to compare this expression to (6.15)–(6.17) to solve for the structure functions. Writing out the components of (6.18) gives,

$$i\big[{}_i^{-}\Delta_j^{+}\big]^{\rm dS}(x;x') = 2\delta_{ij}\mathcal{H}\mathcal{H}'\Big[I[\mathcal{C}_2^{\rm dS}] - \mathcal{C}_1^{\rm dS}\Big] + \partial_i\partial_j' I^2[\mathcal{C}_2^{\rm dS}]\,, \tag{6.19}$$

$$i\big[{}_0^{-}\Delta_i^{+}\big]^{\rm dS}(x;x') = (\partial_i'y)\Big\{\mathcal{H}\Big[\mathcal{C}_1^{\rm dS} - (2-y)\mathcal{C}_2^{\rm dS}\Big] + 2\mathcal{H}'\mathcal{C}_2^{\rm dS}\Big\}\,, \tag{6.20}$$

$$i\big[{}_0^{-}\Delta_0^{+}\big]^{\rm dS}(x;x') = 2(\mathcal{H}^2+\mathcal{H}'^2)\Big[\mathcal{C}_1^{\rm dS} - (2-y)\mathcal{C}_2^{\rm dS}\Big] + \mathcal{H}\mathcal{H}'\Big[-(2-y)\mathcal{C}_1^{\rm dS} + (8-4y+y^2)\mathcal{C}_2^{\rm dS}\Big]. \tag{6.21}$$

Comparing just the (ij) component immediately yields the solution for the two structure functions,

$$\mathcal{C}_1^{\rm dS}(y) = \frac{1}{2\nu H^2}\biggl[-\Bigl(\nu+\frac{1}{2}\Bigr)\mathcal{F}_\nu(y) - \Bigl(1-\frac{\xi}{\xi_s}\Bigr)\frac{\partial}{\partial y}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\biggr]\,, \tag{6.22}$$

$$\mathcal{C}_2^{\rm dS}(y) = \frac{1}{2\nu H^2}\biggl[-\frac{1}{2}\frac{\partial}{\partial y}\mathcal{F}_\nu(y) - \Bigl(1-\frac{\xi}{\xi_s}\Bigr)\frac{\partial^2}{\partial y^2}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\biggr]\,. \tag{6.23}$$

These two de Sitter invariant structure functions, that multiply the two de Sitter invariant tensor structures in (6.18), completely account for all the previous results on the photon two-point functions reported in the literature [60–63]. Nonetheless, it is important to show that this solution correctly reproduces the remaining components. Identity (2.25) implies this is true for the $(0i)$ component. However, a judicious comparison of the (00) components gives,

$$\begin{aligned} i\big[{}_0^{-}\Delta_0^{+}\big](x;x') - i\big[{}_0^{-}\Delta_0^{+}\big]^{\rm dS}(x;x') &= -\frac{\mathcal{H}\mathcal{H}'}{2\nu H^2}\frac{\xi}{\xi_s}\biggl[\Bigl(\nu-\frac{D-3}{2}\Bigr)\mathcal{F}_{\nu+1}(y)\biggr]_{\nu\to\frac{D-3}{2}} \\ &= \xi\times aa'\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{(D-1)\,\Gamma(\frac{D}{2})}\,, \end{aligned} \tag{6.24}$$

upon applying equation (2.20), recurrence identities (2.24) and (2.25), and the limit (2.29). The fact that the de Sitter invariant ansatz (6.18) does not reproduce all the components of the would-be de Sitter invariant propagator means that *there is no de Sitter invariant propagator, except in the limit $\xi\to 0$.*

6.4 Covariantization: de Sitter breaking ansatz

Having discovered that there cannot be a de Sitter invariant solution, we make the following, more general, *Ansatz*,[12]

$$\begin{aligned} i\big[{}_\mu^{-}\Delta_\nu^{+}\big](x;x') = (\partial_\mu\partial'_\nu y)\,\mathcal{C}_1(y,u) + (\partial_\mu y)(\partial'_\nu y)\,\mathcal{C}_2(y,u) \\ + \Bigl[(\partial_\mu y)(\partial'_\nu u)+(\partial_\mu u)(\partial'_\nu y)\Bigr]\mathcal{C}_3(y,u) + (\partial_\mu u)(\partial'_\nu u)\,\mathcal{C}_4(y,u)\,, \end{aligned} \tag{6.25}$$

where $u=\ln(aa')$. Writing out the components of the tensor structures in (6.25),

$$i\big[{}_i^{-}\Delta_j^{+}\big](x;x') = 2\delta_{ij}\mathcal{H}\mathcal{H}'\Bigl\{I[\mathcal{C}_2]-\mathcal{C}_1\Bigr\} + \partial_i\partial'_j I^2[\mathcal{C}_2]\,, \tag{6.26}$$

$$i\big[{}_0^{-}\Delta_i^{+}\big](x;x') = (\partial'_i y)\Bigl\{\mathcal{H}\Bigl[\mathcal{C}_1-(2-y)\mathcal{C}_2+\mathcal{C}_3\Bigr] + 2\mathcal{H}'\mathcal{C}_2\Bigr\}\,, \tag{6.27}$$

$$\begin{aligned} i\big[{}_0^{-}\Delta_0^{+}\big](x;x') =&\, 2(\mathcal{H}^2+\mathcal{H}'^2)\Bigl[\mathcal{C}_1-(2-y)\mathcal{C}_2+\mathcal{C}_3\Bigr] \\ &+ \mathcal{H}\mathcal{H}'\Bigl[-(2-y)\mathcal{C}_1+(8-4y+y^2)\mathcal{C}_2-2(2-y)\mathcal{C}_3+\mathcal{C}_4\Bigr]\,, \end{aligned} \tag{6.28}$$

[12]Strictly speaking we could have made a more general *Ansatz* including a dependence on $v=\ln(a/a')$, but this would lead to the same result.

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results in a form that is straightforward to compare to (6.15)–(6.17). Comparing first the (ij) components yields for the first two structure functions,

$$\mathcal{C}_1(y) = \frac{1}{2\nu H^2}\biggl[-\Bigl(\nu+\frac{1}{2}\Bigr)\mathcal{F}_\nu(y) - \Bigl(1-\frac{\xi}{\xi_s}\Bigr)\frac{\partial}{\partial y}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\biggr]\,, \tag{6.29}$$

$$\mathcal{C}_2(y) = \frac{1}{2\nu H^2}\biggl[-\frac{1}{2}\frac{\partial}{\partial y}\mathcal{F}_\nu(y) - \Bigl(1-\frac{\xi}{\xi_s}\Bigr)\frac{\partial^2}{\partial y^2}\frac{\partial}{\partial \nu}\mathcal{F}_{\nu+1}(y)\biggr]\,, \tag{6.30}$$

and comparing the $(0i)$ component yields,

$$\mathcal{C}_3 = 0\,. \tag{6.31}$$

Lastly, comparing the (00) components yields an unexpected result for the last structure function,

$$\mathcal{C}_4 = -\frac{1}{2\nu H^2}\frac{\xi}{\xi_s}\biggl[\Bigl(\nu-\frac{D-3}{2}\Bigr)\mathcal{F}_{\nu+1}(y)\biggr]_{\nu\to\frac{D-3}{2}} = \xi\times\frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{(D-1)\,\Gamma(\frac{D}{2})}\,, \tag{6.32}$$

which does not vanish! This is in accordance with section 6.3, where we showed that a fully de Sitter invariant form cannot reproduce all the components obtained from mode sums.

The form of the photon two-point function (6.25) with the scalar structure functions (6.29)–(6.32) comprises our solution for the Wightman function. The Feynman propagator is obtained simply by changing every y_{-+} to y_{++}.

6.5 Various limits

Four-dimensional limit. In the $D\to4$ limit we have that $\nu\to\frac{1}{2}$, and $\xi_s\to3$, and that the rescaled propagator functions reduce to,

$$\mathcal{F}_\nu(y) \xrightarrow{D\to4} \frac{H^2}{4\pi^2 y}\,, \qquad \frac{\partial}{\partial y}\frac{\partial}{\partial\nu}\mathcal{F}_{\nu+1}(y) \xrightarrow{D\to4} \frac{H^2}{(4\pi)^2}\biggl[-\frac{3}{y}+\frac{1}{4-y}+\frac{2(6-y)}{(4-y)^2}\ln\Bigl(\frac{y}{4}\Bigr)\biggr]\,. \tag{6.33}$$

Therefore, the photon two-point function (6.25) in $D=4$ is given by,

$$\mathcal{C}_1(y) \xrightarrow{D\to4} \frac{1}{(4\pi)^2}\biggl\{-\frac{4}{y} - \Bigl(1-\frac{\xi}{\xi_s}\Bigr)\biggl[-\frac{3}{y}+\frac{1}{4-y}+\frac{2(6-y)}{(4-y)^2}\ln\Bigl(\frac{y}{4}\Bigr)\biggr]\biggr\}\,, \tag{6.34a}$$

$$\mathcal{C}_2(y) \xrightarrow{D\to4} \frac{1}{(4\pi)^2}\biggl\{\frac{2}{y^2} - \Bigl(1-\frac{\xi}{\xi_s}\Bigr)\biggl[\frac{3}{y^2}+\frac{(12-y)}{y(4-y)^2}+\frac{2(8-y)}{(4-y)^3}\ln\Bigl(\frac{y}{4}\Bigr)\biggr]\biggr\}\,, \tag{6.34b}$$

$$\mathcal{C}_4 \xrightarrow{D\to4} \frac{1}{(4\pi)^2}\times\frac{2\xi}{3}\,. \tag{6.34c}$$

Flat space limit. The three tensor structures from (6.25) in flat space reduce to,

$$(\partial_\mu\partial'_\nu y) \overset{H\to0}{\sim} -2H^2\eta_{\mu\nu}\,, \quad (\partial_\mu y)(\partial'_\nu y) \overset{H\to0}{\sim} -4H^4\Delta x_\mu\Delta x_\nu\,, \quad (\partial_\mu u)(\partial'_\nu u) \overset{H\to0}{\sim} H^2\delta^0_\mu\delta^0_\nu\,, \tag{6.35}$$

while from the power series representation (2.23) we infer that the propagator function reduces to the flat space scalar two-point function, and that its relevant derivatives are also proportional to it,

$$\mathcal{F}_\nu(y) \xrightarrow{H\to0} \frac{\Gamma(\frac{D-2}{2})}{4\pi^{\frac{D}{2}}(\Delta x^2)^{\frac{D-2}{2}}}\,, \qquad \frac{\partial}{\partial y}\frac{\partial}{\partial\nu}\mathcal{F}_{\nu+1}(y) \xrightarrow{H\to0} -\frac{(D-1)}{4}\times\frac{\Gamma(\frac{D-2}{2})}{4\pi^{\frac{D}{2}}(\Delta x^2)^{\frac{D-2}{2}}}\,, \tag{6.36}$$

while the derivative with respect to y reduces to,

$$\frac{\partial}{\partial y} \overset{H\to 0}{\sim} \frac{1}{H^2}\frac{\partial}{\partial(\Delta x^2)} \,. \tag{6.37}$$

Thus the vector two-point function in (6.25) reproduces the correct flat space limit,

$$i\big[{}_\mu^-\Delta_\nu^+\big](x;x') \xrightarrow{H\to 0} \bigg[\eta_{\mu\nu} - \frac{(1-\xi)}{2}\bigg(\eta_{\mu\nu} - (D-2)\frac{\Delta x_\mu \Delta x_\nu}{\Delta x^2}\bigg)\bigg] \frac{\Gamma\big(\frac{D-2}{2}\big)}{4\pi^{\frac{D}{2}}\big(\Delta x^2\big)^{\frac{D-2}{2}}} \,, \tag{6.38}$$

that can be also written in a more familiar form as,

$$i\big[{}_\mu^-\Delta_\nu^+\big](x;x') \xrightarrow{H\to 0} \bigg[\eta_{\mu\nu} - (1-\xi)\frac{\partial_\mu\partial_\nu}{\partial^2}\bigg] \frac{\Gamma\big(\frac{D-2}{2}\big)}{4\pi^{\frac{D}{2}}\big(\Delta x^2\big)^{\frac{D-2}{2}}} \,, \tag{6.39}$$

as a projector acting on the Minkowski space scalar two-point function.

7 Simple observables

The two-point functions worked out in the preceding section are appropriate to use in loop computations. Here we demonstrate their consistency by computing two simple observables at leading orders. The first is the field strength correlator whose leading contribution comes in at tree level. The second is the expectation value of the energy momentum tensor which is an example of a simple one-loop observable where a single two-point function forms the loop.

7.1 Field strength correlator

The tree-level correlator of the field stress tensor is conveniently expressed as antisymmetrized derivatives acting on the Wightman function (6.1),

$$\langle\Omega|\hat{F}_{\mu\nu}(x)\hat{F}_{\rho\sigma}(x')|\Omega\rangle = 4\big(\delta^\alpha_{[\mu}\partial_{\nu]}\big)\big(\delta^\beta_{[\rho}\partial'_{\sigma]}\big) i\big[{}_\alpha^-\Delta_\beta^+\big](x;x') \,. \tag{7.1}$$

It clearly constitutes an observable since the field strength tensor is gauge independent. It is immediately clear that the de Sitter breaking part does not contribute to this quantity on the account of anti-symmetrized derivatives. Acting the derivatives above onto the covariantized representation of the two-point function (6.25) gives,

$$\langle\Omega|\hat{F}_{\mu\nu}(x)\hat{F}_{\rho\sigma}(x')|\Omega\rangle = -\frac{2}{H^2}\bigg[\big(\partial_\mu\partial'_{[\rho}y\big)\big(\partial'_{\sigma]}\partial_\nu y\big)\frac{\partial\mathcal{F}_\nu}{\partial y} + \big(\partial_{[\mu}y\big)\big(\partial_{\nu]}\partial'_{[\sigma}y\big)\big(\partial'_{\rho]}y\big)\frac{\partial^2\mathcal{F}_\nu}{\partial y^2}\bigg] \,. \tag{7.2}$$

The fact that the correlator does not depend on the gauge-fixing parameter ξ reflects that this correlator is indeed an observable. The de Sitter invariance of the correlator reflects the fact that the state is physically de Sitter invariant.[13] In four spacetime dimensions

[13]The phenomenon when the one-point or two-point function breaks the symmetry of the background, but the corresponding energy-momentum tensor does not, is known as *symmetry non-inheritance*, one example of which is discussed in ref. [84] (see also refs. [85, 86]) where it was shown that, even though the scalar one-point function of a massless scalar field sourced by a point charge breaks dilation symmetry, the corresponding energy-momentum tensor does not.

it also becomes obvious that the physical photon couples conformally to gravity. In fact, using (6.33) it follows that in $D\!\to\!4$ the correlator reduces to the flat space result,

$$\begin{aligned}\langle\Omega|\hat{F}_{\mu\nu}(x)\hat{F}_{\rho\sigma}(x')|\Omega\rangle &\xrightarrow{D\to4} \frac{2}{\pi^2}\partial_{\mu]}\partial'_{[\rho}\biggl[\frac{\Delta x_{\sigma]}\Delta x_{[\nu}}{(\Delta x^2)^2}\biggr]\\ &= \frac{2}{\pi^2(\Delta x^2)^2}\biggl[\eta_{\mu[\rho}\eta_{\sigma]\nu} - 4\eta_{\alpha[\mu}\eta_{\nu][\sigma}\eta_{\rho]\beta}\frac{\Delta x^\alpha\Delta x^\beta}{\Delta x^2}\biggr]\,. \end{aligned} \tag{7.3}$$

7.2 Energy-momentum tensor

The energy-momentum tensor of the photon field is defined as a variation of the action with respect to the metric tensor. Two definitions are thus possible since we can either consider the gauge-invariant action (3.1), or the gauge-fixed action (3.14) that in addition contains the gauge-fixing part (1.1). The two definitions give the same answer at the level of expectation value, as they should.

Gauge-invariant energy-momentum tensor. The energy-momentum tensor defined from the gauge-invariant action,

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \Bigl(\delta^\rho_\mu\delta^\sigma_\nu - \frac{1}{4}g_{\mu\nu}g^{\rho\sigma}\Bigr)g^{\alpha\beta}F_{\rho\alpha}F_{\sigma\beta}\,, \tag{7.4}$$

is manifestly gauge-independent as it depends on the field strength tensor only. All of its components consist only of transverse fields and of the secondary first-class constraint [65]. This is why we need not worry about operator ordering of constraints when constructing the operator associated with the observable. It is defined by Weyl-ordering the products, and consequently its expectation value is,

$$\langle\Omega|\hat{T}_{\mu\nu}(x)|\Omega\rangle = \Bigl(\delta^\rho_{(\mu}\delta^\sigma_{\nu)} - \frac{1}{4}g_{\mu\nu}g^{\rho\sigma}\Bigr)g^{\alpha\beta}\times\frac{1}{2}\langle\Omega|\bigl\{\hat{F}_{\rho\alpha}(x),\hat{F}_{\sigma\beta}(x)\bigr\}|\Omega\rangle\,. \tag{7.5}$$

Computing this calls for a dimensionally regulated coincidence limit of the field strength correlator (7.2). The tensor structures in this limit reduce to,

$$(\partial_\mu y)\xrightarrow{x'\to x}0\,,\qquad (\partial_\nu y)\xrightarrow{x'\to x}0\,,\qquad (\partial_\mu\partial'_\nu y)\xrightarrow{x'\to x}-2H^2g_{\mu\nu}\,, \tag{7.6}$$

while the only relevant derivative of the propagator function is inferred from (2.23),

$$\frac{\partial\mathcal{F}_\nu}{\partial y}\xrightarrow{x'\to x}-\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D\!-\!1)}{4\,\Gamma\bigl(\frac{D+2}{2}\bigr)}\,. \tag{7.7}$$

This results in a finite coincidence limit of the field strength correlator in $D\!\to\!4$,

$$\langle\Omega|\hat{F}_{\mu\nu}(x)\hat{F}_{\rho\sigma}(x')|\Omega\rangle\xrightarrow{x'\to x}\frac{H^D}{(4\pi)^{\frac{D}{2}}}\frac{2\,\Gamma(D\!-\!1)}{\Gamma\bigl(\frac{D+2}{2}\bigr)}\times g_{\mu[\rho}g_{\sigma]\nu}\xrightarrow{D\to4}\frac{H^4}{8\pi^2}\times g_{\mu[\rho}g_{\sigma]\nu}\,, \tag{7.8}$$

which implies a vanishing expectation value of the gauge-invariant energy-momentum tensor.

$$\langle\Omega|\hat{T}_{\mu\nu}|\Omega\rangle = 0\,. \tag{7.9}$$

Gauge-fixed energy-momentum tensor. When defined as a variation of the gauge-fixed action, the classical energy momentum tensor,

$$T^{\star}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_\star}{\delta g^{\mu\nu}} = T_{\mu\nu} + T^{\rm gf}_{\mu\nu} \,, \tag{7.10}$$

contains an additional part descending from the gauge-fixing term (1.1),

$$T^{\rm gf}_{\mu\nu} = -\frac{2}{\xi} A_{(\mu}\nabla_{\nu)}\nabla^\rho A_\rho + \frac{g_{\mu\nu}}{\xi}\Big[A_\rho \nabla^\rho \nabla^\sigma A_\sigma + \frac{1}{2} (\nabla^\rho A_\rho)^2 \Big] . \tag{7.11}$$

Classically this piece vanishes on-shell [65] as its every term contains at least one power of the primary first-class constraint from (3.16). When promoting this quantity to an operator it should be noted that some terms contain longitudinal and temporal components of the vector potential. This is why special attention needs to be given to operator ordering. Products containing constraints should not be Weyl-ordered, but rather written in a way that the non-Hermitian constraint operator $\hat{K}(\vec{x})$, defined in sections 3.3 and 4.4, should be on the right of the product, and its conjugate on the left of the product. This is conveniently accomplished by using the decomposition of Hermitian constraints in terms of he non-Hermitian ones from (3.26),

$$\hat{T}^{\rm gf}_{00} = -a^{2-D} \Big[\hat{K}_2^\dagger \hat{A}_0 + \hat{A}_0 \hat{K}_2 + (\partial_k \hat{K}_1^\dagger) \hat{A}_k + \hat{A}_k (\partial_k \hat{K}_1) + \frac{\xi a^{4-D}}{2} \hat{\Psi}_1 \hat{\Psi}_1 \Big] , \tag{7.12a}$$

$$\hat{T}^{\rm gf}_{0i} = -a^{2-D} \Big[(\partial_i \hat{K}_1^\dagger) \hat{A}_0 + \hat{A}_0 (\partial_i \hat{K}_1) + \hat{K}_2^\dagger \hat{A}_i + \hat{A}_i \hat{K}_2 \Big] , \tag{7.12b}$$

$$\begin{aligned} \hat{T}^{\rm gf}_{ij} = -a^{2-D} \Big[& 2 (\partial_{(i} \hat{K}_1^\dagger) \hat{A}_{j)} + 2 \hat{A}_{(i} (\partial_{j)} \hat{K}_1) + \delta_{ij} \Big(\hat{K}_2^\dagger \hat{A}_0 + \hat{A}_0 \hat{K}_2 \\ & - (\partial_k \hat{K}_1^\dagger) \hat{A}_k - \hat{A}_k (\partial_k \hat{K}_1) - \frac{\xi a^{4-D}}{2} \hat{\Psi}_1 \hat{\Psi}_1 \Big) \Big] . \end{aligned} \tag{7.12c}$$

For the terms containing only products of the Hermitian constraint operators this ordering is immaterial since the non-Hermitian constraint and its conjugate commute. From (7.12a)–(7.12c) then immediately follows that,

$$\langle \Omega | \hat{T}^{\rm gf}_{\mu\nu} | \Omega \rangle = 0 \,, \tag{7.13}$$

for all physical states satisfying subsidiary constraints (3.24). Thus, the expectation value of the energy momentum tensor for a physically de Sitter invariant state vanishes, and it is immaterial which one of the two definitions, (7.4) or (7.10), one uses to obtain this result. This mirrors the property that the two definitions give the same answer on-shell.

In refs. [75, 87, 88] the question of operator ordering of the gauge-fixed energy momentum tensor operator was not addressed, and consequently the results reported there contradict the requirement (7.13). Recently the same issue was tackled in [76], but again without addressing the operator ordering. They concluded that it has to vanish, but only after regularization and renormalization implemented by adiabatic subtraction. While we agree with the conclusion, the rationale is quite different. One either needs to consider operator ordering carefully, leading identically to (7.13), or one can obtain the answer (7.13) by Weyl-ordering operators and introducing compensating Faddeev-Popov ghost fields, as we have done in a companion letter [66]. The latter approach is consistent with other findings for the energy-momentum tensor expectation value, see e.g. [89, 90].

8 Discussion

Photon propagators are essential building blocks for any quantum loop computation involving massless vector fields. Here we are concerned with photon propagators in de Sitter space, which is a maximally symmetric idealization of more realistic slow-roll primordial inflation. The expectation for maximally symmetric spaces, such as de Sitter, is that the general covariant gauge is conceptually and practically the simplest and most convenient one to use,[14] on the account of two reasons:

(i) it produces maximally symmetric two-point functions allowing for preservation of manifest covariance at intermediate steps of the computation, and

(ii) it contains a free gauge-fixing parameter defining a one-parameter family of covariant gauges, that can be used to check computed observables that cannot depend on it.

Both of these expectations are of great utility in de Sitter space loop computations. The first provides an organizational principle for computations that are notoriously more difficult than their flat space counterparts. The second is particularly useful for studying gauge dependence and quantum observables in inflation, the understanding of which has still not reached maturity, especially for perturbative quantum gravity [92].

In this work we have shown that the two commonly held expectations stated above are in general not consistent with each other. It is possible for the photon propagator to be de Sitter invariant only in the Landau gauge $\xi\!\to\!0$, in which case we reproduce the result due to Tsamis and Woodard [61]. For all other values of the gauge-fixing parameter it is not possible to maintain de Sitter symmetry of the propagator, while simultaneously satisfying the equations of motion *and* subsidiary conditions. It is the latter that was not checked for the covariant gauge photon propagators reported in the literature previously [65], and it is responsible for breaking of the de Sitter symmetry. The de Sitter breaking part pertains to the pure gauge sector of the free theory, and in that sense it has no physical content for non-interacting photons. However, when interactions are considered this is not as straightforward, and gauge-fixing has to be implemented correctly in order to guarantee correct results for loop corrections.

Our main result is the photon two-point function in the general covariant gauge on the expanding Poincaré patch of de Sitter space,[15] that takes the following covariantized form,

$$i\big[{}_{\mu}^{-}\Delta_{\nu}^{+}\big](x;x') = \big(\partial_\mu\partial'_\nu y\big)\,\mathcal{C}_1(y) + \big(\partial_\mu y\big)\big(\partial'_\nu y\big)\,\mathcal{C}_2(y) + \big(\partial_\mu u\big)\big(\partial'_\nu u\big)\,\mathcal{C}_4\,, \tag{8.1}$$

[14]There exists a photon propagator in de Sitter in a non-covariant gauge due to Woodard [91], that is simpler than any covariant gauge propagator. This suggests that covariant gauges need not be the simplest choice in de Sitter.

[15]We do not consider here spatially compact global coordinates on de Sitter, where the problem of linearization instability arises [93].

The first two terms comprise the de Sitter invariant part, where both the tensor structures and the scalar structure functions given in (6.29) and (6.30) are constructed from the de Sitter invariant distance y only. This part was already obtained in previous works [60–63]. The contribution of our analysis is the unexpected third term, composed out of the de Sitter breaking tensor structure constructed from variable $u = \ln(aa')$, multiplied by a constant,

$$\mathcal{C}_4 = \xi \times \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{(D-1)\,\Gamma(\frac{D}{2})}\,, \tag{8.2}$$

that vanishes only in the exact gauge limit $\xi \to 0$. This contribution is a homogeneous solution of the equation of motion, and would never be seen by considering a de Sitter invariant *Ansatz* for the two-point function. It indeed is true that the de Sitter invariant part of (8.1) satisfies the equation of motion independently, as was shown in [60–63]. However, the quantum subsidiary conditions from section 3.3 and 4.4. are not satisfied unless the de Sitter breaking term is added. These subsidiary conditions are equivalent to the Ward-Takahashi identity for free two-point functions one obtains from BRST quantization, that we considered in a companion letter [66].

We derived our result (8.1)–(8.2) starting from the first principles of canonical quantization. This required us to first impose the general covariant gauge for the photon field as the multiplier gauge in the classical theory in section 3, which treats all components of the vector field on equal footing, and preserves general covariance. Such gauge fixing implies that the dynamics is given by the gauge-fixed action, and kinematics by the subsidiary first-class constraints. The quantization, outlined in section 4, contains two aspects as well — the gauge-fixed dynamics is quantized by the usual rules of canonical quantization, while the constraints are implemented as conditions on the space of states that physical states have to satisfy. In that respect the construction structurally parallels Gupta-Bleuler quantization, but is formulated only in terms of the canonical structure and is divorced from the symmetries of the theory.

The symmetries of the quantum state are considered separately from quantization in section 5, where we construct the state by requiring it to be physically de Sitter invariant, meaning that the expectation values of de Sitter generators descending from the original gauge invariant action must all vanish. This fixes only the transverse part of the state that describes the physical polarizations only. Then we construct the scalar sector of the state by requiring the full state to be an eigenstate of the de Sitter symmetry generators of the gauge-fixed action. All of this was implemented in momentum space. Nevertheless, the integrals over all the modes have to be performed to obtain the two-point function. When performed, integrals yield a de Sitter breaking two-point function in position space. While somewhat unintuitive, this is not the first example of position-space two-point functions breaking de Sitter invariance that was imposed in momentum space. The best known example is the massless, minimally coupled scalar for which the CTBD mode function is associated to the state that is an eigenstate of all the de Sitter symmetry generators, but that fails to produce an infrared-finite two-point function, implying that it does not exist. For the massless photon we consider here de Sitter breaking is finite.

Our propagator in (8.1)–(8.2) and (6.29)–(6.30) is suitable for perturbative calculations in de Sitter space, for Abelian and non-Abelian theories alike, and can be also used to estimate the perturbative effects of interactions in general inflationary spacetimes.[16] Among the simplest such applications are the field strength correlator and the one-loop energy-momentum tensor discussed in section 7.

We expect that understanding of the gauge field quantization in inflation [65], and the construction of the two-point functions in de Sitter presented in this work and in the companion letter [66], will allow us to tackle the questions faced when constructing propagators in realistic inflationary spacetimes and when investigating the effects of interactions. Already in power-law inflation the situation is more complicated as there is no enhanced symmetry, just the cosmological ones, and the construction of propagators is more involved [67, 83, 94]. The goal is to put such propagators to use and quantify how departures from exact de Sitter space influence the effects of quantum loop corrections. The understanding of this issue is important, but it is still in its early stages [33, 51, 95].

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A Noether currents and charges

Here we supply additional intermediate expressions for section 5.

A.1 Noether currents for gauge invariant action

The global transformations from section 5.1 indeed are symmetries of the gauge invariant action (3.1). However, these are defined up to gauge transformations, which do not carry any information about global de Sitter symmetries. Therefore, there are ambiguities in the definition of global symmetries for the gauge-invariant action. However, there is a convenient way to fix these ambiguities by requiring that the currents associated with the de Sitter symmetries be gauge-invariant off-shell (they are invariant on-shell always, and currents are conserved on-shell only anyway). To this end, it is more convenient to write the transformations (5.2), (5.4), (5.6), and (5.8), respectively, in the following form,

$$A_\mu \to A_\mu - \alpha_i F_{i\mu} \,, \tag{A.1}$$

$$A_\mu \to A_\mu + 2\omega_{ij} x_i F_{j\mu} \,, \tag{A.2}$$

$$A_\mu \to A_\mu + \frac{\alpha}{a} F_{0\mu} - \alpha H x_i F_{i\mu} \,, \tag{A.3}$$

$$A_\mu \to A_\mu - \frac{\theta_i x_i}{a} F_{0\mu} + \biggl[H\theta_j x_j x_i + \frac{\theta_i}{2H} \biggl(\frac{1}{a^2} - 1 - H^2 x_j x_j \biggr) \biggr] F_{i\mu} \,. \tag{A.4}$$

[16] See refs. [41–43] for how to tackle the problem of interactions beyond perturbation theory.

The Noether currents associated with these transformations are now gauge invariant off-shell,

$$(\mathscr{P}_i)^\mu = \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\nu)}(-F_{i\nu}) + \delta^\mu_i \mathscr{L}\,, \tag{A.5}$$

$$(\mathscr{M}_{ij})^\mu = \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\nu)}\Big(2x_{[i}F_{j]\nu}\Big) + \delta^\mu_{[i}x_{j]}\mathscr{L}\,, \tag{A.6}$$

$$(\mathscr{Q})^\mu = \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\nu)}\Big(\frac{1}{a}F_{0\nu} - Hx_iF_{i\nu}\Big) - \Big(\frac{\delta^\mu_0}{a} - \delta^\mu_i H x_i\Big)\mathscr{L}\,, \tag{A.7}$$

$$\begin{aligned}(\mathscr{K}_i)^\mu = {}& \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\nu)}\bigg[-\frac{x_i}{a}F_{0\nu} + Hx_ix_jF_{j\nu} + \frac{1}{2H}\Big(\frac{1}{a^2}-1-H^2x_jx_j\Big)F_{i\nu}\bigg] \\ &+ \bigg[\delta^\mu_0\frac{x_i}{a} - \delta^\mu_j Hx_ix_j - \frac{\delta^\mu_i}{2H}\Big(\frac{1}{a^2}-1-H^2x_jx_j\Big)\bigg]\mathscr{L}\,,\end{aligned} \tag{A.8}$$

where the gauge invariant Lagrangian and its derivative in expressions above are,

$$\mathscr{L} = -\frac{\sqrt{-g}}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\,, \qquad \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\nu)} = -\sqrt{-g}\,g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}\,. \tag{A.9}$$

Whether the conserved currents are defined as off-shell gauge invariant or not, they lead to the same conserved charges, given in (5.9)–(5.12).

A.2 Noether currents for gauge-fixed action

The Noether currents associated with the global transformations from section 5.1 are,

$$(\mathscr{P}^\star_i)^\mu = \frac{\partial \mathscr{L}_\star}{\partial(\partial_\mu A_\nu)}(-\partial_i A_\nu) + \delta^\mu_i \mathscr{L}_\star\,, \tag{A.10}$$

$$(\mathscr{M}^\star_{ij})^\mu = \frac{\partial \mathscr{L}_\star}{\partial(\partial_\mu A_\nu)}\Big(2x_{[i}\partial_{j]}A_\mu + 2\delta_{\mu[i}A_{j]}\Big) + 2\delta^\mu_{[i}x_{j]}\mathscr{L}_\star\,, \tag{A.11}$$

$$(\mathscr{Q}^\star)^\mu = \frac{\partial \mathscr{L}_\star}{\partial(\partial_\mu A_\nu)}\Big[\frac{1}{a}\partial_0 A_\nu - Hx_i\partial_i A_\nu - HA_\nu\Big] - \Big(\frac{\delta^\mu_0}{a} - \delta^\mu_i Hx_i\Big)\mathscr{L}_\star\,, \tag{A.12}$$

$$\begin{aligned}(\mathscr{K}^\star_i)^\mu = {}& \frac{\partial \mathscr{L}_\star}{\partial(\partial_\mu A_\nu)}\bigg[-\frac{x_i}{a}\partial_0 A_\nu + Hx_ix_j\partial_j A_\nu + \frac{1}{2H}\Big(\frac{1}{a^2}-1-H^2x_jx_j\Big)\partial_i A_\nu \\ &\qquad + Hx_iA_\nu - \frac{1}{a}(\delta^0_\nu A_i + \delta^i_\nu A_0) + H(\delta^i_\nu x_jA_j - \delta^j_\nu x_jA_i)\bigg] \\ &+ \bigg[\delta^\mu_0\frac{x_i}{a} - \delta^\mu_j Hx_ix_j - \frac{\delta^\mu_i}{2H}\Big(\frac{1}{a^2}-1-H^2x_jx_j\Big)\bigg]\mathscr{L}_\star\,,\end{aligned} \tag{A.13}$$

where the gauge fixed Lagrangian and its derivative in the expressions above are,

$$\mathscr{L}_\star = \mathscr{L} - \frac{\sqrt{-g}}{2\xi}(g^{\mu\nu}\nabla_\mu A_\nu)^2\,, \qquad \frac{\partial \mathscr{L}_\star}{\partial(\partial_\mu A_\nu)} = \frac{\partial \mathscr{L}}{\partial(\partial_\mu A_\nu)} - \frac{\sqrt{-g}}{\xi}(g^{\rho\sigma}\nabla_\rho A_\sigma)g^{\mu\nu}\,. \tag{A.14}$$

and where the gauge-invariant parts are already given in (A.9). The gauge-fixed conserved charges associated to the gauge-fixed conserved currents are then given in (5.29)–(5.32).

A.3 Scalar-transverse decomposition of charges

Both the gauge invariant charges (5.9)–(5.12) and the gauge fixed charges (5.29)–(5.32) reveal more of their structure when the fields comprising them are decomposed into the transverse and longitudinal parts. In particular, it becomes clear how to order products of field operators when defining quantum symmetry generators as observables in (5.14)–(5.17) and (5.33)–(5.36). These decompositions are for spatial translations,

$$P_i = P_i^T + \int d^{D-1}x \left(\Psi_2 A_i^T \right) , \tag{A.15a}$$

$$P_i^\star = P_i^T + \int d^{D-1}x \left(\Psi_2 A_i^L - \Psi_1 \partial_i A_0 \right) , \tag{A.15b}$$

$$P_i^T = \int d^{D-1}x \left(-\Pi_j^T \partial_i A_j^T \right) , \tag{A.15c}$$

for spatial rotations,

$$M_{ij} = M_{ij}^T + \int d^{D-1}x \left(2x_{[i} A_{j]}^T \Psi_2 \right) , \tag{A.16a}$$

$$M_{ij}^\star = M_{ij}^T + \int d^{D-1}x \left(2\Psi_1 x_{[i} \partial_{j]} A_0 - 2\Psi_2 x_{[i} A_{j]}^L \right) , \tag{A.16b}$$

$$M_{ij}^T = \int d^{D-1}x \left(2x_{[i} F_{j]k}^T \Pi_k^T \right) , \tag{A.16c}$$

for dilations,

$$Q = Q^T + \int d^{D-1}x \left[-\frac{a^{3-D}}{2} \Psi_2 \nabla^{-2} \Psi_2 - H \Psi_2 x_i A_i^T \right] , \tag{A.17a}$$

$$\begin{aligned} Q^\star = Q^T + \frac{1}{a} \int d^{D-1}x \bigg[& -\frac{a^{4-D}}{2} \Psi_2 \nabla^{-2} \Psi_2 - \frac{a^{4-D}}{2} \xi \Psi_1^2 - \Psi_2 A_0 \\ & + \Psi_1 \partial_i A_i^L + \mathcal{H} \Big(\Psi_2 x_i A_i^L + \partial_i \Psi_1 x_i A_0 \Big) \bigg] , \end{aligned} \tag{A.17b}$$

$$Q^T = \int d^{D-1}x \left[\frac{a^{3-D}}{2} \Pi_i^T \Pi_i^T + \frac{a^{D-5}}{2} (\partial_i A_j^T)(\partial_i A_j^T) - H \Pi_i^T (1 + x_j \partial_j) A_i^T \right] , \tag{A.17c}$$

and for spatial special conformal transformations,

$$\begin{aligned} K_i = K_i^T + \int d^{D-1}x \bigg[& \bigg(\frac{a^{3-D}}{2} x_i \Psi_2 a^{3-D} \Pi_i^T + (D-3) H A_i^T \bigg) \frac{1}{\nabla^2} \Psi_2 \\ & + H x_i x_j A_j^T \Psi_2 + \frac{1}{2H} \bigg(\frac{1}{a^2} - 1 - H^2 x_j x_j \bigg) A_i^T \Psi_2 \bigg] , \end{aligned} \tag{A.18a}$$

$$\begin{aligned} K_i^\star = K_i^T + \int d^{D-1}x \bigg[& \bigg(\frac{a^{3-D}}{2} x_i \Psi_2 + a^{3-D} \Pi_i^T + (D-3) H A_i^T \bigg) \nabla^{-2} \Psi_2 - \frac{1}{a} A_i^T \Psi_1 \\ & + \frac{a^{3-D} \xi}{2} x_i \Psi_1 \Psi_1 + \frac{1}{a} x_i A_j^L \partial_j \Psi_1 + \frac{1}{a} x_i A_0 \Psi_2 + (D-1) H x_i A_0 \Psi_1 - H x_i x_j A_j^L \Psi_2 \\ & + H x_i x_j \partial_j A_0 \Psi_1 + \frac{1}{2H} \bigg(\frac{1}{a^2} - 1 - H^2 x_j x_j \bigg) \Big(\partial_i A_0 \Psi_1 - A_i^L \Psi_2 \Big) \bigg] . \end{aligned} \tag{A.18b}$$

$$K_i^T = \int d^{D-1}x \left[-\frac{x_i}{2a}\left(a^{4-D}\Pi_j^T \Pi_j^T + a^{D-4}(\partial_j A_k^T)(\partial_j A_k^T)\right) + Hx_i x_j (\partial_j A_k^T)\Pi_k^T \right.$$
$$\left. + \frac{1}{2H}\left(\frac{1}{a^2} - 1 - H^2 x_j x_j\right)\Pi_k^T \partial_i A_k^T + Hx_j A_j^T \Pi_i^T + Hx_i A_j^T \Pi_j^T - Hx_j A_i^T \Pi_j^T \right], \quad \text{(A.18c)}$$

where the first-class constraints are $\Psi_1 = \Pi_0$ and $\Psi_2 = \partial_i \Pi_i^L$.

B Identities for tensor structures

Checking that the solution for the two-point function (6.25) satisfies the appropriate equation of motion and the subsidiary conditions from section 6.1 is facilitated by the following covariant identities for derivatives,

$$(\nabla_\mu \nabla_\nu y) = H^2 g_{\mu\nu}(2-y)\,, \qquad (\nabla_\mu \nabla_\nu u) = -H^2 g_{\mu\nu} - (\partial_\mu u)(\partial'_\nu u)\,, \qquad \text{(B.1)}$$

and tensor structure contractions,

$$g^{\mu\nu}(\partial_\mu y)(\partial_\nu y) = g'^{\rho\sigma}(\partial'_\rho y)(\partial'_\sigma y) = H^2(4y-y^2)\,, \qquad \text{(B.2a)}$$
$$g^{\mu\nu}(\partial_\mu y)(\partial_\nu \partial'_\rho y) = H^2(2-y)(\partial'_\rho y)\,, \qquad \text{(B.2b)}$$
$$g'^{\rho\sigma}(\partial_\mu \partial'_\rho y)(\partial'_\sigma y) = H^2(2-y)(\partial_\mu y)\,, \qquad \text{(B.2c)}$$
$$g^{\mu\nu}(\partial_\mu \partial'_\rho y)(\partial_\nu \partial'_\sigma y) = 4H^4 g'_{\rho\sigma} - H^2(\partial'_\rho y)(\partial'_\sigma y)\,, \qquad \text{(B.2d)}$$
$$g'^{\rho\sigma}(\partial_\mu \partial'_\rho y)(\partial_\nu \partial'_\sigma y) = 4H^4 g_{\mu\nu} - H^2(\partial_\mu y)(\partial_\nu y)\,. \qquad \text{(B.2e)}$$

These are applicable regardless of the $i\varepsilon$ prescription in the distance functions, except in one relevant case,

$$(\nabla_\mu \nabla_\nu y_{++})\left(\frac{y_{++}}{4}\right)^{-\frac{D}{2}} = H^2 g_{\mu\nu}(2-y_{++})\left(\frac{y_{++}}{4}\right)^{-\frac{D}{2}} + (a^2\delta_\mu^0\delta_\nu^0)\frac{4(4\pi)^{\frac{D}{2}}}{H^{D-2}\Gamma(\frac{D}{2})}\frac{i\delta^D(x-x')}{\sqrt{-g}}\,, \qquad \text{(B.3)}$$

that accounts for how the solution for the photon Feynman propagator produces local terms in (6.5) and (6.7).

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